

INDIAN INSTITUTE OF SCIENCE EDUCATION AND RESEARCH THIRUVANANTHAPURAM

Mathematical Foundations to Data Science (DSC 212)

Assignment Analysis Report

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Overview

This report presents an analytical summary of the centrality characteristics observed in Zachary's Karate Club network. The objective is to study the behaviour of key nodes across repeated graph partitions and to evaluate the influence of community structure on common network metrics such as degree, betweenness, closeness, and clustering coefficients.

Nodes Consistently Remaining Central

The analysis reveals that certain vertices maintain dominant influence irrespective of how the graph is partitioned. Node 33 consistently emerges as the most structurally significant vertex, exhibiting the highest average degree along with strong betweenness values. Its location within the right-side community enables it to act as a major conduit for interactions across groups. Node 0 ranks closely behind, maintaining a high degree as well as substantial betweenness centrality. Positioned near the core of the left-side faction, it serves as both a local hub and an intermediary across multiple neighbourhoods. Node 32 retains considerable importance due to its adjacency to node 33 in a densely interconnected region. Furthermore, nodes 1 and 2 exhibit moderately high centrality values because their positions lie near the inter-community boundary, enabling them to act as connectors between dense substructures within the network.

Why These Nodes Remain Central

Degree centrality remains unchanged under graph partitioning since the edge set of the underlying network is preserved. Therefore, nodes with high degree maintain their prominence across all iterations. Betweenness centrality is influenced by the role a node plays in conveying shortest paths. Vertices positioned near or on community boundaries, such as nodes 0, 33, and their immediate neighbours, continue to mediate interactions between the principal groups, thereby retaining elevated betweenness values. Closeness centrality remains relatively stable for major hubs in this compact graph. These nodes maintain small average path lengths to the rest of the network. Clustering coefficient varies depending on the local density of connections; nodes embedded in tightly knit clusters exhibit higher clustering, while hubs spanning multiple communities generally show lower clustering.

Influence of Community Structure

Upon the first partition, the network naturally divides into two communities corresponding closely to the known factions of the Karate Club. Degree centrality remains unchanged because the structural connections are not modified during splitting. Betweenness centrality may shift depending on whether a node is placed within or between communities; however, the principal hubs consistently maintain bridging roles and therefore retain high betweenness. Closeness centrality experiences only minor variation due to the small size and limited diameter of the graph; path lengths change negligibly under partitioning. Clustering coefficients remain high for vertices inside dense modules, whereas hubs connecting separate communities preserve lower clustering values.

Conclusion

Among the metrics examined, degree centrality is the most robust across repeated partitions because it is directly determined by the fixed adjacency structure. Betweenness, closeness, and clustering coefficients show sensitivity to how communities align, but in this dataset, nodes 0 and 33 consistently dominate across all criteria. Their roles as structural anchors in the network are fully aligned with the well-understood dynamics of the historical Karate Club.