Dual-phase--lag model on generalized magneto-thermoelastic interaction in a functionally graded material

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Abstract:

In this work, we consider the problem of magneto-thermoelastic interactions in a functionally graded material (FGM) under dual-phase--lag model due toin the presence of thermal shock. The generalized thermoelasticity theory with on relaxation time has been employed. The material is assumed to be elastic and functionally graded (FGM) (i.e. material with spatially varying properties). The basic equations have been written in the form of a vector-matrix differential equation in the Laplace transform domain, which is then solved by an eigenvalue approach. Numerical inversion of the transforms is carried out using the Stehfest method. Further, The graphs have been drawn to show the effect of the nonhomogeneity parameter, magnetic field, and dual--phase-lag parameters on displacement, temperature, stress, and strain.

Keywords: Dual-phase lag model, Laplace transform, Functionally graded materials, Magneto-thermoelasticity, Eigenvalue approach.

1.

INTRODUCTION

Functionally graded materials (FGMs) are types of non-homogeneous materials in which the composition changes gradually with a corresponding change in the properties. FGMs are usually

designed to be used under high-temperature environments. Thermal shock loading conditions may be involved with high thermal stresses when sudden heating or cooling takes place. As a result, thermal fracture problems can occur. Therefore, it is significant to analyse the transient thermal fracture behaviour of FGMs. The generalized theories of thermoelasticity, which admit the finite speed of thermal signal, have been the centre of interest of active researchers during the last three decades. Biot[1] established the coupled thermoelastic theory to overcome the first shortcoming in the classical uncoupled theory of thermoelasticity where it predicts two phenomena not compatible with physical observations. The first of such modelling is the extended thermoelasticity theory (LS) of Lord and Shulman[2], who established the generalized thermoelasticity with one relaxation time by postulating a new law of heat conduction to replace the classical Fourier law. Green and Lindsay[3] proposed the temperature rate dependent thermoelasticity (GL) theory with two relaxation times. The theory was extended for anisotropic body by Dhaliwal and Sherief[4]. Tzou[5] proposed the dual-phase-lag (DPL) model, which describes the interactions between phonons and electrons on microscopic level as retarding sources causing a delayed response on the macroscopic scale. The physical meaning and the applicability of the DPL model have been supported by the experimental results[6]. Recently, in context of the generalized dual-phase-lag thermoelasticity theory, Abouelregal et al[7–9] solved many problems in the context of dual-phase-lag model. The uniqueness and reciprocal theorems are proved without the use of Laplace transforms for the dual-phase-lag thermoelasticity theory by El-Karamany and Ezzat[10]. Functionally graded materials (FGMs) are types of non-homogeneous materials that have excellent thermo-mechanical properties to withstand high temperature and also have extensive applications to important structures, such as chemicals plants, pressure vessels, nuclear reactors, aerospace, etc. Das and Kanoria[11] investigated the generalized thermoelastic interaction in a functionally graded solid with a periodically varying heat source. Many researchers have studied the effects of magnetic field by applying the above theories of generalized thermoelasticity using different methods[12–29]. Zenkour and Abbas[30] used Green-Naghdi's theory to study the effect of magnetic field on a fibre-reinforced anisotropic half-space with a thermal shock. Abbas[31] studied the fractional order generalized magneto-thermoelastic medium subjected to a moving heat source using the eigenvalue approach. Abbas and Zenkour[32] investigated the analyses of finite element in magnetothermoelastic

interaction in an infinite FG cylinder. Othman et al[33] [33] used the normal mode method to investigate the initial stress and gravitational effect on the generalized magneto-thermo-microstretch elastic solid for different theories. Lotfy and Othman[34] [34] studied the effect of magnetic field for a mode-I crack on a two-dimensional problem of a ~~fiber~~fibre-reinforced material in the generalized thermoelasticity. Ezzat et al[35] [35] investigated the fractional ultrafast laser-induced magneto-thermoelastic ~~behavior~~behaviour in perfect conducting metal films.

In the current paper, we have applied the eigenvalue approach which was developed in a previous study[36] [36] to solve a problem of dual-phase-lag model on generalized magneto-thermoelastic interaction in a functionally graded material in the presence of thermal shock. Laplace transforms techniques with eigenvalue approach are used to get the general solution for any set of boundary conditions. Inversion of Laplace transforms is obtained numerically, and the variables in physical space–time domain have been shown graphically. Numerical results indicate the effects of magnetic field, non-homogeneity, and dual-phase-lag parameters on all the physical quantities.

