

Project 2 - Booth's Algorithm

Contents of Submission:-

1. Code
 - a. Multiplication and Division
 - b. Documentation inside code
2. Report
 - a. Multiplication
 - i. Algorithm
 - ii. Complexity Analysis
 - iii. Flow Chart
 - iv. Test Cases
 - b. Division
 - i. Algorithm
 - ii. Complexity Analysis
 - iii. Flow Chart
 - iv. Test Cases

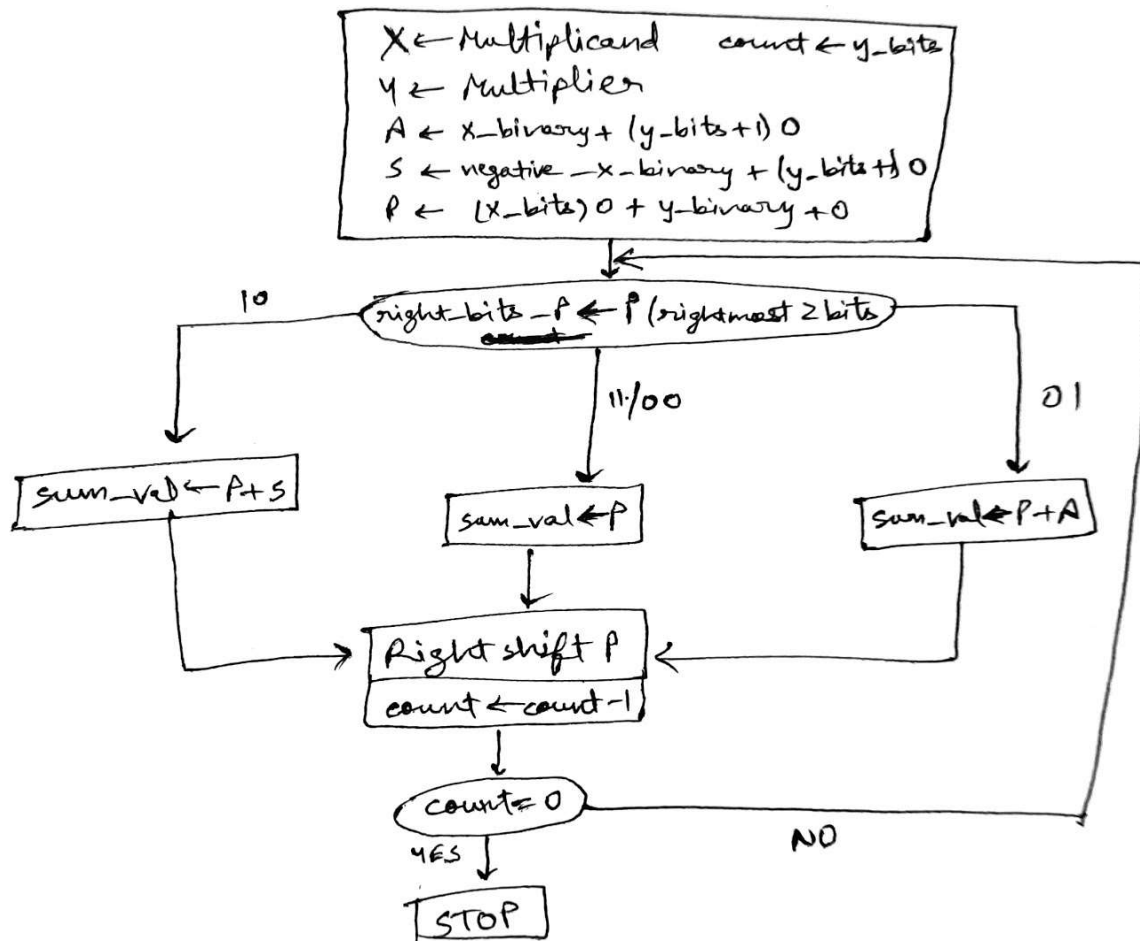
Multiplication

Algorithm used:

1. x and y are the numbers to be multiplied. They are converted to their binary forms. The binary form of $-x$ is also calculated. The number of bits in x and y are represented by x_bits and y_bits .
2. $A = x_binary + (y_bits + 1) 0$
 $S = \text{negative_}x_binary + (y_bits + 1) 0$
 $P = (x_bits) 0 + y_binary + 0$
3. Then the following operations is done y_bits times. The last two bits of P are calculated.
 - a. If they are 01, then we do $P + A$ and do not take into account the extra bits
 - b. If they are 10, then we do $P + S$ and do not take into account the extra bits
 - c. If they are 00 or 11, then we take P as it isThe above value is shifted to the right by one place. P is now equal to this value.
4. After step 3 is completed, remove the rightmost bit in P . This is the final answer.

