Artificial Intelligence

Homework 2

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Solution to Problem 2.1:

Given $\delta = 0.03$ and $\in(M,N,\delta) = \sqrt{\frac{1}{2N}\ln\frac{2M}{\delta}}$

Where M = amount of model complexity,

N = number of samples,

δ = level of confidence.

2.1(a): M =1, $\in \leq 0.05$

$\in(M,N,\delta) = \sqrt{\frac{1}{2N}\ln\frac{2M}{\delta}} \leq \in$

$= \sqrt{\frac{1}{2N}\ln\frac{2M}{\delta}} \leq 0.05$

$= \sqrt{\frac{1}{2N}\ln\left(\frac{2}{0.03}\right)} \leq 0.05$

$= \sqrt{\frac{1}{2N} * 4.199} \leq 0.05$

Squaring on both sides,

$= \frac{1}{2N} * 4.199 \leq 0.0025$

$= 4.199 \leq 0.005N$

$= \frac{4.199}{0.005} \leq N$

$N \geq 840$ examples are needed to make $\in \leq 0.05$

2.1(b): M =100, $\in \leq 0.05$

$\in(M,N,\delta) = \sqrt{\frac{1}{2N}\ln\frac{2M}{\delta}} \leq \in$

$= \sqrt{\frac{1}{2N}\ln\frac{2M}{\delta}} \leq 0.05$

$= \sqrt{\frac{1}{2N} \ln\left(\frac{2*100}{0.03}\right)} \leq 0.05$

$= \sqrt{\frac{1}{2N} \ln\left(\frac{200}{0.03}\right)} \leq 0.05$

$= \sqrt{\frac{1}{2N} * 8.8} \leq 0.05$

Squaring on both sides,

$= \frac{1}{2N} * 8.8 \leq 0.0025$

= $4.4 \leq 0.0025N$

$= \frac{4.4}{0.0025} \leq N$

$N \geq 1760$ examples are needed to make $\in \leq 0.05$

2.1(C): M =10,000, $\in \leq 0.05$

$\in(M,N,\delta) = \sqrt{\frac{1}{2N} \ln\frac{2M}{\delta}} \leq \in$

$= \sqrt{\frac{1}{2N} \ln\frac{2M}{\delta}} \leq 0.05$

$= \sqrt{\frac{1}{2N} \ln\left(\frac{2*10000}{0.03}\right)} \leq 0.05$

$= \sqrt{\frac{1}{2N} \ln\left(\frac{20000}{0.03}\right)} \leq 0.05$

$= \sqrt{\frac{1}{2N} * 13.4} \leq 0.05$

Squaring on both sides,

$= \frac{1}{2N} * 13.4 \leq 0.0025$

$= 13.4 \leq 0.005N$

$= \frac{13.4}{0.005} \leq N$

$N \geq 2680$ examples are needed to make $\in \leq 0.05$

Solution to 2.11:

If $m_H(N) = N+1$, then $m_H(2N) = 2N +1$ and

Given $d_{vc} = 1$, N = 100 training examples,

We use generalization bound to give E_{out} with confidence 90%, $\delta = 0.1$

$E_{out}(g) \leq E_{in}(g) + \sqrt{\frac{8}{N} \ln \frac{4mH(2N)}{\delta}}$

$\leq E_{in}(g) + \sqrt{\frac{8}{N} \ln \frac{4(2N+1)}{0.1}}$

$\leq E_{in}(g) + \sqrt{\frac{8}{100} \ln \frac{4(200+1)}{0.1}}$

$\leq E_{in}(g) + \sqrt{0.08 * \ln \left(\frac{804}{0.1}\right)}$

$\leq E_{in}(g) + \sqrt{0.08 * 8.992}$

$\leq E_{in}(g) + \sqrt{0.08 * \ln \left(\frac{804}{0.1}\right)}$

$\mathbf{E_{out}(g) \leq E_{in}(g) + 0.848}$

For N = 10,000

$E_{out}(g) \leq E_{in}(g) + \sqrt{\frac{8}{N} \ln \frac{4mH(2N)}{\delta}}$

$\leq E_{in}(g) + \sqrt{\frac{8}{N} \ln \frac{4(2N+1)}{0.1}}$

$\leq E_{in}(g) + \sqrt{\frac{8}{10000} \ln \frac{4(20000+1)}{0.1}}$

$\leq E_{in}(g) + \sqrt{\frac{8}{10000} \ln(800040)}$

$\leq E_{in}(g) + \sqrt{\frac{8}{10000} * 13.592}$

$\leq E_{in}(g) + \sqrt{0.0108}$

$\mathbf{E_{out}(g) \leq E_{in}(g) + 0.103}$

Solution to 2.12:

d_{vc}= 10

95% confidence δ = 0.1

∈= 0.05

$$N \geq \frac{8}{\in 2} \ln\left(\frac{4((2N)^{\text{dvc}}+1)}{\delta}\right)$$

$$\geq \frac{8}{(0.05*0.05)} \ln\left(\frac{4((2N)^{10}+1)}{0.1}\right)$$

$$\geq \frac{8}{0.0025} \ln\left(\frac{4096N^{10}+4}{0.1}\right)$$

$$\geq 3200 \ln 40960N^{10} + 40$$

$$\frac{N}{3200} \geq \ln 40960N^{10} + 40$$

'e' on both sides

$$e^{\frac{N}{3200}} \geq e^{\ln 40960N^{10}+40}$$

$$e^{\frac{N}{3200}} \geq 40960N^{10} + 40$$

Integrating on both sides

$$\int e^{\frac{N}{3200}} = \int (40960N^{10} + 40)$$

$$e^{\frac{N}{3200}} + c = \frac{40960N^{11}}{N+1}$$

Method 2:

$$N \geq \frac{8}{0.0025} \ln\left(\frac{4096N^{10}+4}{0.1}\right)$$

If for N = 3000,

$$N \geq \frac{8}{0.0025} \ln\left(\frac{4096*3000^{10}+4}{0.1}\right)$$

$$N \geq 3200 \ln(2.418647 + 49)$$

N ≥ 363871.60

If for N = 4000,

$$N \geq \frac{8}{0.0025} \ln\left(\frac{4096*4000^{10}+4}{0.1}\right)$$

N ≥ 299394.712

If for N = 5000,

$$N \geq \frac{8}{0.0025} \ln\left(\frac{4096 * 5000^{10} + 4}{0.1}\right)$$

$$N \geq 3200 \ln(4e + 41)$$

$$N \geq 3200 * 3.94$$

$$N \geq 12636.16$$