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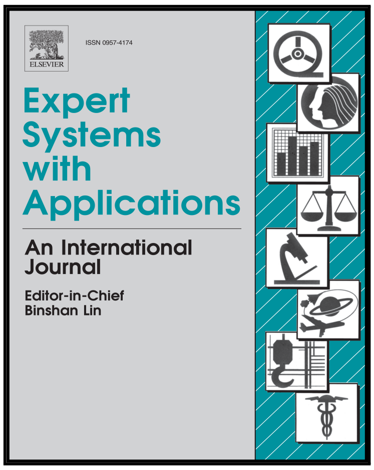


# A fuzzy decision system for money investment in stock markets based on fuzzy candlesticks pattern recognition

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Highlights

- Fuzzy stock market trading recommendation system with fuzzy Japanese candlesticks
- A new capital management fuzzy strategy is proposed to invest money
- The effect of currency devaluation on the market forecasting has been considered
- Applied to the American Nasdaq100 and the Spanish Ibex35 stock markets
- Fuzzy system with capitalization is profitable and efficient support for investors

# A fuzzy decision system for money investment in stock markets based on fuzzy candlesticks pattern recognition

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**Abstract**

This article proposes a novel fuzzy recommendation system for stock market investors. This intelligent decision tool uses fuzzy Japanese candlesticks and includes the effect of currency devaluation on the forecasting. To do so, first the next market session is obtained by a new designed fuzzy forecasting trading system. Then, it is compared to the one obtained by a non-parametric system based on the k-nearest neighbour technique. Finally, an amount of money to be invested is considered using a new capital management fuzzy strategy. The results have been compared to an analogous fuzzy trading system that has the same all-or-nothing investment strategy with risk control, but where this capitalization is not included. Both intelligent decision systems have been applied to two very different stock markets, the American Nasdaq100 and the Spanish Ibex35 markets, using the Buy and Hold investment strategy as benchmark. Results prove that the proposed fuzzy system with capitalization is profitable and presents high stability, and could be a good support system for investors.

**Keywords:** Japanese candlestick; fuzzy trading; stock market forecasting; capital management; investment strategy; recommendation system.

## 1. Introduction

Due to the increasing complexity of stock markets and the many interrelated factors involved in their behaviour, investors have been forced to evolve from conventional techniques to new advanced computational strategies to increase trading profits. For decades, no effort has been spared to improve the market prediction, applying more and more sophisticated tools and new techniques that lately come from the artificial intelligence field, in order to assist in the development of financial decision systems (Roy, Kumar, Sharma, 2014b).

Within the great variety of intelligent techniques that allow to address these complex issues, fuzzy logic has been proved as one of the most successful one due to its capacity to emulate the human reasoning. Indeed, stock markets are characterized as high complex and non-linear systems (Ahmadi et al., 2018), and they deal with uncertainty and vagueness that come not only from the data but also from the very definition of the terms used.

In addition, fuzzy logic presents some useful advantages such as the simplicity of its implementation, the interpretability of the models, and the capacity of showing the reasoning applied at each stage of the decision process. For all of these reasons, fuzzy logic is considered a good approach to deal with these financial decision systems (Zhou & Dong, 2004; Gradojevic & Gençay, 2013; Govindasamy & Thambidurai, 2013, among others).

Regardless of the techniques used for investment, traders need a paradigm for the representation of the market sessions. Time series show the evolution of the share price during a market session. Charts are graphical representations of a series of prices over time. Japanese Candlesticks are a technical analysis tool that traders use to chart and analyse the price movement of securities.

Independent of the representation system chosen, the market forecasting systems can be based on technical indicators, which are mathematical formulas and statistics applied to series of share prices, or on the chartist analysis, which studies the patterns that can be found in stock prices graphically represented.

Systems based on Japanese candles have quickly become the most widespread among investors for several reasons, namely, its simplicity, interpretability, and the significant dimensionality reduction. Indeed, instead of using many technical indicators, they only need four values (open, high, low and close) that summarize a complete market session (Thomsett, 2017).

Despite the great advantage of being able to predict, with greater or less accuracy, the future behaviour of the market by a computational trading system, it only represents a part of a complete trading process. Investments systems are, mainly, decision support systems, which take advantage of forecasting to improve financial markets' profitability. In particular, an important part of such systems is the investment strategy. Efficient capital management and risk control must be part of any intelligent trading proposal. Therefore, the decision system must not only suggest the right moment to buy or sell shares, but also the amount of capital to invest in each operation, as well as to indicate some threshold to limit the risk in case of losses.

Thus, the main contribution of this article is that a complete investment methodology is proposed, to help the investors all along the trading process.

The here proposed trading decision system consists of a fuzzy forecasting system that predicts the future market sessions, represented by Japanese candlesticks. Our system uses fuzzy candles and a lazy learning algorithm (*k-nearest neighbours*, k-NN), and applies fuzzy rules to obtain the values of opening, closing, maximum and minimum prices of the stocks, unlike other works that predict the future market trend.

The obtained prediction is then used to build an investment framework that will give the right time to enter the market, when to exit with profits, and the risk bounds in case of losses. In addition, it calculates the volume to invest taking into account the rate inflation of the market.

This allows a better management of the risk level. Despite using an all-nothing strategy, -we use all the available capital in each trade-, we control the risk by limiting the losses (stop-loss) when exiting the stock market and besides, we also control the number of shares to be bought depending on the risk we are willing to take. That is, if the risk is high, fewer stocks are bought and vice versa.

Thus, the capitalization is included in this financial system. This way, the price variation due to the currency devaluation is taking into account and a better prediction may be obtained.

This complete trading system, including forecasting and capital management, has been applied to two very different scenarios: the Nasdaq-100 market, which is composed of the 100 largest American technology companies registered in the New York Stock Exchange (NYSE), and the main Spanish Stock Exchange Index Ibex35, with the 35 most liquid companies listed on the Spanish Stock Exchange Interconnection System (SIBE) of the four Spanish stock exchanges (Madrid, Barcelona, Bilbao and Valencia). The obtained results are analysed with the statistical measures that are most commonly used by investors to test the performance of financial systems.

The rest of this paper is organized as follows. Section 2 presents the related works found in the literature. Section 3 describes the fuzzy candlestick trading system. Section 4 presents the developed strategy for forecasting and capital management. Results of the proposed fuzzy decision system are discussed and compared with other trading strategies in Section 5. The paper ends with the conclusions and future woks.

## 2. Background

Prediction of stock markets is a major challenge for traders because of the nature of the information to deal with and the complexity of the working scenario, highly nonlinear, with uncertainties and data that vary over time. In this sense, soft computing techniques have been a valuable help to investors' decision making (Hadavandi, Shavandi, & Ghanbari, 2010; Wan, Gong, & Si, 2016; Ravichandra, & Thingom, 2016; Zhong, & Enke, 2017; Göçken et al., 2017). In (Chong, Han, & Park, 2017) authors offer a systematic analysis of the use of deep learning networks for stock market analysis and prediction, applied and tested on frequency intraday stock returns. In (Roy, Kumar, & Sharma, 2014b) a survey on recent literature about Soft Computing, Data-mining and Swarm Intelligence for stock market forecasting is presented. Particularly, fuzzy logic has allowed a significant improvement in financial analysis (Zhou & Dong, 2004; López, Santos, & Montero, 2010; Ijegwa et al., 2014).

There are different approaches to design fuzzy trading decision systems. A first methodology applies fuzzy rules to deal with financial time series. These papers are usually focused on the analysis of some technical indicators. They develop fuzzy rule-based systems for decision support in stock trading and provide a recommendation for selling and buying shares (Govindasamy & Thambidurai, 2013; Naranjo et al., 2015; Chourmouziadis & Chatzoglou, 2016; Lincy & John, 2016; Wan & Si, 2017).

Another research line deals with candlesticks and applies fuzzy logic to the forecasting of the market trend. That is, these papers use crisp Japanese candlesticks but they model the trading process with fuzzy logic, i.e., they apply fuzzy rules to extract knowledge from them (Arévalo et al. 2017). For instance, Kamo and Dagli (2009) present two fuzzy rule-based models, the second one using candlestick patterns to recognize the strength of the market conditions. Both models work with a few simple Japanese candlestick patterns and standard if-then fuzzy rules. Roy, Kumar, and Sharma (2014a) make the prediction of the future market trend using the Hammer candlestick pattern and a fuzzy rule-base with a fuzzy inference mechanism.

A further step consists of working with fuzzy candlestick and standard rule-based decision systems. These fuzzy candlestick patterns may be unknown in advance, and can be obtained from the historical data (Naranjo & Santos, 2016), or are defined by investors, based on traders' previous experience and expert knowledge. The work by Lee, Liu and Chen (2006), where a financial decision support system based on fuzzy candlestick patterns is proposed and developed, has been very inspiring for us. These authors model Japanese candlestick patterns using fuzzy linguistic variables. The fuzzyfication of the candlesticks is similar to ours. This work was further improved in (Dong & Wan, 2009), where a fuzzy decision system was developed based on experience and stock market techniques to determine the buying and selling time. In (Linares, González, & Hernández, 2009) authors generate fuzzy patterns by a linear regression of the average prices of the candlesticks and the market trend. Roy, Sharma, and Kowar (2012), have also investigated the fuzzy candlestick approach, following (Lee, Liu & Chen, 2006). They develop a pattern recognition system for two candlestick patterns, U-Turn and Engulfing, creating models for the white and black candles, and classifying them according to the size of the candle. Because different investors have different interpretations of a candlestick pattern, Lee extended his work to model different parts of a candlestick line with fuzzy linguistic variables to create a fuzzy candlestick pattern (Lee, 2009). The paper by Lan, Zhang, and Xiong (2011), also applies fuzzy logic to Japanese candlesticks by converting the open, close, high and low stock prices into a fuzzy candlestick chart. Using that model, they define and interpret the "symptom sequence" before the reversal point, and then identify the reversal patterns of the candlestick chart. As an example of forecasting with fuzzy candlesticks pattern in a completely different field see (Yu-Chia, 2016).

Finally, our approach not only uses fuzzy logic to model the pattern recognition process but also works with fuzzy candlesticks patterns. The fuzzyfication of the candlesticks patterns follows (Lee, Liu, & Chen, 2006), but it has been extended and modified to take into account information that we consider significant for the final decision. Indeed, we have developed a new and thus completely different fuzzy trading system from the one presented in (Naranjo, Arroyo, & Santos, 2018) where, though using a similar fuzzification of the candle's parameters, different variables were used to adapt three well-known candlestick patterns to fuzzy rules.

Therefore, our work differs from the related papers mentioned above in two main aspects. First, they use a forecasting system to obtain, exclusively, rules which form a binary trading decision system. Our work, following the fuzzyfication method here proposed, establishes a fuzzy forecasting framework to obtain the future market session. On the other hand, these works base their results on the error and deviations in the prediction; however, these results fail to show how the market would behave in a real scenario with an investment strategy. Therefore they do not offer the investor a full picture of the trading system, which is precisely what we want to propose.

## 3. Fuzzy forecasting system based on fuzzy Japanese candles

In stock markets, candlesticks are used to summarize the price evolution of stock or indexes in each session (Nison, 2001). Candlesticks are defined by four parameters: Open (value at the beginning of the trading session or period considered), High (maximum value the stock reaches during the session), Low (minimum value reached during the session), and Close (value at the end of the trading session). The open value can be higher than the close one (and vice versa). Candlesticks represent this fact in two ways. The first one is to fill with colour the candlestick body when the open value is higher than the close one (Figure 1, right (a)) and leave it blank otherwise (Figure 1, left (a)). The second method indicates the open value by a small mark on the left and the close value by a small horizontal line on the right (Figure 1, left (b)) or the opposite (Figure 1, right, (b)).

