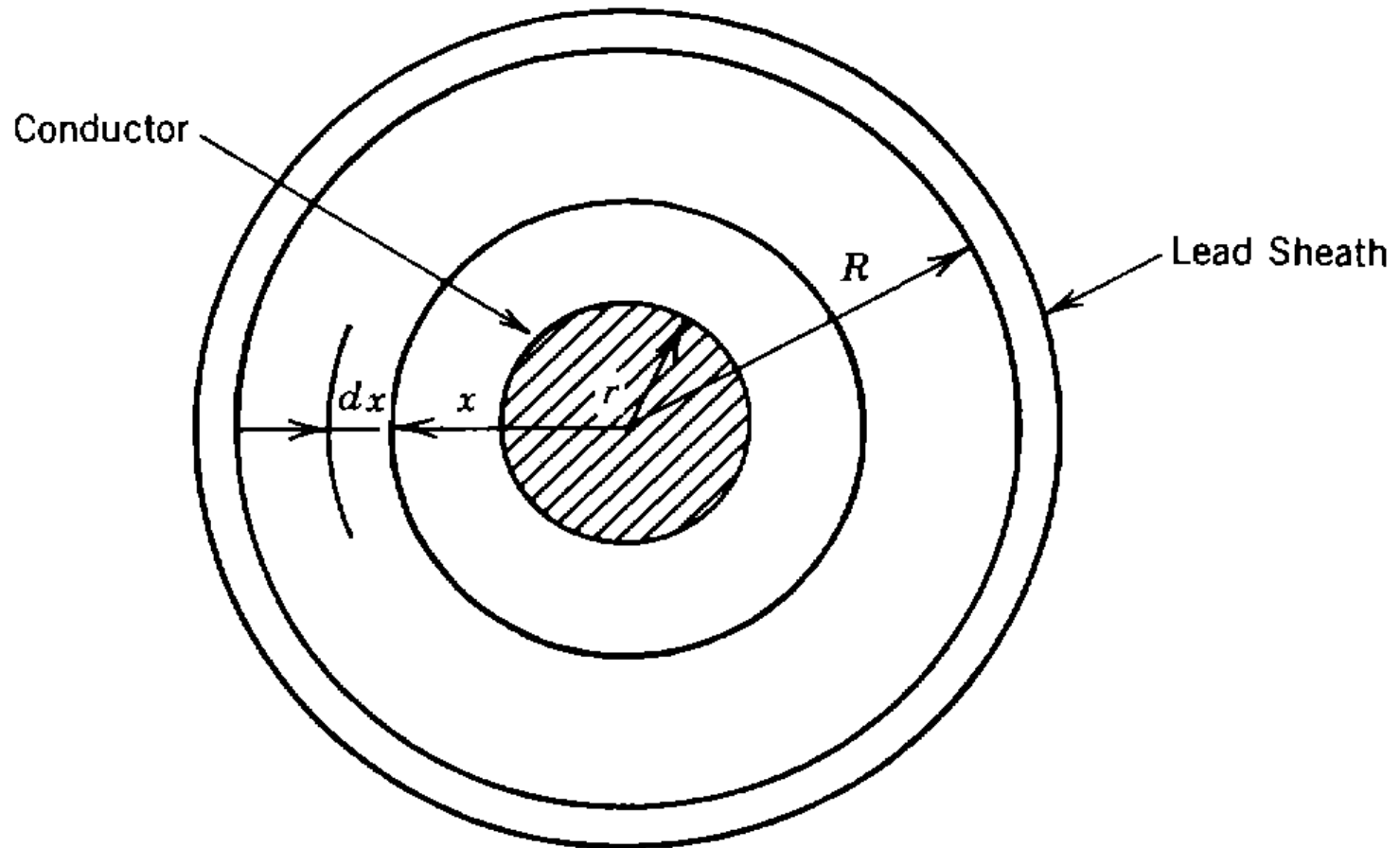


Electrical Characteristics of Cables

- Electric Stress in Single-Core Cables
- Capacitance of Single Core Cables
- Charging Current
- Insulation Resistance of Single- Core Cables
- Dielectric Power Factor & Dielectric Losses
- Heating of Cables: Core loss ;
- Dielectric loss and intersheath loss

Electrical Characteristics of Cables



Electric Stress in Single-Core Cables

$$D = q / (2\pi x)$$

$$E = D / \epsilon = q / (2\pi \epsilon x)$$


q: Charge on conductor surface (C/m)

D: Electric flux density at a radius x (C/m²)

E: Electric field (potential gradient), or electric stress, or dielectric stress.

