

Electrical Machines .

Transformers - Single Phase Transformer - construction & operating principles, emf eqⁿ & phasor diagram, eq. circuit, testing of transformer, efficiency & regulations, parallel operations.

Transformer II - Operating principle, eq vs ckt, open ckt & short ckt test, efficiency regulation, auto core loss test

transformer, comparison with 2 winding transformer on the basis of copper losses & vol^m of copper.

Review of single & 3 phase circuits, magnetic circuits & electromechanical energy conversion principle, basics of static & rotating electrical machines, Emf eqⁿ's

Transformation Ratio $\Rightarrow V_1 I_1 \cos \phi_1 = V_2 I_2 \cos \phi_2$ (Ideal case)

$$\frac{V_1}{V_2} = \frac{I_2}{I_1} = \frac{N_1}{N_2}$$

Transformer is a static device which increase or decrease voltage without changing its frequency.

Faraday's Law - $E = -\frac{d\phi}{dt}$

Prove $E = 4.44 f \phi_{max} N$

Emf equation

For primary side, $E_1 = 4.44 f \phi_{max} N_1$

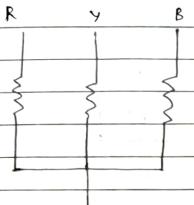
For secondary side, $E_2 = 4.44 f \phi_{max} N_2$.

So, $\frac{E_1}{E_2} = \frac{N_1}{N_2}$

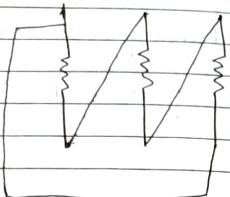
Input Power = Output Power

$$\Rightarrow V_1 I_1 = V_2 I_2, \text{ so, } \frac{I_2}{I_1} = \frac{N_1}{N_2}$$

Construction	Voltage	Power	Phase	Connections
I) Core type	I) Step up	I) Distribution	I) Single Phase	I) Star star One Line + one neutral
II) Shell type	II) Step down	II) Power	II) Star Delta II) Delta Delta (Home)	II) Three Phase
	100 200	In our country, generation voltage as 11 kV. 132 kV, 66 kV, 33 kV, etc.		Three lines 2 One neutral
	100 200	Used for distribution (Multiples of 11)		R Y B - 3 lines B - 1 line Neutral (Black)
		Used for domestic purpose.		11 kV 11 kV 11 kV



Star



Delta

Emf eqⁿ proof : $E_1 = -N_1 \frac{d\phi_1}{dt}$

$$E_1 = -N_1 \frac{d}{dt} (\phi_m \sin \omega t)$$

$$E_1 = -N_1 w \phi_m \cos \omega t = N_1 w \phi_m \sin (\omega t - \pi/2)$$

$$E_{1(\max)} = N_1 w \phi_m \quad (\because \text{Res } \sin(\omega t - \pi/2) = 1)$$

$$E_{1(\max)} = 2\pi f N_1 \phi_m$$

$$E_{1(\text{rms})} = \sqrt{2} \pi f N_1 \phi_m$$

$$E_{1(\text{rms})} = 1.44 f N_1 \phi_m$$

Similarly,

$$E_2 = 1.44 f N_2 \phi_m$$

$$\text{So, } \frac{E_2}{E_1} = \frac{N_2}{N_1}$$

- Core type Transformer : • Magnetic circuit consists of two sections, vertical section called limbs & two horizontal section called yokes.
- Flux leakage is minimized.
- There is minimum use of insulating material due to the pattern of winding.

Shell type transformer : • It consists of one central limb as shown in figure. Both windings are wound on same limb.

- The funcⁿ of outer limbs is to complete the path of low reluctance for magnetic flux.

Advantage - • Gives better support against electromagnetic forces b/w current carrying conductors.

- Provides shorter magnetic path.

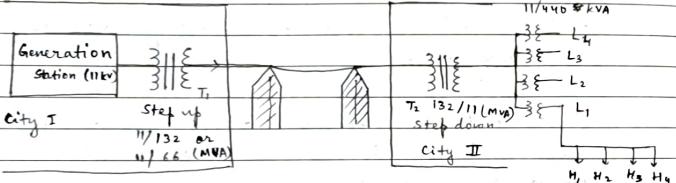
Disadvantage - • Poor natural cooling.

Step up transformer - Primary voltage is lower than secondary voltage. E.g. \rightarrow stepping up of 208 V to 480 V.

Examples - Marine substation transformers used by power companies.