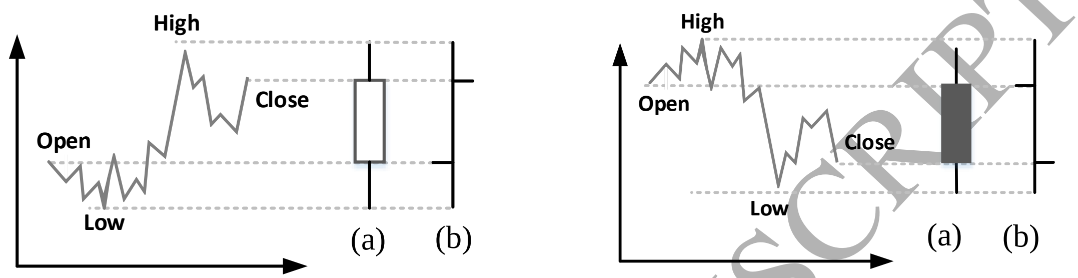


Figure 1. Two representation of a market session by Japanese candlestick theory

In the coloured body representation of the candles, three parts can be distinguished (Figure 2): the body of the candlestick, which represents the variation between the open and close values of the trading session; the upper shadow, which is the price difference between the maximum value (high) and the open or close value (depending on whichever is higher); and the lower shadow, which is given by the price variation between the minimum price (low) and the close or open value (depending on whichever is lower).

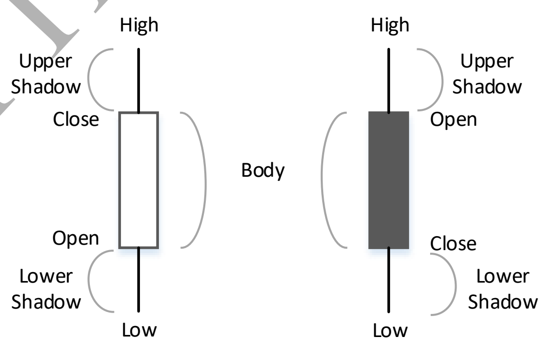


Figure 2. Parameters and parts of the candle

### *3.1 Fuzzy inputs*

As it has just been said, there are three sections in a candlestick (upper shadow, body, and lower shadow). In our case, following (Naranjo & Santos, 2016; Lee, Liu, Chen, 2006), three linguistic variables are considered as inputs of the fuzzy system: Lupper, Llower and Lbody, which represent the length of the shadows (upper and lower) and the body, respectively. The range of the Lbody variable was extended to represent positive and negative values by the body colour (white for positive values and black for negative ones). This extension allows a better representation of a candlestick pattern and therefore it makes easier the pattern recognition and improves the efficiency of its identification.

The formulas used to define these fuzzy inputs are:

$$L_{upper}(t) = 100 \cdot \frac{high(t) - max(open(t), close(t))}{open(t)} \tag{1}$$

$$L_{lower}(t) = 100 \cdot \frac{min(open(t), close(t)) - low(t)}{open(t)} \tag{2}$$

$$L_{body}(t) = 100 \cdot \frac{close(t) - open(t)}{close(t)} \tag{3}$$

In (Lee, Liu, and Chen, 2006), a factor of 100 was used to normalize the values of body and shadows between 0 and 14, to adapt the system to the price fluctuations of the Taiwanese stock market. Nevertheless, in this paper we use a more general approach that can be applied to any stock market. As a consequence, the maximum and minimum values obtained by (1) and (2) are not known in advance. To solve this problem, a correction is applied to the Lupper, Llower and Lbody variables to ensure the obtained values are in the range (0-100). This correction is also used in image processing for stretching the histogram. It is expressed as:

$$g(x) = \frac{f(x) - f(x)_{min}}{f(x)_{max} - f(x)_{min}} \cdot (MAX - MIN) + MIN \tag{4}$$

Where $f(x)_{max}$ and $f(x)_{min}$ are the historical maximum and minimum values of the candlesticks and MAX and MIN are the maximum and minimum values of the interval (in our case, MIN = 0 and MAX = 100). Thus, from now on, variables Lupper, Llower and Lbody include the correction given by (4).

Four trapezoidal membership functions (NULL, SHORT, MIDDLE and LONG) have been assigned to the Lupper and Llower fuzzy variables (Figure 3) for each stock market day.

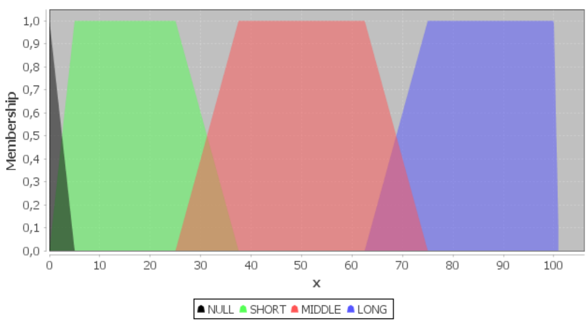


Figure 3. Membership functions of fuzzy input variables Lupper and Llower

For the Lbody fuzzy input variable, seven triangular and trapezoidal membership functions have been used in each trading session: BLACK_LONG, BLACK_MIDDLE, BLACK_SHORT, NULL, WHITE_SHORT, WHITE_MIDDLE, WHITE_LONG (Figure 4).

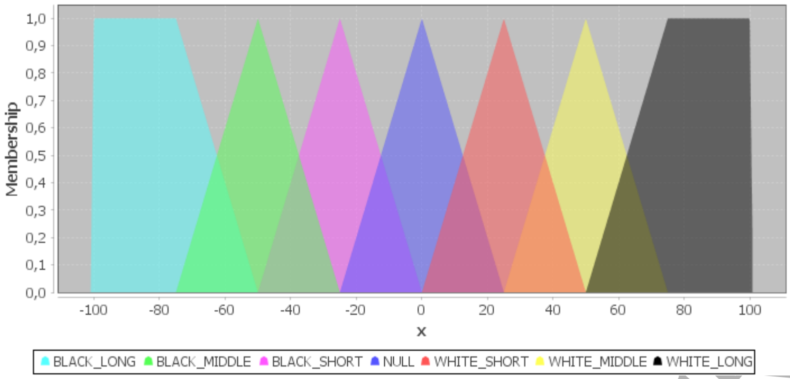


Figure 4. Membership functions of fuzzy input variable Lbody

### *3.2 Fuzzy outputs*

Two variables have been chosen as outputs of the fuzzy system, Rsize and Rpos. These variables refer to the existing relationship between the body size and the total size of the candle (Rsize), and to the position of the body within the candlestick (Rpos). Five triangular membership functions have been assigned to Rpos (DOWN, CENTER_DOWN, CENTER, CENTER_UP, UP) and nine to Rsize (BLACK_MEDIUM_LOW, BLACK_MEDIUM_EQUAL, BLACK_MEDIUM, BLACK_LOW, EQUAL, WHITE_LOW, WHITE_MEDIUM, WHITE_MEDIUM_EQUAL, WHITE_MEDIUM_LOW) (Figure 5 and 6, respectively).

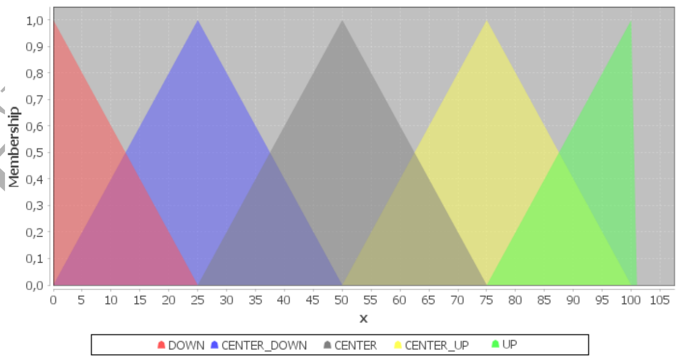


Figure 5. Membership functions of fuzzy output Rpos

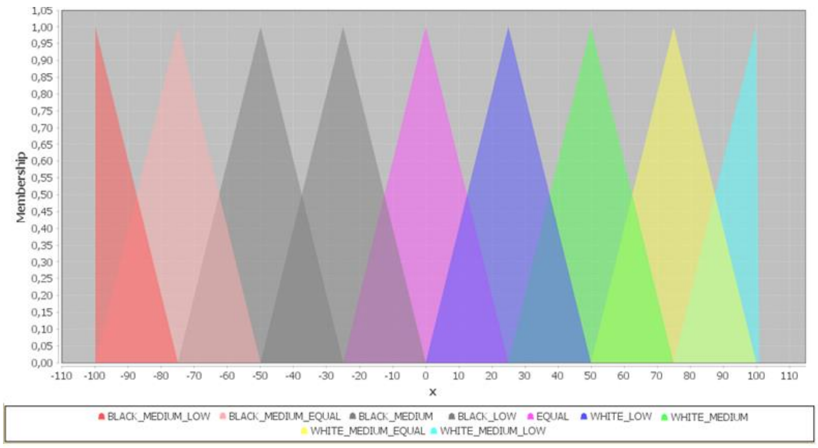


Figure 6. Membership functions of fuzzy output Rsize

### *3.3 Fuzzy rules*

In this Mamdani-type fuzzy inference system, a set of IF-THEN rules have been defined. The product operator has been used to implement the implication and the centroid defuzzyfication method gives the inferred output.

A set of 224 fuzzy rules was obtained from the combination of the above inputs and outputs. It has been generated relating the candle body size to the whole candle itself (Rsize) and its position (Rpos), that is, whether it is in the middle, or in the upper part, etc. Then, we have compared it with the historical set to choose the most similar one. In this sense, there is not expert knowledge regarding the behaviour of the stock market. The rules are based on the representation of the knowledge, i.e., the geometric figures of the candles. Table 1 shows some examples of these rules.

Table 1. Fuzzy rules

| **Lupper** | **Llower** | **Lbody** | **Rsize** | **Rpos** |
|---|---|---|---|---|
| LONG | LONG | WHITE_LONG | WHITE_MEDIUM_EQUAL | CENTER |
| LONG | LONG | WHITE_MIDDLE | WHITE_MEDIUM | CENTER |
| NULL | SHORT | WHITE_LONG | EQUAL | CENTER_UP |
| NULL | MIDDLE | BLACK_MIDDLE | BLACK_MEDIUM_EQUAL | UP |
| SHORT | NULL | LONG | EQUAL | CENTER_DOWN |
| MIDDLE | NULL | BLACK_SHORT | BLACK_MEDIUM_LOW | DOWN |
| … | … | … | … | … |

### *3.4 Fuzzy forecasting system*

Once the results of the two fuzzy output variables ($R_{size}$ and $R_{pos}$) are obtained, they are used as inputs of the prediction system. To implement this forecasting system a nonparametric and lazy classification method, the *k-Nearest Neighbours* (k-NN), has been chosen. The forecasting system basically consists of comparing the candle of the previous available session, characterized by the output variables Rsize and Rpos, with a group of *n* candles, made up with the previous known candlestick and the *n-1* previous ones. The most similar candlestick to the current one is then retrieved. The classification system finds the nearest neighbour, i.e., a 1-NN system has been implemented (*k* = 1).

To determine the similarity between neighbours, the Euclidean distance is applied:

$$\mathrm{D_k}(\{\mathrm{C_{1\ldots n}}\},\{\mathrm{C_{1k\ldots nk}}\}) = \sqrt{(\mathrm{R_{pos1}} - \mathrm{R_{pos1k}})^2 + (\mathrm{R_{size1}} - \mathrm{R_{size1k}})^2 + \ .\ .\ .\ + (\mathrm{R_{posn}} - \mathrm{R_{posnk}})^2 + (\mathrm{R_{sizen}} - \mathrm{R_{sizenk}})^2} \quad (5)$$

Where {$C_1$, $C_2$, …,$C_n$} is the candlestick set used as pattern and the candlesticks {$C_{1k}$, $C_{2k}$,.., $C_{nk}$} are the neighbours to compare with. This operation is repeated *m* times, where *m* is determined by:

$$m = \text{nº previous candles} - n + 1 \quad (6)$$

This way, the Euclidean distances of the Rpos and Rsize values, candle by candle, are computed between the group considered as pattern and the one that is compared with. The first group (pattern) contains as the last candle the one of the current session, starting from session *k* to current session *n*. The second group is the historical set of candles, forming groups of the same number of candles, starting from the first historical session (session 1).