ϵ : Permittivity ($\epsilon = \epsilon_o \cdot \epsilon_r$)

ϵ_r : relative permittivity or dielectric constant.


$$V = \int_r^R E \cdot dx = \frac{q}{2\pi\epsilon} \ln \frac{R}{r}$$

$$E = \frac{q}{2\pi\epsilon \cdot x} = \frac{V}{x \cdot \ln \frac{R}{r}}$$

r: conductor radius.

R: Outside radius of insulation or inside radius of sheath.

V: potential difference between conductor and sheath (Operating voltage of cable).

Dielectric Strength: Maximum voltage that dielectric can withstand before its breakdown.

Average Stress: Is the amount of voltage across the insulation material divided by the thickness of the insulator.

$$E_{\max} = E \text{ at } x = r$$
$$= V/(r \ln R/r)$$

$$E_{\min} = E \text{ at } x = R$$
$$= V/(R \ln R/r)$$

For a given V and R , there is a conductor radius that gives the minimum stress at the conductor surface. In order to get the smallest value of E_{\max} :

$$dE_{\max}/dr = 0$$

$$\ln(R/r) = 1$$

$$R/r = e = 2.718$$

Insulation thickness is:

$$R-r = 1.718 r$$

$$E_{\max} = V/r \quad (\text{as: } \ln(R/r)=1)$$

Where r is the optimum conductor radius that satisfies ($R/r=2.718$)

Example

A single- core conductor cable of 5 km long has a conductor diameter of 2cm and an inside diameter of sheath 5 cm. The cable is used at 24.9 kV and 50 Hz. Calculate the following:

- a- Maximum and minimum values of electric stress.**
- b- Optimum value of conductor radius that results in smallest value of maximum stress.**

a- $E_{\max} = V/(r \ln R/r) = 27.17 \text{ kV/cm}$

$E_{\min} = V/(R \ln R/r) = 10.87 \text{ kV/cm}$

b- Optimum conductor radius r is:

$R/r = 2.718$

$r = R/2.718 = 0.92 \text{ cm}$

The minimum value of E_{\max} :

$= V/r = 24.9/0.92 = 27.07 \text{ kV/cm}$

Example

A single- core conductor cable of 7.5 km long has a conductor diameter of 2.7cm and an inside diameter of sheath 5.6 cm. The cable is used at 33 kV and 50 Hz. Calculate the following:

- a- Maximum and minimum values of electric stress.**
- b- Optimum value of conductor radius that results in smallest value of maximum stress.**

Grading of Cables

Grading of cables means **the distribution of dielectric stress such that the difference between the maximum and minimum electric stress is reduced.**

Therefore, the cable of the same size could be operated at higher voltages or for the same operating voltage, a cable of relatively small size could be used.

1. Capacitance Grading

This method involves the use of **two or more layers of dielectrics having different permittivities**, those with higher permittivity being near the conductor.

$$E_x = q / (2 \pi \epsilon_0 \cdot \epsilon_r \cdot x)$$

The permittivity can be varied with radius x such that (ideal case):