2. BASIC EQUATION

The basic equations of dual-phase-lag thermoelastic model for a FGM in the absence of heat sources and the presence of body forces **F** are considered as that in previous work[10] in [10]. The equations of motion is

$$\sigma_{ij,j} + F_i = \rho \frac{\partial^2 u_i}{\partial t^2}. \tag{1}$$

The equation of heat conduction is

$$\left(1 + \tau_T \frac{\partial}{\partial t}\right)\left(KT_{,i}\right)_{,i} = \left(1 + \tau_q \frac{\partial}{\partial t} + m \frac{\tau_q^2}{2} \frac{\partial^2}{\partial t^2}\right)\left(\rho c_e \dot{T} + \gamma T_0 \dot{e}\right). \tag{2}$$

The constitutive equations are given by

$$\sigma_{ij} = 2\mu e_{ij} + \left[\lambda e - \gamma\left(T - T_0\right)\right]\delta_{ij}, \tag{3}$$

where $e = e_{ii}, \quad i, j = x, y, z.$

Three cases arise:

i) Classical dynamical coupled theory (CT)

$$\tau_T = \tau_q = m = 0,$$

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ii)   Lord and Shulman's theory (LS)

$$\tau_q = \tau > 0, \tau_T = m = 0,$$

iii)   Dual phase model (DPL)

$$0 < \tau_T < \tau_q, m = 1,$$

where ρ is the density of the medium; α_t is the coefficient of linear thermal expansion; λ, μ are the Lame's constants; t is the time; T_0 is the reference temperature; c_e is the specific heat at constant strain; K is the thermal conductivity; T is the temperature; τ_T and τ_q denote the finite times required for thermal equilibrium to be obtained and for effective collisions to take place between the electrons and the phonons, respectively. Tzou called the delay time τ_T as the PL of the temperature gradient and the other delay time τ_q as, the PL of the heat flux; τ is the relaxation time; δ_{ij} is the Kronecker symbol; σ_{ij} are the components of stress tensor; and u_i are the components of displacement vector. Let us consider a functionally graded isotropic thermoelastic body at a uniform reference temperature T_0 of a perfect electrical conductivity permeated by an initial magnetic field $\mathbf{H}_0 = (0, H_0, 0)$. Due to the effect of this magnetic field there arises in the conducting medium an induced magnetic field $\mathbf{h}$ and induced electric field $\mathbf{E}$. Also, there arises a force $\mathbf{F}$ (The Lorentz Force). The Maxwell's equations (in the absence of the displacement, current and charge density) is given by

$$\operatorname{curl} \mathbf{H} = \mathbf{J} \quad, \quad \operatorname{curl} \mathbf{E} = -\frac{\partial \mathbf{B}}{\partial t} \quad, \quad \operatorname{div} \mathbf{B} = 0 \,, \quad \mathbf{B} = \mu^e \mathbf{H}, \tag{4}$$

where $\mathbf{J}$ is the current density vector, $\mathbf{B}$ is the magnetic inductance vector, and μ^e is the magnetic permeability. The above equations are supplemented by Ohm's law and Lorentz force for perfect conducting medium[37] [37]

$$\mathbf{E} = -\mu_e \frac{\partial \mathbf{u}}{\partial t} \times \mathbf{H} \quad, \quad \mathbf{F} = \mathbf{J} \times \mathbf{B}.$$

(5)

Thus, we replace $\lambda, \mu, \gamma, K, \mu^e$ and ρ by $\lambda_o f(X), \mu_o f(X), \gamma_o f(X), K_o f(X), \mu_o^e f(X),$ and $\rho_o f(X)$ where $f(X)$ is the non-dimensionl dimensional function of the space variable $X = (x, y, z)$ and

$\lambda_o, \mu_o, \gamma_o, K_o, \mu_o^e$ and ρ_o are assumed to be constants. Then the Eqequations. (1) to (3) take the following forms

$$f(X)\left[2\mu_o e_{ij} + \left[\lambda_o e - \gamma_o\left(T - T_0\right)\right]\delta_{ij}\right]_{,j} +$$
$$f(X)_{,j}\left[2\mu_o e_{ij} + \left[\lambda_o e - \gamma_o\left(T - T_0\right)\right]\delta_{ij}\right] + F_i = \rho_o f(X)\frac{\partial^2 u_i}{\partial t^2}, \tag{6}$$

$$\left(1 + \tau_T\frac{\partial}{\partial t}\right)\left(K_o f(X)T_{,i}\right)_{,i} = \left(1 + \tau_q\frac{\partial}{\partial t} + m\frac{\tau_q^2}{2}\frac{\partial^2}{\partial t^2}\right)\left(\rho_o f(X)c_e \dot{T} + \gamma_o f(X)T_0 \dot{e}\right), \tag{7}$$

$$\sigma_{ij} = f(X)\left[2\mu_o e_{ij} + \left[\lambda_o e - \gamma_o\left(T - T_0\right)\right]\delta_{ij}\right], \tag{8}$$

