Insertions and deletions in B+ trees

Introduction to Database Design 2011, Lecture 11
Supplement to lecture slides

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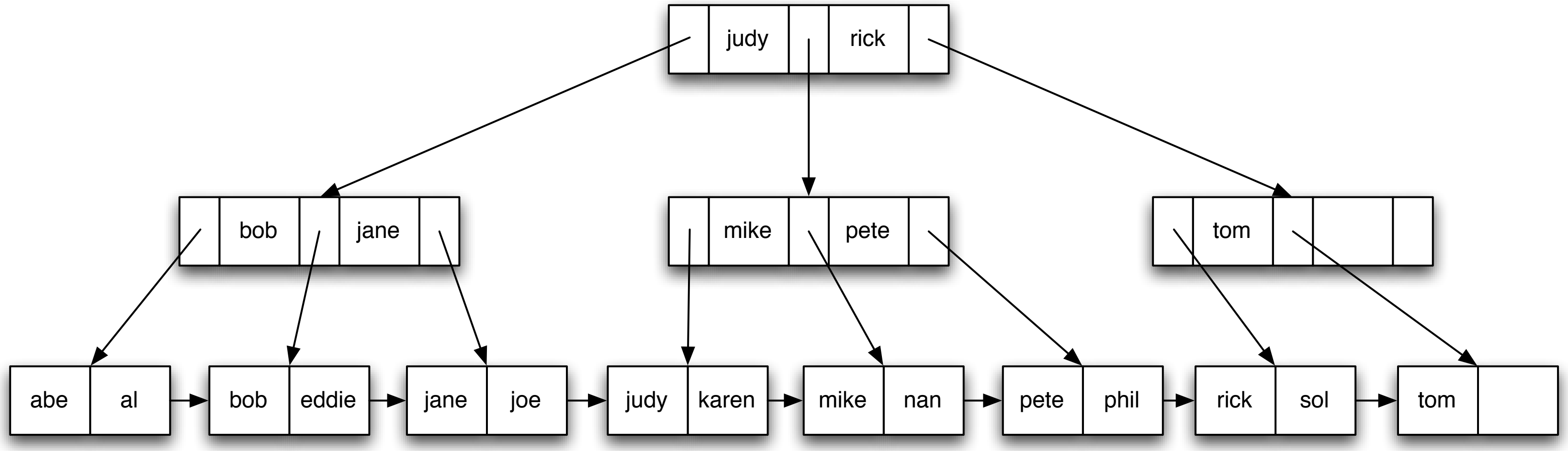
Initial setup