The previous candlestick market sessions are saved in the historical database. The neighbour with the smallest distance (lowest error) will be chosen as the closest one. Therefore the next candlestick, that is, the one of the next session, is the most similar one.

Figure 7 shows the process described above. First, a group of *n* candles is formed (in the example, *n* = 3). It is compared to all the groups of *n* candles in the historical database (*m* comparisons), using the distance given by (5). Then the nearest neighbour (1-NN), i.e., the one with the shortest distance, is chosen (detected pattern). The candle corresponding to the next session of the detected pattern is selected. Finally, in our system the real value of the currency is updated according to the corresponding country inflation and it is included to obtain the prediction.

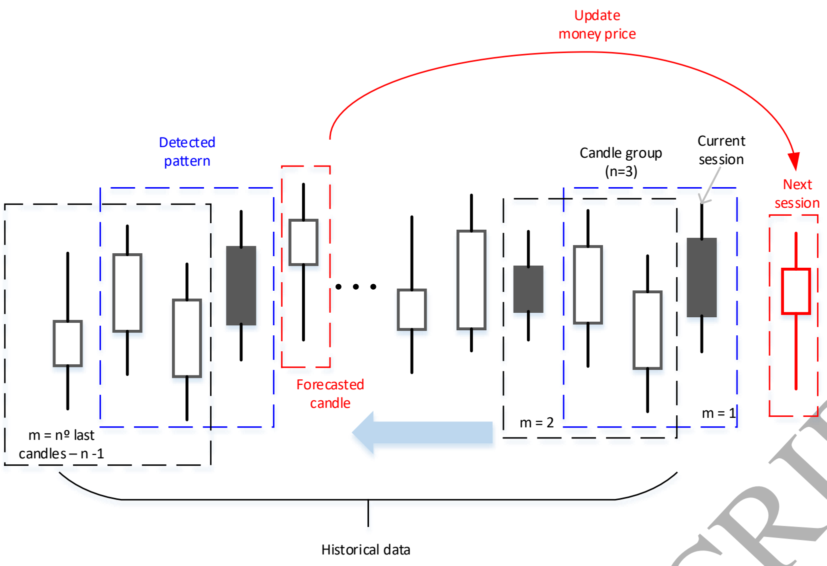


Figure 7. Forecasting using k-NN (1-NN)

Due to the fact that the comparison of fuzzy candles is made by percentages, the currency devaluation over time is not considered. However, when a candle pattern is detected, the next candle is chosen as the best prediction of the future session. To use this prediction at the current time, it would be necessary to update the currency value. That is why this system has been updated using the country's inflation rate, depending on the market on which it is being calculated. It could be then expected that the errors in the prediction will be smaller than without considering the price variation.

## 4. Fuzzy investment strategy

Although most of the studies only implement a forecasting system, this work proposes a complete trading system, including an investment strategy. This will allow us to validate our system in a real scenario, as an expert trader would do.

We have implemented a simple all-or-nothing type investment strategy but limiting the risk.

### *4.1 Entry market conditions and day trading capital*

There is a variety of ways in which a company can enter a stock market. None market entry strategy works for all markets. The trading entry strategy is the most important part of the trade. This is the one time when all of the trading capital is at risk. Two main entries have been considered in this paper, long and short. The terms long and short refer to whether a trade is entered by buying or selling. A long trade is initiated by buying, with the expectation of selling at a higher price in the future and get profits. A short trade is initiated by selling first (before buying), with the expectation of buying the stock back at a lower price and get profits.

In our case, the information provided by our forecasting system is the opening, closing, maximum and minimum values of each session. Thus, *a priori*, either way of entering the market would be efficient, since the predicted market trend is not taken into account. That is why a long entry (bullish) has been chosen.

In order to enter the market, it is also necessary to specify the price to buy the shares and how much capital is going to be invested, or in other words, how many stocks one is planning to buy.

The buy price has been defined as the minimum value (lowest) that results from the prediction minus a safety margin proportional to the average error obtained during the training period (7). The number of shares to buy is calculated by (8):

$$\text{entry condition} = low_{pred} \cdot (1 - 0.5 * error_{low}) \tag{7}$$

$$\text{number of shares} = \left\lfloor \frac{capital}{stoploss} \right\rfloor \tag{8}$$

Where *capital* is all the available capital to day trade at the time of entering the market and *stop-loss* (S/L) is the minimum value of the share price that one is willing to assume as losses.

Other strategies use the *stop-loss* order only to exit a trade. In our case it is also used to calculate the number of shares with which to enter the market. This results in a more efficient risk management strategy, limiting possible losses when the market moves against you.

### *4.2 Exit strategies (stop-loss and take-profit)*

When getting out of a trade, depending on the trading decisions taken, you may lose or make a gain, or even exit the market with zero profit or loss. When talking about exit strategies, the terms take-profit and stop-loss orders refer to these two types of exit, the most common ones. Sometimes these terms are abbreviated as "T/P" and "S/L" by traders. The third case, in which there would be no losses or gains, happens when the share price during the session does not exceed the take-profit threshold (to obtain profits), neither it falls below the stop-loss (to exit with losses). The session closes at the same price it entered the market, because it enters and exits the market in the same session.

A stop-loss is an order placed with a broker to sell a security when it reaches a predetermined price or percentage. That is, when the stock price falls below the pre-set *stop-loss*, the exit is forced in order to limit the losses. *Take-profit*, or limit orders are similar to stop-loss in the sense that they are converted into market orders to sell when the point is reached. The exit point must be set above the current market price, instead of below. That means that the profits are already enough because the share price went up above the pre-set price. Both variables are defined in (9) and (10), respectively:

$$stop - loss = low_{pred} \cdot (1 - 1.5 * error_{low}) \quad (9)$$

$$take - profit = high_{pred} \quad (10)$$

Where $low_{pred}$ y $high_{pred}$ means the minimum and maximum values, respectively, of the forecasted session and $error_{low}$ is the average error during the training period.

With these four variables: entry price, number of shares, stop-loss and take-profit, the behaviour of our forecasting system in the market is completely specified. However, since candles provide four different variables for each session, it is unknown *a priori* which of the two values (stop-loss or take-profit) has been reached first. That is, it is not known which exit strategy has been applied, neither if there were losses (first case) or profits.

Thus, the worst case will be considered. The stop-loss value is checked, considering losses, regardless the maximum stock value. Another possible situation would be that neither of the two values is reached by the real price of the shares, in which case all the shares are sold at the session closing price. The profits or losses will depend on that close value. Figure 8 shows a flow diagram of the trading strategy.

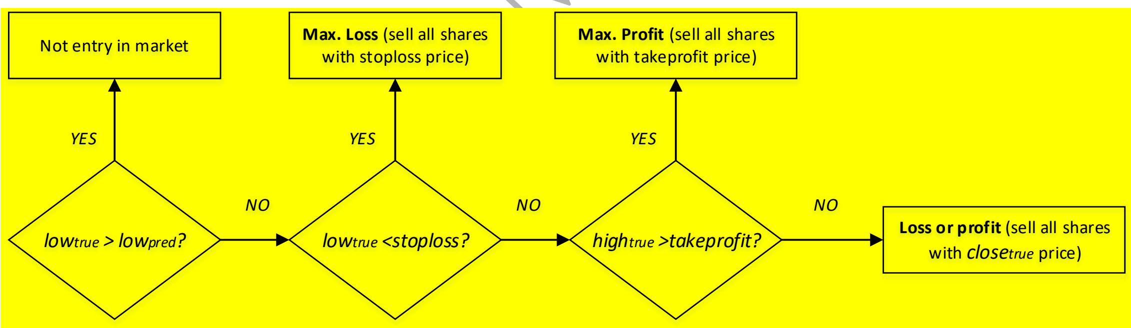


Figure 8. Trading strategy

## 5. Fuzzy trading system validation and results discussion

The proposed trading decision system has been tested on two well differentiated markets, the American Nasdaq-100 (Section 5.1) and the Spanish Ibex35 (Section 5.2). For each of these markets, firstly the stock market is described. Then, the predicted values obtained during the training by this new fuzzy trading system and the one reported in (Naranjo & Santos, 2016), where a fuzzy trading system without capital management was described, are presented. Finally, to make a fair comparison, the same investment strategy (all-or-nothing with risk control) is used in both fuzzy systems, and they are compared with a third one, the naïve Buy and Hold (B&H) strategy, which is used as benchmark. The latter consists of investing all the capital at the beginning of the trading period (buying) and then selling all the shares at the closing price of the previous session. The results are discussed for each particular market. In all the cases, the fuzzy trading systems are proved to be efficient.

The training and validation periods were chosen in order to compare this new proposal with a previous ones, but the validation interval has been extended. We used official data sources for the data and the inflation rate of each country. In particular, the source used to obtain the CPI (Consumer Price Index) of the Spanish Ibex market was the INE[1] (National Institute of Statistics) and the CPI of the Nasdaq market is given by the Bureau of Labor Statistics[2]. Yahoo Finance[3] has been used for share price data.

The period used to measure the inflation rate is the period between the candle (or group of candles) detected as a predictor and the future session to be predicted. That is, if we want to predict the session of March 14, 2013 and our predictor pattern is November 11, 2011, the inflation period is the time between these two dates.

In this work the dividends have not been taken into account because we are analysing the capital gains, that is, the profit. The shares are bought at a specific price and sold at another one.

### *5.1 American Nasdaq-100 market*

The Nasdaq-100 stock market has been chosen to apply the fuzzy trading system. The same 15 companies used in (Naranjo & Santos, 2016) were selected. A natural year is chosen as input (2011), and the next one for the training (2012). During this training the optimal number of candles, $n$, is obtained. The pattern made up of these $n$ candles will be used by the *k-NN* algorithm to minimize the mean error of the prediction. The validation period was only until 2013 in (Naranjo & Santos, 2016) and it has now been extended up to the 30th of April 2015, that is, adding 2014 and 2015.

Table 2. Training and validation period for Nasdaq market

| **Training period** | | | | **Validation period** | | | |
|---|---|---|---|---|---|---|---|
| **Input data** | | **Training** | | **Input data** | | **Validation** | |
| **Start** | **End** | **Start** | **End** | **Start** | **End** | **Start** | **End** |
| 1-Jan-11 | 31-Dec-11 | 1-Jan-12 | 31-Dec-12 | 1-Jan-11 | 31-Dec-12 | 01-Jan-13 | 30-Apr-15 |

[1] https://www.ine.es/en/welcome.shtml
[2] https://www.bls.gov/
[3] https://es.finance.yahoo.com/

Simulations were run with different numbers of neighbour candlesticks, with *n* varying from 1 to 10. To measure the performance of the forecasting system, the error percentage between the predicted and the current candlestick parameters is calculated. That is, the error of the open, high, low and close values is obtained by:

$$\%error = \frac{|P_{TC} - P_{PC}|}{P_{TC}} \cdot 100 \qquad (11)$$

Where $P_{TC}$ and $P_{PC}$ represent the value of the different parameters (open, high, low, close) of the present and the predicted candlesticks, respectively. These error percentages have been calculated using two types of candlesticks. The first one (indicated by subscript 1) refers to those obtained directly as a result of the application of the forecasting system. The second type of candlesticks (indicated by subscript 2) are those which have been obtained matching the open value of the estimated and the current candlestick and, consequently, the candlestick (result of the prediction) has been shifted that difference. That is, prediction candlestick values of the second type are given by:

$$\mathrm{open}_{PC2} = \mathrm{open}_{PT} \qquad (12)$$

$$\mathrm{high}_{PC2} = \mathrm{high}_{PC1} + (\mathrm{open}_{PT} - \mathrm{open}_{PC2}) \qquad (13)$$

$$\mathrm{low}_{PC2} = \mathrm{low}_{PC1} + (\mathrm{open}_{PT} - \mathrm{open}_{PC2}) \qquad (14)$$

$$\mathrm{close}_{PC2} = \mathrm{close}_{PC1} + (\mathrm{open}_{PT} - \mathrm{open}_{PC2}) \qquad (15)$$

Average error percentage, *x*, and standard deviation with the Bessel correction, *s*, have been calculated, where *nc* is the total number of candlesticks for the time interval under consideration.

$$x = \frac{1}{nc} \cdot \sum_1^{nc} \%error_i \qquad s = \sqrt{\frac{1}{nc-1} \cdot \sum_1^{nc} (\%error_i - x)^2} \qquad (16)$$

Table 3 shows the maximum and minimum values of the average error, *x*, and standard deviation, *s*, obtained for the 15 Nasdaq-100 companies (first column: Apple, APPL; Adobe, ADBE; Analog Devices, ADI; and so on), for the best and worst value of *n*. The expression into brackets represents the corresponding candle parameter, i.e., x(o1) means error(open1). That is, o1, h1, l1, and c1 refer to the open, high, low and close percentage value (respectively) of candlestick type 1, and o2, h2, l2, c2 relate to the type 2 candlestick.