Step down transformer - It has primary voltage higher than secondary voltage. E.g. \rightarrow stepping down of 11 kV to 220 V and 440 V.

* Draw a single line diagram showing transmission & distribution lines from generating station to end point.



Q: Why transformer rating is in kVA, not in kW?

Q: Why core is laminated?

Q: Which loss is greater, core or copper? Why?

B/

Problems

Q: A 50 kVA single phase transformer has 600 turns on primary & 40 turns on secondary. The primary winding is connected to 2.2 kV 50 Hz supply. Determine

(i) Secondary voltage at no load.

(ii) Primary & secondary currents at full load.

$$(i) \frac{E_1}{E_2} = \frac{N_1}{N_2} \Rightarrow \frac{600}{40} = \frac{2200}{E_2}$$

$$\therefore E_2 = \frac{2200 \times 40}{2200 + 40} = 33000 \text{ V} = 33 \text{ kVA}$$

$$\Rightarrow E_2 = \frac{2200 \times 220}{3600} = \frac{440}{3} = 146.66 \text{ V.}$$

$$(ii) I_{\text{primary}} = \frac{50 \times 1000}{2.2 \times 1000} = 22.72 \text{ A.}$$

$$I_{\text{secondary}} = \frac{50 \times 1000}{146.66} = 340.92 \text{ A.}$$

Hence assignment.

Q: A single phase 4 kVA transformer has 400 primary turns and 1000 secondary turns. The net cross sectional area of the core is 60 cm^2 . When the primary winding is connected to 500 V 50 Hz supply, calculate -

(i) max. value of ϕ in the core.

(ii) voltage induced in the secondary winding and

(iii) Secondary full load current.

$$B = \frac{\phi}{A}, \quad E = 4.44 \phi f N$$

Q: Which cross winding area is more? HV winding or LV winding?

① Types of losses in transformer -

- Copper loss
- Iron loss

Copper losses (I^2R) depend on current which passes through windings while iron losses depend on voltage and current so, total loss depends on current & voltage which are expressed in volt ampere (VA) and not on load power factor. So, the unit VA, kVA are used and not watt, kW.

When transformers are manufactured, the kind of load is not specified due to variety of applications. The load may be resistive, inductive or capacitive. So, there would be different power factor at secondary side. So, rating is specified in kVA and not in kW.

② Core is laminated to minimize energy losses due to eddy currents as these currents interfere with transfer of energy from primary to secondary side. Due to eddy currents, electrical energy gets wasted in the form of heat.

③ Copper loss is greater.

$$\text{Copper loss} \propto I^2R$$

If current in transformer is more, i.e. load is high, so copper loss is more.

Due to variable load, copper loss is variable. In most cases, copper loss is high.

HOME ASSIGNMENT

$$\begin{aligned} \textcircled{i} \quad E_1 &= 4.44 f \phi_m N_1 \\ \Rightarrow 500 &= 4.44 \times 50 \times B_{\max} \times A_1 \times N_1 \\ \Rightarrow B_{\max} &= \frac{10}{4.44 \times 66 \times 10^{-4} \times 400} \\ \Rightarrow B_{\max} &= \frac{100}{24 \times 4.44} \quad T = 0.9384 T \end{aligned}$$

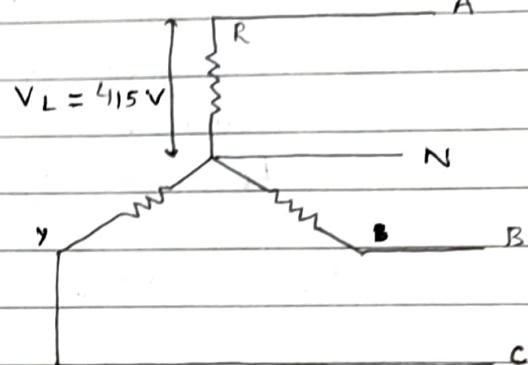
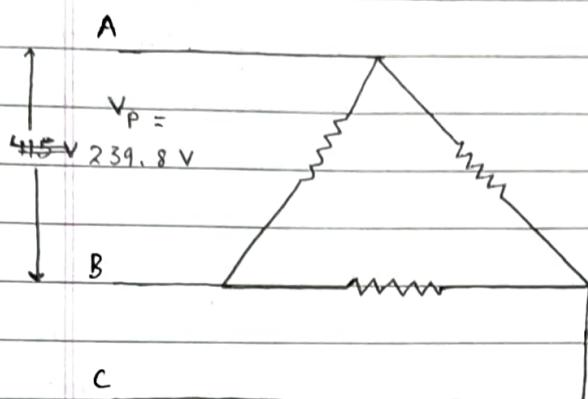
$$\textcircled{ii} \quad \frac{E_1}{E_2} = \frac{N_1}{N_2} \Rightarrow E_2 = \frac{1000}{400} \times 500 = 1250$$

$$\textcircled{iii} \quad P_1 = V_1 I_1, \Rightarrow I_1 = \frac{4 \times 1000}{500} = 8 \text{ A.}$$

$$\Rightarrow \frac{I_2}{I_1} = \frac{N_1}{N_2} \Rightarrow I_2 = \frac{8 \times 4}{10} = 3.2 \text{ A.}$$

