

$$f(n) = 1000 n$$

Find smallest n where:
 $g(n) \geq f(n)$

$$g(n) = 2 n^2$$

$$2n^2 \geq 1000 n$$

$$n \geq 500$$

n	f(n)	g(n)	f(n) / f(n-1)	g(n) / g(n-1)
1	1000	2	-	-
2	2000	8	2	4
3	3000	18	1.5	2.25
4	4000	32	1.33	1.78
5	5000	50	1.2	1.5625

Scalability

- 1) $f(n)$ is $O(g(n)) \Rightarrow g(n)$ is $\Omega(f(n))$
- 2) $f(n)$ is $\Omega(g(n)) \Rightarrow g(n)$ is $O(f(n))$
- 3) $f(n)$ is $\Theta(g(n)) \Leftrightarrow f(n)$ is $O(g(n))$ AND $f(n)$ is $\Omega(g(n))$

To prove: $f(n)$ is $\Theta(g(n)) \Leftrightarrow g(n)$ is $\Theta(f(n))$

$$f(n) \text{ is } \Theta(g(n)) \Rightarrow f(n) \text{ is } O(g(n)) \text{ AND } f(n) \text{ is } \Omega(g(n)) \quad [\text{apply rule 3}]$$

$$\Rightarrow g(n) \text{ is } \Omega(f(n)) \text{ AND } g(n) \text{ is } O(f(n)) \quad [\text{apply rules 1 and 2}]$$

$$\Rightarrow g(n) \text{ is } \Theta(f(n)) \quad [\text{apply rule 3}]$$

$$f(n) = 2n^2 + 4n^3$$

$$= d(n) + e(n)$$

$$= O(n^2 + n^3)$$

$$= O(g(n))$$

$$d(n) = 2n^2 \text{ is } O(n^2)$$

$$e(n) = 4n^3 \text{ is } O(n^3)$$

$$g(n) = O(n^3)$$

$g(n) = n^2 + n^3$
 Now, $g(n)$ is a polynomial of degree 3,
 $g(n)$ is $O(n^3)$.

$$\text{Thus, } f(n) = O(n^3)$$

$$f(n) = 3 \log(n) + \log(\log(n))$$

$$= d(n) + e(n)$$

$$= O(\log(n) + \log(\log(n)))$$

$$= O(2 \log(n))$$

$$= O(b(n))$$

$$d(n) = 3 \log(n) \text{ is } O(\log(n))$$

$$e(n) = \log(\log(n)) \text{ is } O(\log(n))$$

$$b(n) = 2 \log(n) \text{ is } O(\log(n))$$

$$\text{Thus, } f(n) \text{ is } O(\log(n))$$

$$\text{So, } e(n) \text{ is } O(\log(n))$$

$$2n^5 \text{ is } O(n^5)$$

$$n^5 + 3 \text{ is } O(n^5)$$