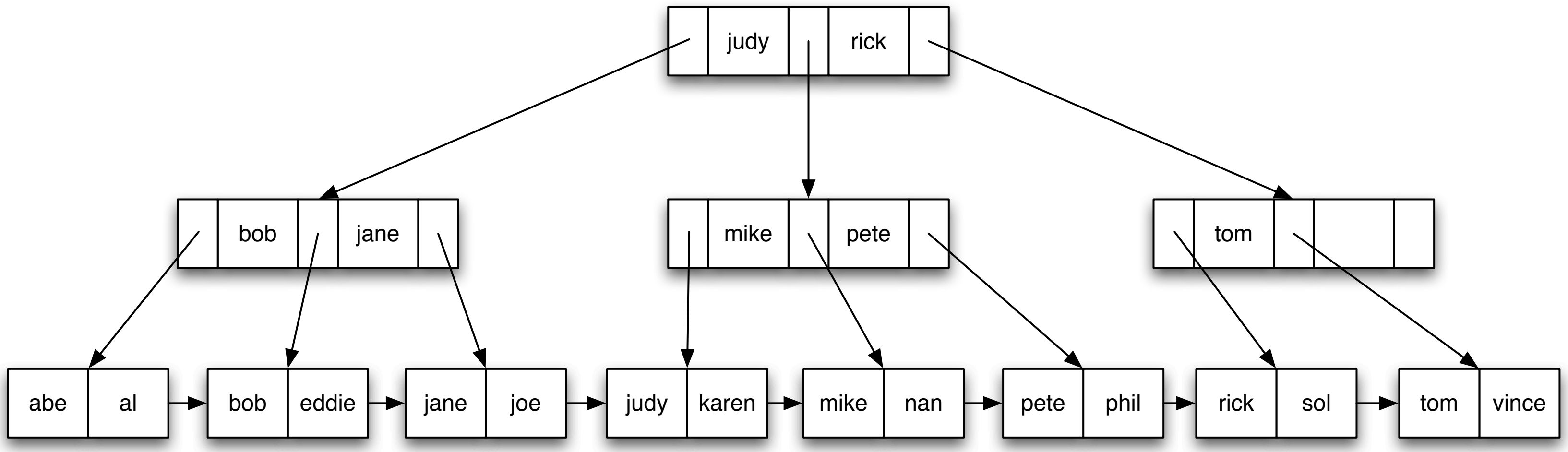
- We consider the B+ tree below



Observations

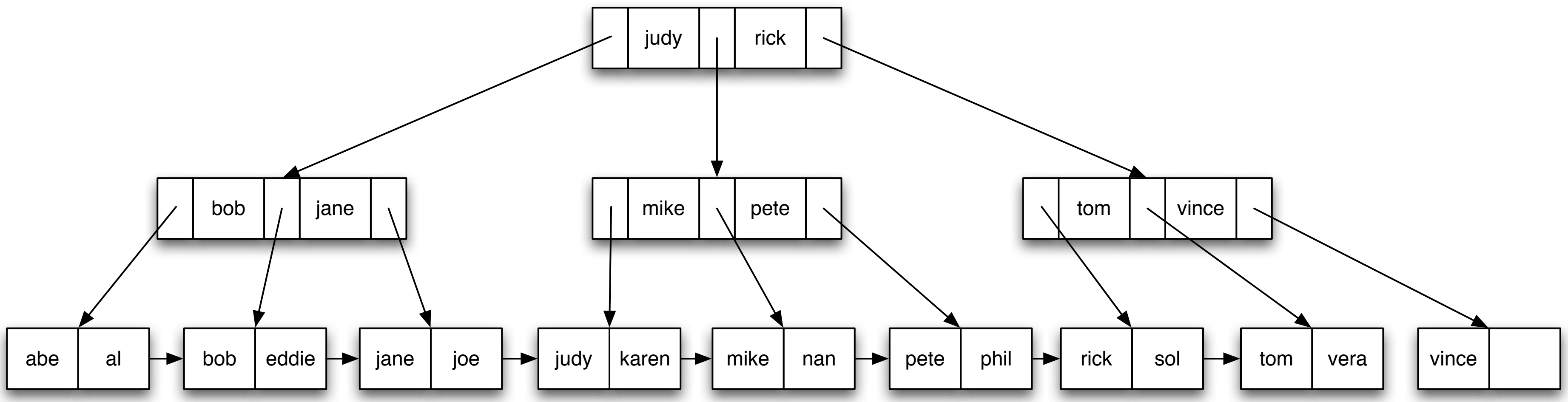
- Observe that the tree has fan out 3
- Invariants to be preserved
    - Leafs must contain between 1 and 2 values
    - Internal nodes must contain between 2 and 3 pointers
    - Root must have between 2 and 3 pointers
    - Tree must be balanced, i.e., all paths from root to a leaf must be of same length

Inserting Vince



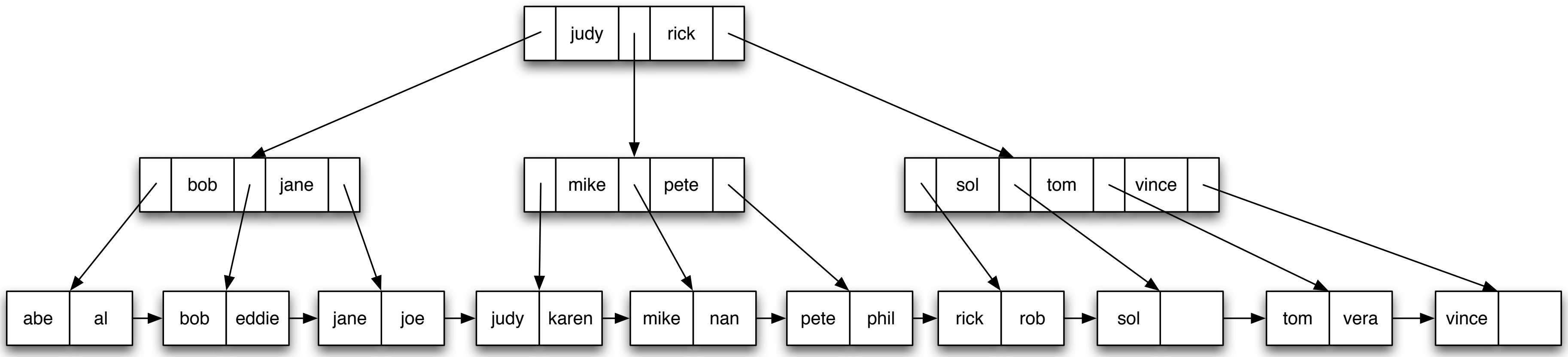
Inserting Vera

- Leaf consisting of tom and vince is split and extra pointer is inserted in parent