Laboratory rating - 3 kVA
 $\left\{ \begin{array}{l} HV - 220V \\ LV - 110V \end{array} \right.$
 25 kVA

Q: Draw Y and Δ connection & write line & phase voltages



For lab ratings,

$$P = 3 \text{ kVA}, \quad HV - 220 \text{ V}.$$

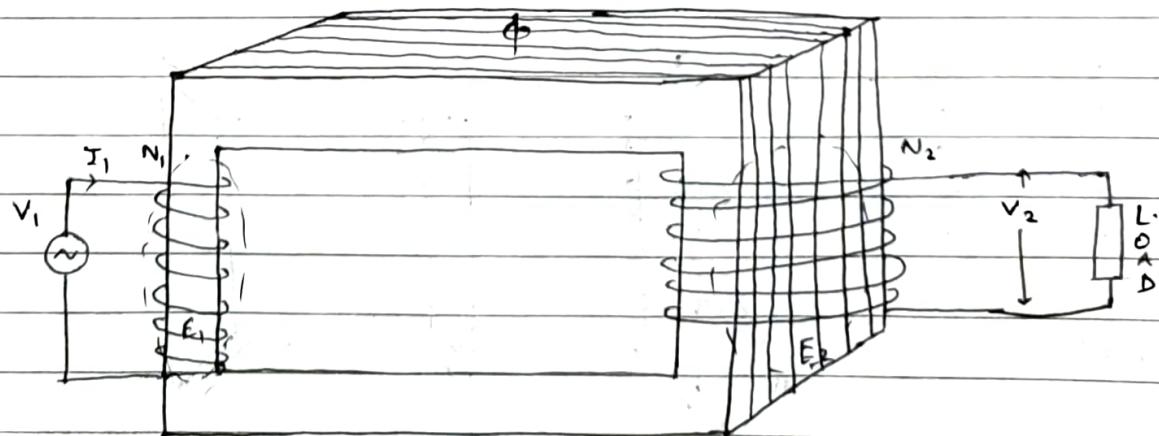
$$I = \frac{3000}{220} = 13.63 \text{ A}.$$

Similarly for Low voltage, $I = \frac{3000}{110} = 27.27 \text{ A}$

LV current is more, so, winding of LV is more.

Q: What will happen when we connect DC supply to transformer?

*



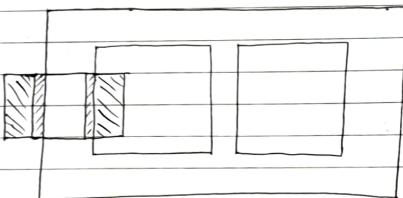
Phasor diagram on no load of ideal & no load transformer

When an alternating voltage V_1 is applied to the primary & alternating flux ϕ is set up in the core, this flux links both the windings & induces emf in them.

$$E_1 = -N_1 \frac{d\phi}{dt}$$

$$E_2 = 4.4V f \phi_m N_2$$

- Q: What is an ideal transformer?
- Total flux is confined in core.
 - No losses
 - Efficiency is 100%.



Core thickness is between 0.18 mm to 0.40 mm.
(For each core).

* Phasor diagram of Transformer on no load condition and ideal condition.

$$\phi = \phi_m \sin \omega t$$

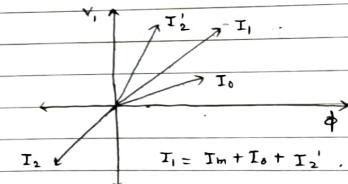
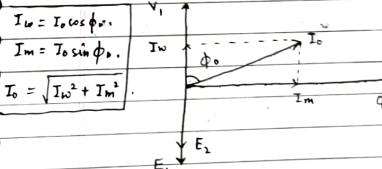
$$E_1 = -N_1 \frac{d\phi}{dt} = E_{1m} \sin(\omega t - \frac{\pi}{2})$$

$$\text{Similarly, } E_2 = E_{2m} \sin(\omega t - \frac{\pi}{2})$$

Practically (No load)
 $I_D = I_m + I_w$

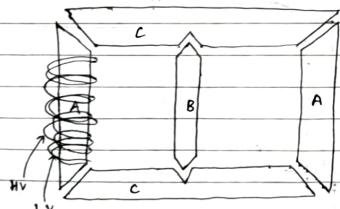
I_w - Magnetic components / Reaction working wattless component -
 I_m - Core loss component / Actual magnet-wattfull component.
-ising .

