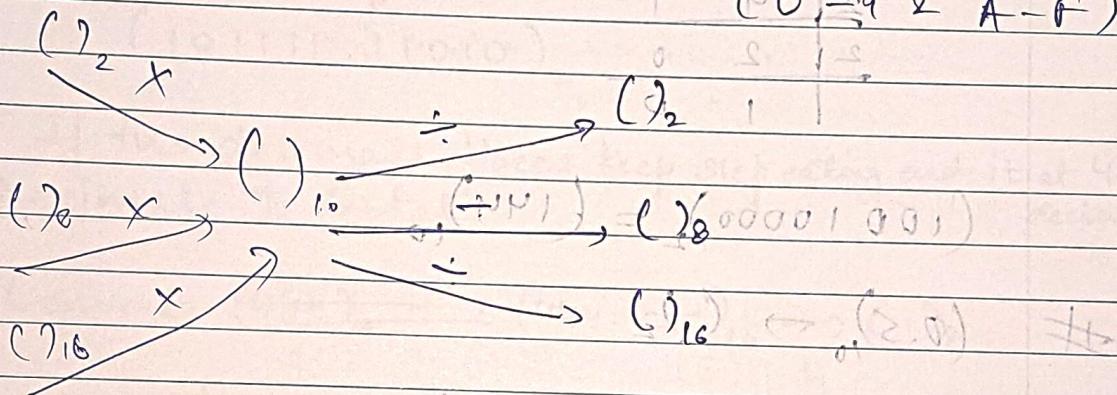


Number System

(S) to (BIN) conversion

- 1) Decimal $\rightarrow (.)_{10} \rightarrow 0-9$
- 2) Binary $\rightarrow (.)_2 \rightarrow 0, 1$
- 3) Octal $\rightarrow (.)_8 \rightarrow 0-7$
- 4) Hexadecimal $\rightarrow (.)_{16} \rightarrow 0-9, 0-A-F$
 $(0-9 \text{ & } A-F)$



Q Convert $(53)_{10}$ to $(.)_2$

$$(01.0) = .1 \cdot 2^0$$

2 53	
2 26 1	
2 13 0	
2 6 1	
2 3 0	
1 0	

$$(.) \leftarrow (.2^0.0) \quad 0 \leftarrow 2^0.0$$

$$1 \leftarrow 0.1 \quad 1 \leftarrow 2.1 \quad 2^0.0$$

$$(53)_{10} = (110101)_2$$

Q Convert $(144)_{10}$ to $(.)_2$

2 144	
2 72 0	
2 36 0	
2 18 0	
2 9 0	
4 1	

Q Convert $(144)_{10}$ to $(\text{?})_2$

$$\begin{array}{r} 2 | 144 \\ 2 | 72 \quad 0 \\ 2 | 36 \quad 0 \\ 2 | 18 \quad 0 \\ 2 | 9 \quad 0 \\ 2 | 4 \quad 1 \\ 2 | 2 \quad 0 \\ 1 \quad 0 \end{array}$$

$$(10010000)_2 = (144)_{10}$$

$(0.5)_{10} \rightarrow (\text{?})_2$

$$\begin{array}{l} 0.5 \times 2 = 1.00 \quad | \\ 0.00 \times 2 = 0.0 \quad (0) \downarrow \end{array}$$

$$(0.5)_{10} = (0.10)_2$$

Q $(0.625)_{10} \rightarrow (\text{?})_2$

$$\begin{array}{ll} 0.625 \times 2 = 1.25 \rightarrow 1 & 1.25 \rightarrow 1 \\ 0.75 \quad 0.75 \rightarrow 0 & 0.5 \rightarrow 0 \\ 1.5 \rightarrow 1 & 1.0 \rightarrow 1 \\ 1 & (100.0 \rightarrow 0) \end{array}$$

$$\Rightarrow (0.625)_{10} = (0.1010)_2$$

Q

Convert $(47.8125)_{10}$ to (D_2) - mixed

$$\begin{array}{r}
 2 | 47 & \quad (0.8125 \times 2 = 1.625 \rightarrow 1) \\
 2 | 23 & 1 \quad 1.625 \rightarrow 1 \\
 2 | 11 & 1 \quad (0.625 \times 2 = 1.25 \rightarrow 1) \\
 2 | 5 & 1 \quad 1.25 \rightarrow 1 \\
 2 | 2 & 1 \quad 1.0 \rightarrow 1 \\
 \hline
 & 1 & 0
 \end{array}$$

$(101111.110010)_2$

If the decimal places keep repeating end it at 4-s
Decimal to Octal $\leftarrow (2751)$ decimal

Q

Convert $(444.456)_{10}$ to (D_8)

$$\begin{array}{r}
 8 | 444 & 0.456 \times 8 = 3.648 \rightarrow 3 \\
 8 | 55 & 4 \quad (3.648 \rightarrow 3) \\
 8 | 6 & 7 \quad 5.184 \rightarrow 5 \\
 \hline
 & 0 & 6 \quad 1.472 \rightarrow 1 \\
 & & 2.8216 \quad 2.8216 \rightarrow 2 \\
 & & 0.674351361 & 1.664 \rightarrow 1
 \end{array}$$

Q

Convert $(235)_{10}$ to (D_{16})

$$\begin{array}{r}
 16 | 235 \\
 16 | 14 & B
 \end{array}$$

$$(235)_{10} = (EB)_{16} + (1)_{16}$$

Binary to Decimal

$$(1010)_2 \rightarrow ()_{10}$$

$$1(2^3) + 0(2^2) + 1(2^1) + 0(2^0)$$

Q Convert $(1011110)_2 \rightarrow ()_{10}$

$$\begin{aligned} & 2+4+8+2+4+8+(160+64+16+8+4+2+1) \\ & = 94 \end{aligned}$$

Q $(1275)_8 \rightarrow ()_{10}$

$$\begin{aligned} & 5 + 7(8^0) + 7(8^1) + 2(64) + 5(128) \\ & 5 + 7(8) + 2(64) + 512 \end{aligned}$$

