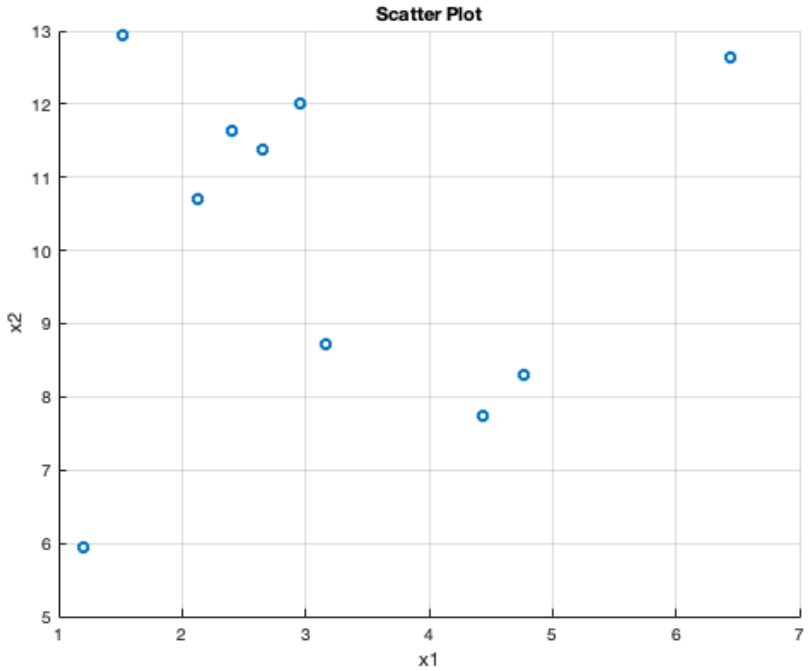
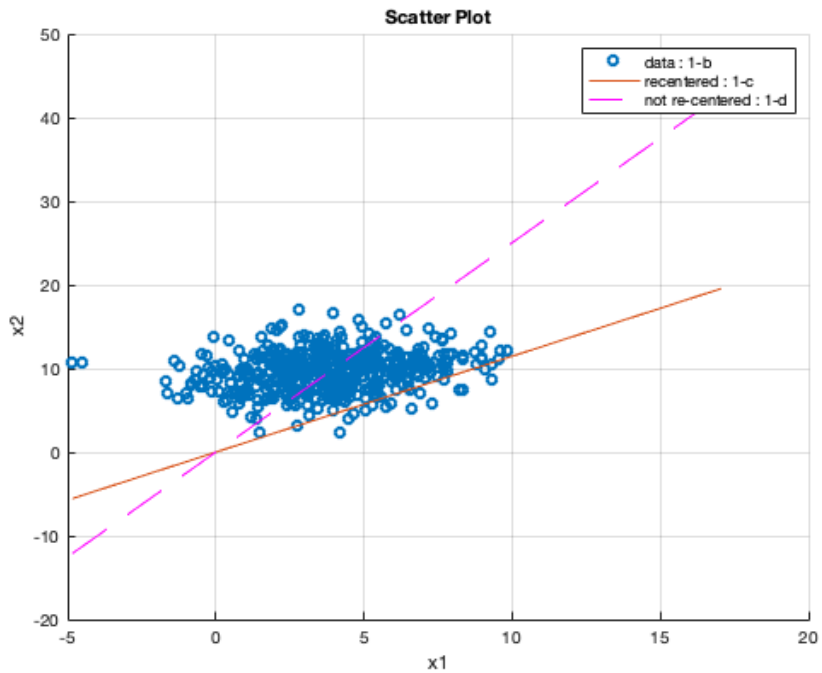
```
% Homework 5 - Question 1

disp('1a)')
mu = [4 ;10];
W = [1; 1];
sigma_sq = 4;
zi = 0;
M = 10;
X = mvnrnd(W*zi+mu, sigma_sq*eye(2), M);
figure(1);
scatter(X(:,1), X(:,2));
xlabel('x1');
ylabel('x2');
title('Scatter Plot');
grid on
snapnow;

disp('1b)')
N = 50;
z = normrnd(0, 1, [N,1]);
X = [];
for i=1:N
    mu = [4 ;10];
    W = [1; 1];
    sigma_sq = 4;
    zi = z(i);
    M = 10;
    X = [X; mvnrnd(W*zi+mu, sigma_sq*eye(2), M)];
end
figure(2);
scatter(X(:,1), X(:,2));
grid on
xlabel('x1');
ylabel('x2');
title('Scatter Plot');


disp('1c)')
X_new= X - mean(X);
n = size(X_new,1);
S = 1/n * (X_new' * X_new);
[V, ~] = eigs(S, 1);
hold on
tx = [min(X(:,1)) max(X(:,2))];
ty = V(2)*tx/V(1);
plot(tx, ty);
grid on

disp('1d)')
n = size(X,1);
S = 1/n * (X' * X);
[V, ~] = eigs(S, 1);
tx = [min(X(:,1)) max(X(:,2))];
ty = V(2)*tx/V(1);
plot(tx, ty, '--m');

legend('data : 1-b', 'recentered : 1-c', 'not re-centered : 1-d')
snapnow;
disp(" The variance is not captured properly if data is not re-centered.")

disp('1e)')
mu = [4 ;10];
W = [1; 1];
sigma_sq = 4;
N = 500;
X = zeros(N,2);
for i=1:N
    zi = normrnd(0, 1);
    X(i,:)=mvnrnd(W*zi+mu, sigma_sq*eye(2));
end
figure(5);
scatter(X(:,1), X(:,2));
grid on
xlabel('x1');
```

```
ylabel('x2');
title('Scatter Plot');
snapnow;
disp("In PPCA, we generate zi and then generate Xi - repeat this 500 times. This is different from 1d, where");
disp("zi is randomly generated 50 times and then used to generate Xi. Here zi's are randomly picked 500 times.");
```