No load (Practical). When load is connected



Sol: Low voltage winding can be primary. High voltage winding carries a lower current, hence the resistance of winding of is more.

As the resistance of winding is inversely proportional to the area of cross section, the area of high voltage winding is less compared to low voltage winding.



Stacking factor - It is the ratio of effective cross sectional area of core to the physical cross sectional area of core.

$$\frac{1000 \times 0.3}{200 \times \frac{1}{K}}$$

Soln: A constant DC source connected to the primary side of transformer will have zero emf. The constant current creates zero induced emf.

As a result primary side of transformer will behave short ckt and heavy current will flow through primary which will in turn burn the winding.

Q: why ϕ is the reference?

Principle of Ideal Transformer

- Primary and secondary windings of an ideal transformer has negligible resistance.
- The core has ∞ permeability, so, negligible mmf is required to create the flux.
- Entire flux is confined in two windings. (No leakage).
- There are no losses and efficiency is 100%.

Ideal

- NO losses
- 100% efficiency.
- Zero voltage regulation.
- It is impossible to construct.

Practical

- There are copper loss & core loss.
- Efficiency is approx. 93-98%.
- Voltage regulation is never 0.
- All constructed transformers are practical.

Q: A single ϕ transformer has 1000 turns on the primary & 200 turn on secondary. The no load current is 3A at p.f. of 0.2 lagging. Calc. the -
HW) Primary current & p.f. when secondary current is 280 A at a p.f. of 0.8 lagging.

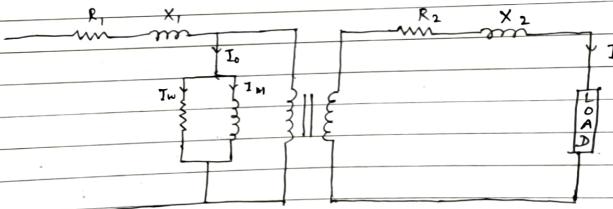
Q: Q) A 1000/1200 V transformer takes 0.3 A at p.f. of 0.2 on open circuit. Find magnetizing & iron loss component of no load primary current.

$$1000 \times 0.3 \times \cos \phi = 0.2 \\ \sin \phi = 0.98$$

$$I_w = 0.3 \times 0.2 = 0.06 \text{ A}$$

$$I_m = 0.3 \times 0.98 = 0.294 \text{ A}$$

* Equivalent circuit of Transformer:



- Q: What is construction, working principle of transformer?
- Q: What is Ideal condition for a transformer?
- Q: Diff b/w Practical & Ideal.
- Q: What are the parameters of eq. ckt of a transformer?
- Q: What is Efficiency?
- Q: What is all day efficiency?
- Q: What is voltage regulation?
- Q: Explain Phasor diagram. (Resistive, Capacitive, Inductive)
For load case

Q: Why polarity test is required & what will happen when we connect with wrong polarity?

★ Testing of a Transformer

- o) Ratio Test.
- i) High voltage winding resistance test (Wheatstone bridge)
- ii) Low voltage winding resistance test (Kelvin double bridge)
- iii) Insulation resistance test. (Megger's) ($M\Omega$)
- ✓ iv) Open circuit test / No load test / Iron loss test.
- ✓ v) Short circuit test / Copper loss / Load loss test.
- vii) Back to Back test.
- viii) Polarity Test.
- viii) DVDF (Double voltage Double frequency) test.
(Withstand test for 1 minute).
- ix) High voltage test for 1 minute.
- x) Swan Burn test.
- xii) Impulse Test xii) Scott Connection Test.