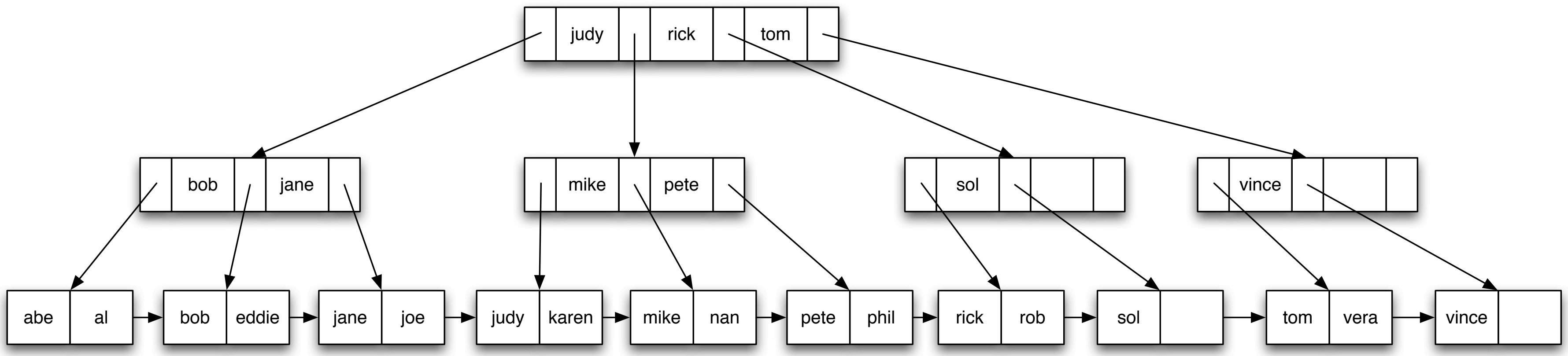
Inserting rob

- Inserting rob is more difficult. We first create a new leaf node and insert it as below
- The node above is temporarily extended to contain 4 pointers



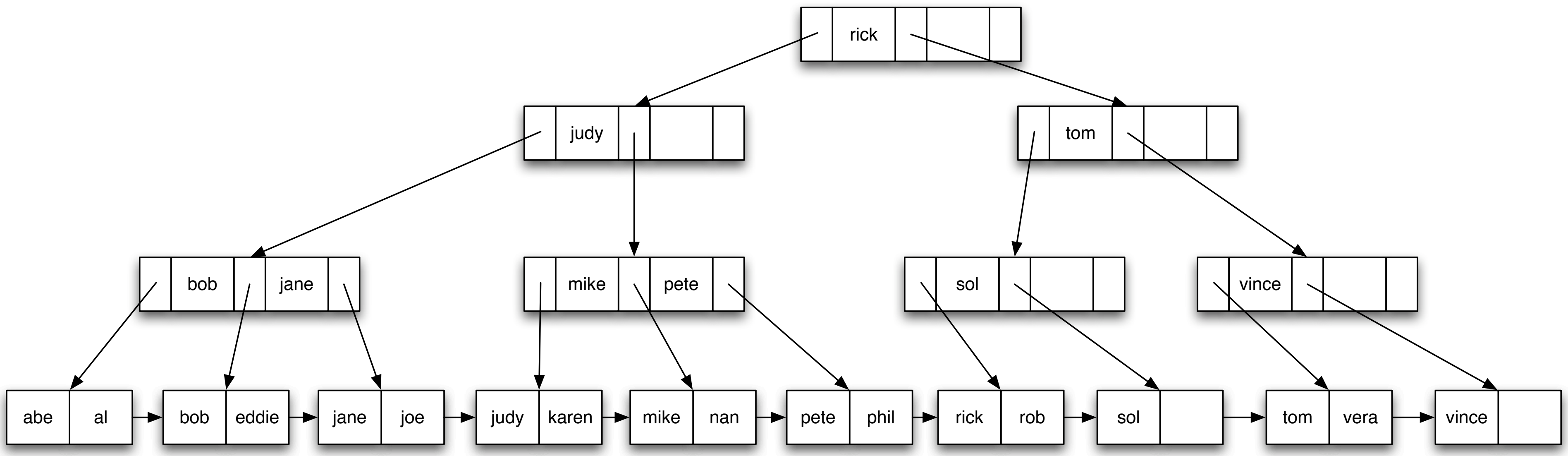
Inserting rob

- The overfull internal node is then split in 2
- The new pointer is inserted into the root node which then becomes overfull