The minimum errors (bolded) are obtained for the ADP (Automatic Data Processing) company for all the variables.

Table 3. Minimum and maximum errors with the new fuzzy proposed system (Nasdaq)

| Company | | x(o1) | s(o1) | x(h1) | s(h1) | x(l1) | s(l1) | x(c1) | s(c1) | x(h2) | s(h2) | x(l2) | s(l2) | x(c2) | s(c2) |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| **AAPL** | **MIN** | 27.40 | 13.53 | 27.29 | 13.59 | 27.26 | 13.58 | 27.20 | 13.66 | 0.72 | 0.73 | 0.92 | 0.90 | 1.34 | 1.13 |
| | **MAX** | 29.47 | 14.77 | 29.40 | 14.89 | 29.32 | 14.63 | 29.30 | 14.77 | 0.77 | 0.86 | 1.03 | 1.02 | 1.45 | 1.37 |
| **ADBE** | **MIN** | 7.36 | 6.32 | 7.35 | 6.30 | 7.63 | 6.45 | 7.60 | 6.43 | 0.82 | 0.69 | 0.82 | 0.65 | 1.35 | 1.02 |
| | **MAX** | 8.37 | 7.35 | 8.27 | 7.22 | 8.50 | 7.50 | 8.45 | 7.38 | 0.94 | 0.79 | 0.95 | 0.82 | 1.52 | 1.16 |
| **ADI** | **MIN** | 5.48 | 4.50 | 5.36 | 4.33 | 5.60 | 4.56 | 5.43 | 4.49 | 0.84 | 0.70 | 0.75 | 0.62 | 1.21 | 0.97 |
| | **MAX** | 6.27 | 5.04 | 6.24 | 4.84 | 6.56 | 5.08 | 6.42 | 5.02 | 0.95 | 0.81 | 0.87 | 0.74 | 1.44 | 1.15 |
| **ADP** | **MIN** | **4.86** | **3.68** | **4.83** | **3.67** | **4.99** | **3.78** | **4.93** | **3.75** | **0.47** | **0.38** | **0.47** | **0.43** | **0.79** | **0.59** |
| | **MAX** | 7.10 | 4.83 | 7.00 | 4.71 | 7.27 | 4.87 | 7.12 | 4.69 | 0.51 | 0.51 | 0.56 | 0.57 | 0.87 | 0.79 |
| **ADSK** | **MIN** | 14.65 | 10.35 | 14.58 | 10.16 | 14.68 | 10.60 | 14.57 | 10.36 | 1.24 | 1.12 | 1.30 | 1.32 | 2.01 | 1.64 |
| | **MAX** | 16.71 | 11.99 | 16.40 | 11.88 | 16.75 | 12.13 | 16.43 | 12.01 | 1.48 | 1.42 | 1.45 | 1.64 | 2.25 | 2.04 |
| **AKAM** | **MIN** | 12.45 | 11.40 | 12.34 | 11.46 | 12.41 | 11.64 | 12.35 | 11.60 | 1.05 | 0.92 | 1.13 | 0.99 | 1.69 | 1.37 |
| | **MAX** | 17.68 | 14.96 | 17.51 | 14.97 | 17.79 | 15.10 | 17.56 | 15.00 | 1.20 | 1.23 | 1.31 | 1.27 | 1.98 | 1.72 |
| **ALTR** | **MIN** | 14.78 | 10.90 | 14.95 | 10.91 | 14.76 | 10.97 | 14.88 | 11.19 | 1.20 | 1.12 | 1.26 | 1.04 | 1.88 | 1.57 |
| | **MAX** | 17.79 | 14.20 | 17.85 | 14.19 | 17.81 | 14.05 | 17.97 | 14.09 | 1.38 | 1.38 | 1.40 | 1.20 | 2.16 | 1.70 |
| **ALXN** | **MIN** | 16.89 | 14.26 | 16.91 | 14.15 | 17.04 | 14.34 | 17.03 | 14.19 | 1.00 | 0.92 | 1.16 | 1.14 | 1.67 | 1.38 |
| | **MAX** | 21.02 | 16.21 | 20.83 | 16.18 | 21.18 | 16.17 | 20.90 | 16.15 | 1.20 | 1.11 | 1.26 | 1.27 | 1.86 | 1.55 |
| **AMAT** | **MIN** | 13.05 | 11.87 | 12.92 | 11.71 | 12.92 | 11.68 | 12.96 | 11.76 | 0.90 | 0.72 | 1.01 | 0.93 | 1.49 | 1.18 |
| | **MAX** | 16.29 | 13.99 | 16.26 | 14.06 | 16.14 | 13.92 | 16.23 | 14.11 | 1.11 | 1.09 | 1.20 | 1.32 | 1.88 | 1.67 |
| **AMGN** | **MIN** | 19.81 | 10.19 | 19.78 | 10.20 | 19.88 | 10.20 | 19.84 | 10.18 | 0.62 | 0.54 | 0.54 | 0.52 | 0.96 | 0.71 |
| | **MAX** | 22.11 | 10.75 | 22.10 | 10.70 | 22.17 | 10.75 | 22.20 | 10.69 | 0.70 | 0.59 | 0.61 | 0.59 | 1.08 | 0.83 |
| **AMZN** | **MIN** | 11.84 | 7.85 | 11.73 | 7.72 | 11.81 | 7.99 | 11.65 | 7.85 | 0.94 | 0.78 | 0.92 | 0.84 | 1.54 | 1.28 |
| | **MAX** | 13.10 | 9.39 | 13.03 | 9.18 | 13.13 | 9.28 | 13.04 | 9.16 | 1.08 | 1.04 | 1.06 | 0.96 | 1.78 | 1.48 |
| **ATVI** | **MIN** | 5.62 | 3.88 | 5.46 | 3.83 | 5.50 | 3.87 | 5.41 | 3.94 | 0.88 | 0.69 | 0.93 | 0.80 | 1.36 | 1.05 |
| | **MAX** | 6.12 | 4.46 | 6.04 | 4.49 | 6.07 | 4.60 | 6.11 | 4.66 | 0.98 | 0.84 | 1.08 | 0.92 | 1.55 | 1.22 |
| **AVGO** | **MIN** | 7.88 | 5.82 | 7.81 | 5.62 | 8.08 | 5.97 | 7.88 | 5.76 | 1.06 | 1.02 | 1.28 | 1.17 | 1.87 | 1.61 |
| | **MAX** | 9.50 | 6.77 | 9.33 | 6.55 | 9.75 | 6.82 | 9.60 | 6.67 | 1.34 | 1.25 | 1.42 | 1.41 | 2.19 | 1.88 |
| **BBBY** | **MIN** | 12.12 | 8.06 | 12.06 | 8.03 | 12.08 | 8.11 | 12.03 | 8.02 | 0.86 | 0.82 | 0.85 | 0.87 | 1.35 | 1.15 |
| | **MAX** | 14.11 | 9.04 | 14.08 | 9.11 | 14.16 | 9.05 | 14.18 | 9.09 | 0.94 | 0.95 | 0.93 | 0.94 | 1.49 | 1.27 |
| **BIDU** | **MIN** | 15.13 | 11.78 | 15.21 | 11.75 | 14.98 | 11.95 | 15.05 | 11.86 | 1.24 | 1.04 | 1.43 | 1.39 | 2.11 | 1.57 |
| | **MAX** | 16.51 | 14.07 | 16.37 | 14.21 | 16.43 | 14.22 | 16.50 | 14.39 | 1.54 | 1.54 | 1.83 | 1.84 | 2.48 | 1.96 |

Table 4 shows the same performance measures with the system proposed in (Naranjo & Santos, 2016). In this case, the minimum values are obtained for the ADI company with fuzzy candles type 1 and for ADP company with fuzzy candles type 2.

Table 4. Minimum and maximum errors with (Naranjo & Santos, 2016) (Nasdaq)

