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Sub Code:- EES 551

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Department:- Electrical Engineering

Semester:- 5

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Experiment No:- 6

Title :- Experiment on DC motor speed control with root locus design approach

Objective :-

(i) To design PI controller and also lag compensator for DC motor speed control

(ii) Develop MATLAB code in ".m" file to synthesize the controller parameters.

(iii) Verify the controller performance in MATLAB-Simulink

Theory :-

Transfer function of an armature voltage controlled DC motor is :-

$$P(s) = \frac{\Omega(s)}{V(s)} = \frac{K}{(Js+b)(Ls+R)+K^2} \left[\frac{rad/sec}{V}\right]$$

PI controller :-

Transfer function:- $C_{PI}(s) = K_P + \frac{K_I}{s}$

PD controller :-

Transfer function: $C_{PD}(s) = K_P + K_D s$

PID controller :-

Transfer function

$$C_{PID}(s) = \left(K_{P_1} + \frac{K_I}{s}\right)\left(K_{P_2} + K_D s\right)$$
$$= \bar{K}_P + \frac{\bar{K}_I}{s} + \bar{K}_D s$$

Lag compensator:-

(i) Produces output having phase lag than the input

$$G_c(S) = \frac{1}{\alpha}\left[\frac{S + 1/T}{S + \frac{1}{\alpha T}}\right] ; \alpha > 1$$

(ii) Pole is nearer to origin

(iii) It reduces steady state error

Given parameters of DC motor:-

$J = 0.01\ kg\,m^2/s^2$ $\quad b = 0.1\ Nms$

$K = K_e = K_t = 0.01\ Nm/Amp$, $R = 1\,\Omega$, $L = 0.5\,H$

$\therefore$ Transfer function, $P(s) = \dfrac{K}{(Js+b)(Ls+R)+K^2}$

$$= \frac{0.01}{(0.01S+0.1)(0.5S+1)+0.01^2}$$

$$\Rightarrow P(s) = \frac{2}{S^2+12S+20.02}$$

$$\Rightarrow P(s) = \frac{2}{(S+2.0025)(S+9.9975)}$$

Requirements:-

(i) Settling time, $t_s < 2s$

(ii) Maximum Overshoot, $\%\ M_p < 5\%$

(iii) Steady state error, $e_{ss} < 1\%$

Given,

$$\% M_p = 5\% = e^{\frac{-\pi \xi}{\sqrt{1-\xi^2}}} \times 100\%.$$

$$\Rightarrow \xi = \frac{-\ln(M_p)}{\sqrt{\pi^2 + \ln^2(M_p)}} = \frac{-\ln(0.05)}{\sqrt{\pi^2 + \ln^2(0.05)}}$$

$$\Rightarrow \boxed{\xi = 0.6901}$$

Damping ratio

$$t_s = \frac{4}{\xi \omega_n} \quad \{\text{for } 2\% \text{ tolerance band}\}$$

$$\Rightarrow \omega_n = \frac{4}{\xi t_s} = \frac{4}{0.6901 \times 2}$$

$$\Rightarrow \boxed{\omega_n = 2.8981}$$

Natural frequency

Now

$$\cos^{-1}(\xi) = \cos^{-1}(0.6901) = 46.362^\circ$$

Break away point

$$1 + G(s) H(s) = 0$$

$$1 + \frac{K}{(s+2)(s+10)} = 0$$

$$\Rightarrow K = -(s+2)(s+10)$$

$$\Rightarrow \frac{dK}{ds} = -[2s+12]$$

$$\frac{dK}{ds} = -[2s+12] = 0$$

$$\Rightarrow s = -6$$

Desired close loop poles are :-

$$s = -\xi\omega_n \pm j\,\omega_n\sqrt{1-\xi^2}$$

$$s_{1,2} = -2 \pm j\,2.0973$$

From the root locus dominant poles are

$-6.0429 \pm j6.25$

Proportional gain, $K_1 = \dfrac{1}{\left|G(s)\right|_{s=-6.0429+j6.25}}$

$\Rightarrow K_1 = 27.52$

Given steady state error, $e_{ss} = 1\%$.