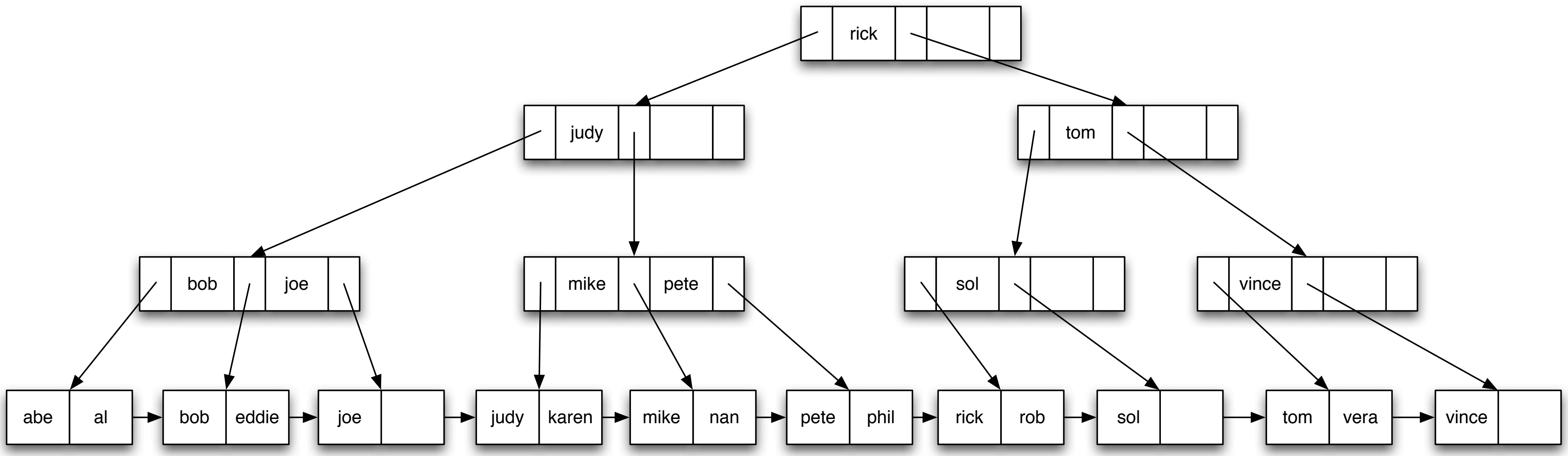
Inserting rob

- Finally the overfull root is split in 2
- At this point the tree satisfies the requirements of slide 3 and so the insertion procedure ends

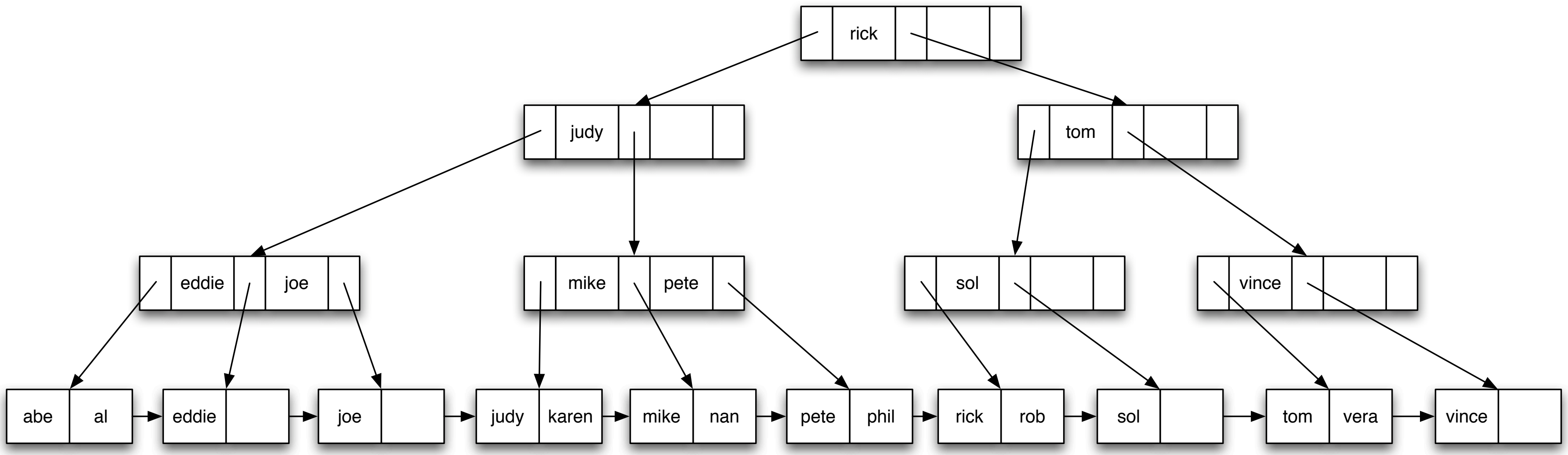


Deleting jane

- Is straight forward
- Note that the node above the leaf where jane was deleted must also be updated