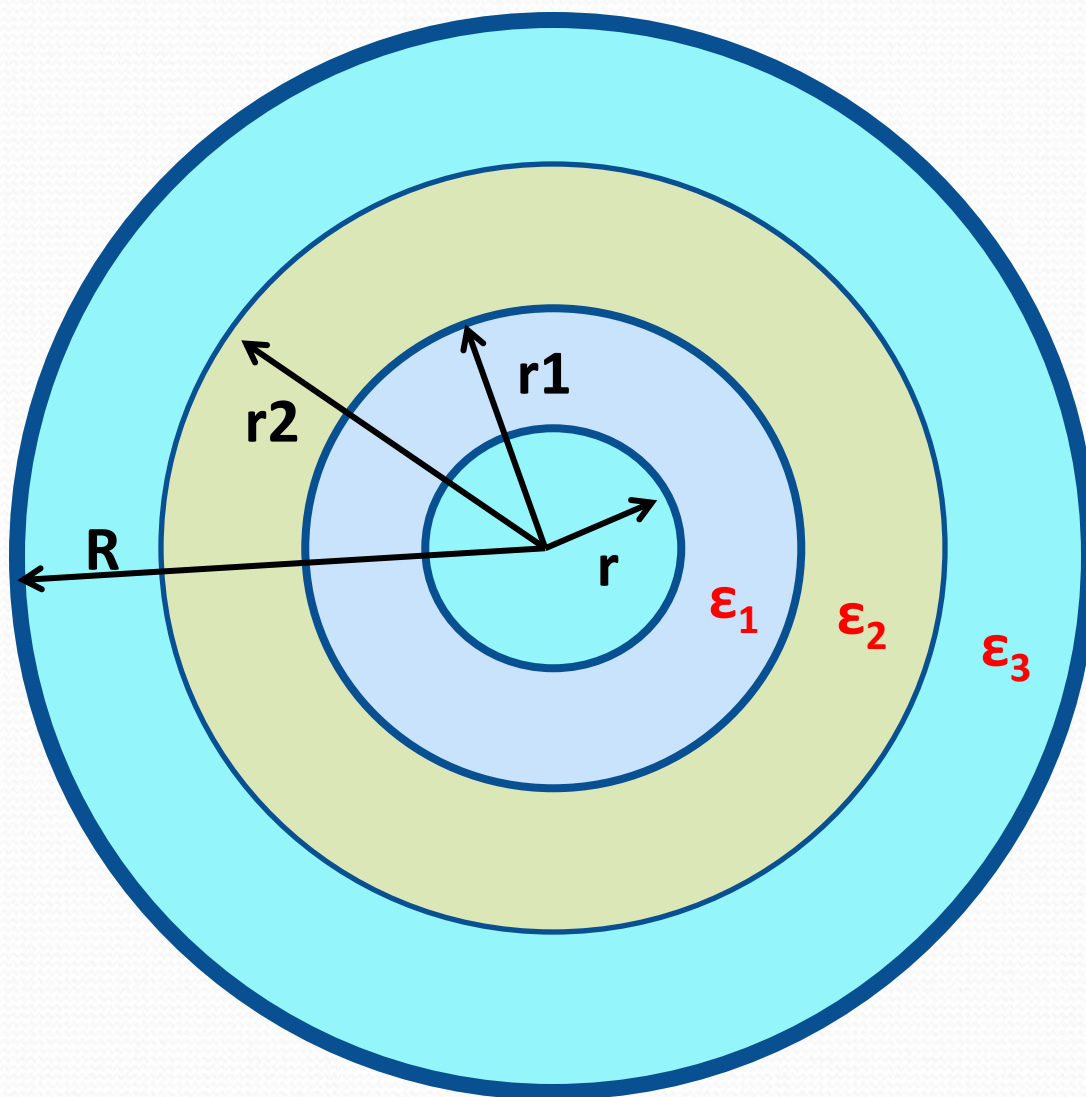
$$\epsilon_r = k/x$$

Then $E_x = q / (2 \pi \epsilon_0 \cdot k)$

E_x is constant throughout the thickness of insulation.