Q $(A27B)_{16} \rightarrow ()_{10}$

$$11(16^0) + 11(16^1) + 5(16^2) + 4(16^3) + 11(16^4) = 41,595$$

Q $(1010)_2 \rightarrow ()_{10}$

$$0 \times 1 + 1 \times 2 + 0 \times 4 + 1 \times 8$$

Q $(32.12)_8 \rightarrow ()_{10}$

$$2(1) + 3(8) = 26.15625$$

Q) $(AF23.75)_{16} \rightarrow (?)_{10} \rightarrow (01110110110)_{2}$

$$3(1) + 2(16) + 15(16^2) + 10(16^3) = 44835.45703125$$

Q) $(2356.25)_{10} \rightarrow (?)_2$

$$(?) \leftarrow (0101010)$$

2	2356	$0.25 \times 2 = 0.5 \rightarrow 0$
2	1178	$(00100100100)_{2} + (00100100100)_{2} + (00100100100)_{2} \rightarrow 1$
2	589	$0.0 \rightarrow 0$
2	294	1
2	147	0
2	73	1
2	36	1
2	18	$(1001) \leftarrow (0010101010)_{2}$
2	9	0
2	4	1
2	2	0
2	1	0

$$(?) \leftarrow (01110110110111) \rightarrow 100100110100.010$$

Q) $(5623.123)_{10} \rightarrow (?)_{16}$

16	5623	$0.123 \times 16 \rightarrow 1.968 \rightarrow 1$
16	351	7
16	21	F
	1	S

$$(010101011.111010100)$$

15F7. 1F7C

Q) $(0110110.1110)_2 \rightarrow (\)_{10}$, D.S. + 0.25 + 0.125

$$\begin{aligned} & 0(1) + 1(2) + 1(4) + 0(8) + 1(16) + 0(32) + 0(64) \\ & = 2 + 4 + 16 + 32 \\ & = 22.875 \end{aligned}$$

Q) $(7252)_8 \rightarrow (\)_{10}$

$$\begin{aligned} & 1 \cdot 2(1) + 5(8) + 22(64) + 7(512) \\ & = 3754 \end{aligned}$$

Binary to Octal

$$\begin{array}{c} 010(10100)_2 \rightarrow (124)_8 \\ \text{---} \\ 4 \ 2 \ 2 \quad 4 \ 2 \ 1 \quad 2 \ 2 \ 1 \ 2 \ 1 \\ (001 \ 010 \ 100)_2 \end{array}$$

Q) Convert $(11101101.10\underbrace{1110}_2)_2 \rightarrow (\)_8$

$$\begin{array}{c} 011 \ 1011 \ 01 \quad 101 \quad 110 \\ \text{---} \\ 4 \ 2 \ 1 \quad 4 \ 2 \ 1 \quad 4 \ 2 \ 1 \\ (355.56)_8 \end{array}$$

Q) $(127.652)_8 \rightarrow (\)_2$

$$(001010111.110101010)_2$$

Q

$$(2x)_b = (34)_8$$

$$2 \times b + x(1) \Rightarrow 3(\cancel{2}) + 3(b) + 4(8)$$

$$16 + x \Rightarrow 24 + 4$$

$$24 + x = 12 + 8 + 24 + 16$$

$$x = 16$$

Q

$$(211)_b = (152)_8$$

$$2(64) + 8 + 1 = 121(64) + 5(8) + 2$$

$$= 106 + 3(16) + 2$$

$$2(x^2) + x + 1 = 106$$

$$2x^2 + x = 105$$

$$2x^2 + x - 105 = 0$$

$$49(2) + 7 - 105$$

$$\rightarrow 98 + 7 - 105 = 0$$

or radix

D Determine ~~def~~ of number in each operation

$$(\sqrt{41})_b = (5)_b$$

$$\overbrace{4(b) + 1}^{\text{1 0 0 1 0 1}} = 5$$

$$\rightarrow 4 + 6(b^1) + 4(b^2) + 1 = (\cancel{2}\cancel{5})_b 25(b^0)$$

$$= 4b + 1 = 25$$

$$\Rightarrow 4b = 24$$

$$\Rightarrow b = 8$$

Q

$$23 + 44 + 14 + 32 = 123$$

$$\Rightarrow 2(b) + 3 + 4(b) + 4 + 1(b) + (3)(b) + 2 + 4 \\ \Rightarrow 2(b^2) + 2(b) + 3$$

$$\Rightarrow 10b + 8b = 2b^2 + 2b + 3 \rightarrow 22b = 2b^2 + 2b + 3$$

$$\Rightarrow 10b - 8b + 9 = 2b^2$$

$$\Rightarrow 2b^2 - 8b - 9 = 0$$

$$b^2 - 4b - 4.5 = 0$$

$$b^2 - 5b + 1b - 5 = 0$$

$$b(b - 5) + 1(b - 5) = 0$$

$$\Rightarrow (b + 1)(b - 5) = 0$$

$$\Rightarrow b = 5, -1$$

Arithmetic Operator

C) Addn

$$21 + 18 = 39$$

$$21 + 18 = 39$$

C) Subn

$$21 - 18 = 3$$

C) Multn

$$21 \times 18 = 378$$

C) Division

Binary Addition

Q

$$\begin{array}{r} 10111 \\ 01101 \\ \hline 100100 \end{array}$$

$$\begin{array}{r} 111111 \\ 01011101 \\ + 0111001 \\ \hline 101011110 \end{array}$$

$$\begin{array}{r} 01001101 \\ 1011100 \\ \hline 01011101 \\ 1011100 \\ \hline 11110110 \\ 10110 \end{array}$$

Subtraction

Q Subtract 0110 from 1010

$$\begin{array}{r} 1010 \\ - 0110 \\ \hline 0100 \end{array}$$

Ans: 0100

(0100 → 4)

Signed & Unsigned Nos.

Unsigned (True)

A B C 3bit

0 0 0 → 0

0 0 1 → 1

0 1 0 → 2

0 1 1 → 3

1 0 0 → 4

1 0 1 → 5

1 1 0 → 6

1 1 1 → 7

Range → $(0 + 2^n)$

Signed (True, -ve) 1 → -ve 0 → +ve

A B C 3bit

0 0 0 → 0

0 0 1 → 1

0 1 0 → 2

0 1 1 → 3

0 1 1 0 0 0 1 1 0 → 0

1 0 0 0 0 0 1 1 0 → -1

1 0 0 1 0 0 1 1 0 → -2

1 0 1 1 0 0 1 1 0 → -3

Range $(-(2^n) + 1 \text{ to } 2^n - 1)$

Binary to Hexadecimal

$$(1010\ 0110)_2 \rightarrow (?)_{16}$$

8 4 2 1 8 4 2 1

$\rightarrow (A6)_{16}$

$$\text{Convert: } (11.1100110101)_2 \rightarrow (?)_{16}$$

8 4 2 1 8 4 2 1

$\rightarrow (F35)_{16}$

Q Convert $(BA37.25)_{16} \rightarrow (?)_2$

$$(1011\ 1010\ 0011'0111.0010\ \cancel{0101})_2$$

1's Complement

$$(1010)_2 \rightarrow 0101$$

2's complement

$$\begin{array}{r} 1010 \\ \text{---} \\ + 0101 \\ \hline \end{array}$$

$$\begin{array}{r} 000110101 \\ \text{---} \\ + 0111001010 \\ \hline \end{array}$$