$$\frac{1}{1+K_{pn}} = \frac{1}{100}$$

$\Rightarrow K_{pn} = 99$

K_p for DC motor speed control with gain is -

$$K_{po} = \lim_{s \to 0} K_1 G(s) = \frac{27.52 \times 2}{20.02}$$

$\Rightarrow \boxed{K_{po} = 2.749}$

Let, the transfer function of lag compensator be :-

$$G(s) = K_1\left[\frac{s+Z_c}{s+P_c}\right]$$

$$\frac{Z_c}{P_c} = \frac{K_{pn}}{K_{po}} = \frac{99}{2.749}$$

Let $P_c = 0.05$ then,

$$Z_c = \frac{99}{2.749} \times 0.05$$

$$Z_c = 1.8$$

$\therefore$ Transfer function of Lag compensator is:-

$$G(s) = 27.52\left[\frac{S+1.8}{S+0.05}\right]$$

<u>Design of PI controller for DC. motor speed control:-</u>

Transfer function of DC motor speed control is:-

$$G(s) = \frac{2}{s^2 + 12s + 20.02}$$

$\xi = 0.6901$ and $\omega_n = 2.8981$

dominant poles are $-6.042 \pm j6.25$

$$K_1 = 27.52$$

Let the transfer function of PI Controller is

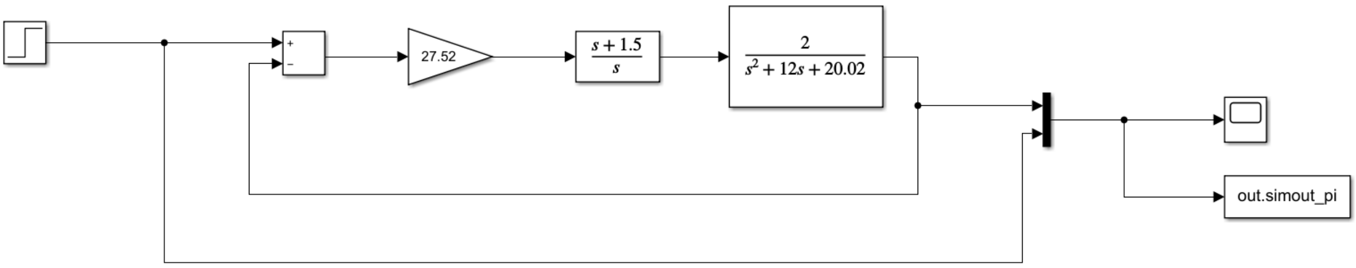
$$G_{PI}(s) = K_1\left[\frac{S+Z}{S}\right]$$

Let $Z = 1.5$

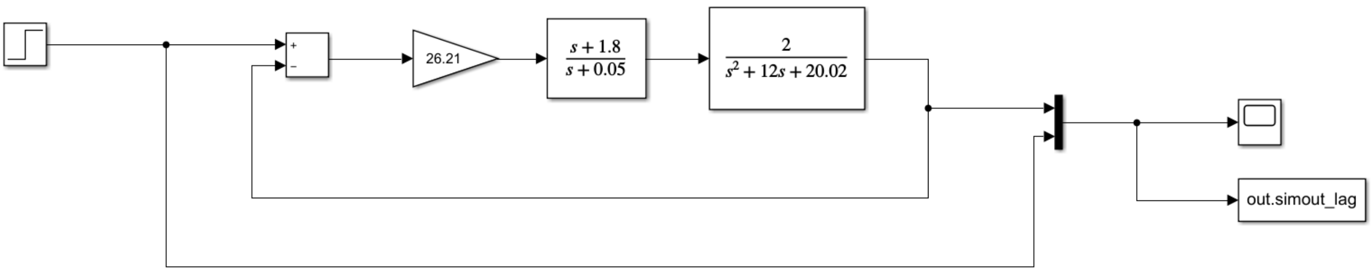
$$\boxed{G_{PI}(s) = 27.52\left[\frac{S+1.5}{S}\right]}$$

MATLAB-Simulink Block Diagrams: -

1. PI controller design –



2. Lag compensator design –



MATLAB Codes (".m" files) –

PI.m | PI2.m | uncomp.m | gain.m

```
J=0.01;
b=0.1;
K=0.01;
R=1;
L=0.5;
num=(27.5*K);
den=[(J*L) (J*R+L*b) ((b*R)+K^2)];
rlocus(num,den)
sgrid
sgrid(0.69,2.89)
title('Root locus without a controller')
[kp,poles]=rlocfind(num,den)
[numc,denc]=cloop(kp*num,den,-1);
t=0:0.01:3;
step(numc,denc,t)
title('Step response with gain')
```