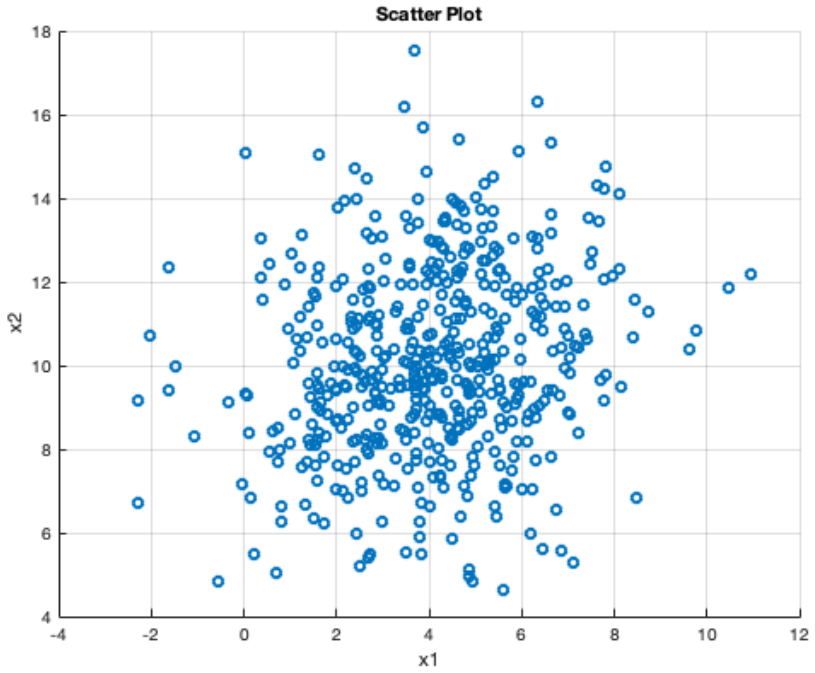
1a)



1b)
1c)
1d)



The variance is not captured properly if data is not re-centered.
1e)




```
In PPCA, we generate zi and then generate Xi - repeat this 500 times. This is different from 1d, where
zi is randomly generated 50 times and then used to generate Xi. Here zi's are randomly picked 500 times.
```

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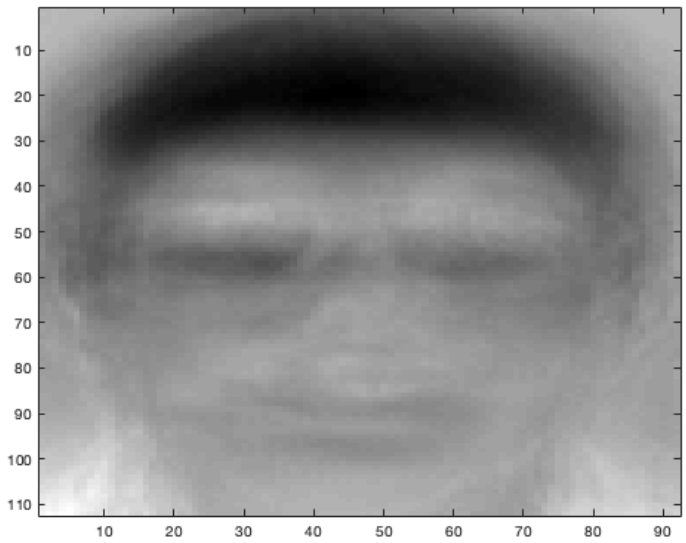
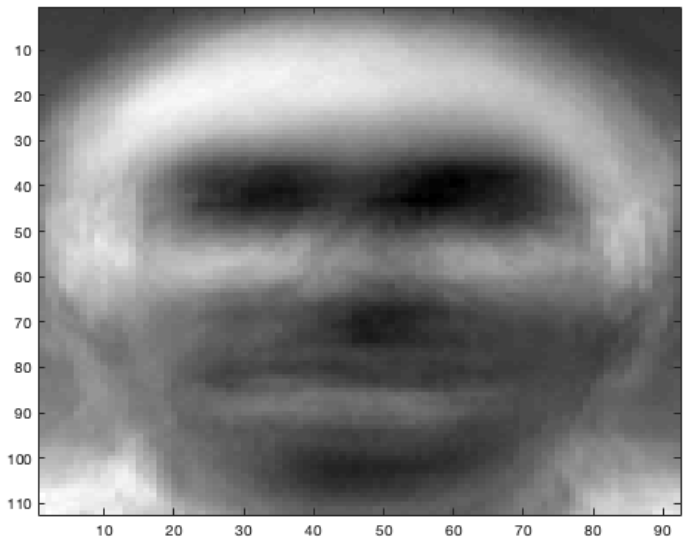
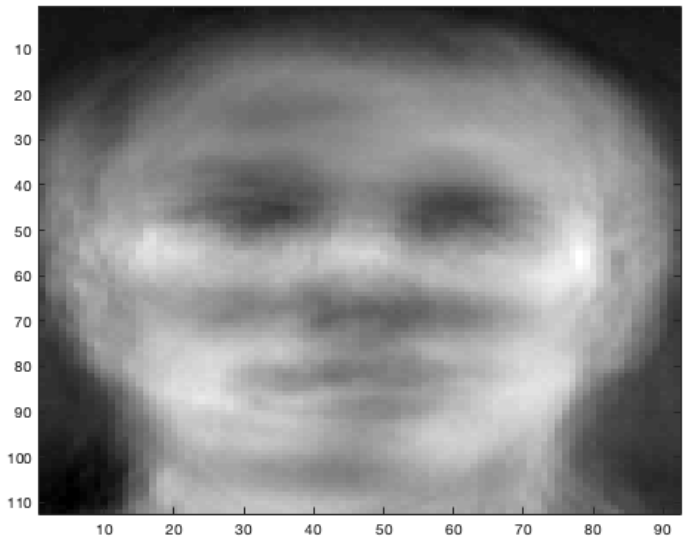
```
% Homework 5 - Question 2
X_train = csvread('X_train.csv');
Y_train = csvread('Y_train.csv');
%figure(1);colormap(gray);
%imagesc(reshape(X(10,:),112,92));
disp('2a)')
M = 112;
N = 92;
X_train_new = X_train - mean(X_train);
n = size(X_train_new,1);
S = 1/n * (X_train_new' * X_train_new);
[V, ~] = eigs(S, 10);

for i=1:10
    figure();colormap(gray);
    imagesc(reshape(V(:,i), M, N));
    snapnow;
end
disp("Characteristics captured by eigenfaces :-");
fprintf("1 - Most of the Hair \n 2 - eyes and nose \n 3 - eyes \n 4 - Hair \n 5 - Right side of face");
fprintf("6 - Hair, eyes and mouth \n 7 - Cheeks \n \n 8 - One eye \n 9 - Chin \n 10 - eyebrows, nose and mustache, ears, chin \n");

disp('2b)');
D = eigs(S, M*N);
plot(D);xlim([0 100]);
title('Elbow Plot');
snapnow;
elbow = 20;
fprintf('Elbow is approximately found at q=%d \n', elbow);
var = (sum(D(1:elbow))*100)/sum(D);
fprintf("Percentage of variance explained = %f \n", var);

disp('2c)');
X_test = csvread('X_test.csv');
Y_test = csvread('Y_test.csv');
[V, ~] = eigs(S, 50);
X_P_train = X_train_new*V;
X_test_new = X_test - mean(X_test);
X_P_test = X_test_new*V;
correctly_classified = 0;
Y_pred=zeros(size(X_P_test,1));
for i=1:size(X_P_test,1)
    dist = zeros(size(X_train,1),1);
    for j=1:size(X_train,1)
        dist(j)=norm(X_P_train(j,:)'-X_P_test(i,:)');
    end
    [~,ind] = min(dist);
    Y_pred(i) = Y_train(ind);
    if Y_test(i)==Y_pred(i)
        correctly_classified = correctly_classified+1;
    else
        figure
        colormap(gray);
        imagesc(reshape(X_test(i,:), M, N));
        title('Test Image')
        snapnow;
        figure
        colormap(gray);
        imagesc(reshape(X_train(ind,:), M, N));
        title('Train Image')
        snapnow;
    end
end

fprintf("Correctly classified Fraction with PCA = %.2f%% \n", (correctly_classified*100)/size(X_test,1));
correctly_classified = 0;
disp('2d)');
for i=1:size(X_test,1)
    dist = zeros(size(X_train,1),1);
    for j=1:size(X_train,1)
        dist(j)=norm(X_train(j,:)'-X_test(i,:)');
    end
    [~,ind] = min(dist);
    Y_pred(i) = Y_train(ind);
    if Y_test(i)==Y_pred(i)
        correctly_classified = correctly_classified+1;
    else
        figure
        colormap(gray);
        imagesc(reshape(X_test(i,:), M, N));
        title('Test Image')
        snapnow;
        figure
        colormap(gray);
        imagesc(reshape(X_train(ind,:), M, N));
        title('Train Image')
```

```
        snapnow;
    end
end
fprintf("Correctly classified Fraction without PCA = %.2f%% \n", (correctly_classified*100)/size(X_test,1));
```

