Problem 1

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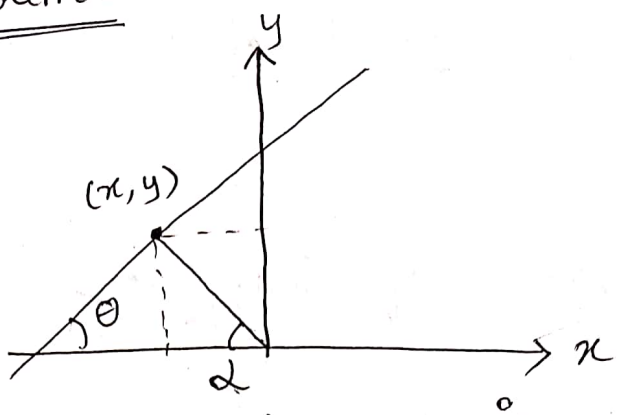

Equation of line is: $x\sin\theta - y\cos\theta + p = 0$

$$\Rightarrow \quad p = -x\sin\theta + y\cos\theta$$

$$= (\sqrt{x^2+y^2})\left(\frac{-x}{\sqrt{x^2+y^2}}\sin\theta + \frac{y}{\sqrt{x^2+y^2}}\cos\theta\right)$$

$$p = a\sin(\theta - \alpha) \qquad -(1)$$

Where, $a = \sqrt{x^2+y^2}$, and $\alpha = \tan^{-1}\left(\frac{-y}{x}\right)$

General form of sinusoidal equation is given by

$$f(x) = a\sin(bx - c) + d, \qquad -(2)$$

Comparing (1) & (2)

$f(x) = p \qquad x = \theta$

$a = a$

$bx = \theta \Rightarrow b = 1$

$c = \alpha$

$d = 0$

amplitude $= |a| = \sqrt{x^2+y^2}$

phase shift $= \frac{c}{b} = \frac{\alpha}{1} = \alpha = \tan^{-1}\left(\frac{-y}{x}\right)$

period $= \frac{2\pi}{b} = \frac{2\pi}{1} = 2\pi$

Period of the sinusoid is a constant and doesn't vary with (x, y).

By substituting different values of (x_i, y_i) we

$$a_i = \sqrt{x_i^2 + y_i^2}$$

$$\alpha_i = \tan^{-1}\left(\frac{-y_i}{x_i}\right)$$

Both a and α vary. Hence, by substituting in sinusoid eq (1)

$$\rho = a_i \sin(\theta - \alpha_i)$$

we get different sinusoids