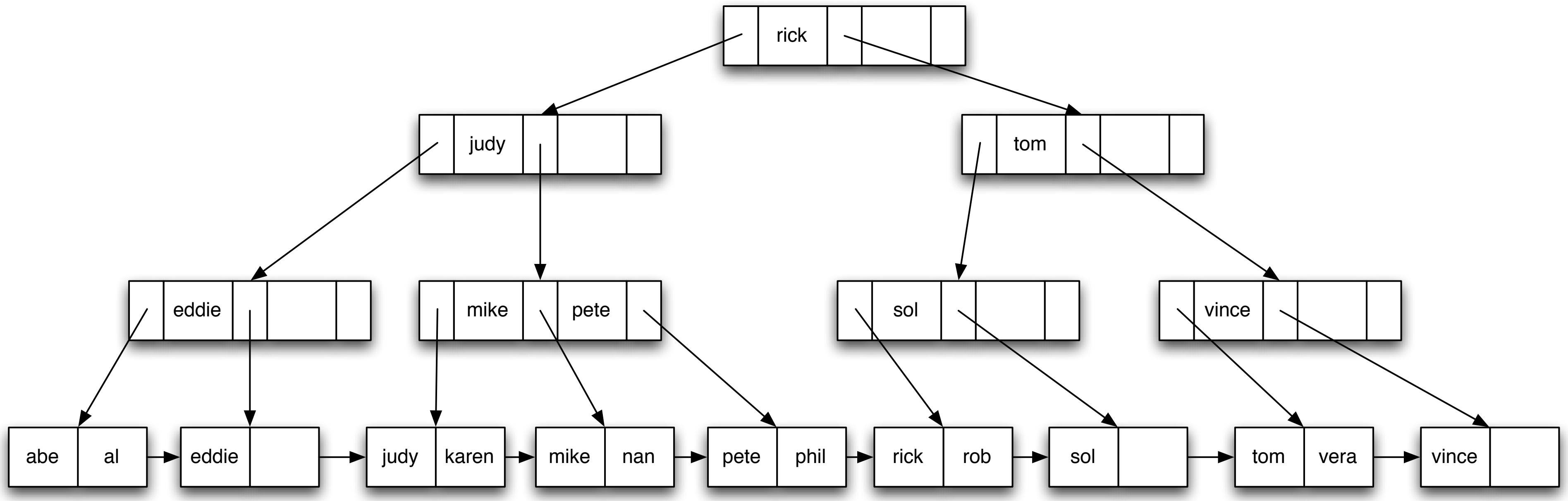


Deleting bob



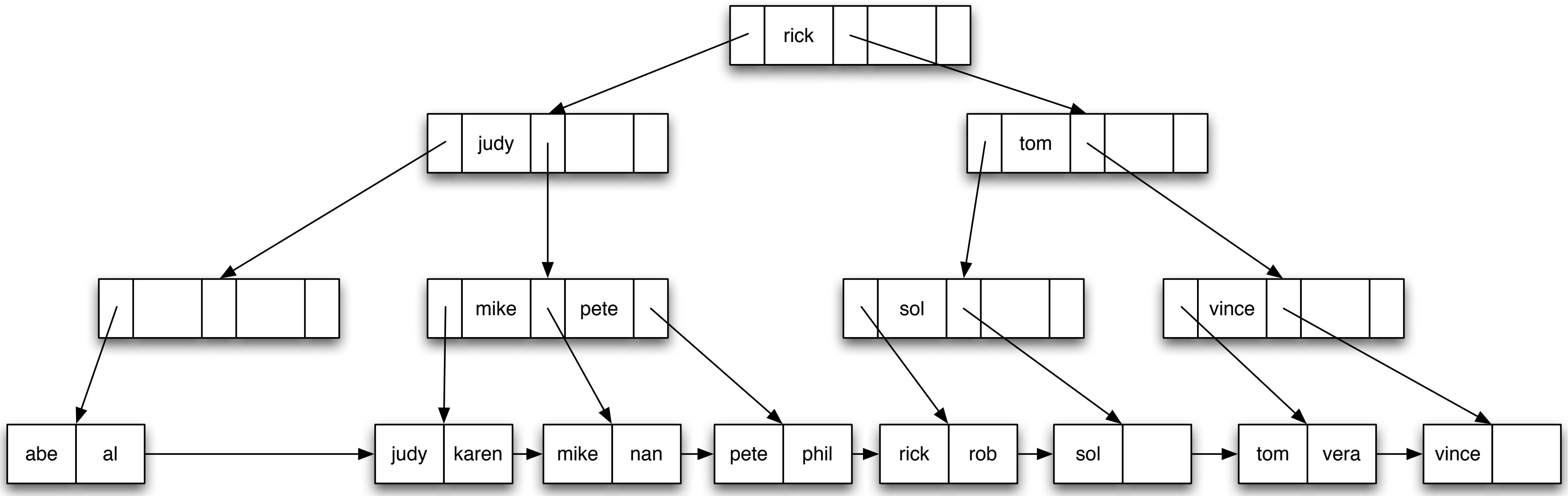
Deleting joe

- Leads to a leaf being deleted and the parent being updated



Deleting eddie

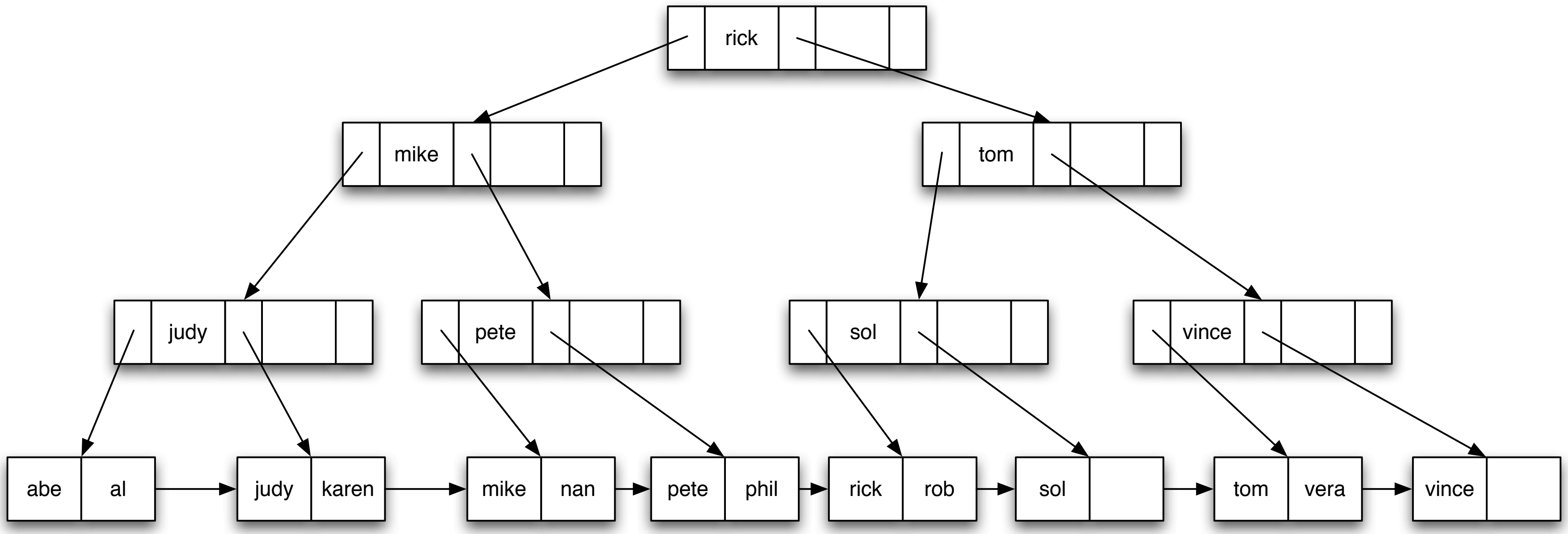