Subtraction using 1's complement

Q Use 1's complement method to perform A-B
where $A = 1010$, $B = 1001$

Ans 1's complement of $B = \cancel{+}0110$

Adding A and 1's complement = $\cancel{+}00110110$

If carry is generated it will be added $\begin{array}{r} + 0110 \\ \hline 0000 \end{array}$

If carry is not generated 2 take 1's complement. $\begin{array}{r} + 1 \\ \hline 0001 \end{array}$
2 add -ve ahead of it.

Q

Subtract 1010 from 1000, by 1's compliment

$$A = 1000$$

$$B = 0101$$

$$A+B = \underline{1000}$$

$$\underline{+ 0101}$$

$$\underline{\underline{1101}} \Rightarrow 0010$$

$$\text{Ans} \Rightarrow -(0010)_2$$

Q

Find the binary equivalent of -18 if the number is represented in

- (i) sign magnitude
- (ii) 1's compliment
- (iii) 2's compliment

2 18	$-(10010)_2$
2 9 0	(1000010)
2 4 1	
2 2 0	
1 0	

$$\Rightarrow 00010010 \rightarrow 10$$

$$\Rightarrow 10010010 \rightarrow -18$$

$$00010010 \rightarrow 11101101 \quad (\text{1's compliment})$$

$$\underline{11101101} + 1 \quad (\text{2's compliment})$$

Q $(54)_2 - (87)_2$ use 1's complement method.

$$\begin{array}{r}
 \begin{array}{c|cc}
 2 & 54 \\
 \hline
 2 & 27 & 0 \\
 \hline
 2 & 13 & 1 \\
 \hline
 2 & 6 & 1 \\
 \hline
 2 & 3 & 0 \\
 \hline
 & 1 & 1 \\
 \hline
 (110110) & &
 \end{array}
 \quad
 \begin{array}{c|cc}
 2 & 87 \\
 \hline
 2 & 43 & 1 \\
 \hline
 2 & 21 & 1 \\
 \hline
 2 & 10 & 1 \\
 \hline
 2 & 5 & 0 \\
 \hline
 & 2 & 1 \\
 \hline
 & 1 & 0 \\
 \hline
 & &
 \end{array}
 \end{array}$$

$$\text{ans: } 110110_2 - 1010111_2 = 1011000_2$$

$$\begin{array}{r}
 \cancel{0}01001 \\
 110110 \\
 + \cancel{1}0101000 \\
 \hline
 1011110
 \end{array}$$

$$\begin{array}{r}
 \cancel{0}01001 \rightarrow 01001 \\
 \hline
 0100001 = (0100001)_2
 \end{array}$$

Subtraction using 2's complement

Step 1 Take 2's complement of the second no.

Step 2 Add the two

Step 3 If carry generated discard it.

Step 4 Take 2's complement of the answer.
If carry isn't generated

$$\begin{array}{r}
 1010 - 1100 \\
 \hline
 0110
 \end{array}$$

$$\begin{array}{r}
 + 1 \\
 \hline
 0111
 \end{array}$$

$$\begin{array}{r}
 + 1010 \\
 \hline
 0001
 \end{array}$$

Q

Subtract $10010 - 1010$

$$\begin{array}{r}
 0101 \\
 + 1 \\
 \hline
 0110 \\
 + 1000 \\
 \hline
 \cancel{\underline{\text{1111}}} \\
 \underline{\text{0110}} \\
 0001 \\
 + 1 \\
 \hline
 \underline{(0010)}_2
 \end{array}$$

Q

 $(235) - (324)$

$$\begin{array}{r}
 2 | 235 \\
 2 | 117 \\
 2 | 58 \\
 2 | 29 \\
 2 | 14 \\
 2 | 7 \\
 2 | 3 \\
 1 | 1
 \end{array}
 \quad
 \begin{array}{r}
 2 | 324 \\
 2 | 167 \\
 2 | 88 \\
 2 | 41 \\
 2 | 20 \\
 2 | 10 \\
 2 | 5 \\
 2 | 1
 \end{array}
 \quad
 \begin{array}{r}
 2 | 324 \\
 2 | 162 \\
 2 | 81 \\
 2 | 408 \\
 2 | 20 \\
 2 | 10 \\
 2 | 5 \\
 1 | 0
 \end{array}$$

$(11101011)_2$

$$(10+001110)_2$$

$$0+011000+$$

$$\begin{array}{r}
 + 1 \\
 \hline
 0+0110010
 \end{array}$$

$$101000100$$

$$\begin{array}{r}
 010110010 \\
 + \cancel{\underline{\text{11101101}}} \\
 \hline
 110011101
 \end{array}$$

Q Adding $-(20)_{10}$ & $-(18)_{10}$ using 1's complement

$$\begin{array}{r} 2 | 20 \\ \hline 2 | 10 \\ \hline 2 | 5 \\ \hline 2 | 2 \\ \hline 1 \end{array}$$

$$(10100)_2 \Rightarrow (\underline{\underline{01011}})_2 = (00010100)$$

$$18 \rightarrow (10010)_2 = (00010010)$$

$$(01101)_2 = (11101101)$$

$$\begin{array}{r} \cancel{1} \cancel{1} \cancel{1} \\ \cancel{0} \cancel{1} \cancel{0} \cancel{1} \\ + 01101 \\ \hline \cancel{1} \cancel{0} \cancel{1} \cancel{0} \end{array}$$

$$\begin{array}{r} 1 \cancel{1} \cancel{1} \cancel{1} \cancel{1} \\ + 11101101 \\ \hline 011011000 \end{array}$$