PI.m | PI2.m | uncomp.m | gain.m

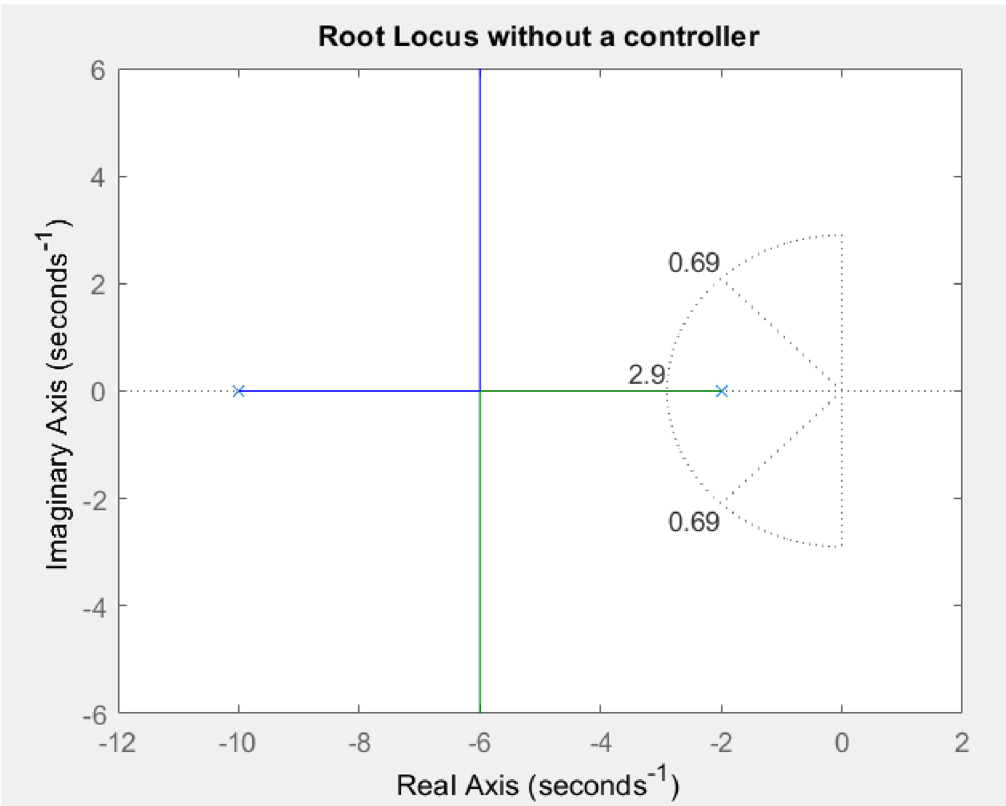
```
z1=3.29;
p1=0;
numa=[1 z1];
dena=[1 p1];
numb=conv(num,numa);
denb=conv(den,dena);
rlocus(numb,denb);
sgrid
sgrid(0.69,0)
title('Root Locus with PI controller')
[kp, poles]=rlocfind(numb,denb)
[numc,denc]=cloop(kp*numb,denb,-1);
t=0:0.01:3;
step(numc,denc,t)
title('Step response with a PI controller')
```

