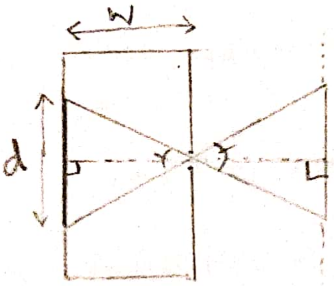
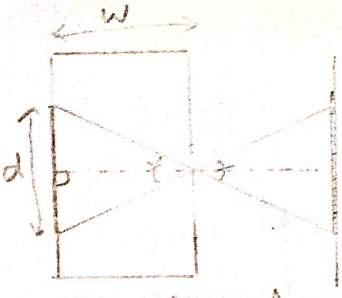
AKSHATA BHAT
abhat6@wisc.edu

CS766 - HOMEWORK 1

a)



Top view of pinhole camera



side view of pinhole camera

By extrapolating it can be proved that it is a circle when the plane is parallel to plane of projection

The shape of the image of the disk is a circle.

b) We know that

$$\frac{Area_i}{Area_o} = m^2 \Rightarrow Area_i \sim m^2 \qquad [\because \text{Image magnification}]$$

also $|m| = \frac{\|d_i\|}{\|d_o\|}$, $m = \frac{i}{o}$ i is image distance, o is object distance

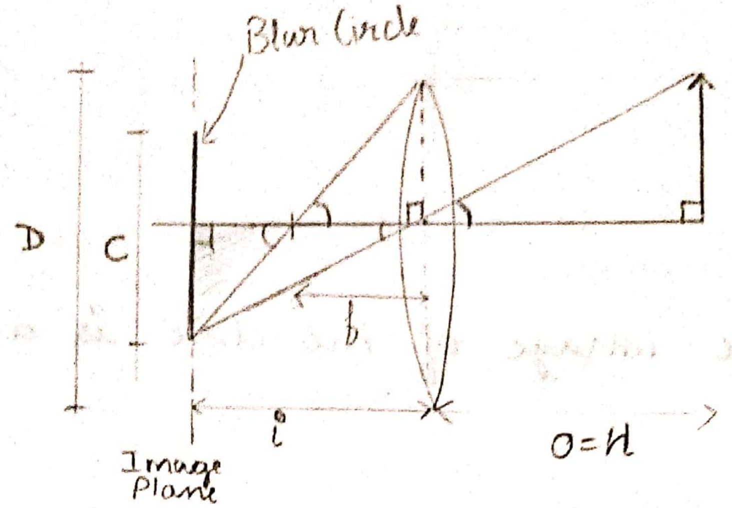
$$\Rightarrow m \propto \frac{1}{o}$$

Given, $o_1 = 1m$, $Area_{i_1} = 1mm^2$

if $o_2 = 2m$, $m \downarrow \frac{1}{2}$, $Area_i \uparrow \left(\frac{1}{2}\right)^2 \Rightarrow \boxed{Area_i = \frac{1}{4} mm^2}$

c) The perspective projection of a sphere on a pinhole camera will result in a circle, ellipse, parabolas or hyperbolas. The boundary formed by the pinhole and the circle on the sphere will be a cone. Then the perspective projection is equivalent to intersecting this cone with image plane, which results in conic section. (This is considering the fact that the center of projection and the sphere are on the same side of image plane in a pinhole camera).

(2)



An object at distance h forms a sharp image at distance i

Using similarity of triangles,

$$\frac{i-f}{C/2} = \frac{f}{D/2}$$

$$i = \frac{fC}{D} + f \quad — (1)$$

Also,

$$\frac{H}{D/2} = \frac{i}{C/2}$$

$$H = \frac{D}{C} i \quad — (2)$$

Substituting (1) in (2), we get :-

$$H = \frac{D}{C}\left(\frac{fC}{D} + f\right)$$

$$= \frac{Df}{C} + f$$

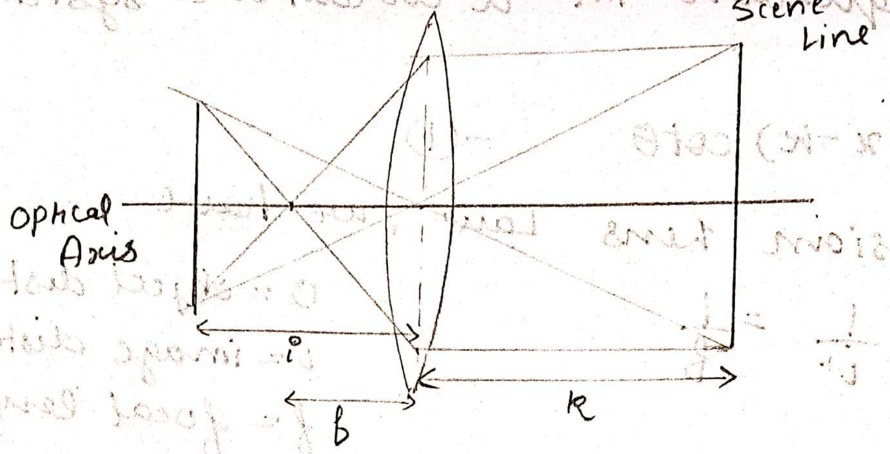
Hence $H = \frac{f^2}{NC} + f$

We know that $D = \frac{f}{N}$ where N is the F-number of lens

Scanned by CamScanner

③

a)