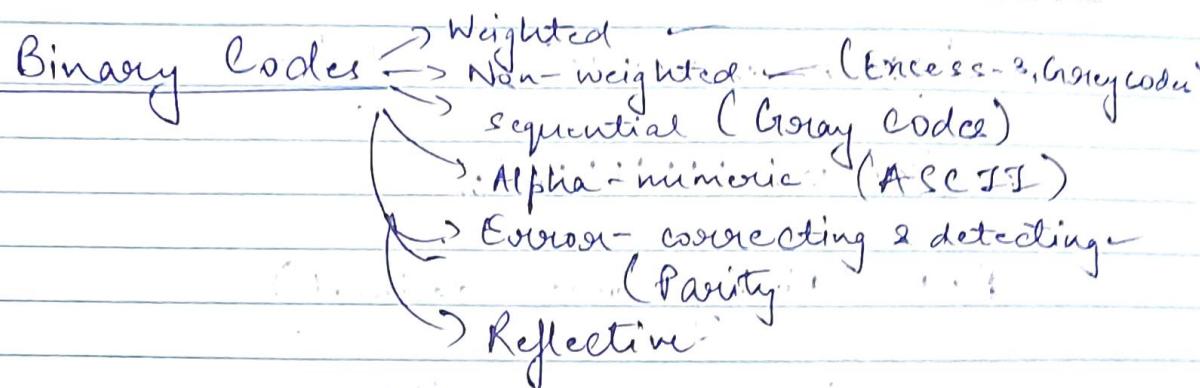
$$\begin{array}{r} 111011001 \\ + 110110010 \\ \hline 00100110 \end{array}$$

Q $(23.75)_{16} \rightarrow (.)_8$

$$(00010\ 0011\ 1010\ 1100\ 0101\ 1001)_2$$

$$216368$$

$$21675.262$$



Non-weighted

Excess-3:

$$(123)_{10} \rightarrow \begin{array}{r} 0001 \\ + 0011 \\ \hline 0100 \end{array}$$

$$\begin{array}{r} 2 | 123 \\ 2 | 381 \\ 2 | 190 \\ 2 | 91 \\ 2 | 45 \\ 2 | 22 \\ 2 | 10 \end{array}$$

$$0001 + 0011 = 0100$$

$$\boxed{(123)_{10} \rightarrow (0100)_{Excess-3}}$$

$$0001 + 0011 = 0100$$

$$\boxed{(123)_{10} \rightarrow (0100)_{Excess-3}}$$

$$0001 + 0011 = 0100$$

$$\boxed{(123)_{10} \rightarrow (0100)_{Excess-3}}$$

• Gray Codes

These are also called cyclic codes.

Convert $(101011)_G \rightarrow ()_2$ Steps $()_G \rightarrow ()_2$

Diagonal XOR of the bits.

$$(101011)_G \rightarrow ()_2$$

$$()_2 \rightarrow ()_G$$

XOR adjacent bits

$$1010 = (1+0)010, \downarrow$$

$$(001100101)_2$$

$$(1010101)_G$$

$$(1100010101011)_2 \rightarrow (?)_G$$

$$(?)_G (101001111110)_G$$

BCD (Binary Coded decimal)

Binary	BCD
0	↙
1	↙
10	1010

1000 0100 1000

1100 1100 1100

0110 1010 0010

& grey code

Q Give Binary, BCD, excess-3 of the follo.

→ 1) 5

2) 8

3) 14

$$(5)_{10} \rightarrow (0101)_G, \rightarrow (0101)_BCD \rightarrow (1000)_G$$

$$\text{Excess-3} \quad \text{BCD} \quad \text{Grey code}$$

0101

+ 0011

1000

Binary	BCD	Excess-3	Grey code
8	1000	1000	1101
14	1110	00010100	11011100

$$14 : (1110 00010100)_{BCD} (01000111)_{Excess-3} (11011100)_{Grey code}$$

Rules in BCD addition

- I If sum is equal to or less than 9 and carry is 0 then it is valid BCD.
- II If sum is greater than 9 with carry == 0 we add 6 (0110) to make it valid.
- III If sum <= 9 with carry = 1, we add 6 to make it valid BCD.

Ex:

9	1001	A A A
+ 8	1000	B A S A
	<u>0001 00001</u>	→ invalid
	+ 0110	0 0 1
	<u>0001 01111</u>	→ 17 0 1 0
		0 0 0

II Add $(57)_{10}$ & $(26)_{10}$

57	0 1010110	(0.1010110 = 57)
+ 26	00100010	
	<u>0 1111101</u>	
		(0.1111101 = 83)
		S.C.A = (0.1).A
		10000 00111 = 83
		A + S = 341

III Add $(569)_{10}$ & $(687)_{10}$ in BCD

569	0101 0110 1001	(010101101001 = 569)
687	0110 1000 0111	(011010000111 = 687)
1256	<u>1011 1111 0000</u>	
		(101111110000 = 1256)

Boolean Algebra \rightarrow Subtraction, division

Addition $\rightarrow + \rightarrow A+B$

Multiplication $\rightarrow \times \rightarrow A \cdot B$

$\rightarrow 1+1=0$ without carrying

Truth Table

AND

A	B	AB	A	B	A+B
1	1	1	100	0000	0
1	0	0	001	1	1
0	1	0	111	010	1
0	0	0	1	1	1

NOT $\Rightarrow 0 \rightarrow 1 (\bar{A}, A')$

A	\bar{A}
1	0
0	1

(1) Commutative

$$A \cdot B = B \cdot A$$

$$A+B = B+A$$

(2) ~~+~~ Associative law

$$A \cdot (B \cdot C) = (A \cdot B) \cdot C$$

$$A+(B+C) = (A+B)+C$$

(3) Distributive

$$A \cdot (B+C) = A \cdot B + A \cdot C$$

$$A+(B \cdot C) = (A+B) \cdot (A+C)$$

(4) De Morgan's

$$\bar{A} \cdot \bar{B} = \bar{A} + \bar{B}$$

$$\bar{A} + \bar{B} = \bar{A} \cdot \bar{B}$$

A	B	$A \cdot B$	$\bar{A} \cdot B$	\bar{A}	\bar{B}	$\bar{A} + \bar{B}$
1	1	1	0	0	0	0
1	0	0	1	0	1	1
0	1	0	1	1	0	1
0	0	0	1	1	1	1

Duality Principle

$\rightarrow 0$

$0 \rightarrow 1$

$$(A + (B \cdot C)) + D \cdot (A + B) = 1$$

$\cdot \rightarrow +$

$$AB + \overline{A}B = B$$

$+ \rightarrow \cdot$

$$AB + (A + B) \cdot \overline{AB} = AB$$

$$AB + A\overline{B} = AB$$

AND Laws

$$A \cdot 0 = 0$$

$$A \cdot 1 = A$$

$$A \cdot A = A$$

$$A \cdot \overline{A} = 0$$

$$((A+B) + (\overline{A}+B)) \cdot 0 = ((\overline{A}) + A) \cdot 0 = 0$$

A	\overline{A}	$A \cdot \overline{A}$	$(A + \overline{A}) \cdot A + \overline{A}$
1	0	0	1 + 0 = 1
0	1	0	0 + 1 = 1