$$\text{Soln: } \frac{I_p}{I_s} = \frac{N_s}{N_p} \Rightarrow I_p = \frac{N_s \times I_s}{N_p} = \frac{200 \times 280}{1000}$$

$$\Rightarrow I_p = 56 \text{ A.}$$

$$\cos \phi_2 = 0.8, \cos \phi_0 = 0.2, \sin \phi_2 = 0.6.$$

$$\sin \phi_0 = 0.98.$$

$$I_1 \cos \phi_1 = I_2 \cos \phi_2 + I_0 \cos \phi_0 = (56 \times 0.8) + (3 \times 0.2) = 45.4 \text{ A.}$$

$$I_1 \sin \phi_1 = I_2 \sin \phi_2 + I_0 \sin \phi_0 = (56 \times 0.6) + (3 \times 0.98) = 36.54 \text{ A.}$$

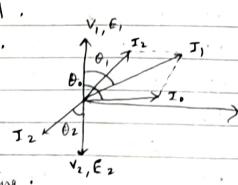
$$I_1 = \sqrt{(45.4)^2 + (36.54)^2} = 58.3 \text{ A.}$$

$$I_M = I_0 \sin \phi_0 = 3 \times 0.6 = 1.8 \text{ A.}$$

$$\tan \phi_1 = \frac{36.54}{45.4} = 0.805$$

$$\phi_1 = 38^\circ.$$

$$\text{P.f.}, \cos \phi_1 = \cos 38^\circ = 0.78 \text{ lagging.}$$



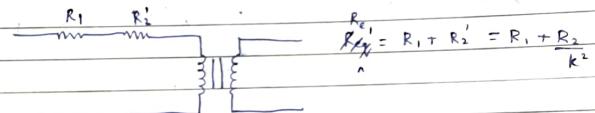
Soln: $N_1 = 1000, N_2 = 200$ Equivalent circuit of transformer:
It is a graphical representation of a transformer ckt in which the resistance & leakage reactance are imagined to be external to the winding.

Now transfer primary winding resistance to secondary by adding R_2' in primary in such way that Power absorbed by R_2' when carrying I_1 = Power absorbed by R_2 when carrying I_2 .

$$I_1^2 R_1' = I_2^2 R_2$$

$$\Rightarrow R_1' = \left(\frac{I_2}{I_1} \right)^2 R_2 = \frac{R_2}{k^2}$$

$$K = \frac{I_1}{I_2} = \frac{V_2}{V_1}$$

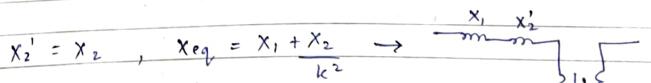


Similarly, we can transfer primary resistance to secondary side.

$$I_2^2 R_1' = I_1^2 R_1$$

$$R_1' = \left(\frac{I_1}{I_2} \right)^2 R_1 = k^2 R_1$$

$$R_{eq2} = R_2 + R_1' = R_2 + k^2 R_1$$

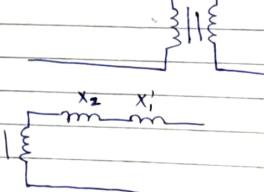


$$X_1' = k^2 X_1$$

$$X_{eq2} = X_2 + X_1' k^2 \rightarrow$$

$$Z_{eq1} = R_{eq1} + X_{eq1} \text{ (in primary side)}$$

$$Z_{eq2} = R_{eq2} + X_{eq2} \text{ (in secondary side).}$$



$V_1 = E_1 + I_1 Z_1$ (Supply voltage to primary winding).

V_2 = load voltage

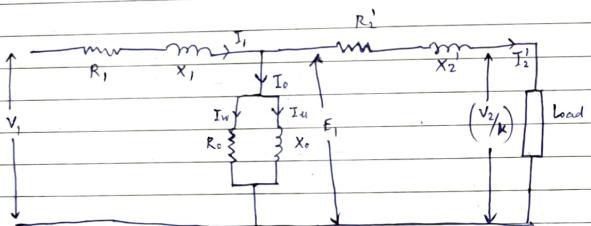
$$E_2 = V_2 + I_2 Z_2$$

$R_0 \rightarrow$ No load resistance represents iron & core losses
and I_{M0} is supplied by the core loss.

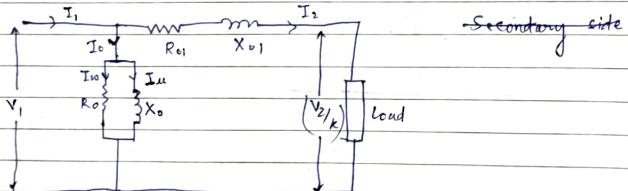
$X_0 \rightarrow$ No load inductance represents a loss free coil
and I_{M0} passes through X_0 .

