classmate

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Ex 2.7 : $\beta_n = \alpha / \bar{o}_n$, $Q_{n+1} = Q_n + \beta_n (R_n - Q_n)$

$$\bar{o}_n = \bar{o}_{n-1} + \alpha(1 - \bar{o}_{n-1}) \text{ for } n \geq 1, \ \bar{o}_0 = 0$$

To show : There is no initial bias →

$$\bar{o}_1 = \bar{o}_0 + \alpha(1 - \bar{o}_0) = \alpha$$
$$\beta_1 = \alpha/\alpha = 1$$

$$Q_2 = Q_1 + \beta_1 (R_1 - Q_1)$$
$$= Q_1 + 1(R_1 - Q_1) = R_1$$

Thus, step function $\beta_n = \alpha / \bar{o}_n$ is free from initial bias.

To show : recency-weighted average.
lets calculate β_n in terms of α →

$$\bar{o}_n = \bar{o}_{n-1} + \alpha(1 - \bar{o}_{n-1})$$
$$= \bar{o}_{n-1}(1-\alpha) + \alpha$$
$$= (1-\alpha)^2 \bar{o}_{n-2} + \alpha + \alpha(1-\alpha).$$
$$= (1-\alpha)^n \bar{o}_0 + \sum_{i=0}^{n-1} \alpha(1-\alpha)^i$$
$$= (1-\alpha)^n \bar{o}_0 + \frac{\alpha(1-(1-\alpha)^n)}{\alpha}$$
$$= (1-\alpha)^n (\bar{o}_0 - 1) + 1$$
$$= 1 - (1-\alpha)^n.$$

$$\beta_n = \frac{\alpha}{1-(1-\alpha)^n}$$, we can clearly see that as n increases β_n decreases.
and also $\alpha \leq \beta_n < 1$ for $n > 1$

Now, $Q_{n+1} = Q_n + \beta_n (R_n - Q_n)$

$$= Q_n(1-\beta_n) + R_n$$

$$= R_n + (1-\beta_n) Q_{n-1}((1-\beta_{n-1}) + R_{n-1})$$

$$= Q_{n-1}(1-\beta_n)(1-\beta_{n-1}) + \beta_n R_n + \beta_{n-1} R_{n-1} (1-\beta_n)$$

$$= \underbrace{Q_1 \prod_{i=1}^{n} (1-\beta_i)}_{1} + \underbrace{\sum_{i=1}^{n} \beta_i R_i \prod_{j=i+1}^{n} (1-\beta_j)}_{2}$$

~~Not~~

Note that 1 will be zero because $1-\beta_1 = 0$, ($\because$ $\beta_1 = 1$).

~~No when considering 2 R_n is weighted by α product of n terms where~~

Now when we consider ~~R_n~~ 2 term, R_i is weighted by ~~β_n and product~~ by a product $n-i$ terms where each term is ~~less than~~ = between $[0, 1)$ and of same order thus as ~~we will incr~~ i will increase $n-i$ will decrease thus the weight of R_i will increase

Here, it is a recency weight average.