Inversion Laws $(A + \overline{A} + \overline{AB}) = 1$

$$\overline{\overline{A}} = A$$

$$A \cdot \overline{B} + (\overline{A} \cdot B) = (\overline{A} + B) \cdot (\overline{B} + A) =$$

$$\text{① } (1) A + \overline{A} \cdot B = \overline{A} + (A + \overline{A}) \cdot (A + B) = 1 \cdot (A + B)$$

$$(2) (A + B) \cdot (\overline{A} + C) = A + (B \cdot C)$$

$$(A + \overline{A}) \cdot (A + B)$$

$$\text{Simplify } y = \text{Prove } ((A + \overline{B}) + AB)(A + \overline{B})(\overline{A}B) = 0$$

$$((A + \overline{B}) + AB)(A + \overline{B})(\overline{A}B) = 0$$

$$(A + (\overline{B} + AB)) \cdot (A + \overline{B}) \cdot (\overline{A}B) = 0 \quad \text{Acc to associative}$$

$$(1 + \overline{A} + A) \cdot (A + \overline{B}) \cdot (\overline{A}B) = 0$$

$$(A + (\overline{B} + AB)) \cdot (A\overline{B} + \overline{B}\overline{A}B) = 0$$

$$(A + (\overline{B} + AB)) \cdot (0 \cdot B + 0 \cdot \overline{A}) = 0$$

Q Simplify $y = (AB + C) \cdot (AB + D)$

$$\begin{aligned} y &= AB(AB + D) + C(AB + D) \\ &= AB \cdot AB + ABD + CAB + CD \\ &= AB(1 + D) + CD \\ &= AB + CD \end{aligned}$$

Q Simplify $A \cdot \bar{B} \cdot C + \bar{A} \cdot B \cdot C + ABC$

$$\begin{aligned} &\Rightarrow C(A \cdot \bar{B} \cdot 1 + \bar{A} \cdot B + AB) \\ &\Rightarrow C(A(\bar{B} + B) + \bar{A}B) \\ &\Rightarrow C(A \cdot 1 + \bar{A}B) \\ &\Rightarrow C(A + (\bar{A}B)) \Rightarrow C((A + \bar{A}) + (A + B)) \\ &\Rightarrow C(0 + A + B) \\ &\Rightarrow C(A + B) \\ &= AB + BC \end{aligned}$$

Q Simplify $y = (\overline{AB} + \bar{A} + AB) = \overline{AB} \cdot \bar{A} + \overline{AB}$

$$\begin{aligned} &= AB \cdot A \cdot \overline{AB} \\ &= (\overline{AB} + A + \overline{AB}) = (A \cdot B \cdot \bar{A}) + \bar{B} \cdot A \cdot \bar{B} \\ &\Rightarrow A(\bar{B} + 1) + \overline{AB} = 0 \cdot B + 1 \cdot A \\ &\Rightarrow \overline{AB} + A - A + \overline{AB} = 1 + 0 \\ &\Rightarrow (\overline{A} + \bar{B}) \cdot (A + \bar{B}) \end{aligned}$$

Q Simplify $y = \overline{ABC} + \overline{ABC} + A\overline{BC} + ABC$

$$\begin{aligned} &= \bar{C}(\overline{AB} + \overline{AB} + A\bar{B} + AB) \\ &= \bar{C}(A(\bar{B} + B) + A(\bar{B} + B)) \quad \text{As } \bar{B} + B = 1 \\ &= \bar{C}(\bar{A} + A) \quad \text{As } \bar{A} + A = 1 \end{aligned}$$

$$= \bar{C} + \bar{C}A$$

$$= (\bar{C} + 1)A$$

$$\begin{aligned}
 \Theta \quad & \text{Simplify } y = ABC + A\bar{B}C + A\bar{B}\bar{C} \\
 & = A(CB + \bar{B}C + B\bar{C}) \\
 & = A(C(B + \bar{B}) + B\bar{C}) \\
 & = A(C + B\bar{C}) \\
 & = A(C + B)(C + \bar{C}) \quad \{ C + (B\bar{C}) = (C + B)(C + \bar{C}) \} \\
 & = AC(B + C)
 \end{aligned}$$

$$\begin{aligned}
 \Theta \quad & \text{Simplify } y = \bar{A}C(\bar{A}\bar{B}\bar{D}) + \bar{A}\bar{B}\bar{C}\bar{D} + A\bar{B}C \\
 & = \bar{A} \cdot C (\bar{A} + \bar{B} + \bar{D}) + \bar{A}\bar{B}\bar{C}\bar{D} + A\bar{B}C \\
 & = \bar{A} \cdot C (A + \bar{B} + D) + \bar{A}\bar{B}\bar{C}\bar{D} + A\bar{B}C \\
 & = \bar{A} \cdot C \cdot A + \bar{A} \cdot C \cdot B + \bar{A} \cdot C \cdot D + \bar{A}\bar{B}\bar{C}\bar{D} + A\bar{B}C \\
 & = \bar{A} (CB + CD + B\bar{C}\bar{D}) + A\bar{B}C \\
 \Rightarrow & \bar{A} (CB + D(C + (B\bar{C}))) + A\bar{B}C \\
 \Rightarrow & \bar{A} (CB + D(C + B), C + \bar{C})) + A\bar{B}C \\
 \Rightarrow & \bar{A} (CB + D(C + DB)) + \cancel{\bar{A}B} A\bar{B}C \\
 \Rightarrow & \bar{A} \cdot C (A + \bar{B} + \bar{D}) + \bar{A} B \bar{C} \bar{D} + A\bar{B}C \\
 \Rightarrow & \bar{A} C (\bar{A} + \bar{B} + \bar{D}) + \bar{A} B \bar{C} \bar{D} + A\bar{B}C \\
 \Rightarrow & \bar{A} \bar{B} C + \bar{A} \bar{D} C + \bar{A} B \bar{C} \bar{D} + A\bar{B}C \\
 \Rightarrow & \bar{B} C (\bar{A} + A) + \bar{A} \bar{D} [AC + (B\bar{C})] \\
 \Rightarrow & \bar{B} C + \bar{A} \bar{D} [(C + B), C + \bar{C})] \\
 \Rightarrow & \bar{B} C + \bar{A} \bar{D} (C + B)
 \end{aligned}$$