Equivalent circuit referred to primary side:

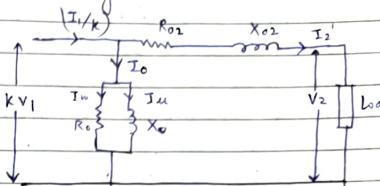
$$E_2' = \frac{E_2}{k}, \quad V_2 = \frac{V_2}{k}, \quad R_2' = \frac{X_2}{k^2}, \quad X_2' = \frac{X_2}{k^2}$$



$$R_{01} = R_1 + R_2' \quad , \quad X_{01} = X_1 + X_2'$$



Secondary side.



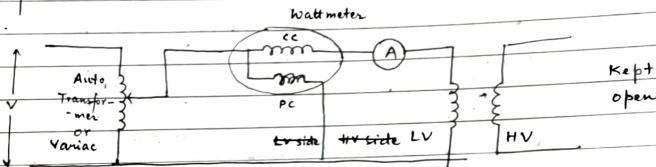
$$R_1' = k^2 R_1$$

$$X_1' = k^2 X_1$$

$$R_{02} = R_1' + R_2$$

$$X_{02} = X_1' + X_2$$

* Open circuit test.



Rating -

$$\begin{cases} 3 \text{ kVA} \\ 5 \text{ kVA} \\ 100 \text{ kVA} \end{cases} \rightarrow \begin{cases} I = 13.63 \text{ A} \\ I = 22.7 \text{ A} \\ I = 9.09 \text{ A} \end{cases}$$

Primary

$$I_{LV} = 27.27 \text{ A}$$

$$I_{LV} = 45.45 \text{ A}$$

$$I_{HV} = 227.27 \text{ A}$$

$$I_{HV} = 13.63 \text{ A}$$

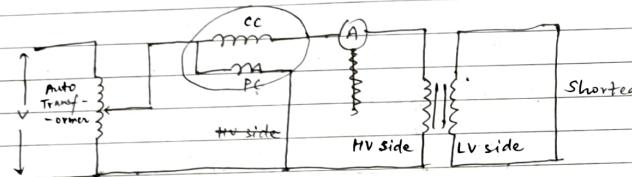
$$I_{HV} = 22.72 \text{ A}$$

$$I_{HV} = 9.09 \text{ A}$$

The aim of this test is to determine the iron losses of the transformer. This test is performed at normal supply voltage & frequency. The primary winding

(preferable LV side) is supplied with a normal voltage at normal frequency i.e. 50 Hz & the other winding (HV side) is kept open. Iron loss & no load current will be very small so small ammeter & voltmeter of low range are connected in the primary. No load current will be approximately 3-4% of rated current.

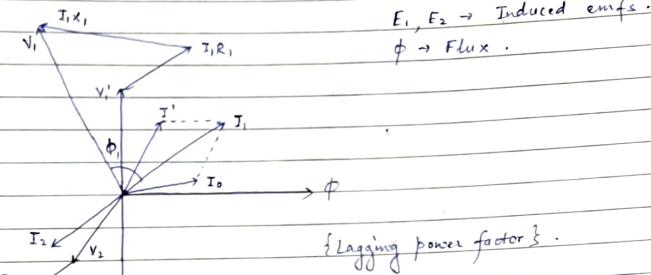
* Short circuit test.



In this test primary winding (HV side) is supplied the rated / full load current with reduced voltage at normal frequency. The reduced voltage is called impedance voltage & this impedance voltage is 3-8% of rated voltage.

H.W solutions :

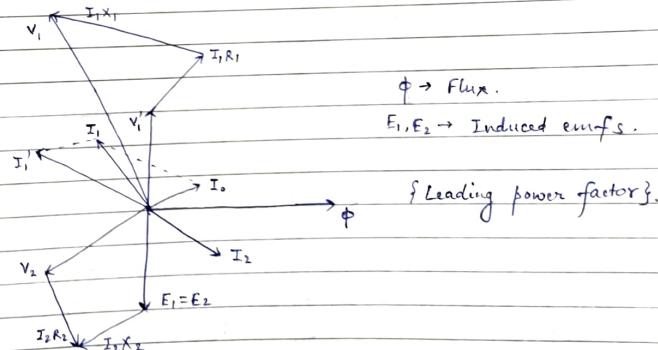
(i) Phasor diagram of Transformer on Inductive Load.



$E_1, E_2 \rightarrow$ Induced emfs.
 $\phi \rightarrow$ Flux.

{Lagging power factor}.

(ii) Phasor diagram of Transformer on Capacitive Load.

