Ex 2.7 → $\beta_n = \frac{\alpha}{\bar{o}_n}$

$$\bar{o}_n = \bar{o}_{n-1} + \alpha(1 - \bar{o}_{n-1}) \text{ — (1) with } \bar{o}_0 = 0$$

$$Q_{n+1} = Q_n + \beta(R_n - Q_n)$$
$$= Q_n + \frac{\alpha_n}{\bar{o}_n}(R_n - Q_n)$$
$$= Q_n\left(1 - \frac{\alpha_n}{\bar{o}_n}\right) + \frac{\alpha_n}{\bar{o}_n}(R_n) \text{ — (2)}$$

by (1) & (2)

$$Q_{n+1} = Q_n\left(1 - \frac{\alpha}{\bar{o}_{n-1} + \alpha(1 - \bar{o}_{n-1})}\right) + \frac{R_n\alpha}{\bar{o}_{n-1} + \alpha(1 - \bar{o}_{n-1})}$$

$$Q_{n+1}\left(\bar{o}_{n-1} + \alpha(1 - \bar{o}_{n-1})\right) = Q_n\left(\bar{o}_{n-1} - \alpha\bar{o}_{n-1}\right) + R_n\alpha.$$

$$Q_{n+1}\left(\underbrace{\bar{o}_{n-1} + \alpha(1 - \bar{o}_{n-1})}_{\bar{o}_n}\right) = \alpha R_n + Q_n\bar{o}_{n-1}(1-\alpha)$$

$$\bar{o}_n Q_{n+1} = \alpha R_n + Q_n\bar{o}_{n-1}(1-\alpha).$$

$\cancel{Q_{n+1}} =$

$$\bar{o}_n Q_{n+1} = \alpha R_n + (1-\alpha)\left(\alpha R_{n-1} + Q_{n-1}\bar{o}_{n-1}(1-\alpha)\right)$$
$$= \alpha R_n + \alpha(1-\alpha)R_{n-1} + Q_{n-1}\bar{o}_{n-1}(1-\alpha)^2$$

$$\bar{o}_n Q_{n+1} = (1-\alpha)^n \bar{o}_0 Q_1 + \sum_{i=1}^{n} \alpha(1-\alpha)^{n-1} R_i$$

since, $\bar{o}_0 = 0$, thus

$$\bar{o}_n Q_{n+1} = \sum_{i=1} \alpha(1-\alpha)^{n-1} R_i$$

Thus we can clearly see, that $Q_{n+1}$ is independent of $Q_1$ and is recency weighted average.