$$r < r_1 < r_2$$

$$\epsilon_1 > \epsilon_2 > \epsilon_3$$



In the figure shown

$$\text{At } x=r \quad E_{\max 1} = q / (2 \pi \epsilon_0 \cdot \epsilon_1 r)$$

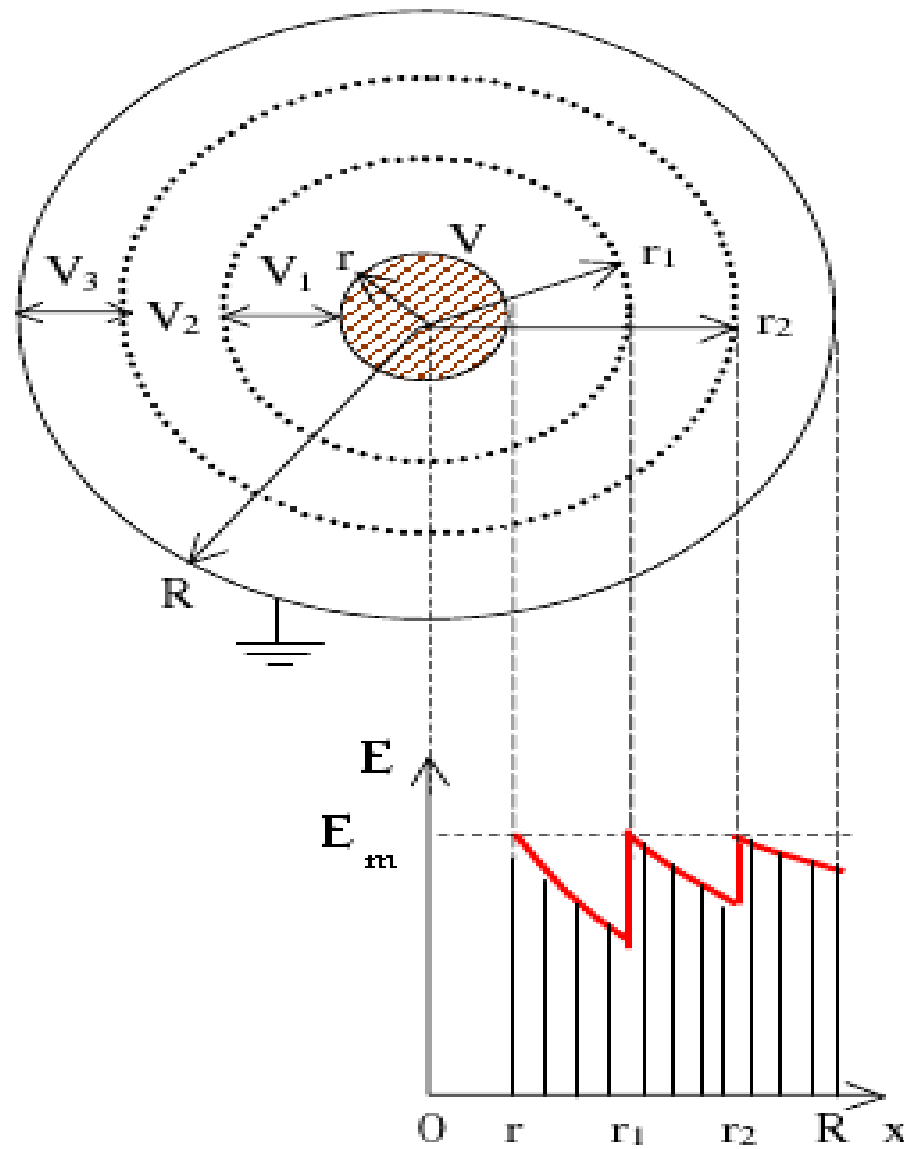
$$\text{At } x=r_1 \quad E_{\max 2} = q / (2 \pi \epsilon_0 \cdot \epsilon_2 r_1)$$

$$\text{At } x=r_2 \quad E_{\max 3} = q / (2 \pi \epsilon_0 \cdot \epsilon_3 r_2)$$

If all the three dielectrics are operated at the same maximum electric stress ($E_{\max 1} = E_{\max 2} = E_{\max 3} = E_{\max}$), then:

$$(1 / \epsilon_1 r) = (1 / \epsilon_2 r_1) = (1 / \epsilon_3 r_2)$$

$$\epsilon_1 r = \epsilon_2 r_1 = \epsilon_3 r_2, \quad \text{get } r_1, r_2$$



The operating voltage V is:

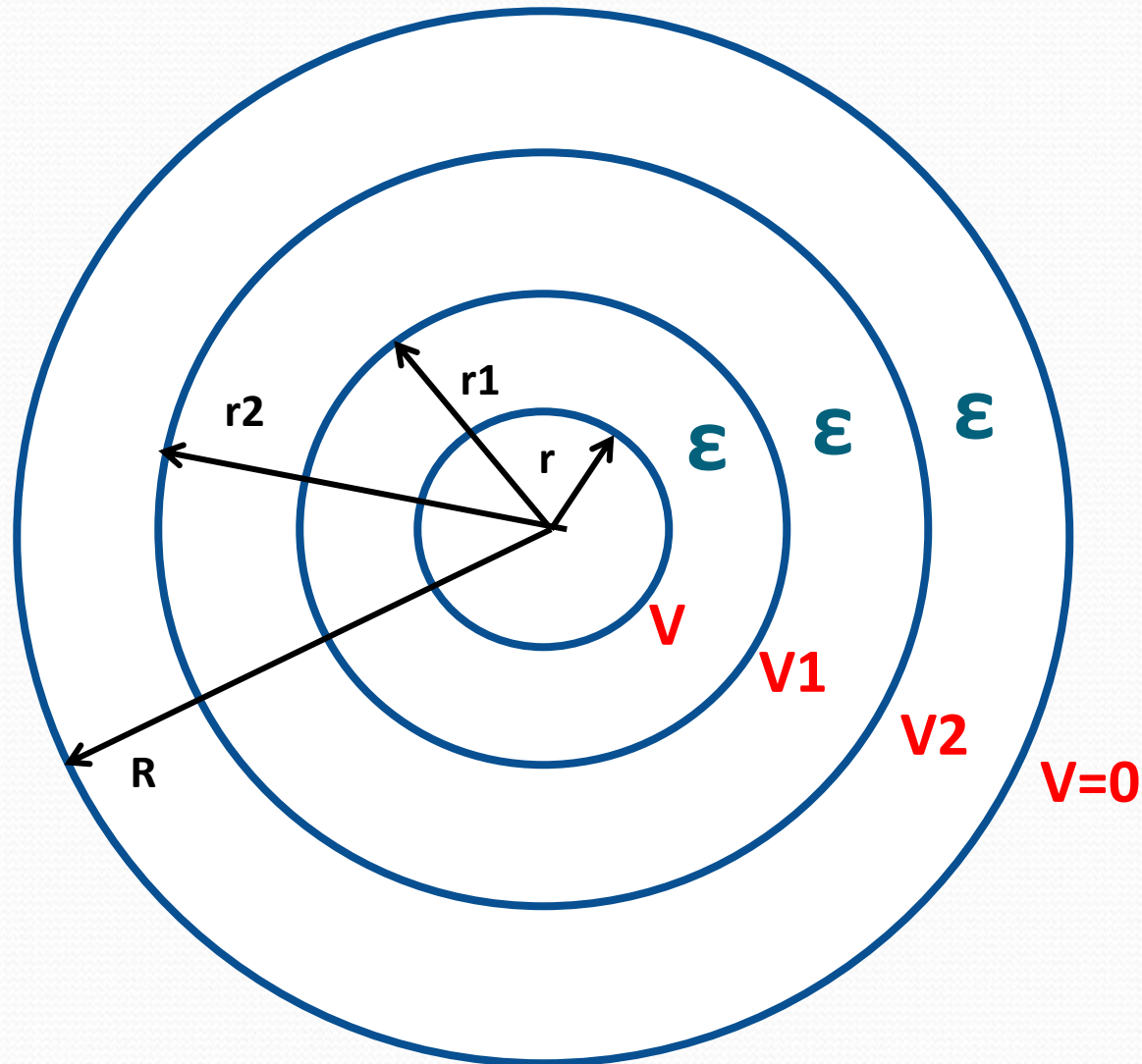
$$\begin{aligned} V &= \int_r^{r_1} E_x \cdot dx + \int_{r_1}^{r_2} E_x \cdot dx + \int_{r_2}^R E_x \cdot dx \\ &= \frac{q}{2\pi\epsilon_o\epsilon_1} \ln \frac{r_1}{r} + \frac{q}{2\pi\epsilon_o\epsilon_2} \ln \frac{r_2}{r_1} + \frac{q}{2\pi\epsilon_o\epsilon_3} \ln \frac{R}{r_2} \\ V &= E_{\max} \left[r \ln \frac{r_1}{r} + r_1 \ln \frac{r_2}{r_1} + r_2 \ln \frac{R}{r_2} \right] \end{aligned}$$