| Company | | x(o1) | s(o1) | x(h1) | s(h1) | x(l1) | s(l1) | x(c1) | s(c1) | x(h2) | s(h2) | x(l2) | s(l2) | x(c2) | s(c2) |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| **AAPL** | **MIN** | 28.53 | 12.49 | 28.41 | 12.59 | 28.46 | 12.43 | 28.34 | 12.51 | 0.74 | 0.76 | 0.88 | 0.86 | 1.34 | 1.10 |
| | **MAX** | 31.93 | 14.3 | 31.98 | 14.42 | 31.98 | 14.24 | 32.05 | 14.36 | 0.83 | 0.93 | 1.00 | 0.96 | 1.48 | 1.39 |
| **ADBE** | **MIN** | 6.42 | 5.07 | 6.37 | 5.19 | 6.49 | 5.20 | 6.46 | 5.22 | 0.81 | 0.67 | 0.84 | 0.68 | 1.35 | 1.00 |
| | **MAX** | 8.19 | 7.69 | 8.27 | 7.67 | 8.35 | 7.91 | 8.34 | 7.81 | 1.08 | 0.81 | 1.01 | 0.86 | 1.77 | 1.31 |
| **ADI** | **MIN** | **4.55** | **3.41** | **4.51** | **3.41** | **4.68** | **3.57** | **4.63** | **3.57** | 0.8 | 0.69 | 0.75 | 0.66 | 1.21 | 0.94 |
| | **MAX** | 5.64 | 4.77 | 5.46 | 4.65 | 5.72 | 4.94 | 5.56 | 4.82 | 0.89 | 0.74 | 0.87 | 0.73 | 1.39 | 1.06 |
| **ADP** | **MIN** | 6.21 | 3.99 | 6.08 | 3.91 | 6.31 | 4.02 | 6.17 | 3.92 | **0.49** | **0.39** | **0.46** | **0.43** | **0.82** | **0.64** |
| | **MAX** | 7.98 | 5.87 | 7.74 | 5.65 | 7.99 | 5.80 | 7.77 | 5.57 | 0.57 | 0.51 | 0.54 | 0.54 | 0.88 | 0.73 |
| **ADSK** | **MIN** | 14.96 | 11.01 | 14.73 | 10.77 | 15.09 | 11.06 | 14.85 | 10.9 | 1.21 | 1.14 | 1.31 | 1.20 | 1.94 | 1.62 |
| | **MAX** | 16.72 | 11.79 | 16.52 | 11.56 | 16.64 | 12.10 | 16.54 | 11.83 | 1.39 | 1.25 | 1.49 | 1.70 | 2.25 | 1.92 |
| **AKAM** | **MIN** | 14.75 | 11.8 | 14.60 | 11.55 | 14.83 | 11.92 | 14.65 | 11.69 | 1.07 | 0.92 | 1.19 | 1.06 | 1.80 | 1.43 |
| | **MAX** | 23.18 | 16.83 | 24.03 | 17.74 | 23.59 | 17.09 | 24.36 | 17.91 | 2.08 | 2.06 | 1.4 | 1.43 | 2.57 | 2.21 |
| **ALTR** | **MIN** | 13.77 | 11.63 | 13.86 | 11.68 | 13.77 | 11.58 | 13.96 | 11.74 | 1.11 | 1.04 | 1.17 | 1.02 | 1.96 | 1.50 |
| | **MAX** | 20.29 | 14.43 | 20.13 | 14.32 | 20.18 | 14.27 | 20.1 | 14.17 | 1.34 | 1.30 | 1.36 | 1.17 | 2.09 | 1.74 |
| **ALXN** | **MIN** | 16.88 | 13.57 | 16.64 | 13.45 | 16.91 | 13.61 | 16.68 | 13.44 | 1.00 | 0.89 | 1.09 | 1.09 | 1.65 | 1.40 |
| | **MAX** | 21.83 | 15.95 | 21.61 | 15.87 | 21.85 | 16.02 | 21.54 | 16.02 | 1.20 | 1.15 | 1.43 | 1.27 | 1.86 | 1.62 |
| **AMAT** | **MIN** | 14.91 | 12.48 | 14.91 | 12.5 | 14.82 | 12.40 | 14.95 | 12.51 | 0.98 | 0.86 | 0.94 | 0.85 | 1.60 | 1.27 |
| | **MAX** | 18.00 | 13.98 | 17.9 | 14.1 | 18.16 | 14.05 | 17.97 | 14.1 | 1.10 | 1.06 | 1.17 | 1.25 | 1.82 | 1.55 |
| **AMGN** | **MIN** | 20.04 | 9.94 | 20.03 | 9.86 | 20.10 | 9.87 | 20.11 | 9.85 | 0.65 | 0.56 | 0.57 | 0.57 | 0.99 | 0.82 |
| | **MAX** | 21.30 | 11.29 | 21.24 | 11.3 | 21.34 | 11.34 | 21.35 | 11.38 | 0.69 | 0.62 | 0.63 | 0.65 | 1.11 | 0.88 |
| **AMZN** | **MIN** | 12.65 | 8.58 | 12.55 | 8.56 | 12.73 | 8.53 | 12.55 | 8.57 | 0.90 | 0.84 | 0.85 | 0.73 | 1.54 | 1.23 |
| | **MAX** | 14.25 | 9.62 | 14.20 | 9.43 | 14.37 | 9.64 | 14.24 | 9.48 | 1.06 | 0.99 | 1.04 | 0.89 | 1.72 | 1.45 |
| **ATVI** | **MIN** | 5.27 | 3.95 | 5.34 | 3.85 | 5.29 | 3.96 | 5.38 | 3.78 | 0.82 | 0.69 | 0.95 | 0.81 | 1.39 | 1.08 |
| | **MAX** | 5.69 | 4.37 | 5.63 | 4.37 | 5.75 | 4.47 | 5.73 | 4.48 | 0.93 | 0.88 | 1.08 | 0.95 | 1.53 | 1.25 |
| **AVGO** | **MIN** | 7.92 | 6.00 | 7.76 | 5.97 | 8.15 | 6.21 | 7.89 | 6.09 | 1.08 | 0.96 | 1.17 | 1.04 | 1.90 | 1.49 |
| | **MAX** | 11.54 | 7.38 | 11.56 | 7.57 | 11.94 | 7.59 | 11.53 | 7.38 | 1.31 | 1.21 | 1.44 | 1.35 | 2.09 | 1.71 |
| **BBBY** | **MIN** | 12.58 | 8.18 | 12.57 | 8.19 | 12.66 | 8.20 | 12.65 | 8.13 | 0.82 | 0.77 | 0.85 | 0.78 | 1.31 | 1.10 |
| | **MAX** | 16.08 | 9.45 | 16.11 | 9.49 | 16.11 | 9.45 | 16.14 | 9.46 | 0.92 | 0.87 | 0.99 | 0.96 | 1.50 | 1.24 |
| **BIDU** | **MIN** | 14.36 | 11.72 | 14.36 | 11.68 | 14.19 | 11.64 | 14.34 | 11.6 | 1.30 | 1.12 | 1.65 | 1.57 | 2.21 | 1.79 |
| | **MAX** | 15.98 | 14.16 | 16.04 | 14.2 | 15.89 | 14.18 | 16.01 | 14.21 | 1.49 | 1.37 | 1.87 | 1.94 | 2.50 | 2.00 |

As it can be seen if we compare Table 3 and Table 4, the new fuzzy trading system improves the results in 243 cases out of the 420 possible ones, representing 57.86%. Indeed, 8 times the error in the opening value *x(o1)* is smaller and the same happens with the standard deviation *s(o1)* in 19 cases in comparison to the previous fuzzy system. But despite this improvement provided by the new proposal, the errors are still large.

It is possible to reduce the errors using candles type 2, where the off-set has been eliminated. In this case, regarding the error and deviation, the results improve in 98 out of 180 cases (54.44%). As a matter of fact, considering only the mean errors the system gets better results in 52 out of 90 possible cases (57.77%).

Therefore, in order to validate both systems the same investment strategy during the same training/validation period, same portfolio of securities, capital, and all other factors that could influence it, are considered. This way it is possible to determine the errors that are only due to the forecasting system. In this case the value of *n* that minimizes the errors in experiment 2 is selected by,

$$n_{opt} = Min(x(h2) + x(l2)) \tag{17}$$

The results now obtained with the new fuzzy decision system are shown in Table 5, whereas Table 6 shows the results of the forecasting system proposed in (Naranjo & Santos, 2016).

Table 5. Validation results with the new proposed fuzzy decision system (Nasdaq)

| Co. | NetProfit | MaxDD | Total trades | Trades + | Trades - | Average trade | Avg profit trade | Avg loss trade | Avg profit/ avg loss |
|---|---|---|---|---|---|---|---|---|---|
| AAPL | 602.58 | -6.49 | 63 | 31 | 32 | 0.02 | 0.26 | -0.20 | 1.28 |
| ADBE | 3830.63 | **-3.84** | 239 | 126 | 113 | 0.28 | 1.22 | -0.77 | 1.60 |
| ADI | 2802.08 | -6.51 | 214 | 115 | 99 | 0.26 | 1.22 | -0.85 | 1.42 |
| ADP | 589.03 | -6.00 | 221 | 99 | 122 | 0.04 | 0.53 | -0.36 | 1.46 |
| ADSK | 3058.88 | -5.30 | 172 | 94 | 78 | 0.42 | 1.84 | -1.29 | 1.43 |
| AKAM | 903.46 | -7.61 | 184 | 86 | 98 | 0.09 | 1.22 | -0.91 | 1.35 |
| ALTR | 1277.65 | -7.20 | 156 | 80 | 76 | 0.24 | 1.79 | -1.39 | 1.29 |
| ALXN | **5139.92** | -15.07 | 263 | 139 | 124 | 0.13 | 0.80 | -0.61 | 1.30 |
| AMAT | 3163.69 | -6.22 | 231 | 121 | 110 | 0.74 | 3.92 | -2.76 | 1.42 |
| AMGN | 1915.22 | -16.52 | 278 | 130 | 148 | 0.06 | 0.58 | -0.40 | 1.46 |
| AMZN | 3167.43 | -6.67 | 219 | 114 | 105 | 0.05 | 0.25 | -0.17 | 1.45 |
| ATVI | 1087.06 | -10.46 | 229 | 104 | 125 | 0.31 | 4.25 | -2.97 | 1.43 |
| AVGO | -229.57 | **-17.90** | 262 | 97 | 165 | 0.00 | 1.21 | -0.71 | 1.71 |
| BBBY | 2262.88 | -5.52 | 182 | 98 | 84 | 0.19 | 0.83 | -0.57 | 1.47 |
| BIDU | 5104.52 | -12.49 | 229 | 120 | 109 | 0.12 | 0.70 | -0.52 | 1.37 |
| Total | **34675.45** | **-0.83** | 3142 | 1554 | 1588 | | | | |

From Tables 5 and 6 the following conclusions can be drawn. The new proposed fuzzy forecasting system achieves greater profits (*Net profit*) for this American market than the proposed in (Naranjo & Santos, 2016), namely 38.53% versus 32.81%. Regarding the success rate, both systems have obtained similar results, 49.46% vs 50.08%. However, the new proposal presents higher market entries due to a better accuracy in the prediction, resulting in an increase in profits.

Table 6. Validation results with the fuzzy system (Naranjo & Santos, 2016) (Nasdaq)

| Co | NetProfit | MaxDD | Trades total | Trades + | Trades - | Average trade | Avg profit trade | Avg loss trade | Avg profit/ avg loss |
|---|---|---|---|---|---|---|---|---|---|
| AAPL | 558.31 | -6.92 | 151 | 69 | 82 | 0.00 | 0.18 | -0.15 | 1.18 |
| ADBE | 4438.65 | **-6.49** | 236 | 128 | 108 | 0.32 | 1.27 | -0.81 | 1.56 |
| ADI | 1963.14 | -9.02 | 196 | 101 | 95 | 0.21 | 1.20 | -0.85 | 1.41 |
| ADP | 1557.08 | -6.96 | 212 | 101 | 111 | 0.10 | 0.63 | -0.39 | 1.65 |
| ADSK | **4505.65** | -8.30 | 192 | 102 | 90 | 0.53 | 2.14 | -1.31 | 1.64 |
| AKAM | 523.30 | **-17.50** | 183 | 86 | 97 | 0.06 | 1.19 | -0.94 | 1.26 |
| ALTR | 747.43 | -6.81 | 135 | 69 | 66 | 0.17 | 1.74 | -1.47 | 1.18 |
| ALXN | 5306.02 | -15.63 | 267 | 143 | 124 | 0.12 | 0.71 | -0.56 | 1.25 |
| AMAT | 1481.89 | -11.37 | 221 | 110 | 111 | 0.34 | 3.23 | -2.53 | 1.27 |
| AMGN | 1899.03 | -14.34 | 266 | 131 | 135 | 0.06 | 0.53 | -0.40 | 1.33 |
| AMZN | 751.60 | -9.13 | 211 | 98 | 113 | 0.01 | 0.19 | -0.15 | 1.28 |
| ATVI | 894.30 | -15.92 | 228 | 109 | 119 | 0.30 | 4.10 | -3.19 | 1.29 |
| AVGO | 1660.68 | -8.85 | 224 | 109 | 115 | 0.16 | 1.47 | -1.10 | 1.35 |
| BBBY | 1156.91 | -9.60 | 108 | 60 | 48 | 0.14 | 0.74 | -0.61 | 1.23 |
| BIDU | 2086.02 | -14.47 | 215 | 109 | 106 | 0.06 | 0.60 | -0.48 | 1.23 |
| **Total** | **29530.01** | **-0.77** | 3045 | 1525 | 1520 | | | | |

The Drawdown (DD) is a risk indicator of a portfolio. It represents the reduction of capital after a series of losing trades. This is normally calculated by the difference between a relative peak in capital minus a relative trough. The drawdown, also known as the regression of the results curve, is a way of measuring the risk of our trading system. Maximum Drawdown (Max DD) is the maximum of the calculated drawdowns. Traders normally note this indicator as a percentage of their trading account. The best and worse DD values have been boldfaced in Table 5 and 6. The average *max DD* is slightly higher in the new system than in (Naranjo & Santos, 2016) (-0.83% versus -0.77%). Nevertheless, both values are low and can be assumed by any investor, which does not justify the decrease of the profits obtained.

Other performance measures of the trading system are also given in those Tables, such as the number of total, positive (+) and negative (-) trades. The Avg trade means the average profit from all of the trades. It is defined as the amount of money earned or loss in all the trades divided by the number of trades. The

Avg profit trade is the total profit divided by the number of winning trades. The Avg loss trade represents the total losses divided by the number of losing trades. Finally, the Avg profit/avg loss returns a ratio that compares the average profit and loss trades. A value greater than 1 means that the profit trades are better than the loss ones. If this result is smaller than 1, it means that you need more profit operations than loss trades to have a profitable trading system.