2a)



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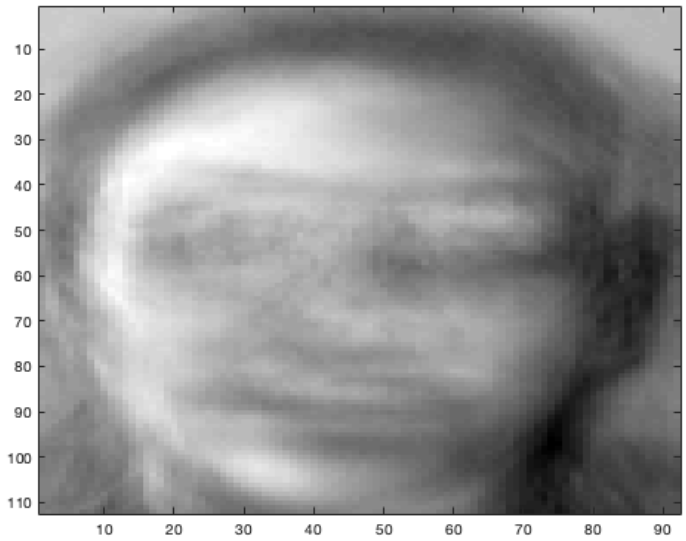
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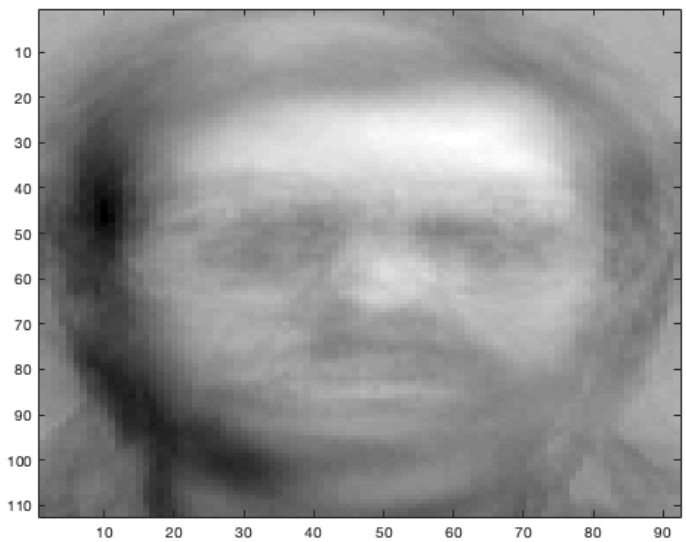
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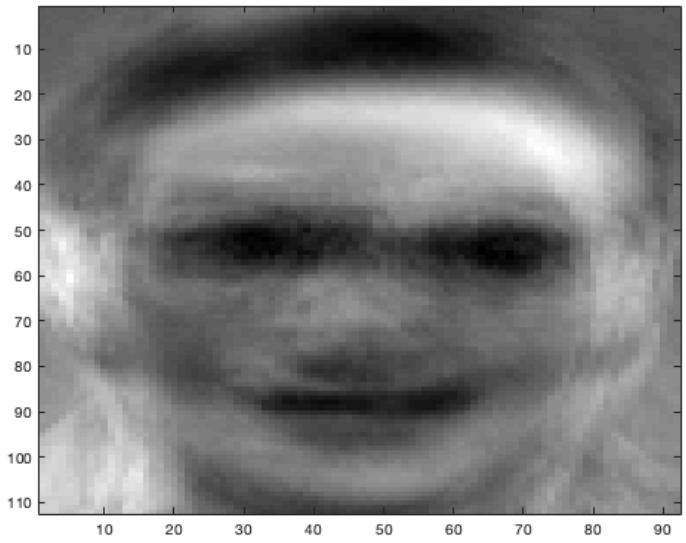
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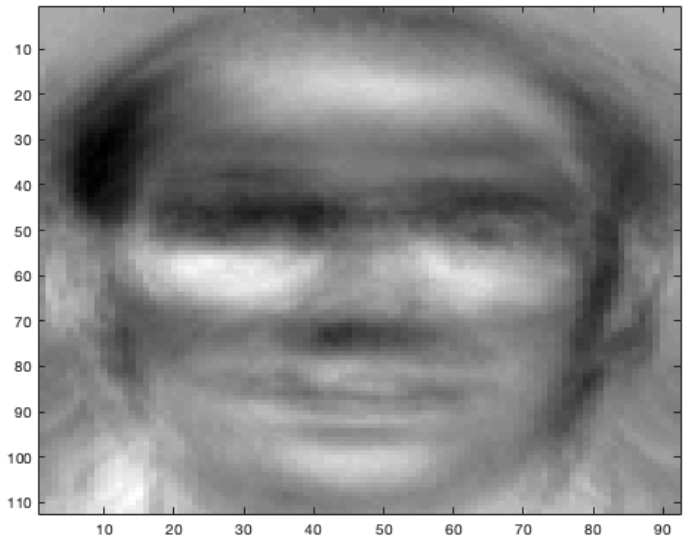