$$\begin{aligned}
 \Theta \quad & \text{If } AB + \bar{A}B \rightleftharpoons C \quad \text{Show, } \bar{A}C + \bar{A}C = B \\
 & \bar{C} = \bar{AB} + \bar{A}B \\
 & = (\overline{AB}) \cdot (\overline{AB}) \\
 & = (\bar{A} + B)(A + \bar{B}) = \bar{A}A + \bar{A}\bar{B} + BA + B\bar{B} \\
 & = \bar{A}\bar{B} + AB \\
 & A(\bar{A}\bar{B} + AB) + \bar{A}(\bar{A}\bar{B} + \cancel{AB}) \\
 \Rightarrow & A\bar{A}\bar{B} + AAB + \cancel{A}\bar{A}\bar{B} + \bar{A}\bar{A}B \\
 \Rightarrow & AB + \bar{A}B = B \\
 \Rightarrow & B(A + \bar{A}) = B \\
 \Rightarrow & B = B
 \end{aligned}$$

LOGIC GATES

Basic gates:

- NOT: $\rightarrow A \rightarrow \bar{A}$
- OR: $\oplus A + B \rightarrow A + B$
- AND: $\wedge A \cdot B \rightarrow A \cdot B$
- NOT-OR: $\oplus \bar{A} + B \rightarrow \bar{A} + B$

Universal:

$$A + B \rightarrow C \rightarrow \bar{A} + \bar{B} \rightarrow \bar{A} \cdot \bar{B}$$

$$\bar{A} + \bar{B} \rightarrow C \rightarrow \bar{A} \cdot \bar{B} \rightarrow A \cdot B$$

$$\bar{A} \cdot \bar{B} \rightarrow C \rightarrow \bar{A} + \bar{B} \rightarrow A + B$$

Exclusive Gates

$$A \oplus B = ((\bar{A} \cdot B) + (\bar{B} \cdot A)) \rightarrow A \oplus B$$

$$A \oplus B = (\bar{A} \cdot \bar{B}) + (\bar{A} \cdot B) + (A \cdot \bar{B}) \Rightarrow A \oplus B = \bar{A} \cdot \bar{B} + A \cdot B$$

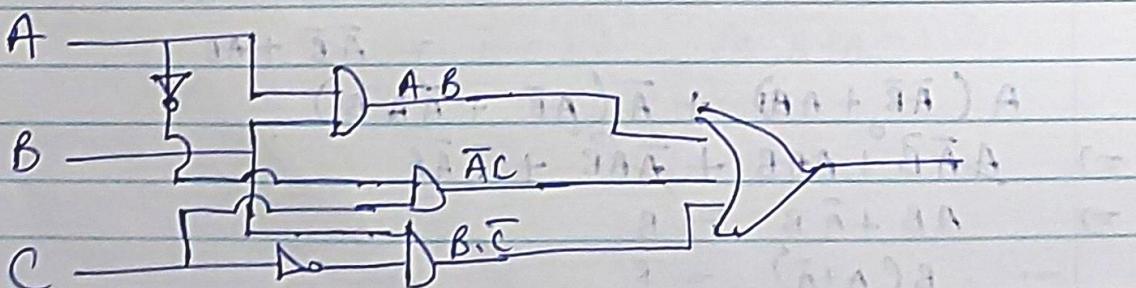
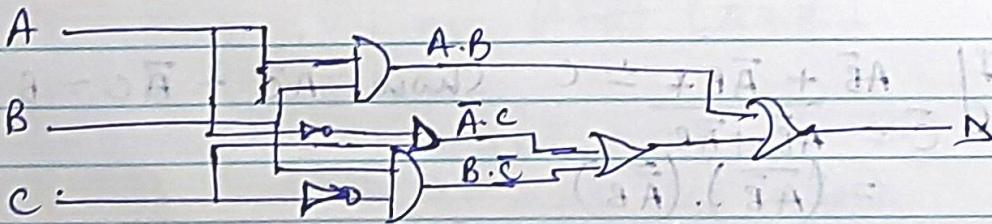
$$A \oplus B = (\bar{A} \cdot \bar{B}) + (\bar{A} \cdot B) + (A \cdot \bar{B}) \Rightarrow A \oplus B = \bar{A} \cdot \bar{B} + A \cdot B$$

$$A \oplus B = (\bar{A} \cdot \bar{B}) + (\bar{A} \cdot B) + (A \cdot \bar{B}) \Rightarrow A \oplus B = \bar{A} \cdot \bar{B} + A \cdot B$$

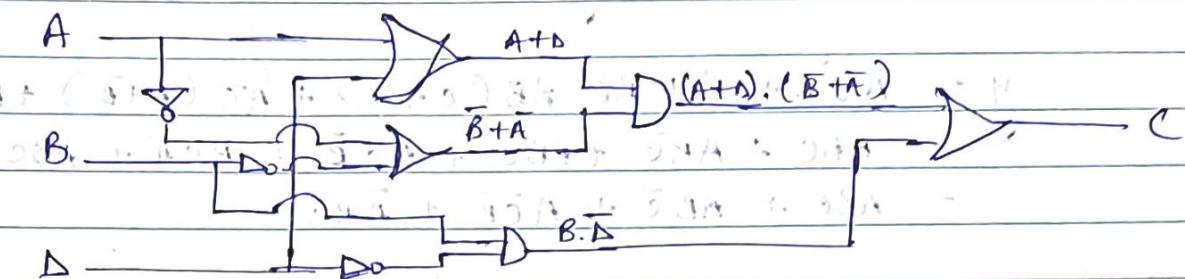
$$A \oplus B = (\bar{A} \cdot \bar{B}) + (\bar{A} \cdot B) + (A \cdot \bar{B}) \Rightarrow A \oplus B = \bar{A} \cdot \bar{B} + A \cdot B$$

Q) Draw logic diagram for the given equation

$$AB + \bar{A}c + B\bar{C}$$



$$(ii) (A + D)(\bar{B} + \bar{A}) + B.\bar{D}$$



SOP and POS form

$$\text{SOP} \rightarrow AB + B\bar{D} + \bar{B}C + \bar{B}\bar{A} + \bar{D}A =$$

$$\text{POS} \rightarrow (A + B)(B + \bar{D})(\bar{B} + C) + \bar{D}A =$$

In case of minterms or SOP we complement the variables where zero exists.