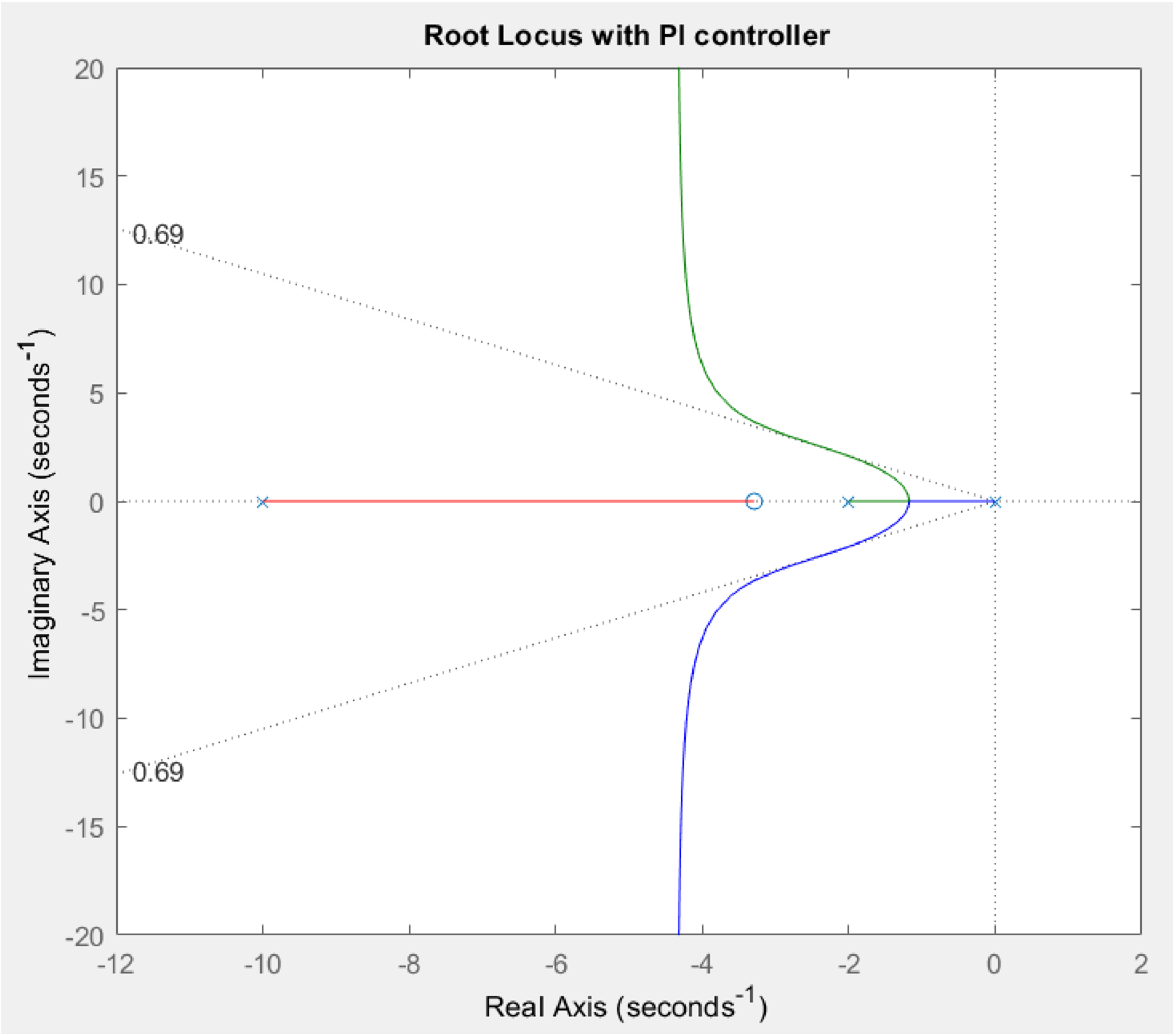
PI.m | PI2.m | uncomp.m | gain.m

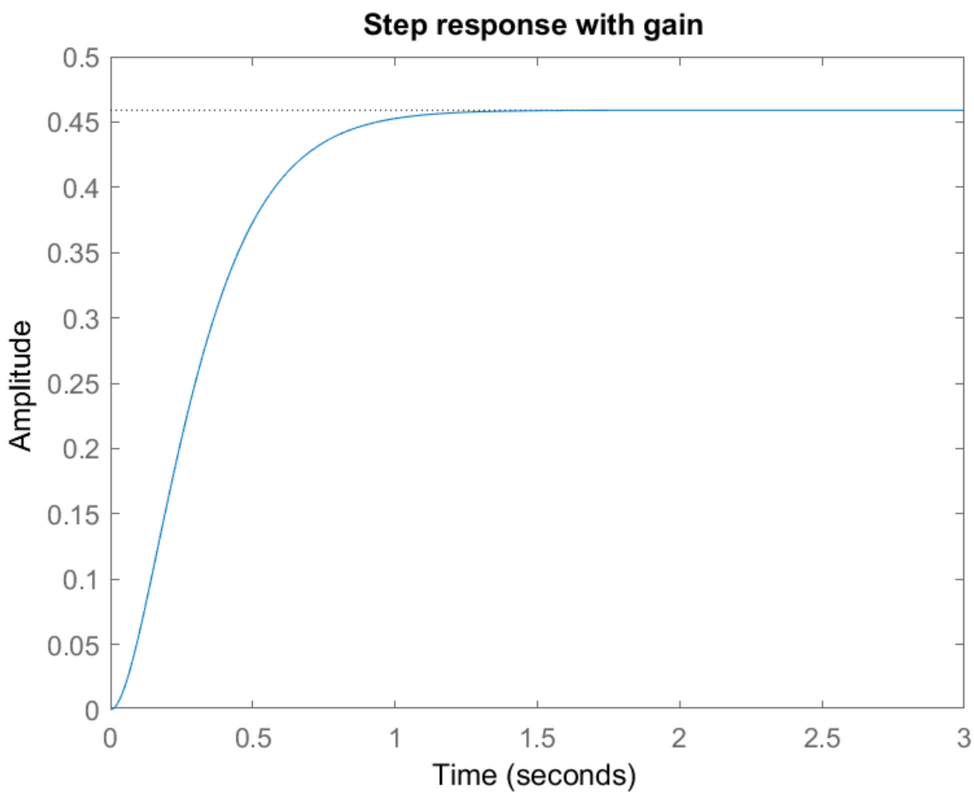
```
z1=1.8;
p1=0.05;
numa=[1 z1];
dena=[1 p1];
numb=conv(num,numa);
denb=conv(den,dena);
rlocus(numb,denb);
sgrid
sgrid(0.69,2.98)
title('Root Locus with lag compensator')
[kp,poles]=rlocfind(kp*numb,denb,-1)
t=0:0.01:3;
step(numc,denc,t)
title('Step response with a lag compensator')
```