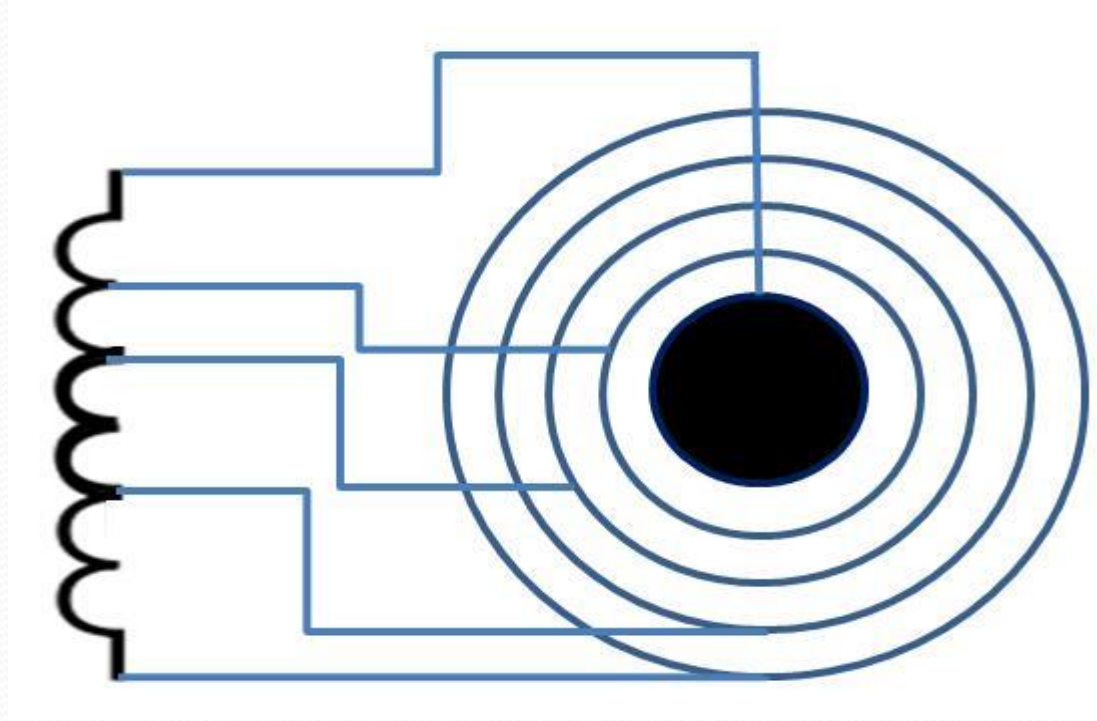
Cable Capacitance

$$C = \frac{q}{V}$$

$$= \frac{2\pi\epsilon_0}{\frac{1}{\epsilon_1} \ln \frac{r_1}{r} + \frac{1}{\epsilon_2} \ln \frac{r_2}{r_1} + \frac{1}{\epsilon_3} \ln \frac{R}{r_2}}$$

2. Intersheath Grading





Intersheath Grading is a method of creating uniform voltage gradient across the insulation by means of separating the insulation into two or more layers by thin conductive strips. **These strips are kept at different voltage levels through the secondary of a transformer.**

In this method only one dielectric is used but the dielectric is separated into two or more layers by thin metallic intersheaths.