DIGITS	BINARY	SOP (minterm)	POS (maxterm)
0	000	$\bar{A}\bar{B}\bar{C}$	$A + B + C$
1	001	$\bar{A}\bar{B}C$	$A + B + \bar{C}$
2	010	$\bar{A}B\bar{C}$	$A + \bar{B} + C$
3	011	$\bar{A}BC$	$A + \bar{B} + \bar{C}$
4	100	$A\bar{B}\bar{C}$	$\bar{A} + B + C$
5	101	$A\bar{B}C$	$\bar{A} + B + \bar{C}$
6	110	$AB\bar{C}$	$\bar{A} + \bar{B} + C$
7	111	ABC	$\bar{A} + \bar{B} + \bar{C}$

$$\text{Minterms} \rightarrow \Sigma m(2, 3, 5) \rightarrow \bar{A}\bar{B}\bar{C} + \bar{A}\bar{B}C + A\bar{B}\bar{C}$$

$$\text{Maxterms (POS)} \rightarrow \Pi M(2, 3, 5) \rightarrow (A + \bar{B} + C) + (A + \bar{B} + \bar{C}) + (\bar{A} + B + \bar{C})$$

Q Convert the expression $y = AB + A\bar{C} + B.C$ into standard SOP form

$$\begin{aligned}
 y &= A[B + \bar{C}] + BC, AB(C + \bar{C}) + AC(B + \bar{B}) + BC(A + \bar{A}) \\
 &= ABC + ABC\bar{C} + ABC\bar{C} + A\bar{C}\bar{B} + BC\bar{A} + ABC \\
 &= ABC + AB\bar{C} + A\bar{C}\bar{B} + \bar{A}BC
 \end{aligned}$$

Q Convert $y = A + B.C + ABC$ into SOP

$$\begin{aligned}
 &= A(B + \bar{B}) + B.C(A + \bar{A}) + ABC \\
 &= AB + A\bar{B} + BCA + \bar{A}BC + ABC \\
 &= ABC + A\bar{B}C + A\bar{B}C + \bar{A}BC + \bar{A}BC + ABC \\
 &= ABC + A\bar{B}C + A\bar{B}C + \bar{A}BC + \bar{A}BC
 \end{aligned}$$

Q Convert $y = (A+B)(A+C)(B+\bar{C})$ into standard POS.

$$= (A + B + C\bar{C})(A + C + B\bar{B})(B + \bar{C} + A\bar{A})$$

ERROR DETECTING & CORRECTING

Parity Bits \rightarrow Even
 \rightarrow Odd

3 + P Bits	Binary	Even Parity	Odd Parity
3 + 0	000	000	1
3 + 1	001	100	0
3 + 2	010	010	1
3 + 3	011	111	0
4	100		
5	101		
6	110		
7	111		

Q If we have transmitted data 0110, then find no. of parity bits with even parity.

Ans

$$0110 \rightarrow n=4, k=1 \text{ (odd number)}$$

$$2^k \geq n+k+1 \quad \begin{matrix} \text{even} \\ \text{no. of parity bits} \end{matrix}$$

$$\Rightarrow 2^k \geq 4+1+1 \quad \begin{matrix} \text{odd number} \\ 2^2 > 2^1 + 2^0 \end{matrix}$$

$$\Rightarrow k=2 \quad P_1, P_2, P_3, P_4$$

Encoder posn	P ₃	P ₂	P ₁ \rightarrow Parity	0	1	1	P ₃ =0	P ₂ =P ₁
0	0	0	0	0	1	1	0	0
1	0	0	1	1	1	0	1	1
2	0	1	0	1	0	1	0	1
3	0	1	1				P ₂ = 2, 3, 6, 7 $\Rightarrow P_2=1$	
4	1	0	0				Codeword P ₂ , 0, 0 follows 8	
5	1	0	1				P ₃ = 4, 5, 6, 7 $\Rightarrow P_3=0$	
6	1	1	0				P ₃ = 1, 1, 0, 0, 2	
7	1	1	1					

Q At receiver end, the received codeword is 0110111. Find the bit which erroneous with even parity.

$$C_1 = 4, 5, 6, 7 \quad C_1=0$$

$$0 \ 1 \ 1 \ 0$$

$$C_2 = 2, 3, 6, 7 \quad C_2=1$$

0 1 1 0 1 1 0 \rightarrow 0 has even parity, so group A is high

$$C_3 = 1, 3, 5, 7 \quad C_3=1$$

0 1 1 0 1 1 0 \rightarrow 1 has even parity, so group B is low

$$C_1, C_2, C_3$$

0, 1, 1, 0, 1, 1, 0 \rightarrow 1 is odd, so find 1 state

Unit - 2

Date: _____

Minimization of switching funcⁿ

Karnaugh Map (K-Map)

Method to simplify Boolean expression:

→ Algebraic Expression

→ K-Map

→ D-Map

(SOP)

Karnaugh Map (K-Map)

① 2 variables $\rightarrow A \bar{B} + \bar{A} B$

A	\bar{A}	A
B	0	1
\bar{B}	1	0

② 3 variables ($2^3 = 8$ cells)

ABC		$\bar{B}\bar{C}$	$\bar{B}C$	$B\bar{C}$	BC
A		00	01	11	10
A		0	1	3	2
0	0	0	1	3	2
1	0	4	5	7	6

③ 4-variables ($2^4 = 16$ cells)

•	1	3	2
4	5	7	6
12	13	15	14
8	9	11	10

Pair : A group of 2 adjacent '1's & '0's

Quad : A group of 4 adjacent '1's & '0's

Octate : Group of 8 adjacent '1's & '0's.

$$\Theta \quad y = \sum m(1, 3, 2, 7) \\ \hookrightarrow \text{SOP} \quad \hookrightarrow 0-7 \Rightarrow 2^3 \Rightarrow A, B, C$$

	$\bar{B}C$	$\bar{B}\bar{C}$	$B\bar{C}$	BC
\bar{A}	1	0	1	1
A	0	1	1	0
	1	1	1	0

$$\bar{A}C + \bar{A}B + BC$$

 Σ_m

$$\Theta \quad y = \sum m(0, 1, 2, 5, 13, 15)$$

get simplified expression.