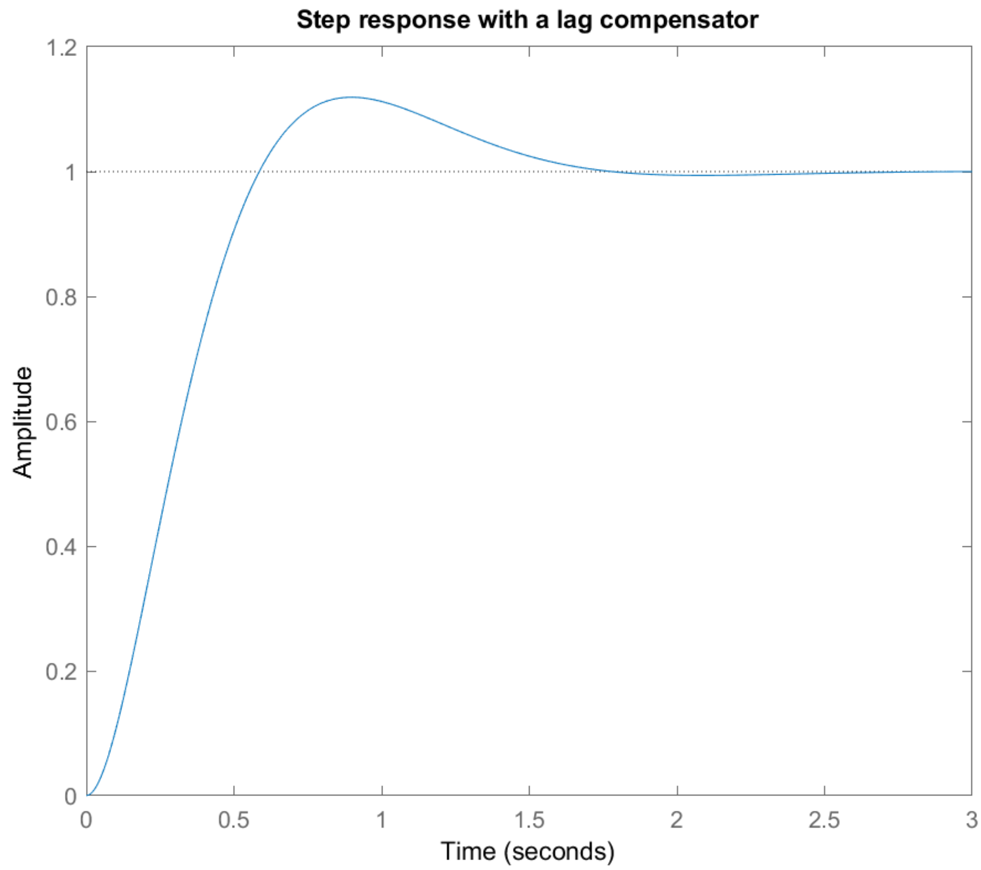
```
J=0.01;
b=0.1;
K=0.01;
R=1;
L=0.5;
ts=2;
z=-log(0.05)/sqrt(pi^2+(log(0.05))^2)
wn=4/(z*ts)
num=K;
den=[ (J*L) [(J*R)+(L*b)] ((b*R)+K^2)];
rlocus(num,den);
sgrid(z,0);
sgrid(z,wn);

title('Root Locus without a controller');
[kp,poles]=rlocfind(num,den);
[numc,denc]=cloop(kp*num,den,-1);
t=0:0.01:3;
step(numc,denc,t)
title('Step response with gain')
```

