

## DAA ASSIGNMENT

Q1- Asymptotic notation describe the growth rate of algorithms as their input size approaches infinity.  
Commonly used notations are.

(i) Big O ( $O$ )  $\rightarrow$  Represents an upper bound on the growth rate.

Ex  $\rightarrow$  If an algorithm has a time complexity of  $O(n)$ , it means the worst case running time grows linearly with the input size  $n$ .

(ii) Omega ( $\Omega$ )  $\rightarrow$  represents a lower bound on the growth rate. If an algorithm has a time complexity of  $\Omega(n^2)$ , it means the worst case time running grows at least quadratically with the input size  $n$ .

(iii) Theta ( $\Theta$ )  $\rightarrow$  represents both upper and lower bounds, indicating a tight bound on the growth rate. If an algorithm has a time complexity of  $\Theta(n)$ , it means the worst case running time grows linearly, and there is well defined constant factor.

Q2  $\rightarrow$   $\text{for } (int\ i = 0; i < n; i++)$   
 $\uparrow$   
 $\equiv$   
 $3$

Taking log both sides

$$\log_2 n = (K-1) \log_2 2 \Rightarrow \log_2 n = K-1$$

$$K = \log_2 n + 1$$

$$TC \Rightarrow O(\log_2 n)$$

Q 3-  $T(n) = \begin{cases} 3T(n-1) & \text{if } n > 0, \text{ otherwise } 1 \end{cases}$

$$T(n) = 3T(n-1) \quad \text{put } n = n-1$$

$$T(n-1) = 3T(n-2) \quad \text{put } n = n-2$$

$$T(n-2) = 3T(n-3)$$

And so on

$$T(K) = 3^K T(n-K)$$

$$\text{put } n-K=0, \Rightarrow n=K$$

$$T(n) = 3^n T(0)$$

$$T(n) = 3^n$$

Q 4-  $T(n) = \begin{cases} 2T(n-1) - 1 & \text{if } n > 0, \text{ otherwise } 1 \end{cases}$

$$T(n) = 2T(n-1) - 1 \quad \text{--- (1)}$$

$$\text{put } n = n-1 \text{ in (1)} \quad T(n-1) = 2T(n-2) - 1$$

$$\text{put } n = n-2 \text{ in (1)} \quad T(n-2) = 2T(n-3) - 1$$

If we expand this, we get

$$T(n) = 2^K T(n-K) - K$$

We will continue this until  $n-K=0, K=n$

$$T(n) = 2^n T(0) - n$$

$$T(n) = 2^n - n$$

Time complexity  $\Rightarrow 2^n$

(5) -  $\text{int } i=0, S=1;$

$\text{while}(S < 2^n)$

{

$i++;$

$S = S + i;$

$\text{put}(\text{"#"});$

}

$i = 1 \quad 2 \quad 3 \quad 4 \quad 5$

$S = 1 \quad 3 \quad 6 \quad 10 \quad 15$

$$S = \frac{1(i+1)}{2}$$

⑥ void function (int n)  
{

int i, count = 0;

for (int i = 1; i \* i <= n; i++)

count++;

}

i = 1 2 3 4

i<sup>2</sup> = 1 4 9 16

complexity =  $O(\sqrt{n})$

⑦ void function (int n)  
{

int i, j, k, count = 0;

for (i = n/2; i <= n; i++)

for (j = 1; j <= n; j = j \* 2)

for (k = 1; k <= n; k = k \* 2)

count++

}

$\frac{n}{2} \times \log_2(n) \times \log_2(n)$

complexity =  $O(n \log^2(n))$

⑧ function (int n)  
{

if (n <= 1) return;

for (i = 1; i <= n; i++)

for (j = 1; j <= n; j++)

put("#");

}

function(n-3);

T(n) =  $O(n^2 + T(n-3))$

$T \leq O(n^2)$



09- Void function (int n)

```
{
    for (i = 1; i <= n; i++) {
        for (j = 1; j <= n; j = j + i) {
            put( "4" );
        }
    }
}
```

i = 1, 2, 3, 4, ...

j = 1, 3, 6, 10, ...

i = 1, n times

i = 2, n/2 times

i = 3, n/3 times

1 + n/2 + n/3 + ... + 1

Complexity =  $O(n \log n)$

10-  $n^k$  ( $k \geq 1$ )  $c^n$  ( $c > 1$ )

$c^n$  grows faster than  $n^k$ .