- Leads to deletion of a leaf
- At this point the parent becomes underfull



Deleting eddie

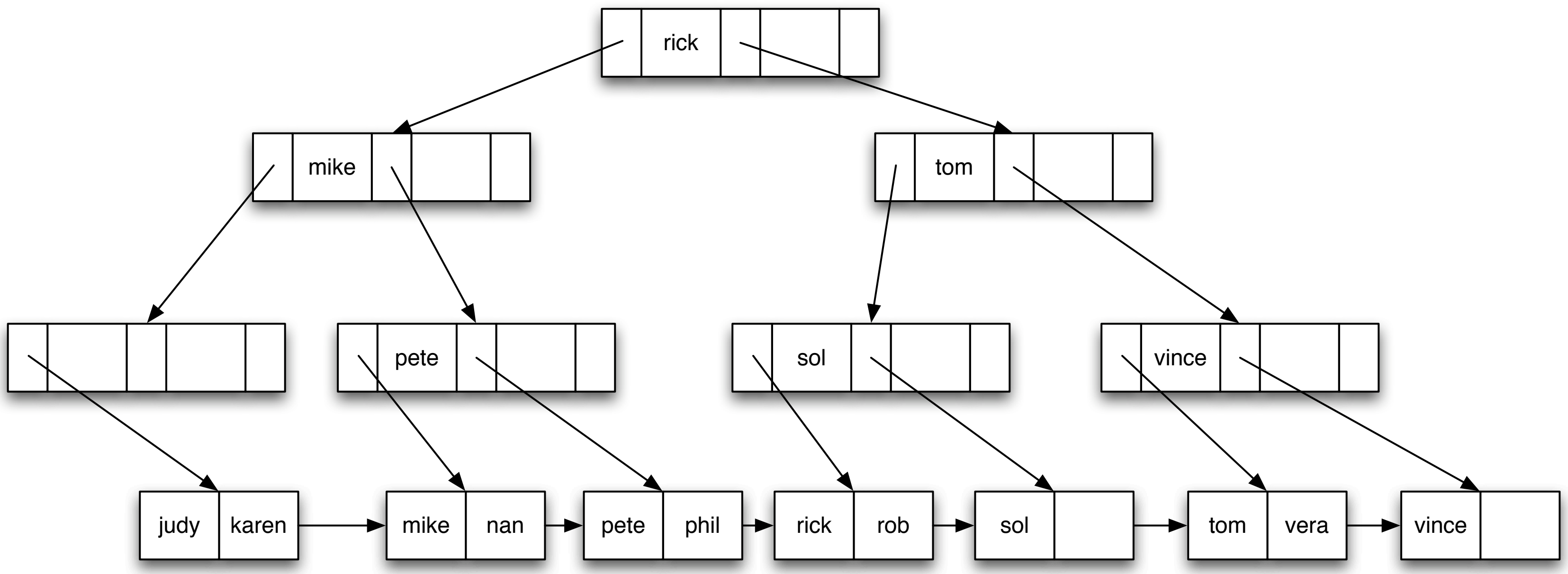
- When a node becomes underful the algorithm will try to **redistribute** some pointers from a neighbouring sibling to it.