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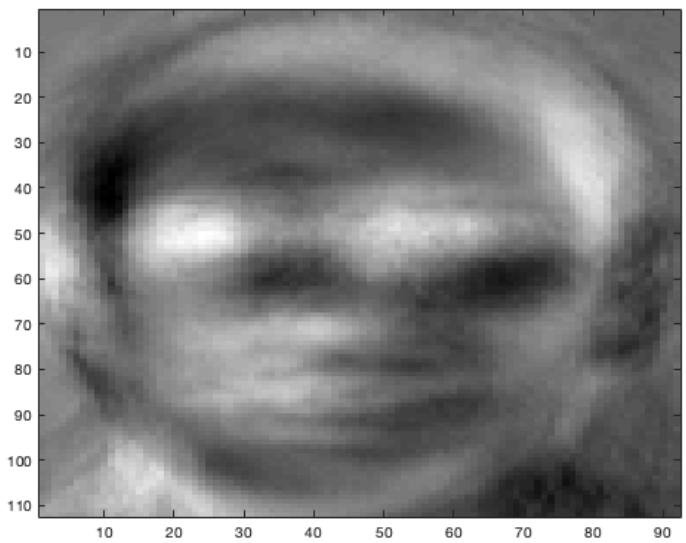
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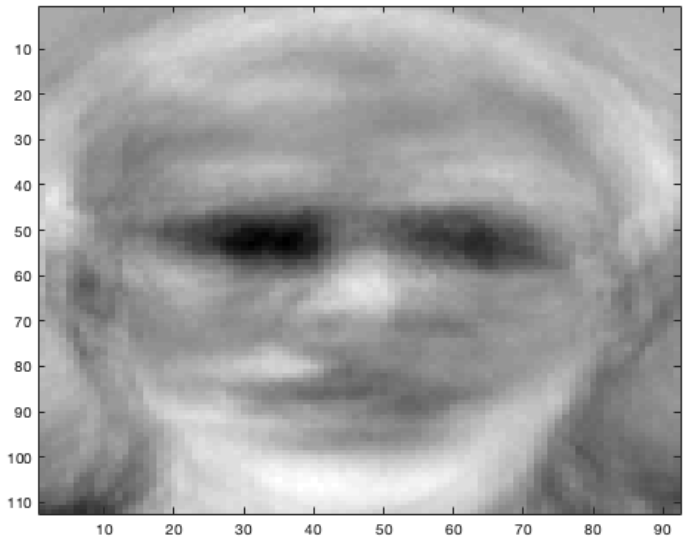
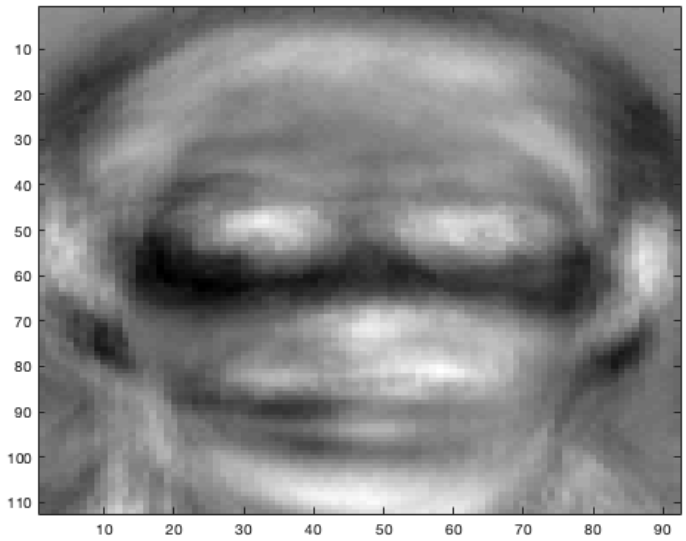
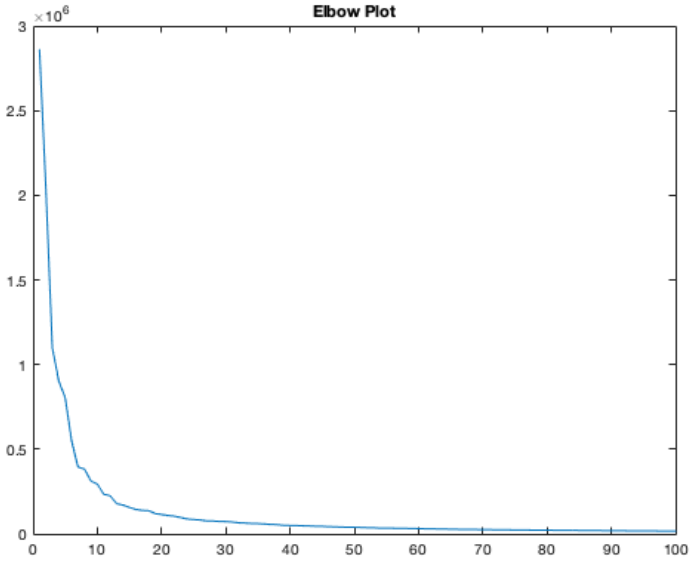
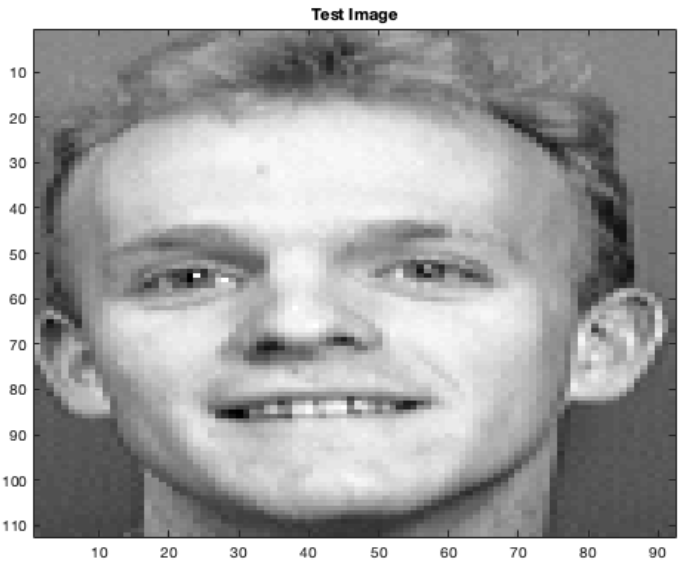