3. FORMULATION OF THE PROBLEM

We consider the problem of a thermoelastic half-space $(x \geq 0)$ where the x-axis is taken perpendicular to the bounding plane of the half-space pointing inwards. We shall consider a one-dimensional disturbance of the medium, so that the displacement vector **u** and the temperatures field T can be expressed in the following forms:

$$\mathbf{u} = (u(x,t),0,0) \ , \ T = T(x,t). \tag{9}$$

Then, from the Equations. (4) and (5) we get observe that Lorentz force has only one component F_x in the x-direction.

$$F_x = \mu^e H_0^2 \frac{\partial e}{\partial x}. \tag{10}$$

We assumed that the properties of material depend only on x, i.e., we take $f(X)$ as $f(x)$. So, in the theory of the generalized thermoelasticity based on dual-phase--lag model, the governing equations of the perfect conductor can be written as

$$f(x)\left[\left(\lambda_o + 2\mu_o + \mu_o^e H_0^2\right)\frac{\partial^2 u}{\partial x^2} - \gamma_o\frac{\partial T}{\partial x}\right] + \frac{\partial f(x)}{\partial x}\left[\left(\lambda_o + 2\mu_o\right)\frac{\partial u}{\partial x} - \gamma_o T\right] = \rho_o f(x)\frac{\partial^2 u}{\partial t^2},$$
(11)

$$\left(1 + \tau_T\frac{\partial}{\partial t}\right)\left(f(x)\frac{\partial^2 T}{\partial x^2} + \frac{\partial f(x)}{\partial x}\frac{\partial T}{\partial x}\right) = f(x)\left(1 + \tau_q\frac{\partial}{\partial t} + m\frac{\tau_q^2}{2}\frac{\partial^2}{\partial t^2}\right)\left(\rho_o c_e\frac{\partial T}{\partial t} + \gamma_o T_0\frac{\partial^2 u}{\partial t\partial x}\right), \tag{12}$$

$$\sigma_{xx} = f(x)\left[\left(\lambda_o + 2\mu_o\right)\frac{\partial u}{\partial x} - \gamma_o\left(T - T_0\right)\right].$$

(13)

We define the following dimensionless quantities

$$(x', u') = \frac{c}{\chi}(x, u), \ T' = \frac{T - T_0}{T_0}, \left(t', \tau_q', \tau_T'\right) = \frac{c^2}{\chi}\left(t, \tau_q, \tau_T\right), \sigma'_{xx} = \frac{\sigma_{xx}}{\lambda_o + 2\mu_o}.$$

Hwhere $c^2 = \dfrac{\lambda_o + 2\mu_o}{\rho_o}$ and $\chi = \dfrac{K_o}{\rho_o c_e}$.

Upon introducing the above quantities in Equations. (11)—(13), and after suppressing the primes, we obtain

$$f(x)\left(\left(1 + R_H\right)\frac{\partial^2 u}{\partial x^2} - \beta\frac{\partial T}{\partial x}\right) + \frac{\partial f(x)}{\partial x}\left(\frac{\partial u}{\partial x} - \beta T\right) = f(x)\frac{\partial^2 u}{\partial t^2},$$

(14)

$$\left(1 + \tau_T\frac{\partial}{\partial t}\right)\left(f(x)\frac{\partial^2 T}{\partial x^2} + \frac{\partial f(x)}{\partial x}\frac{\partial T}{\partial x}\right) = f(x)\left(1 + \tau_q\frac{\partial}{\partial t} + m\frac{\tau_q^2}{2}\frac{\partial^2}{\partial t^2}\right)\left(\frac{\partial T}{\partial t} + \varepsilon\frac{\partial^2 u}{\partial t\partial x}\right),$$

(15)

$$\sigma_{xx} = f(x)\left[\frac{\partial u}{\partial x} - \beta T\right],$$

(16)

where $\beta = \dfrac{T_0\gamma_o}{\lambda_o + 2\mu_o}, \ \varepsilon = \dfrac{\gamma_o}{\rho_o c_e}, R_H = \dfrac{c_A^2}{c^2}$ and $c_A = H_0\sqrt{\dfrac{\mu_o^e}{\rho_o}}$ is the Alfven wave velocity of the medium.