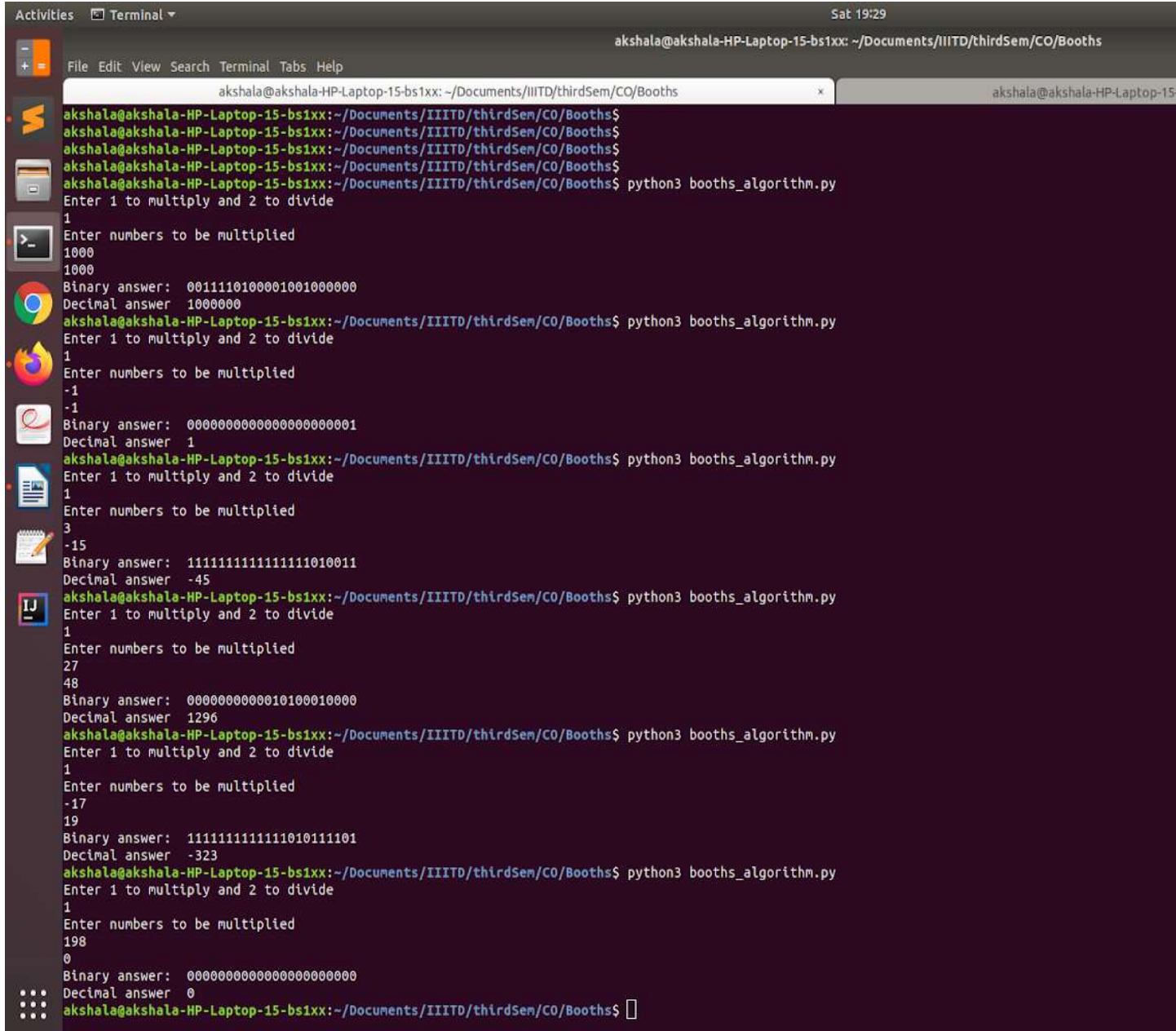
Time Complexity: $O(n^2)$

There is a while loop which runs y_bits time. This complexity can be taken as $O(n)$. Inside the while loop conversions are being done from decimal to binary and vice versa. This complexity is also $O(n)$. Therefore the overall complexity can be taken as $O(n^2)$.