Analysing the *avg profit/avg loss* values, which indicates how good the winning operations are on average with respect to the average losers, both systems have obtained values above one, proving to be winning systems in the long term.

Moreover, both systems have been compared to the Buy and Hold (B&H) investment strategy that is commonly used as benchmark in stock markets. It consists of investing all the capital at the beginning of the trading period (buying) and then selling all the shares at the closing price of the previous session. Figure 9 shows the evolution of the invested capital (in terms of variations of the share prices over time) for the B&H and the two fuzzy trading systems under study with the closing price of each session.

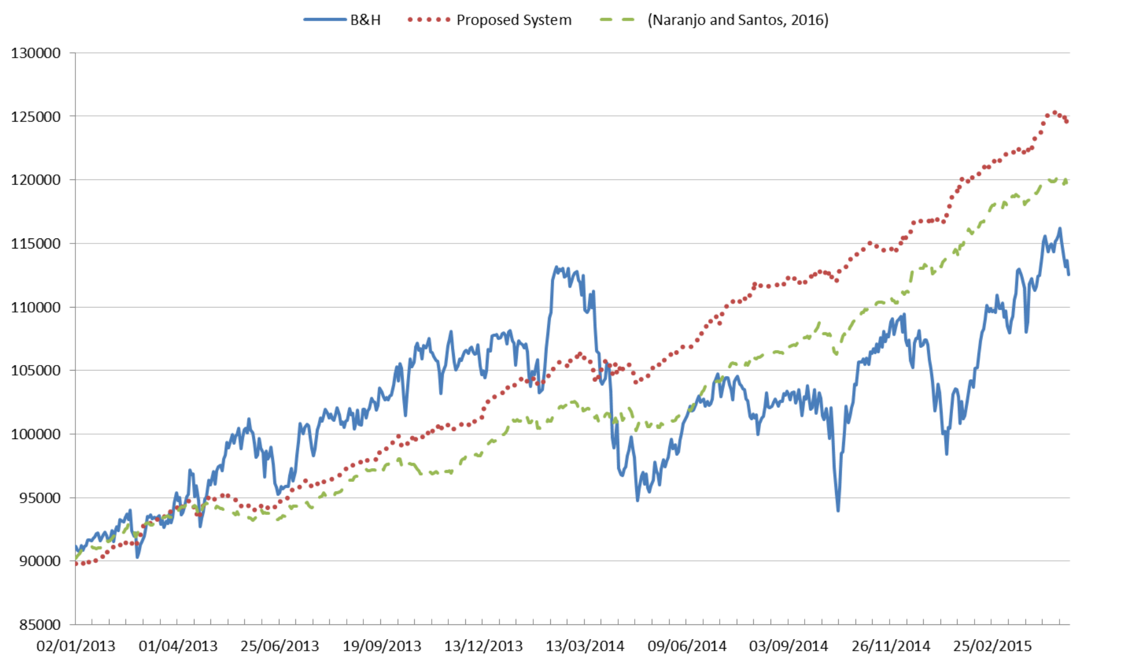


Figure 9. Evolution of the capital for the validation period with the three strategies (Nasdaq market)

The B&H strategy (blue line) gets the lowest profit (25.04%). Thus, regarding the *max DD*, which graphically can be spotted around the 17th October 2013 (a great fall of capital), the value obtained with this B&H technique, 16.97%, would be considered too high for most of the investors, who would have exited the market at that stage to limit the losses with a smaller profit.

It is interesting to note that the new fuzzy trading system (red dotted line), despite its slight improvement during the training period, gets much better during validation. This is because in this period the effect of currency depreciation takes place over more than four natural years (two years from the historical data and over two validation years), while the training covered only two years (one year from the historical database and the other one of training). This effect can be observed in the first part of Figure 9, where the capital obtained by the system proposed in (Naranjo & Santos, 2016) (green dashed line) surpasses the new one. However, as the sessions proceed, it is not capable of maintaining the same profit rate, suffering from streaks of losses with slowly recovery. This effect will be taken into account for the next experiments.

In conclusion, in spite of having moderate benefits, given the low risk, these fuzzy trading systems can be considered efficient.

### *5.2 Spanish Ibex35 market*

The same trading systems have been applied to the Spanish Ibex35 market. It is worth noting that the Ibex35 is a non-exclusive market, i.e., in contrast to Nasdaq, the companies belong to different sectors: banks, technology, construction, real estate, airport, telecommunications, electricity and gas, etc. The tests will be carried out with a portfolio of 15 companies with the highest trading volume out of the 35 companies that are included in this index.

Another difference with the previous experiments is that the validation period now comprises about 2 years taken from the historical database (Table 7). This will allow us to verify the stability of the investment systems and how the currency devaluation affects the results.

Table 7. Training and validation period for Spanish Ibex35

| **Training period** | | | | **Validation period** | | | |
|---|---|---|---|---|---|---|---|
| **Input data** | | **Training** | | **Input data** | | **Validation** | |
| **Start** | **End** | **Start** | **End** | **Start** | **End** | **Start** | **End** |
| 1-Jan-11 | 31-Dec-11 | 1-Jan-12 | 31-Dec-12 | 1-Jan-11 | 31-Dec-13 | 1-Jan-14 | 20-Sep-15 |

The same performance measures than before have been calculated for this stock market, applying the new fuzzy system (Table 8) and, with the purpose of comparison, the trading system presented in (Naranjo & Santos, 2016), Table 9. Again, for both systems, working with fuzzy candles type 2 gives smaller errors than with type 1 candles.

Table 8. Minimum and maximum errors with the new fuzzy proposed system (Ibex35)

| Company | | x(o1) | s(o1) | x(h1) | s(h1) | x(l1) | s(l1) | x(c1) | s(c1) | x(h2) | s(h2) | x(l2) | s(l2) | x(c2) | s(c2) |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| **AAPL** | **MIN** | 16.49 | 13.09 | 16.40 | 12.89 | 16.64 | 12.98 | 16.51 | 12.88 | 0.99 | 0.90 | 1.13 | 0.98 | 1.55 | 1.29 |
| | **MAX** | 17.99 | 14.70 | 17.81 | 14.44 | 18.02 | 14.63 | 17.94 | 14.49 | 1.10 | 1.00 | 1.26 | 1.23 | 1.76 | 1.47 |
| **ADBE** | **MIN** | 53.55 | 39.58 | 52.49 | 38.07 | 54.34 | 39.98 | 53.06 | 38.71 | 1.59 | 1.64 | 1.87 | 1.66 | 2.69 | 2.38 |
| | **MAX** | 66.55 | 44.97 | 65.21 | 43.69 | 68.15 | 46.42 | 66.62 | 44.81 | 1.94 | 1.95 | 2.22 | 1.90 | 3.38 | 2.91 |
| **ADI** | **MIN** | 22.80 | 18.32 | 22.32 | 18.17 | 22.89 | 18.88 | 22.47 | 18.59 | 1.30 | 1.15 | 1.37 | 1.12 | 1.98 | 1.65 |
| | **MAX** | 31.10 | 21.71 | 30.60 | 21.27 | 31.49 | 22.03 | 30.96 | 21.47 | 1.41 | 1.27 | 1.62 | 1.38 | 2.47 | 1.84 |
| **ADP** | **MIN** | 35.66 | 26.14 | 35.43 | 25.55 | 36.44 | 26.96 | 36.14 | 26.37 | 1.51 | 1.38 | 1.74 | 1.55 | 2.63 | 2.21 |
| | **MAX** | 38.64 | 28.34 | 38.24 | 27.78 | 39.54 | 29.33 | 39.04 | 28.57 | 1.75 | 1.59 | 1.89 | 1.80 | 2.97 | 2.54 |
| **ADSK** | **MIN** | 24.35 | 19.45 | 24.03 | 19.08 | 24.68 | 19.60 | 24.29 | 19.03 | 1.34 | 1.29 | 1.67 | 1.47 | 2.51 | 2.17 |
| | **MAX** | 29.34 | 22.26 | 28.83 | 21.63 | 29.67 | 22.66 | 29.20 | 22.21 | 1.83 | 1.80 | 1.89 | 1.82 | 2.88 | 2.51 |
| **AKAM** | **MIN** | 36.39 | 28.29 | 35.90 | 27.85 | 36.61 | 28.68 | 36.09 | 28.37 | 1.69 | 1.60 | 2.01 | 1.80 | 3.06 | 2.71 |
| | **MAX** | 40.41 | 30.29 | 39.80 | 29.75 | 41.04 | 30.99 | 40.43 | 30.52 | 2.09 | 2.22 | 2.22 | 1.92 | 3.57 | 3.40 |
| **ALTR** | **MIN** | 40.12 | 30.85 | 40.04 | 30.28 | 40.89 | 31.53 | 40.78 | 30.93 | 1.42 | 1.27 | 1.69 | 1.83 | 2.50 | 2.12 |
| | **MAX** | 52.07 | 34.25 | 51.56 | 33.56 | 53.24 | 35.23 | 52.47 | 34.45 | 1.67 | 1.64 | 2.05 | 1.94 | 2.78 | 2.41 |
| **ALXN** | **MIN** | 26.55 | 20.06 | 26.39 | 19.98 | 26.62 | 20.39 | 26.54 | 20.10 | 1.31 | 1.08 | 1.45 | 1.24 | 2.18 | 1.70 |
| | **MAX** | 29.46 | 23.01 | 29.29 | 22.69 | 29.64 | 23.32 | 29.37 | 22.93 | 1.41 | 1.28 | 1.63 | 1.40 | 2.35 | 1.87 |
| **AMAT** | **MIN** | 58.54 | 46.44 | 58.29 | 45.87 | 59.45 | 47.37 | 59.28 | 46.86 | 2.07 | 2.10 | 2.07 | 1.79 | 3.44 | 3.11 |
| | **MAX** | 72.27 | 50.90 | 71.29 | 49.89 | 73.69 | 51.96 | 72.79 | 50.91 | 2.45 | 2.83 | 2.79 | 2.49 | 4.24 | 3.98 |
| **AMGN** | **MIN** | 112.30 | 92.64 | 111.31 | 90.61 | 113.41 | 93.51 | 112.30 | 91.42 | 2.79 | 2.91 | 3.73 | 3.53 | 4.81 | 4.02 |
| | **MAX** | 162.28 | 108.83 | 160.88 | 107.01 | 164.79 | 110.77 | 163.06 | 108.82 | 3.78 | 4.14 | 4.53 | 4.49 | 6.23 | 5.28 |
| **AMZN** | **MIN** | 36.30 | 23.34 | 35.86 | 22.64 | 36.72 | 23.75 | 36.36 | 23.14 | 1.37 | 1.37 | 1.53 | 1.50 | 2.31 | 2.05 |
| | **MAX** | 44.43 | 26.87 | 43.78 | 26.17 | 45.08 | 27.46 | 44.52 | 26.78 | 1.52 | 1.60 | 1.86 | 1.82 | 2.54 | 2.45 |
| **ATVI** | **MIN** | 37.98 | 27.72 | 37.33 | 27.03 | 38.28 | 28.29 | 37.70 | 27.62 | 1.46 | 1.25 | 1.66 | 1.38 | 2.43 | 1.90 |
| | **MAX** | 48.79 | 31.36 | 48.10 | 30.53 | 49.54 | 31.83 | 48.94 | 31.27 | 1.64 | 1.50 | 1.88 | 1.70 | 2.71 | 2.22 |
| **AVGO** | **MIN** | **18.85** | **12.14** | **18.79** | **12.26** | **18.72** | **12.06** | **18.72** | **12.12** | **0.84** | **0.76** | **0.97** | **0.84** | **1.36** | **1.04** |
| | **MAX** | 20.83 | 13.42 | 20.78 | 13.63 | 20.85 | 13.44 | 20.82 | 13.53 | 1.04 | 0.94 | 1.05 | 0.95 | 1.54 | 1.24 |
| **BBBY** | **MIN** | 20.70 | 19.18 | 20.42 | 18.85 | 21.24 | 19.68 | 20.81 | 19.39 | 1.55 | 1.51 | 1.64 | 1.39 | 2.45 | 2.15 |
| | **MAX** | 23.67 | 20.65 | 23.29 | 20.30 | 24.37 | 21.32 | 24.04 | 20.92 | 1.90 | 1.91 | 1.85 | 1.60 | 2.90 | 2.65 |
| **BIDU** | **MIN** | 45.48 | 41.82 | 45.16 | 41.37 | 45.62 | 42.56 | 45.19 | 41.87 | 1.40 | 1.27 | 1.90 | 1.68 | 2.84 | 2.23 |
| | **MAX** | 55.74 | 44.30 | 55.41 | 44.00 | 55.92 | 44.86 | 55.53 | 44.71 | 1.73 | 1.52 | 2.21 | 2.24 | 3.17 | 2.85 |