4. EXPONENTIAL VARIATION OF NON-HOMOGENEITY

We consider $f(x) = e^{nx}$, where n is a non-dimensional parameter. So, Equations. (14), (15), and (16) take the forms

$$\left(1 + R_H\right)\frac{\partial^2 u}{\partial x^2} - \beta\frac{\partial T}{\partial x} + n\left[\frac{\partial u}{\partial x} - \beta T\right] = \frac{\partial^2 u}{\partial t^2},$$

(17)

$$\left(1 + \tau_T\frac{\partial}{\partial t}\right)\left(\frac{\partial^2 T}{\partial x^2} + n\frac{\partial T}{\partial x}\right) = \left(1 + \tau_q\frac{\partial}{\partial t} + m\frac{\tau_q^2}{2}\frac{\partial^2}{\partial t^2}\right)\left(\frac{\partial T}{\partial t} + \varepsilon\frac{\partial^2 u}{\partial t\partial x}\right),$$

(18)

$$\sigma_{xx} = e^{nx}\left[\frac{\partial u}{\partial x} - \beta T\right].$$

(19)

5. APPLICATION

We assume the initial conditions of the problem to be taken as

$$u(x,0) = \frac{\partial u(x,0)}{\partial t} = \frac{\partial^2 u(x,0)}{\partial t^2} = 0, \quad T(x,0) = \frac{\partial T(x,0)}{\partial t} = \frac{\partial^2 T(x,0)}{\partial t^2} = 0. \tag{20}$$

We consider the problem of a thick plate of finite high h. We ~~C~~choos~~eing~~ the x-axis perpendicular to the surface of the plate with the origin coinciding with the lower plate. The lower plate is traction free and is subjected to a thermal shock, ~~with traction free while~~ and the upper plate, which is also traction free, ~~is~~ kept at zero temperature: ~~with traction free as~~

$$\sigma_{xx}(0,t) = 0, \ T(0,t) = T_1 H(t), \tag{21}$$

$$\sigma_{xx}(h,t) = 0, \ T(h,t) = 0, \tag{22}$$

where $H(t)$ denotes the Heaviside unit step function.

6. GOVERNING EQUATIONS IN THE LAPLACE TRANSFORM DOMAIN

Applying the Laplace transform for Eq~~uations~~. (17)–(22) defined by the formula

$$\bar{f}(s) = L[f(t)] = \int_0^\infty f(t) e^{-st} dt. \tag{23}$$

Hence, we obtain the following system of differential equations

$$(1 + R_H) \frac{d^2 \bar{u}}{dx^2} - \beta \frac{d\bar{T}}{dx} + n \left[\frac{d\bar{u}}{dx} - \beta \bar{T} \right] = s^2 \bar{u}, \tag{24}$$

$$(1 + \tau_T s) \left(\frac{d^2 \bar{T}}{dx^2} + n \frac{d\bar{T}}{dx} \right) = \left(s + \tau_q s^2 + m \frac{\tau_q^2}{2} s^3 \right) \left(\bar{T} + \varepsilon \frac{d\bar{u}}{dx} \right),$$

$$\tag{25}$$

$$\bar{\sigma}_{xx} = e^{nx} \left[\frac{d\bar{u}}{dx} - \beta \bar{T} \right], \tag{26}$$

$$\bar{T}(0,s) = \frac{T_1}{s}, \ \bar{\sigma}_{xx}(0,s) = 0, \tag{27}$$

$$\bar{\sigma}_{xx}(h,s) = 0, \ \bar{T}(h,s) = 0. \tag{28}$$

The vector-matrix differential equation of Eqs. ~~equations~~ (24) and (25) can be written as follows[36, 38, 39] ~~[36, 38, 39]~~

$$\frac{d\vec{V}}{dx} = A\vec{V}, \tag{29}$$

where $\vec{V} = \left[\overline{u} \ \overline{T} \ \dfrac{d\overline{u}}{dx} \ \dfrac{d\overline{T}}{dx} \right]^T$ and $A = \begin{bmatrix} 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \\ a_{31} & a_{32} & a_{33} & a_{34} \\ 0 & a_{42} & a_{43} & a_{44} \end{bmatrix}$,

with $a_{31} = \dfrac{s^2}{1+R_H}$, $a_{32} = \dfrac{n\beta}{1+R_H}$, $a_{33} = \dfrac{-n}{1+R_H}$, $a_{34} = \dfrac{\beta}{1+R_H}$,

$$a_{42} = s\frac{1 + s\tau_q + ms^2 \frac{1}{2}\tau_q^2}{1 + s\tau_T}, \ a_{43} = \varepsilon a_{42}, \ a_{44} = -n.$$

7. SOLUTION OF THE VECTOR-MATRIX DIFFERENTIAL EQUATION

Let us now proceed to solve Eq. equation (29) by the eigenvalue approach proposed by[36] [36].