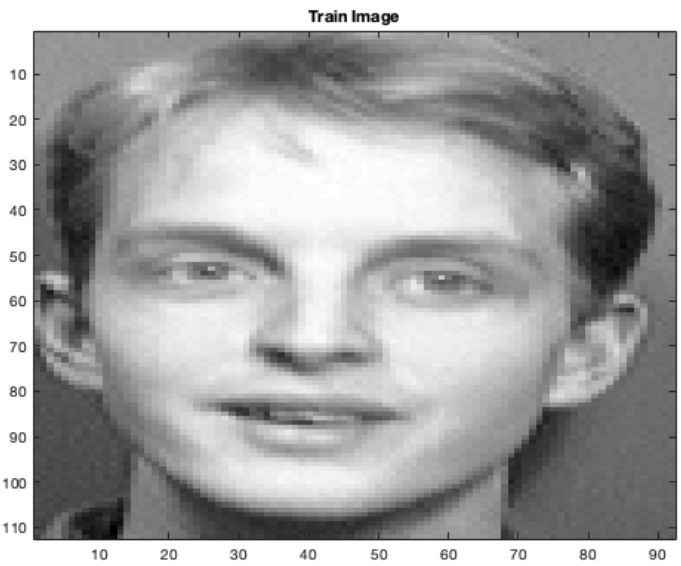


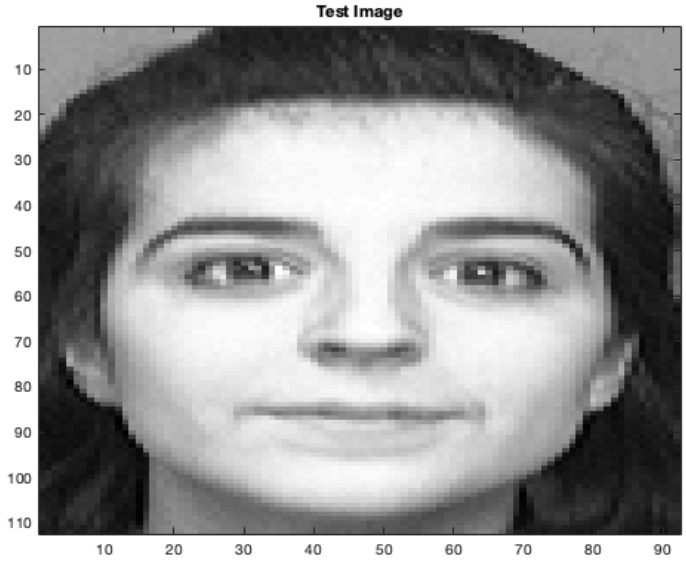

```
Characteristics captured by eigenfaces :-
1 - Most of the Hair
 2 - eyes and nose
 3 - eyes
 4 - Hair
 5 - Right side of face6 - Hair, eyes and mouth
 7 - Cheeks

 8 - One eye
 9 - Chin
 10 - eyebrows, nose and mustache, ears, chin
2b)
```