Table 9. Minimum and maximum errors with the trading system (Naranjo & Santos, 2016) (Ibex35)

| Company | | x(o1) | s(o1) | x(h1) | s(h1) | x(l1) | s(l1) | x(c1) | s(c1) | x(h2) | s(h2) | x(l2) | s(l2) | x(c2) | s(c2) |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| **AAPL** | **MIN** | **17.20** | **12.81** | **16.95** | **12.65** | **17.28** | **12.93** | **17.19** | **12.95** | 0.99 | 0.91 | 1.10 | 0.88 | 1.51 | 1.15 |
| | **MAX** | 21.99 | 15.10 | 21.60 | 14.91 | 21.91 | 15.11 | 21.79 | 15.00 | 1.56 | 1.30 | 1.22 | 1.08 | 2.21 | 1.74 |
| **ADBE** | **MIN** | 57.45 | 38.91 | 56.52 | 38.17 | 58.42 | 40.07 | 57.25 | 39.25 | 1.65 | 1.77 | 1.88 | 1.55 | 2.88 | 2.43 |
| | **MAX** | 66.14 | 45.32 | 64.93 | 44.39 | 67.75 | 46.96 | 66.38 | 45.74 | 2.02 | 2.16 | 2.19 | 1.96 | 3.19 | 2.98 |

| | | | | | | | | | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| **ADI** | **MIN** | 22.75 | 18.88 | 22.42 | 18.46 | 23.03 | 19.18 | 22.81 | 18.88 | 1.21 | 1.01 | 1.43 | 1.14 | 1.98 | 1.58 |
| | **MAX** | 31.31 | 21.17 | 30.72 | 20.85 | 31.69 | 21.41 | 31.15 | 21.09 | 1.33 | 1.16 | 1.62 | 1.35 | 2.24 | 1.86 |
| **ADP** | **MIN** | 35.55 | 26.02 | 35.15 | 25.44 | 35.87 | 26.54 | 35.57 | 26.00 | 1.49 | 1.34 | 1.71 | 1.52 | 2.58 | 2.13 |
| | **MAX** | 40.45 | 28.24 | 39.90 | 27.49 | 41.36 | 29.06 | 40.75 | 28.01 | 1.82 | 1.67 | 2.15 | 1.89 | 3.33 | 2.69 |
| **ADSK** | **MIN** | 23.30 | 19.24 | 23.01 | 19.19 | 23.65 | 19.65 | 23.36 | 19.52 | 1.40 | 1.41 | 1.71 | 1.49 | 2.57 | 2.07 |
| | **MAX** | 29.29 | 24.87 | 29.01 | 24.08 | 29.89 | 25.18 | 29.57 | 24.50 | 1.65 | 1.58 | 1.96 | 1.79 | 3.00 | 2.61 |
| **AKAM** | **MIN** | 35.96 | 26.12 | 35.44 | 25.90 | 36.31 | 26.85 | 35.92 | 26.72 | 1.74 | 1.66 | 2.03 | 1.75 | 2.99 | 2.49 |
| | **MAX** | 43.74 | 30.09 | 43.31 | 29.48 | 44.74 | 30.68 | 44.18 | 30.22 | 2.37 | 2.50 | 2.26 | 2.04 | 3.97 | 3.26 |
| **ALTR** | **MIN** | 40.58 | 31.01 | 40.15 | 30.47 | 41.36 | 31.65 | 40.96 | 31.08 | 1.36 | 1.43 | 1.64 | 1.62 | 2.31 | 1.93 |
| | **MAX** | 50.81 | 35.71 | 50.06 | 35.04 | 52.16 | 37.01 | 51.26 | 36.02 | 1.71 | 1.65 | 1.93 | 1.92 | 2.74 | 2.53 |
| **ALXN** | **MIN** | 28.06 | 19.87 | 27.79 | 19.82 | 28.23 | 20.19 | 28.05 | 19.89 | 1.22 | 1.05 | 1.35 | 1.08 | 1.99 | 1.57 |
| | **MAX** | 36.93 | 22.34 | 36.45 | 22.02 | 37.19 | 22.64 | 36.72 | 22.30 | 1.36 | 1.20 | 1.55 | 1.30 | 2.18 | 1.71 |
| **AMAT** | **MIN** | 58.13 | 44.56 | 57.72 | 44.19 | 58.75 | 45.36 | 58.48 | 44.78 | 1.82 | 1.69 | 2.28 | 1.99 | 3.32 | 2.72 |
| | **MAX** | 69.04 | 51.18 | 67.93 | 50.39 | 70.33 | 51.95 | 69.24 | 51.28 | 2.40 | 3.07 | 2.72 | 2.41 | 3.92 | 3.71 |
| **AMGN** | **MIN** | 121.03 | 96.63 | 119.95 | 95.41 | 122.56 | 98.72 | 121.61 | 98.21 | 2.81 | 2.70 | 3.78 | 3.60 | 4.90 | 4.09 |
| | **MAX** | 171.68 | 115.97 | 169.28 | 112.92 | 174.49 | 117.05 | 171.67 | 114.28 | 3.25 | 3.72 | 4.83 | 4.63 | 5.44 | 4.83 |
| **AMZN** | **MIN** | 37.48 | 24.78 | 36.97 | 24.05 | 38.02 | 25.39 | 37.51 | 24.62 | 1.31 | 1.22 | 1.41 | 1.22 | 2.15 | 1.83 |
| | **MAX** | 45.45 | 28.67 | 44.80 | 28.19 | 46.19 | 29.74 | 45.49 | 29.25 | 1.52 | 1.45 | 1.68 | 1.66 | 2.37 | 2.19 |
| **ATVI** | **MIN** | 37.17 | 27.41 | 36.50 | 26.92 | 37.74 | 27.83 | 37.19 | 27.61 | 1.38 | 1.20 | 1.55 | 1.41 | 2.26 | 1.85 |
| | **MAX** | 47.70 | 30.59 | 47.00 | 30.12 | 48.35 | 30.74 | 47.69 | 30.41 | 1.57 | 1.45 | 1.85 | 1.86 | 2.52 | 2.21 |
| **AVGO** | **MIN** | 18.78 | 12.28 | 18.91 | 12.40 | 18.59 | 12.28 | 18.75 | 12.34 | **0.84** | **0.80** | **0.94** | **0.85** | **1.30** | **1.12** |
| | **MAX** | 23.24 | 13.47 | 23.08 | 13.50 | 23.22 | 13.46 | 23.06 | 13.49 | 0.96 | 0.93 | 1.03 | 0.98 | 1.54 | 1.30 |
| **BBBY** | **MIN** | 19.86 | 18.47 | 19.77 | 18.59 | 20.34 | 18.89 | 20.21 | 19.00 | 1.61 | 1.75 | 1.59 | 1.34 | 2.68 | 2.24 |
| | **MAX** | 23.96 | 21.24 | 23.53 | 20.83 | 24.66 | 21.91 | 24.15 | 21.37 | 1.84 | 2.02 | 1.80 | 1.58 | 2.87 | 2.85 |
| **BIDU** | **MIN** | 46.60 | 42.62 | 46.23 | 42.07 | 47.00 | 42.89 | 46.45 | 42.31 | 1.34 | 1.12 | 1.74 | 1.48 | 2.40 | 2.01 |
| | **MAX** | 66.57 | 52.62 | 65.47 | 51.55 | 66.16 | 52.28 | 65.19 | 51.08 | 1.58 | 1.48 | 2.63 | 2.17 | 3.56 | 2.76 |

In this market, according to Table 8 and 9, there has been an improvement in 218 out of the 420 cases (51.90%). Smaller errors and mean deviations were obtained in 33 out of the 60 cases (55%). However, as for the Nasdaq market, the prediction errors are still high. Again when removing the offset (type 2 candles), the errors are now reduced in 74 of the 180 cases (41.11%). This fact is interesting, and can confirm the hypothesis suggested before regarded the Nasdaq market, that is to say, that despite the fact that in more than half of the cases (58.89%) the results are slightly worse during the training, the system improves over the validation period.

To prove this statement, the validation results of the two fuzzy trading systems under study are shown in Tables 10 and 11. The value of *n*, as in the previous experiments, has been selected to minimize the sum of the average minimum and maximum errors.

Table 10. Validation results with the newly proposed fuzzy decision system (Ibex35)

| Co. | Net_Profit | Max DD | Trades total | Trades + | Trades - | Average trade | Avg profit trade | Avg loss trade | Avg profit/ avg loss |
|---|---|---|---|---|---|---|---|---|---|
| **ABE** | 1497.01 | -9.05 | 156 | 82 | 74 | 0.58 | 3.86 | -3.06 | 1.26 |
| **ACS** | 1674.06 | -10.55 | 155 | 80 | 75 | 0.38 | 2.57 | -1.96 | 1.31 |
| **ACX** | 1453.28 | **-16.32** | 177 | 84 | 93 | 0.77 | 8.44 | -6.15 | 1.37 |
| **ANA** | 4565.86 | -11.69 | 143 | 86 | 57 | 0.49 | 1.68 | -1.30 | 1.29 |
| **BBVA** | 1937.89 | -6.76 | 129 | 65 | 64 | 1.78 | 10.82 | -7.40 | 1.46 |
| **BKT** | 2238.07 | -8.78 | 135 | 74 | 61 | 2.64 | 16.22 | -13.83 | 1.17 |
| **CABK** | 335.17 | -11.36 | 175 | 71 | 104 | 0.48 | 17.60 | -11.20 | 1.57 |
| **ELE** | 3082.41 | -6.15 | 140 | 90 | 50 | 0.94 | 3.21 | -3.16 | 1.02 |
| **FCC** | 1921.77 | -10.88 | 160 | 77 | 83 | 0.89 | 10.74 | -8.25 | 1.30 |
| **GAM** | **4971.89** | -11.89 | 196 | 108 | 88 | 2.51 | 13.80 | -11.34 | 1.22 |
| **IBE** | 1228.15 | **-2.65** | 42 | 31 | 11 | 5.06 | 11.00 | -11.68 | 0.94 |
| **IDR** | 634.49 | -14.85 | 134 | 68 | 66 | 0.49 | 6.99 | -6.22 | 1.13 |
| **ITX** | 570.61 | -3.62 | 42 | 27 | 15 | 0.29 | 1.49 | -1.88 | 0.79 |
| **MAP** | 3280.12 | -4.18 | 131 | 80 | 51 | 8.21 | 28.02 | -22.85 | 1.23 |
| **MTS** | 705.20 | -10.44 | 115 | 56 | 59 | 0.70 | 9.40 | -7.55 | 1.24 |
| **Total** | **30095.98** | **-1.23** | 2030 | 1079 | 951 | | | | |