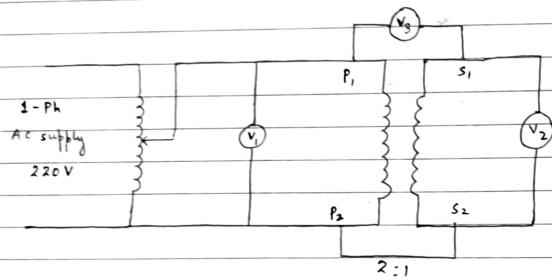
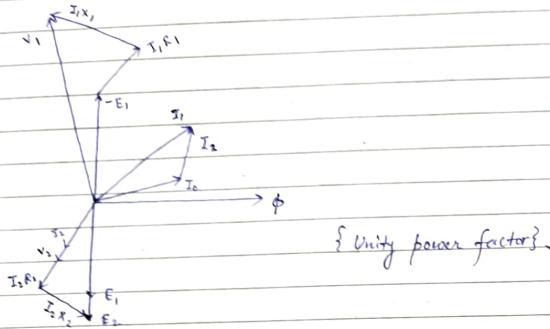


$\phi \rightarrow$ Flux.

$E_1, E_2 \rightarrow$ Induced emfs.

{Leading power factor}.

(ii) Phasor diagram of Transformer on Resistive Load.



Polarity Test.

We should know when we connect a transformer to AC supply then at particular instant 1 terminal of the primary wrt other terminal is positive or at higher potential. The purpose of polarity test is to ensure that all single phase devices (switch, fuse, circuit breaker, MCB, etc.) are certainly connected in the lines. For parallel operation of two or more than two transformers to increase the capacity of the transformers.

- P_1 and S_1 are of the same polarity.

$$V_1 = 100V, V_2 = 50V$$

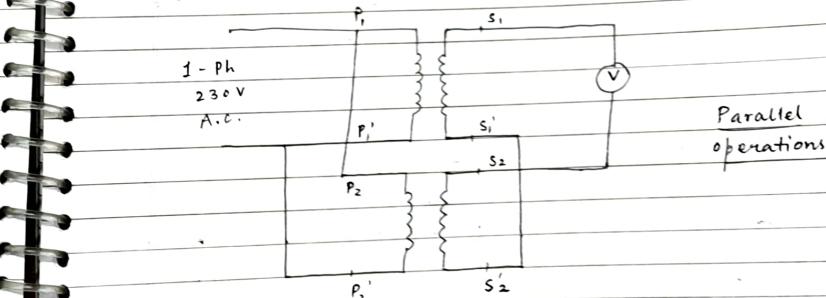
$$V_3 = 100 - 50 = 50V$$

- P_1 and S_1 are of opposite polarity.

$$V_1 = 100V, V_2 = 50V$$

$$V_3 = 150V. (100 + 50)$$

We can conclude that if $V_3 <$ applied voltage, then the terminal voltage S_1 should be marked +ve whereas, if $V_3 >$ applied voltage, then the terminal S_1 should be marked -ve.



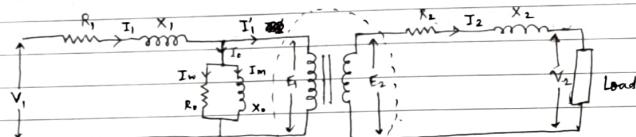
The two transformers to be connected in parallel should be connected in a manner that their primaries & secondaries have same polarity. This can be done by conducting a polarity test.

- Switch on the supply.
- Note the reading in the voltmeter connected on the secondary side. It may be either 0 or twice the secondary terminal voltage.
- If voltmeter reads 0, the correct polarity has been connected.

- v) If voltmeter reads double, then its secondary opposite polarity has been connected. Then terminal S_1 should be connected to S_2' & S_1' should be connected to S_2 .

* Transformer referred to primary & secondary sides.

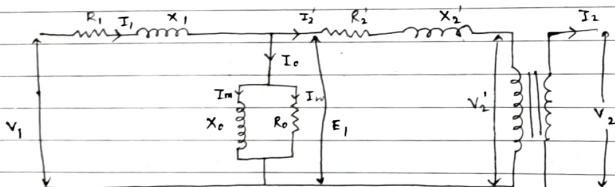
Equivalent circuit of Transformer -



Ideal Transformer

$$\frac{E_2}{E_1} = \frac{N_2}{N_1} \quad \left| \quad \frac{I_1'}{I_2} = \frac{N_2}{N_1} \right.$$

