

# Realization Problems on Reachability Sequences

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Matthew Dippel, Ravi Sundaram, **Akshar Varma**

Northeastern University, Boston

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# Overview

1 The Reachability Realization Problem

2 Our Results

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Our results show an interesting interplay between the local property of degree and the global property of reachability.

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		<b>Out-degree</b>	
		Unbounded	Bounded
<b>In-Degree</b>	Unbounded (DAGs)		
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		Out-degree	
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In-Degree	Unbounded (DAGs)	Linear-time	NP-Complete
	Bounded (Trees)	NP-Complete	NP-Complete

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		Out-degree	
		Unbounded	Bounded
In-Degree	Unbounded (DAGs)	Linear-time	$(O(\log n), O(\log n))$
	Bounded (Trees)	$(O(\log n), O(\log n))$	$(O(\log n), O(\log n))$

foo bar



foo **bar** a what?

A diagram consisting of a rectangular box containing the word "bar". To the left of the box is the word "foo". Above and to the right of the box is the phrase "a what?". A thin black line originates from the top-right corner of the box and extends diagonally upwards and to the right, ending just below the "a" in "a what?".