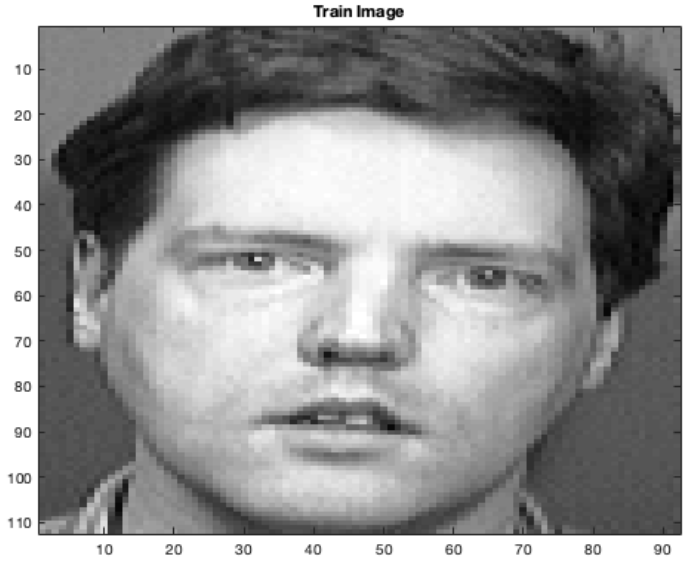



```
Elbow is approximately found at q=20
Percentage of variance explained = 70.266018
2c)
```

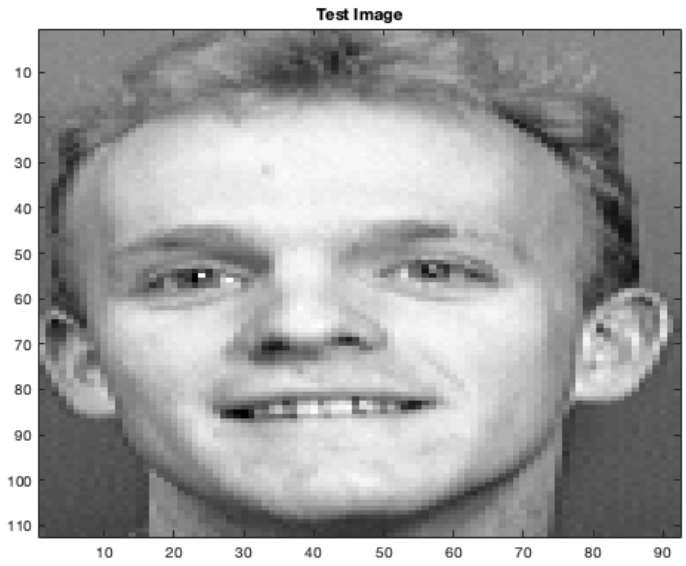









```
Correctly classified Fraction with PCA = 95.00%
2d)
```


Test Image
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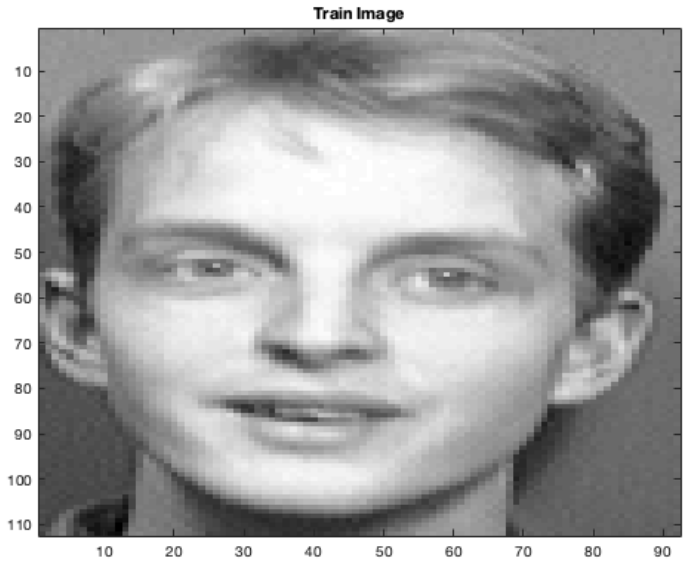
Train Image
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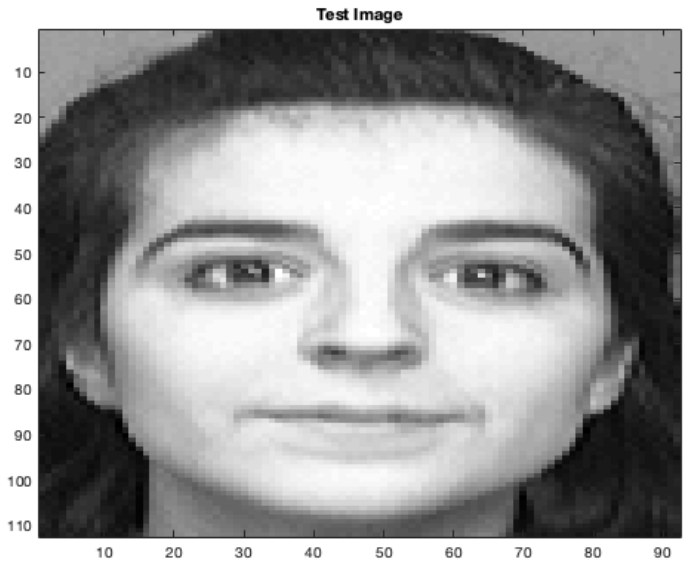
Test Image
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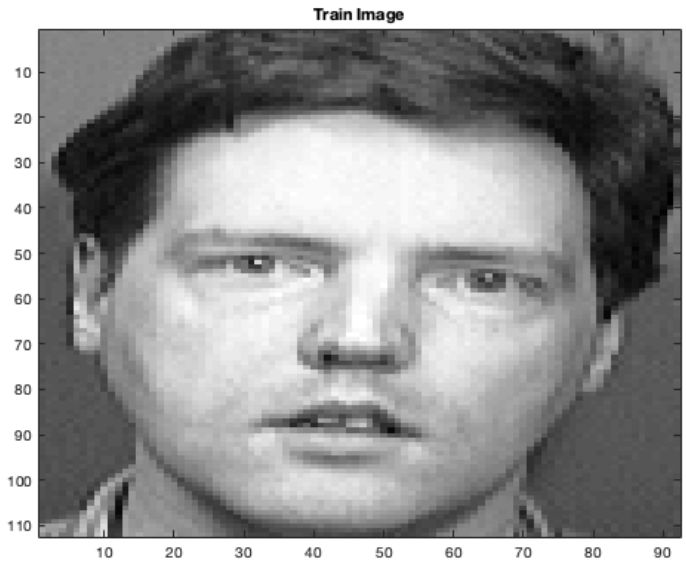