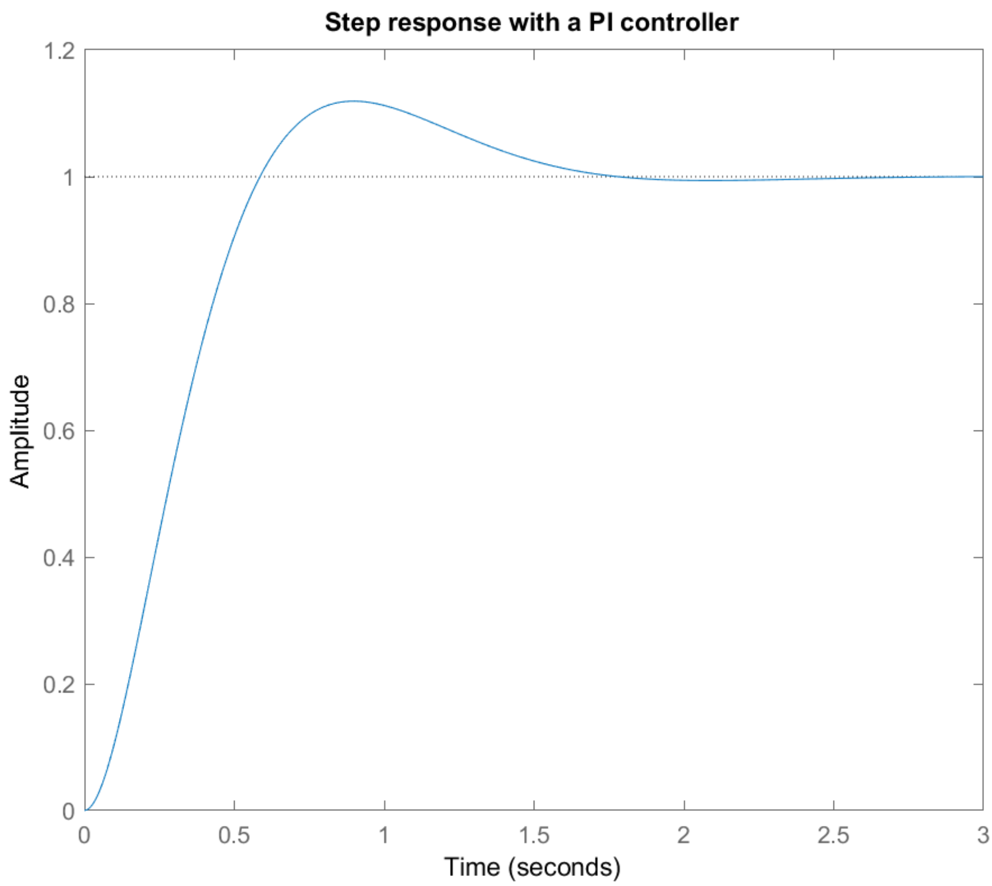
Results –




Root Locus with PI controller
Imaginary Axis (seconds^{-1})
Real Axis (seconds^{-1})
0.69
0.69
20
15
10
5
0
-5
-10
-15
-20
-12
-10
-8
-6
-4
-2
0
2


Step response with gain
Amplitude
Time (seconds)
0.5
0.45
0.4
0.35
0.3
0.25
0.2
0.15
0.1
0.05
0
0.5
1
1.5
2
2.5
3


Step response with a lag compensator
Amplitude
Time (seconds)
1.2
1
0.8
0.6
0.4
0.2
0
0
0.5
1
1.5
2
2.5
3


Step response with a PI controller
Amplitude
Time (seconds)
1.2
1
0.8
0.6
0.4
0.2
0
0
0.5
1
1.5
2
2.5
3

Conclusion:-

In this experiment, we designed PI controller and LAG compensator and have used DC Motor Speed Control as our model. We developed MATLAB code in m file to synthesize control parameters. The purpose of designing is to get desired values of maximum overshoot, steady state error and settling time. We have found the necessary transfer function and used root locus technique for further analysis. Then we have observed the step response of DC Motor with and without PI controller and with lag compensation.