$$\Rightarrow y = (0, 1, 2, 5, 13, 15)$$

	\bar{AB}	$\bar{A}\bar{B}$	AB	$A\bar{B}$
\bar{CD}	1	0	1	1
$\bar{C}\bar{D}$	4	1	5	7
CD	12	13	11	14
$C\bar{D}$	8	9	11	10

$$\bar{A}\bar{B}\bar{C}\bar{D}$$

Θ Minimize the following: $y = \sum m(1, 2, 9, 10, 11, 14, 15)$
 & draw logic circuit for the realized

	\bar{AB}	$\bar{A}\bar{B}$	AB	$A\bar{B}$
\bar{CD}	0	1	1	1
$\bar{C}\bar{D}$	4	5	7	6
CD	12	13	15	14
$C\bar{D}$	8	9	11	10

$$\bar{A}\bar{B}\bar{C} + \bar{A}\bar{B}B$$

$$\begin{aligned} y &= BD + A\bar{B}\bar{C} + \bar{A}B\bar{C} \\ &= B(D + \bar{A}B) + \bar{C}(A\bar{B} + \bar{A}B) \\ &= BD + \bar{C}(A \oplus B) \end{aligned}$$

A

B

C

D

$$\text{Q} \quad y = (m_{4,2} + m_5 + m_8 + m_9 + m_{11} + m_{12} + m_{13}) + m_{15}$$

AB	CD	$\bar{C}\bar{D}$	CD	$\bar{C}D$	$\bar{C}\bar{D}$
$\bar{A}\bar{B}$	0	1	3	2	
$A\bar{B}$	1	0	8	6	
$A\bar{B}$	1	1	13	15	
AB	1	2	19	11	10
$\bar{A}B$	1	3	18	14	

$$y = B\bar{D} + C\bar{B} + A\bar{B}\bar{D}$$

A

B

C

K-Map (P.O.C)

2 variable

B	A	B	\bar{B}
A	$(A+B)$	$(A+\bar{B})$	
\bar{A}	$(B+A)$	$(\bar{A}+\bar{B})$	

~~(2)~~ 8-variable: ~~3 input variables with 2 don't care~~

$$A \setminus BC\bar{B}+C \quad B+\bar{C} \quad \bar{B}+\bar{C} \quad B+C \quad \text{f}(1,0,1,0,1,2,1,0)$$

A	1 0	0 1	0 1	1 2	
\bar{A}	0 1	1 0	1 1	1 0	

$$3A + 2A + A = f$$

~~(3)~~ 8-variable: $y = \text{CATK}(0, 4, 5, 7, 10, 11, 14, 15)$

	CD	C+D	C+D	$\bar{C}+\bar{D}$	$\bar{C}+\bar{D}$	$\bar{C}+D$	$\bar{C}+D$
AB	0 0	0 1	1 0	1 1	0 0	0 1	1 0
$A+B$	0 0	0 1	1 0	1 1	0 0	0 1	1 0
$A+\bar{B}$	0 1	0 0	0 1	1 0	1 1	1 0	0 1
$\bar{A}+B$	1 1	1 0	0 1	0 0	1 1	1 0	0 1
$\bar{A}+\bar{B}$	1 0	0 0	0 0	1 1	1 0	0 1	0 0

$$y = (\bar{A}+B+C) \cdot (\bar{A}+\bar{C}) \cdot \bar{B} \bar{A} + \bar{A} \bar{B} + \bar{A} \bar{C} = f$$

K-Map with don't care cond'n (X or d)

In case of K-map it is not always true that the cells containing ones (SOP) will contain zeroes becuz some combinations of input variables don't occur. In such situations we have freedom to assume to have a zero or one as output for each of these combinations. These conditions are called don't care cond'n.

Q Minimise the expression $y = \sum m(0,1,5,9,13,14,15) + d(3,4,7,10,11)$

AB	CD	$\bar{C}\bar{D}$	$\bar{C}D$	$C\bar{D}$	CD
$\bar{A}\bar{B}$	0	1	1	0	1
$\bar{A}B$	4	1	0	1	0
$A\bar{B}$	12	1	1	1	1
AB	8	1	0	0	1

$$y = D + AC + \bar{A}\bar{C}$$

Q Simplify $y = (m_1, m_4, m_8, m_{12}, m_{13}, m_{15}) + d(3,14)$

AB	CD	$\bar{C}\bar{D}$	$\bar{C}D$	$C\bar{D}$	CD
$\bar{A}\bar{B}$	0	1	1	0	1
$\bar{A}B$	1	1	0	1	0
$A\bar{B}$	12	1	1	1	1
AB	8	1	0	0	1

$$y = B\bar{C}\bar{D} + A\bar{B}B + \bar{A}\bar{B}D + \bar{A}\bar{A}\bar{C}\bar{D} + \bar{A}\bar{B}$$

5 variable K-map $\Rightarrow 32$ cells \therefore $m(0,2,5,7,13,15,18,20,21,23,28,29,31)$

BC	DE	$\bar{D}\bar{E}$	$\bar{D}E$	$D\bar{E}$	DE
$\bar{B}\bar{C}$	0	1	1	1	1
$\bar{B}C$	4	1	1	1	0
$B\bar{C}$	12	1	1	1	1
BC	8	1	1	1	0

BC	DE	$\bar{D}\bar{E}$	$\bar{D}E$	$D\bar{E}$	DE
$\bar{B}\bar{C}$	16	17	17	19	18
$\bar{B}C$	20	21	23	21	22
$B\bar{C}$	29	30	31	32	30
BC	24	25	26	27	28

Q Simplify $j(ABCDE) = \sum m(0,2,5,7,13,15,18,20,21,23,28,29,31)$

A	$\bar{D}\bar{E}$	$\bar{D}E$	$D\bar{E}$	DE
$\bar{B}\bar{C}$	0	1	1	1
$\bar{B}C$	4	1	1	0
$B\bar{C}$	12	1	1	1
BC	8	1	1	0

BC	DE	$\bar{D}\bar{E}$	$\bar{D}E$	$D\bar{E}$	DE
$\bar{B}\bar{C}$	16	17	17	19	18
$\bar{B}C$	20	21	23	21	22
$B\bar{C}$	120	121	123	121	120
BC	24	25	27	26	28

$$y = CE + \bar{B}\bar{C}\bar{D}\bar{E} + \bar{A}\bar{B}\bar{C}\bar{E} + ACD$$

$$\underline{\Theta} \quad f(ABCDE) = \Sigma (0, 1, 7, 9, 11, 13, 15, 16, 17, 23, 25, 27)$$

0	1	3	2
4	8	7	6
12	13	15	16
8	9	11	10

16	17	19	18
20			