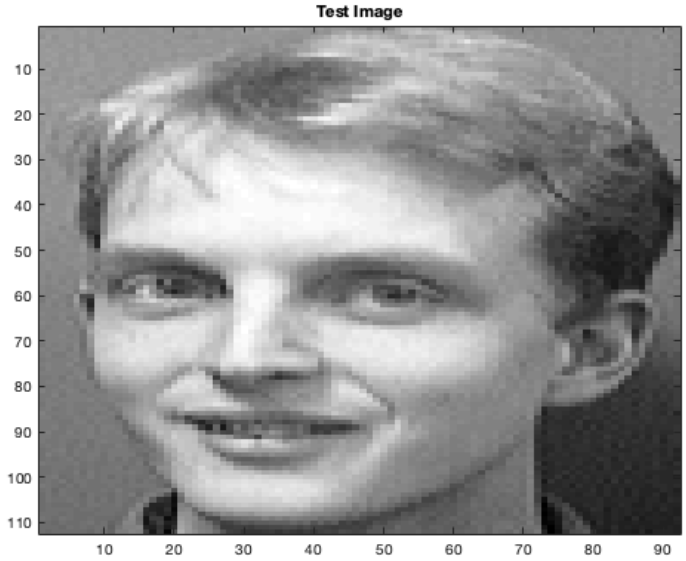

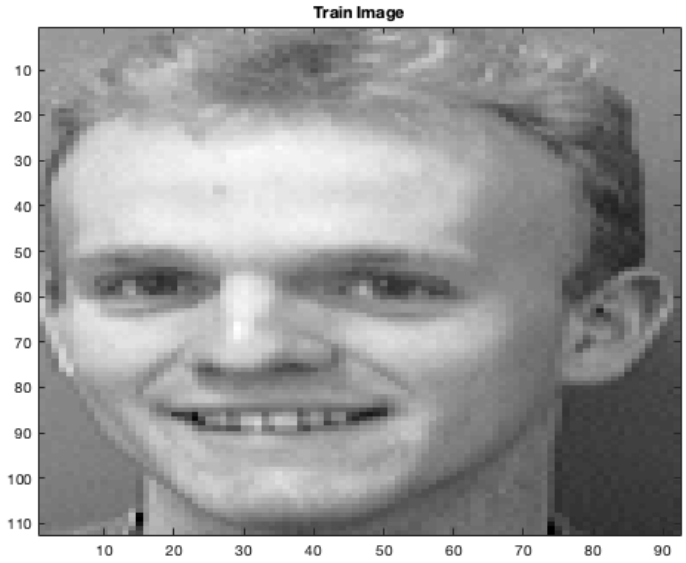


```
Correctly classified Fraction without PCA = 92.50%
```