- Since this is possible in this case we do it



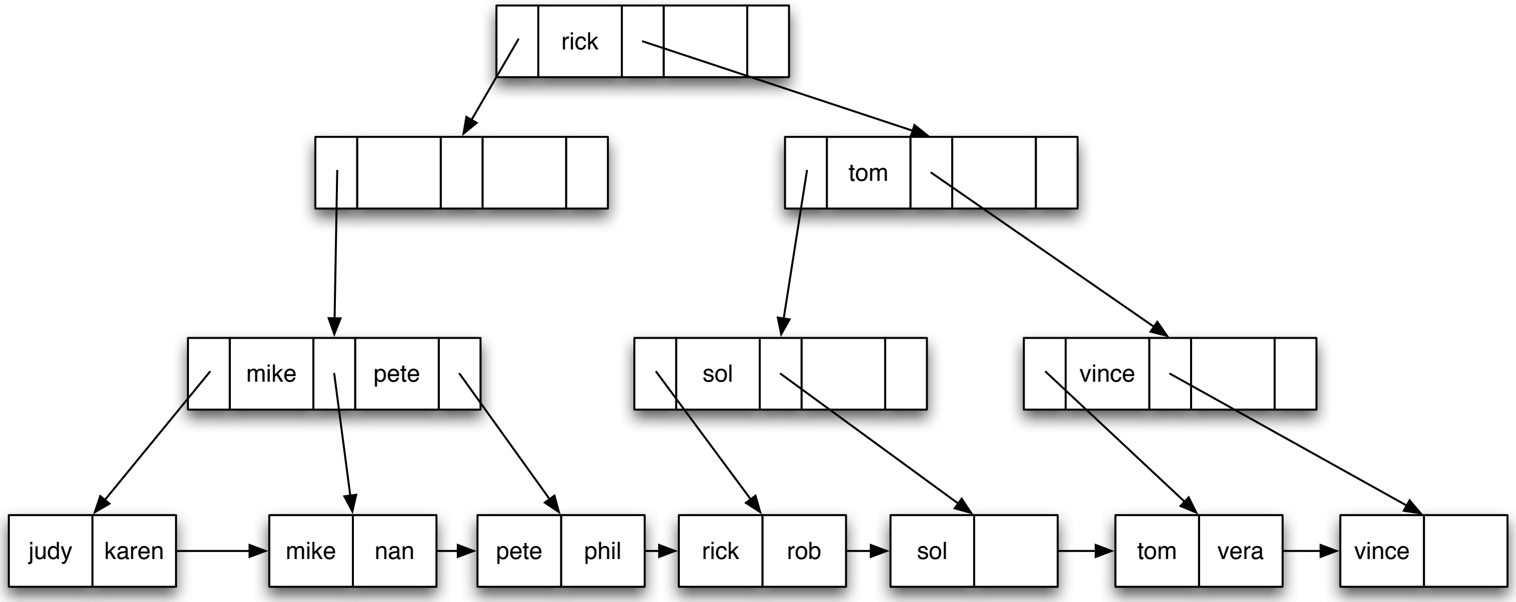
Deleting abe and al

- Leads to deletion of a leaf
- This makes the parent underfull.
- We cannot redistribute pointers again since this will make the neighbour underfull