Flow Chart

BOOTH'S ALGORITHM MULTIPLICATION

Test Cases



```

Activities  Terminal  Sat 19:29
akshala@akshala-HP-Laptop-15-bs1xx: ~/Documents/IIITD/thirdSem/CO/Booths
File Edit View Search Terminal Tabs Help
akshala@akshala-HP-Laptop-15-bs1xx: ~/Documents/IIITD/thirdSem/CO/Booths
akshala@akshala-HP-Laptop-15-bs1xx:~/Documents/IIITD/thirdSem/CO/Booths$
akshala@akshala-HP-Laptop-15-bs1xx:~/Documents/IIITD/thirdSem/CO/Booths$
akshala@akshala-HP-Laptop-15-bs1xx:~/Documents/IIITD/thirdSem/CO/Booths$
akshala@akshala-HP-Laptop-15-bs1xx:~/Documents/IIITD/thirdSem/CO/Booths$ python3 booths_algorithm.py
Enter 1 to multiply and 2 to divide
1
Enter numbers to be multiplied
1000
1000
Binary answer: 0011110100001001000000
Decimal answer 1000000
akshala@akshala-HP-Laptop-15-bs1xx:~/Documents/IIITD/thirdSem/CO/Booths$ python3 booths_algorithm.py
Enter 1 to multiply and 2 to divide
1
Enter numbers to be multiplied
-1
-1
Binary answer: 000000000000000000000001
Decimal answer 1
akshala@akshala-HP-Laptop-15-bs1xx:~/Documents/IIITD/thirdSem/CO/Booths$ python3 booths_algorithm.py
Enter 1 to multiply and 2 to divide
1
Enter numbers to be multiplied
3
-15
Binary answer: 11111111111111010011
Decimal answer -45
akshala@akshala-HP-Laptop-15-bs1xx:~/Documents/IIITD/thirdSem/CO/Booths$ python3 booths_algorithm.py
Enter 1 to multiply and 2 to divide
1
Enter numbers to be multiplied
27
48
Binary answer: 0000000000010100010000
Decimal answer 1296
akshala@akshala-HP-Laptop-15-bs1xx:~/Documents/IIITD/thirdSem/CO/Booths$ python3 booths_algorithm.py
Enter 1 to multiply and 2 to divide
1
Enter numbers to be multiplied
-17
19
Binary answer: 11111111111010111101
Decimal answer -323
akshala@akshala-HP-Laptop-15-bs1xx:~/Documents/IIITD/thirdSem/CO/Booths$ python3 booths_algorithm.py
Enter 1 to multiply and 2 to divide
1
Enter numbers to be multiplied
198
0
Binary answer: 000000000000000000000000
Decimal answer 0
akshala@akshala-HP-Laptop-15-bs1xx:~/Documents/IIITD/thirdSem/CO/Booths$

```

Division Algorithm - Restoring Division Algorithm

Registers Required:-

1. Div - Stores the value of the divisor.
2. Rem - Stores the value of Remainder.
3. Quo - Stores the value of Quotient
4. Restore - Stores the Restoration Bit

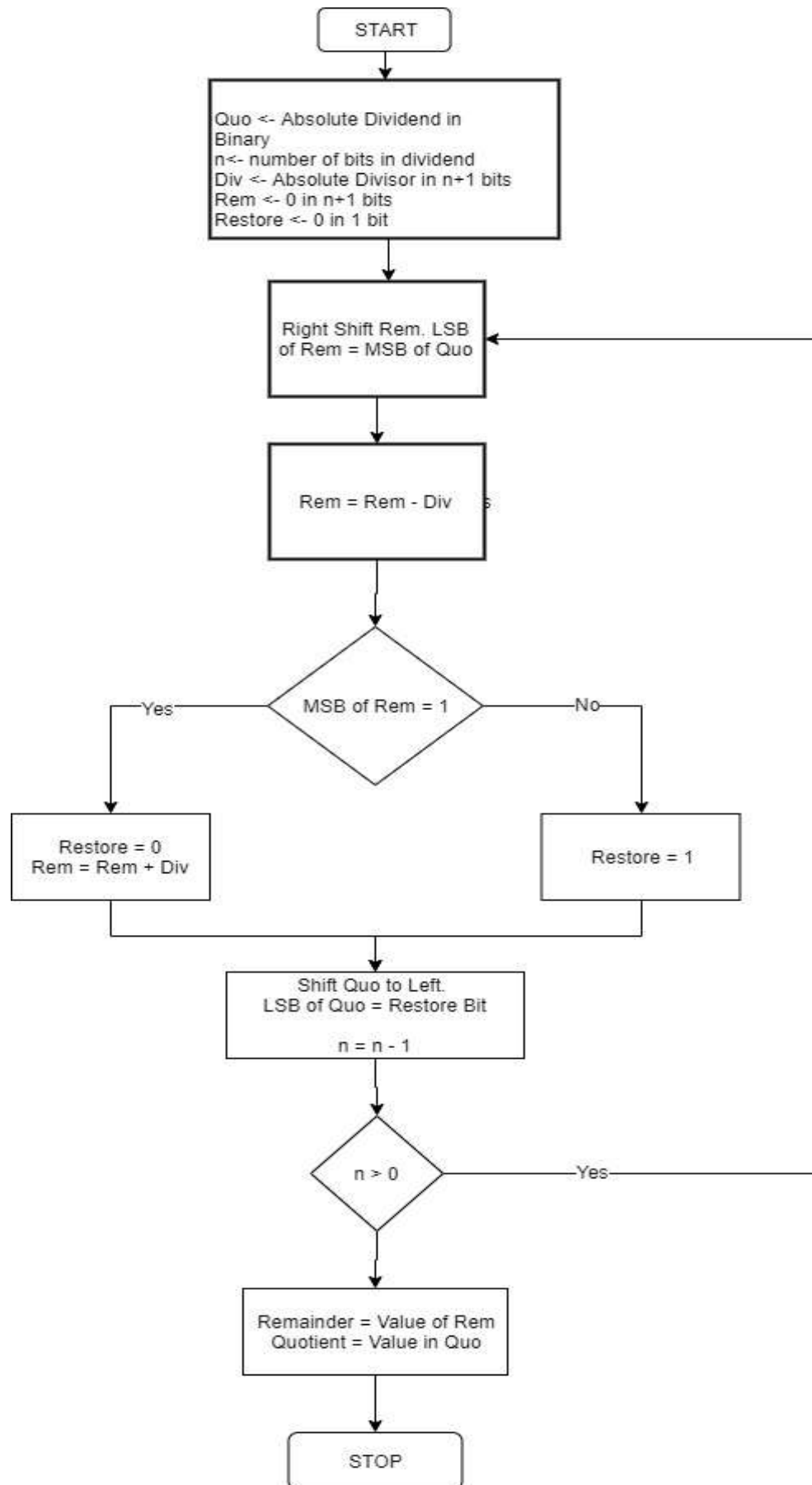
Algorithm used:

1. Initialize the registers with the following values :
 - a. Store absolute value of dividend in binary format in Quo. Let the minimum number of bits required to represent dividend be n .
 - b. Initialize Div with the value of the absolute value of divisor represented in binary format in $n + 1$ bits.
 - c. Initialize Rem with 0 represented in $n + 1$ bits.
 - d. Initialise the Restore with 0.
2. Repeat the following steps for n number of times :
 - a. Shift the bits of Rem to left. MSB of Rem is discarded. LSB of Rem = MSB of Quo.
 - b. Subtract the value of Div from Rem. $\text{Rem} = \text{Rem} - \text{Div}$
 - c. If MSB of Rem = 1 :
 - i. Restore bit is 0.
 - ii. Value of Rem = Value of Rem before step (b)
 - Else:
 - i. Restore bit is 1.
 - d. Shift the bits of Quo to left. Discard MSB and LSB = Restore bit.
3. Quotient is the value of Quo, Remainder is the value of Rem.
4. Signed numbers are handled in manner handled in the python programming language:
 - a. In the case of negative divisors. Remainder is reported in negative. For example. For $(18) / (-5)$, Quotient = -4 , Remainder = -2 and **not** Quotient = -3 and Remainder = 3

Time Complexity - $O(n^2)$:

' n ' is the minimum number of bits required to represent dividend. The algorithm loops n times and in each loop (i.e. step 2) Shifting Rem takes $O(n)$, Shifting of Quo takes $O(n)$ times and Subtraction(Div - Rem) takes $O(n)$. Therefore inside each loop $O(n)$ and the number of loops = n give Time Complexity = $O(n^2)$.

Flow Chart :



Test Cases :

1.

```
Enter the value of dividend and enter
600
Enter the value of divisor and enter
200
Quotient is  3
Remainder is  0
```
2.

```
Enter the value of dividend and enter
502
Enter the value of divisor and enter
0
ERROR : Division by 0
```
3.

```
Enter the value of dividend and enter
600
Enter the value of divisor and enter
201
Quotient is  2
Remainder is  198
```
4.

```
Enter the value of dividend and enter
600
Enter the value of divisor and enter
-201
Quotient is  -3
Remainder is  -3
```
5.

```
Enter the value of dividend and enter
-600
Enter the value of divisor and enter
201
Quotient is  -3
Remainder is  3
```
6.

```
Enter the value of dividend and enter
-600
Enter the value of divisor and enter
-201
Quotient is  2
Remainder is  -198
```
7.

```
Enter the value of dividend and enter
0
Enter the value of divisor and enter
502
Quotient is  0
Remainder is  0
```
8.

```
Enter the value of dividend and enter
592
Enter the value of divisor and enter
1
Quotient is  592
Remainder is  0
```

```
Enter the value of dividend and enter  
-1050  
Enter the value of divisor and enter  
25  
Quotient is  -42  
Remainder is  0
```

9.