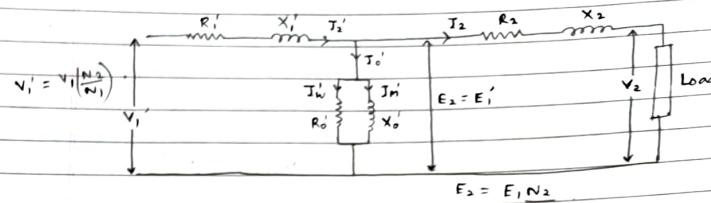
Equivalent circuit of Transformer referred to Primary.



$$R_2' = R_2 \left(\frac{N_1}{N_2} \right)^2, \quad X_2' = X_2 \left(\frac{N_1}{N_2} \right)^2$$

$$V_2' = V_2 \frac{N_1}{N_2}$$

Equivalent circuit of Transformer referred to secondary



$$R_1' = R_1 \left(\frac{N_2}{N_1} \right)^2$$

$$X_1' = X_1 \left(\frac{N_2}{N_1} \right)^2$$

$$R_0' = R_0 \left(\frac{N_2}{N_1} \right)^2$$

$$I_2' = I_1 \frac{N_1}{N_2}, \quad I_0' = I_0 \frac{N_1}{N_2}$$

$$X_M' = X_M \left(\frac{N_2}{N_1} \right)^2$$

* efficiency (η)

$$\eta = \frac{\text{Output}}{\text{Output} + \text{losses}} \quad \text{and} \quad \eta = \frac{\text{Input} - \text{losses}}{\text{Input}}$$

$$\text{Primary input power} = V_1 I_1 \cos \phi_1$$

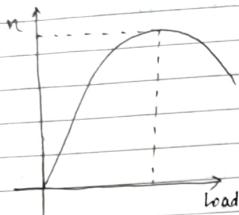
$$\text{Primary copper loss} = I_1^2 R_1$$

$$W_c = W_h + W_e$$

$$\eta = 1 - \frac{\text{losses}}{\text{Input}} = 1 - \frac{I_1^2 R_1}{V_1 I_1 \cos \phi_1} = 1 - \frac{I_1^2 R_1}{V_1^2 \cos^2 \phi_1}$$

$$\frac{d\eta}{dI_1} = \frac{V_1 I_1 \cos \phi_1 - I_1^2 R_1 - W_c}{V_1^2 I_1^2 \cos^2 \phi_1}$$

$$\frac{d\eta}{dI_1} = 0 \Rightarrow W_c - I_1^2 R_1 = 0, \Rightarrow W_c = I_1^2 R_1$$



When Load is 0, $\eta = 0$.
As we increase load, η also increases upto certain point. It becomes maxm then starts decreasing after that point.

Q: A 220/440V 10 kVA 50 Hz 1-φ transformer at full load copper loss is 120 W. If it has $\eta = 98\%$ at full load at p.f. = 1. Determine -

- Iron losses.
- What would be the η at half full load & 0.8 pf. lagging.

Q: A 50 kVA transformer has 98% η at full load 0.8 p.f. and ~~96.9%~~ 96.9% η at 1/4 full load 1 p.f. Determine iron loss & full load cu loss.

$$i) \eta_i = \frac{(x \times \text{full load VA}) \times \text{p.f.}}{(x \times \text{full load VA} \times \text{p.f.}) + P_i + x^2 P_c}$$

$$i) \eta_i = \frac{1 \times 10000 \times 1}{(1 \times 10000 \times 1) + P_i + 120}$$

$$0.9898 = \frac{10000}{10000 + 10120 + P_i}$$

$$P_i = 84 \text{ W.}$$

$$ii) \eta = \frac{\frac{1}{2} \times 10000 \times 0.8}{(\frac{1}{2} \times 10000 \times 0.8) + P_i + \frac{1}{4} P_c}$$

$$\eta = \frac{4000}{4000 + P_i + 0.25 P_c}$$

$$\text{Taking } P_i = 84 \quad \eta = \frac{4000}{4000 + 84 + 30} \\ = 0.9722$$

$$\eta \cdot 100 = 97.22 \cdot 100$$

$$2) 0.98 = \frac{1 \times 50000 \times 0.8}{(1 \times 50000 \times 0.8) + P_i + P_c}$$

$$0.98 P_i + 0.98 P_c = 40000 - 39200 \\ P_i + P_c = 816.32$$

$$0.969 = \frac{\frac{1}{4} \times 50000 \times 1}{(\frac{1}{4} \times 50000 \times 1) + P_i + \frac{1}{16} P_c}$$

$$0.969 = \frac{12500}{12500 + P_i + \frac{P_c}{16}}$$