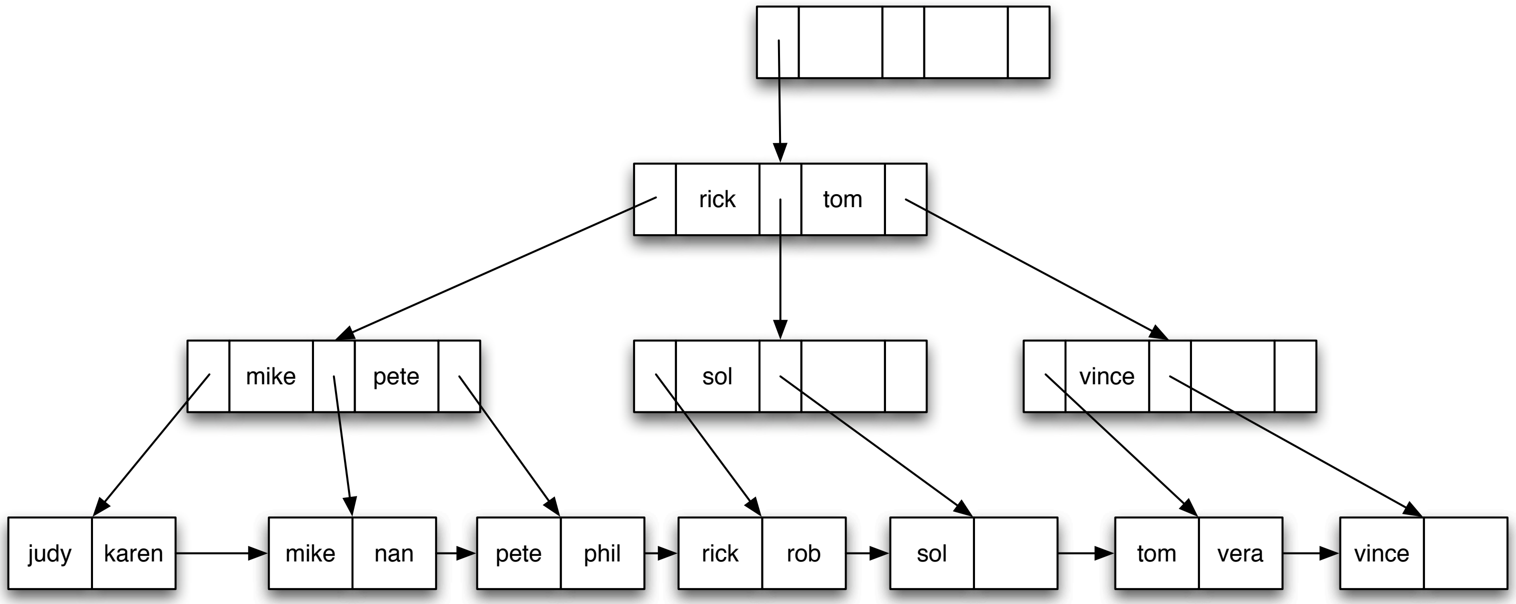
Deleting abe and al

- Instead we must **merge** with the neighbouring sibling
- But this makes the parent underfull



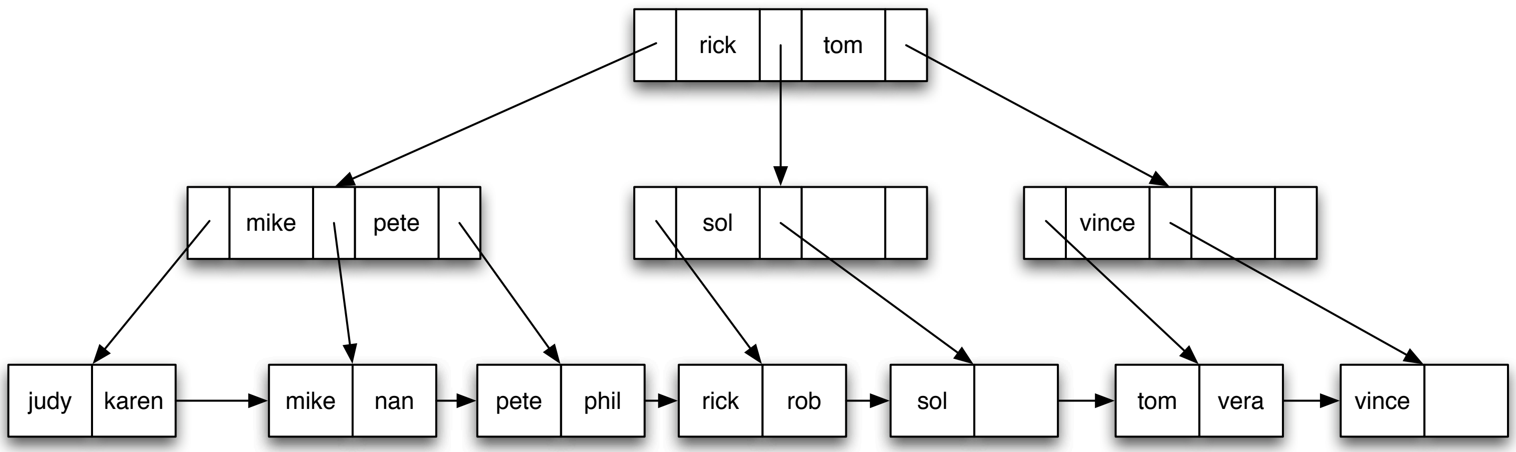
Deleting abe and al

- Since we can not solve this problem by redistributing pointers we must merge siblings again



Deleting abe and al

- Since the root is underfull it can be deleted
- The resulting tree satisfies the requirements and so the deletion algorithm ends



General remarks

- When a node becomes underfull the algorithm will try to redistribute pointers from the neighbouring sibling either on the left or the right
- If this is not possible, it should merge with one of them
- The value held in an internal node or the root should always be the smallest value appearing in a leaf of the subtree pointed to by the pointer after the value