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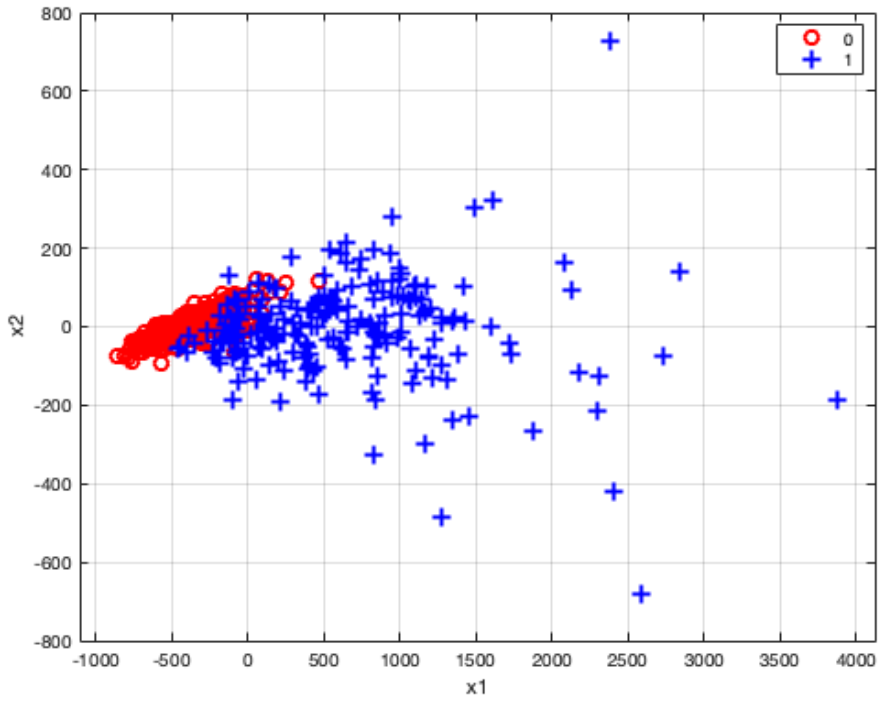
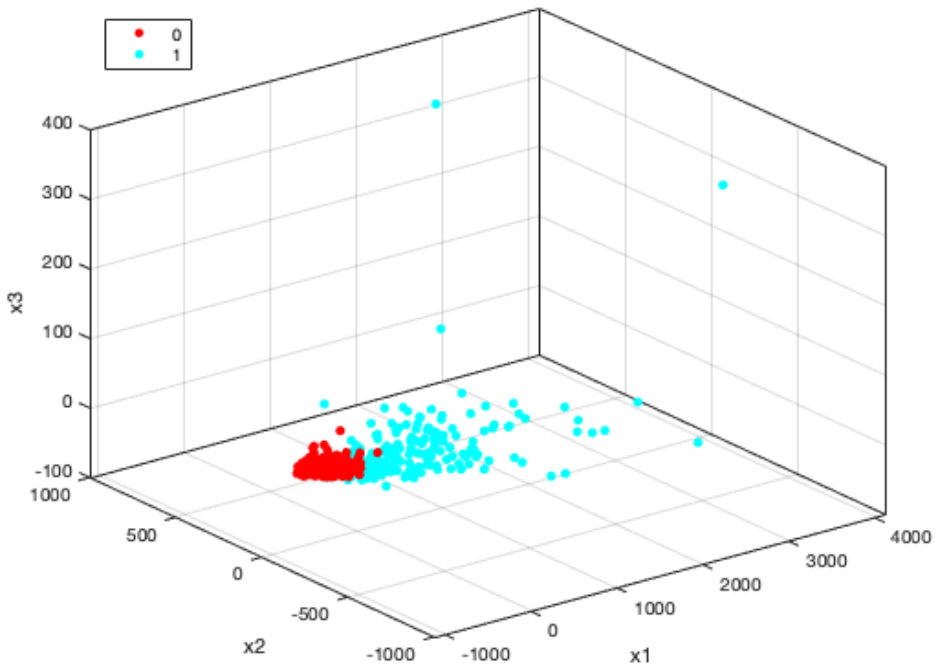
```
% Homework 5 Question 3
data = readtable('bc_wisc.csv');
data = data.Variables;
y = data(:, 2);
X = data(:,3:end);
disp('3a)')
B_glm = glmfit(X,y,'binomial');
disp('Maximum iteration exceeded - failed to converge - due to high dimension')
disp('3b)')
X_new= X - mean(X);
n = size(X_new,1);
S = 1/n * (X_new' * X_new);
[V, D] = eigs(S, 30);
D = diag(D);
lambda = [];
for q=2:10
    fprintf("q=%d , Percentage of variance explained =%.2f%% \n", q, (sum(D(1:q))*100)/sum(D));
end

X_P = X_new*V;
disp('3c)')
figure(1);
gscatter(X_P(:,1), X_P(:,2), y, 'rb', 'o+', 8, 'on', 'x1', 'x2');
grid on
xlabel('x1');
ylabel('x2');
snapnow;

figure(2);
h = gscatter(X_P(:,1), X_P(:,2), y);
grid on
xlabel('x1');
ylabel('x2');
zlabel('x3');

gu = unique(y);
x3 = X_P(:,3);
for k = 1:numel(gu)
      set(h(k), 'ZData', x3( y == gu(k) ));
end
view(3);
snapnow;

ind = 1:size(X,1);
disp('Row indices of outliers');
disp(ind(X_P(:,3)>250));

disp('3d)')
for q=2:10
    X_new = X_P(:,1:q);
    B_glm = glmfit(X_new,y,'binomial');
    X_test = [ones(size(y)), X_new];
    y_pred = X_test*B_glm>=0;
    num_correct_predictions = (sum(y_pred==y)*100)/size(y_pred,1);
    fprintf("q=%d, Percentage of correct predictions=%.2f%% \n", q, num_correct_predictions);
end
disp("Percentage of correct predictions increases slightly when we increase q. This is expected");
disp("Most of the variance is captures by q=2. However, when we capture more variance by increasing q,");
disp(" accuracy slightly increases.");
```

```
3a)
Warning: Iteration limit reached.
Maximum iteration exceeded - failed to converge - due to high dimension
3b)
q=2 , Percentage of variance explained =99.82%
q=3 , Percentage of variance explained =99.98%
q=4 , Percentage of variance explained =99.99%
q=5 , Percentage of variance explained =100.00%
```

```
q=6 , Percentage of variance explained =100.00%
q=7 , Percentage of variance explained =100.00%
q=8 , Percentage of variance explained =100.00%
q=9 , Percentage of variance explained =100.00%
q=10 , Percentage of variance explained =100.00%
3c)
```






```
Row indices of outliers
    68   256

3d)
q=2, Percentage of correct predictions=93.21%
q=3, Percentage of correct predictions=93.04%
q=4, Percentage of correct predictions=95.54%
```

```
q=5, Percentage of correct predictions=95.36%
q=6, Percentage of correct predictions=95.36%
q=7, Percentage of correct predictions=94.64%
q=8, Percentage of correct predictions=95.36%
q=9, Percentage of correct predictions=95.71%
q=10, Percentage of correct predictions=96.07%
Percentage of correct predictions increases slightly when we increase q. This is expected
Most of the variance is captures by q=2. However, when we capture more variance by increasing q,
 accuracy slightly increases.
```

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