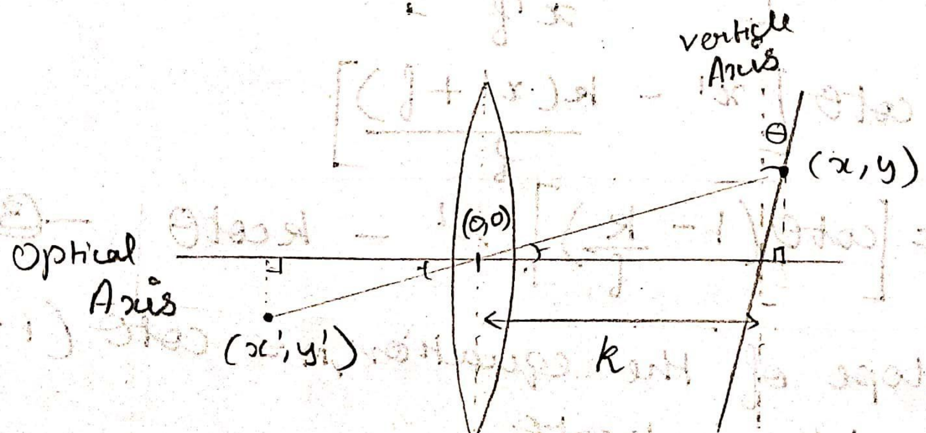
From Guassian lens law, we know that

$\frac{1}{i} + \frac{1}{o} = \frac{1}{f}$, where i is the image distance and o is the object distance f is the focal length

$$\frac{1}{i} + \frac{1}{k} = \frac{1}{f}$$

$$\boxed{i = \frac{kf}{k-f}}$$

b)



Consider a point (x, y) on the scene line as shown in the figure above

(x', y') is its corresponding image.

using line equation in a coordinate system we have

$$y = (x - k)\cot\theta \qquad -①$$

From Guassian Lens Law, we have

$$\frac{1}{o} + \frac{1}{i} = \frac{1}{f}$$

o = object distance
i = image distance
f = focal length

$$\Rightarrow \frac{1}{x} + \frac{1}{-x'} = \frac{1}{f}$$

$$x = \frac{x'f}{x'+f} \qquad -②$$

From similarity of triangles

$$\frac{-y'}{-x'} = \frac{y}{x}$$

$$\Rightarrow \frac{y'}{x'} = \frac{(x-k)\cot\theta}{x} = \cot\theta\left[1 - \frac{k}{x}\right] \qquad \therefore \text{substituting ①}$$

$$\frac{y'}{x'} = \cot\theta\left[1 - \frac{k(x'+f)}{x'f}\right] \qquad \therefore \text{substituting ②}$$

$$y' = \cot\theta\left[x' - \frac{k(x'+f)}{f}\right]$$

$$\boxed{y' = \left[\cot\theta\left(1 - \frac{k}{f}\right)\right]x' - k\cot\theta} \qquad -③$$

Here, slope of the equation, $m = \cot\theta\left(1 - \frac{k}{f}\right)$

c = y-intercept $= -k\cot\theta$

Eq ③ follows equation line $y = mx + c$

Now we need to prove that the image is tilted we need to prove, slope $\neq \infty$

since $\theta \neq 0$

$\cot\theta \neq \infty$

$$\text{slope} = \cot\theta\left(1-\frac{k}{f}\right)$$

also $k \neq \infty$

$\therefore 1-\frac{k}{f} \neq \infty$

also, we need to prove that slope $\neq 0$

since $\theta \neq 90$

$\cot\theta \neq 0$

and $1-\frac{k}{f} \neq 0$, as $f \neq \infty$

c)

Slope of the line interms of ϕ, is given by $\tan(90+\phi) = -\cot\phi$

From eq ③ in b).

$$\text{slope} = \cot\theta\left(1-\frac{k}{f}\right)$$

Equating the above two equations

$$-\cot\phi = \cot\theta\left(1-\frac{k}{f}\right)$$

$$\frac{\tan\phi}{\tan\theta} = \frac{-1}{1-\frac{k}{f}} = \frac{f}{k-f}$$

$$\boxed{\tan\phi = \frac{f}{k-f}\tan\theta}$$