Table 11. Validation results with the fuzzy system (Naranjo & Santos, 2016) (Ibex35)

| Co. | Net_Profit | Max DD | Trades total | Trades + | Trades - | Average trade | Avg profit trade | Avg loss trade | Avg profit/ avg loss |
|---|---|---|---|---|---|---|---|---|---|
| **ABE** | 673,39 | -7,93 | 157 | 77 | 80 | 0,27 | 3,56 | -2,89 | 1,23 |
| **ACS** | 2159,85 | -9,32 | 164 | 84 | 80 | 0,45 | 2,68 | -1,90 | 1,41 |
| **ACX** | 2770,99 | -13,74 | 172 | 94 | 78 | 1,52 | 8,68 | -7,12 | 1,22 |
| **ANA** | 2749,13 | -15,29 | 154 | 81 | 73 | 0,30 | 1,73 | -1,28 | 1,34 |
| **BBVA** | 2693,31 | -7,62 | 137 | 76 | 61 | 2,23 | 9,67 | -7,05 | 1,37 |
| **BKT** | 1383,68 | -15,25 | 146 | 76 | 70 | 1,59 | 14,72 | -12,67 | 1,16 |
| **CABK** | 1480,93 | -9,31 | 160 | 82 | 78 | 2,11 | 19,39 | -16,06 | 1,21 |
| **ELE** | **4665,51** | -2,82 | 129 | 83 | 46 | 1,76 | 4,28 | -2,79 | 1,53 |
| **FCC** | -468,20 | **-19,39** | 146 | 62 | 84 | -0,26 | 8,60 | -6,79 | 0,27 |
| **GAM** | 896,04 | -18,64 | 202 | 101 | 101 | 0,48 | 10,34 | -9,39 | 1,10 |
| **IBE** | 1772,67 | **-1,51** | 30 | 26 | 4 | 10,24 | 13,03 | -7,95 | 1,64 |
| **IDR** | 345,42 | -14,86 | 127 | 63 | 64 | 0,29 | 7,39 | -6,70 | 1,10 |
| **ITX** | 871,53 | -6,66 | 62 | 37 | 25 | 0,23 | 1,59 | -1,78 | 0,89 |
| **MAP** | 2112,33 | -7,87 | 128 | 71 | 57 | 5,25 | 24,68 | -18,95 | 1,30 |
| **MTS** | 2021,65 | -13,39 | 101 | 51 | 50 | 2,49 | 11,97 | -7,19 | 1,67 |
| **Total** | **26128,24** | **-0,91** | 2015 | 1064 | 951 | | | | |

First, it should be noted that both systems obtain results similar to the previous experiments, despite having more historical data, and even more important, with a completely different market. In particular, 33.44% net profit (new system) vs. 29.03% by the system proposed in (Naranjo & Santos, 2016). With respect to the maximum DD, both systems obtain low values and, as in the previous experiments, the new fuzzy decision system obtains a slightly superior value than the previous one. The best and worse DD values have been boldfaced. Again, as it happened with Nasdaq100 market, both trading fuzzy systems gave values above 1 for the *avg profit/avg loss* variable, confirming the fact that they are long-term winning systems for this market as well. In addition, the effect observed in the Nasdaq market is verified in this experiment, that is to say, despite obtaining in some way high average error in more than half of the cases, the proposed system obtains greater benefits.

Both systems have been compared with the B&H benchmark. The evolution of the three strategies over time for the Ibex35 index is shown in Figure 10.

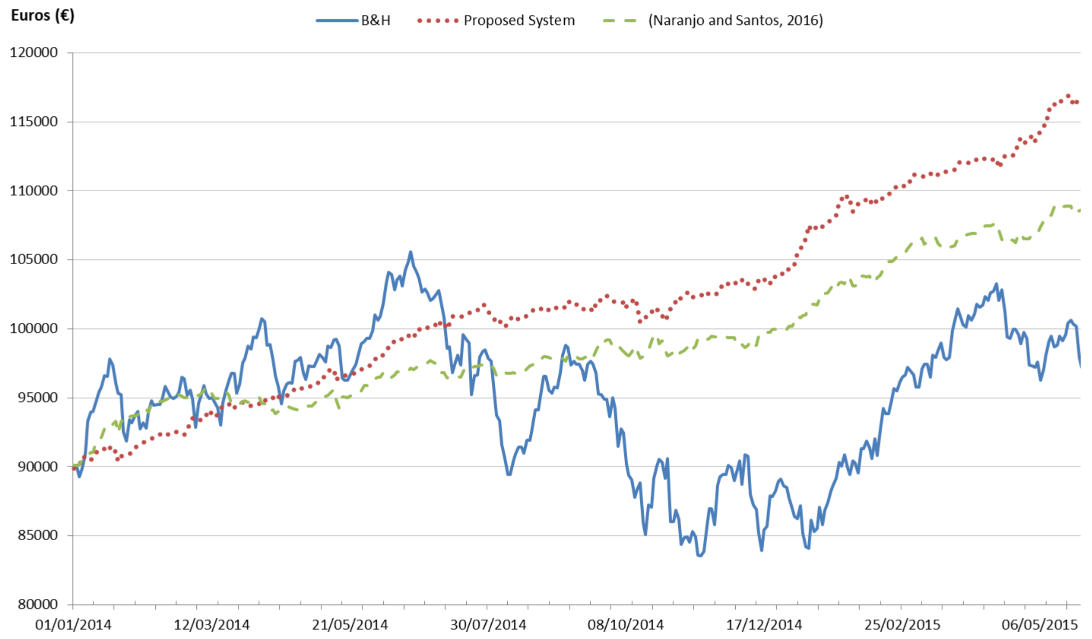


Figure 10. Evolution of the capital for the validation period with the three strategies (Ibex35 market)

For this Spanish stock market and for the selected period, it is possible to observe a completely different behaviour than the one on the Nasdaq100 market. In the American market there was a clearly bullish start with a sideway trend during the first part of the period. However, in this case, although initially there was a weak bullish trend, it is followed by a bearish behaviour. In this case, the B&H investment strategy (blue line) has been the worst strategy while the fuzzy strategies got more profits.

In addition, the maximum DD can be found around mid-November, with a value of -18.47%, approximately double that the one obtained with the Nasdaq100 market. Nevertheless, with the Spanish market this risk increment has not resulted in an increase of profits but more losses. As in the Nasdaq market, the system proposed in (Naranjo & Santos, 2016) (green dashed line) obtains greater profits during the first part of the validation time interval (Figure 10), being unable to maintain that growth rate and suffering losing streak. On the contrary, the new proposed fuzzy decision system (red dotted line) keeps a constant growth rate.

Both fuzzy investment systems, despite presenting a bearish behaviour for this period and taking into account that the investment entry was long-term (bullish), have achieved considerable profits and, even more, with a limited risk. Therefore, they have been proved to present a stable behaviour even for very different markets, independently of the market trend.

Finally, it is worth noting that the trading costs can also influence the results. This is again something that depends on the market and on the time the trading is done. There are some fixed costs and others that depend on the broker. In this work trading costs have not been explicitly considered as they are the same for all the proposed strategies and thus, the comparison is still fair. Indeed, the number of shares is quite similar for all the analysed strategies.

### *5.3 Performance measure of the investment: Sharpe ratio*

We have also considered a performance measure that combines the return and risk measure, in this case, the Sharpe ratio, to assess the investment strategy. The higher its value, the better the profitability is in relation to the risk taken.

$$Sharpe\ Ratio = \frac{R_i - R_f}{\sigma_i} \tag{18}$$

Where $R_i$ is the return, $R_f$ is the risk-free rate and $\sigma_i$ is the variability or standard deviation of the returns.

The United States 10-Year Bond was selected for the Nasdaq market, with risk-free rate 2.04%, and for the Ibex market, the Spanish 10-Year Bond, 1.45% (https://www.treasury.gov and http://www.tesoro.es, respectively). The Sharpe ratio for both markets is shown in Table 12.

Results show that, for the Nasdaq market, the three strategies present a similar performance, although the B&H strategy is slightly better. However, for the Ibex stock market, the B&H one provides the worst results, being the strategy proposed in Naranjo & Santos (2016) the best one.

Table 12. Sharpe ratio for Nasdaq and Ibex35 markets

| | Nasdaq | | | Ibex35 | | |
|---|---|---|---|---|---|---|
| | B&H | Proposed | (Naranjo & Santos, 2016) | B&H | Proposed | (Naranjo & Santos, 2016) |
| $R_i$ | 25,04 | 38,31 | 33,08 | 8,80 | 33,44 | 29,03 |
| $R_f$ | 2,04 | 2,04 | 2,04 | 1,45 | 1,45 | 1,45 |
| Dev | 5,63 | 9,17 | 7,96 | 5,26 | 8,56 | 6,76 |
| **Sharpe Ratio** | **4,08** | **3,98** | **3,87** | **1,40** | **3,74** | **4,08** |

Despite the popularity of this index, it has some drawbacks that must be taken into account to discuss its results. First of all, the Sharpe ratio penalizes extreme results since what is actually being measured is the volatility and not the risk. There is no distinction between positive and negative deviations, the deviation of the profitability is the measure of the risk. That is why this ratio penalizes, for example, a trading system that has the same number of trades than another one but that gets more profits. That explains the different sharpe ratio of the here proposed fuzzy investment system and the intelligent trading one (Naranjo & Santos, 2016).

This index does not work well when the returns are autocorrelated. That is, if two systems obtain the same profit with the same deviation, but the first system alternates losses and profits and the second one first obtains only losses and then profits, both systems would give same Sharpe ratio. Nevertheless, from a risk point of view, the second one would be riskier, with a higher Max DD. This effect can be observed in the results obtained by the Nasdaq market, where a smaller deviation is obtained with the B&H strategy. But according to figure 9, this strategy gives a higher Max DD that would not be acceptable by most of the investors.

Despite the above limitations, the Sharpe ratio is a very popular index for measuring profitability and risk; however, it must be considered with other performance measures in order to select the best strategy.

## 6. Conclusions and future work.

In this paper a complete investment system has been developed and applied to different stock markets. Fuzzy Japanese candlesticks have been used to define a forecasting system. A capital management strategy that takes into account the inflation rate and the currency devaluation has been also implemented.

The intelligent financial decision system includes the entry and exit strategies, the volume of shares to invest, the open and close process, and a risk control strategy.

The obtained results are promising. Despite having implemented a simple investment strategy based on the information provided by the fuzzy forecasting system, profits have been obtained for two different markets, Nasdaq and Ibex35, with a relative low level of risk.

One of the main conclusions is that this approach allows us to show the importance of currency devaluation and how it influences a financial forecasting system that is based on historical data.

From the investors' point of view, the proposed decision system meets several desired characteristics: making profits without high risk and, at the same time, a stable behaviour over time and on different markets.

As future work, we plan to apply fuzzy distances (Naranjo & Santos, 2018) to use fuzzy logic in all the steps of the trading system, and see how working with fuzzy measures would affect its performance.

Another future research line would be to include other type of information to improve the forecasting. Our approach is currently based on technical analysis, that is, the prediction is only based on previous candles. Considering exogenous inputs could be relevant for stock market trading in order to get better predictions. A possible extension could be to add an indicator of the market trend prediction or even of the changes of the trend. That is, if the market trend is bullish, bearish, or reversal points. This sort of so called sentiment indicator, that comes from the fundamental analysis, shows how investors feel about the market, or a particular company, sector, or business, etc. These indicators bring together various factors such as inflation, or macroeconomic and political trends, among others. This type of indicators can be used as support for investment decisions (Pinto, Schnitman, & Reis, 2018; Kelly & Ahmad, 2018).

We also think that it could be interesting to prove if our proposal is statistically superior to alternative investment strategies and if it does not work due to chance, carrying out a hypothesis contrasting.

Finally, another interesting future work will be to design a more efficient portfolio management. In this paper we have invested in each of the securities independently, with a fraction of the total capital. It would be worthy to optimize the investment portfolio and the capital allocation, maybe applying adaptive optimization methods to deal with the uncertainty (Rodríguez-Blanco, Sarabia, & de Prada, 2018).

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