$$E_{\max 1} = (V - V_1) / (r \cdot \ln(r_1 / r))$$

$$E_{\max 2} = (V_1 - V_2) / (r_1 \cdot \ln(r_2 / r_1))$$

$$E_{\max 3} = V_2 / (r_2 \cdot \ln(R / r_2))$$

For the same maximum electric strength:

$$(r_1 / r) = (r_2 / r_1) = (R / r_2) = \alpha$$

$$R / r = \alpha^3$$

$$\text{Then: } (V - V_1) / (r \cdot \ln \alpha) = (V_1 - V_2) / (r_1 \cdot \ln \alpha) = (V_2 / r_2 \cdot \ln \alpha)$$

$$(V - V_1) / r = (V_1 - V_2) / r_1 = V_2 / r_2$$

If the cable does not have any intersheath, the maximum stress is:

$$E_{\max} = V / (r \cdot \ln(R/r))$$

The intersheath radius can be found from

$$R/r = \alpha^3$$

$$(r_1/r) = (r_2/r_1) = (R/r_2) = \alpha$$

The voltages V_1 , V_2 can be found from:

$$(V - V_1)/r = (V_1 - V_2)/r_1 = V_2/r_2$$

Difficulties of Grading

a-Capacitance grading :

- 1- non-availability of materials with widely varying permittivities.
- 2- The permittivities of materials will be change with time, so the electric field distribution may change and lead to insulation breakdown.

b- Intersheath Grading

- 1- Damage of intersheaths during laying operation.**
- 2- The charging current that flows through the intersheath for long cables result in overheating.**
- 3- The setting of proper voltages of intersheaths.**

Example

A single core cable for 53.8 kV has a conductor of 2cm diameter and sheath of inside diameter 5.3 cm. It is required to have two intersheaths so that stress varies between the same maximum and minimum values in three layers of dielectric. Find the positions of intersheaths, maximum and minimum stress and voltages on the intersheaths. Also, find the maximum and minimum stress if the intersheaths are not used.

$$R/r = a^3$$

$$a = 1.384$$

$$(r_1/r) = (r_2/r_1) = (R/r_2) = a$$

$$r_1 = 1.384 \text{ cm}, \quad r_2 = 1.951 \text{ cm}$$

$$(V - V_1)/(r \cdot \ln a) = (V_1 - V_2)/(r_1 \cdot \ln a) = (V_2/r_2 \cdot \ln a)$$

$$\begin{aligned} (V - V_1)/(1 \cdot \ln a) &= (V_1 - V_2)/(1.384 \cdot \ln a) \\ &= (V_2/1.915 \cdot \ln a) \end{aligned}$$

$$V = 53.8 \text{ kV}$$

$$V_1 = 41.3 \text{ kV}, V_2 = 23.94 \text{ kV}$$

$$E_{\max} = (V - V_1) / (r \cdot \ln a) = 38.46 \text{ kV/cm}$$

$$E_{\min} = (V - V_1) / (r_1 \cdot \ln a) = 27.79 \text{ kV/cm}$$

If Intersheaths are not used:

$$E_{\max} = V / (r \cdot \ln(R/r)) = 55.2 \text{ kV/cm}$$

$$E_{\min} = V / (R \cdot \ln(R/r)) = 20.83 \text{ kV/cm}$$

Example

Find the maximum working voltage of a single core cable having two insulating materials A and B and the following data. conductor radius 0.5 cm, inside sheath radius 2.5cm. The maximum working stress of A 60 kV/cm, maximum working stress of B 50 kV/cm, relative permittivities of A and B, 4 and 2.5 respectively.

$$60 = (q / 2\pi\epsilon_0 \cdot \epsilon_A r)$$

$$q / (2\pi\epsilon_0) = 120$$

$$50 = (q / 2\pi\epsilon_0 \cdot \epsilon_B r_1) = 120 / (2.5 r_1)$$

$$r_1 = 0.96 \text{ cm}$$

$$\begin{aligned} V &= q \cdot \ln(r_1 / r) / (2\pi\epsilon_0 \cdot \epsilon_A) + q \cdot \ln(R / r_1) / (2\pi\epsilon_0 \cdot \epsilon_B) \\ &= (120 / 4) \cdot \ln(0.96 / 0.5) + (120 / 2.5) \cdot \ln(2.5 / 0.96) \\ &= 65.51 \text{ kV} \end{aligned}$$