The characteristic equation of the matrix A takes the form

$$a_{31}a_{42} + \left(a_{33}a_{42} - a_{32}a_{43} + a_{31}a_{44}\right)\lambda + \left(a_{33}a_{44} - a_{31} - a_{42} - a_{34}a_{43}\right)\lambda^2 - \left(a_{33} + a_{44}\right)\lambda^3 + \lambda^4 = 0. \tag{30}$$

The roots of the characteristic Eq. equation (30) which are also the eigenvalues of matrix A are of the form $\lambda = \lambda_1, \ \lambda = \lambda_2, \lambda = \lambda_3, \ \lambda = \lambda_4,$.

The eigenvector $\vec{X} = \left[x_1, x_2, x_3, x_4\right]^T$, corresponding to eigenvalue λ can be calculated as:

$$x_1 = a_{32} + a_{34}\lambda \ , \ x_2 = -a_{31} + \left(\lambda - a_{33}\right)\lambda \ , \ x_3 = \lambda x_1 \ , \ x_4 = \lambda x_2.$$

(31)

From Eq. equation (30), we can easily calculate the eigenvector $\vec{X}_j$, corresponding to eigenvalue $\lambda_j, j = 1, 2, 3, 4$. For further reference, we shall use the following notations:

$$\vec{X}_1 = \left[\vec{X}\right]_{\lambda=\lambda_1}, \quad \vec{X}_2 = \left[\vec{X}\right]_{\lambda=\lambda_2}, \quad \vec{X}_3 = \left[\vec{X}\right]_{\lambda=\lambda_3}, \quad \vec{X}_4 = \left[\vec{X}\right]_{\lambda=\lambda_4}. \tag{32}$$

The solution of Eq. equation (29) can be written as follows:

$$\vec{V} = \sum_{j=1}^{4} B_j \vec{X}_j e^{\lambda_j x} = B_1\vec{X}_1 e^{\lambda_1 x} + B_2\vec{X}_2 e^{\lambda_2 x} + B_3\vec{X}_3 e^{\lambda_3 x} + B_4\vec{X}_4 e^{\lambda_4 x}, \tag{33}$$

where $B_1, B_2, B_3,$ and B_4 are constants to be determined from the boundary conditions of the problem. Thus, the field for the variables x and s can be written as:

$$\bar{u}(x,s) = \sum_{j=1}^{4} B_j x_3^j e^{\lambda_j x}, \tag{34}$$

$$\bar{T}(x,s) = \sum_{j=1}^{4} B_j x_4^j e^{\lambda_j x}, \tag{35}$$

$$\bar{\sigma}_{xx}(x,s) = \sum_{j=1}^{4} \left(\lambda_j x_3^j - \beta x_4^j \right) B_j e^{(\lambda_j + n)x}. \tag{36}$$

To complete the solution we have to know the constants $B_1, B_2, B_3,$ and B_4. By using the boundary conditions (27) and (28) we obtain

$$\begin{pmatrix} B_1 \\ B_2 \\ B_3 \\ B_4 \end{pmatrix} = \begin{pmatrix} R_{11} & R_{12} & R_{13} & R_{14} \\ R_{21} & R_{22} & R_{23} & R_{24} \\ R_{31} & R_{23} & R_{33} & R_{34} \\ R_{41} & R_{42} & R_{43} & R_{44} \end{pmatrix}^{-1} \begin{pmatrix} 0 \\ 0 \\ \dfrac{T_1}{s} \\ 0 \end{pmatrix}, \tag{37}$$

where

$$R_{11} = \lambda_1 x_3^1 - \beta x_4^1, \quad R_{12} = \lambda_2 x_3^2 - \beta x_4^2, \quad R_{13} = \lambda_3 x_3^3 - \beta x_4^3, \quad R_{14} = \lambda_4 x_3^4 - \beta x_4^4,$$

$$R_{21} = \left(\lambda_1 x_3^1 - \beta x_4^1 \right) e^{\lambda_1 h}, \quad R_{22} = \left(\lambda_2 x_3^2 - \beta x_4^2 \right) e^{\lambda_2 h},$$

$$R_{23} = \left(\lambda_3 x_3^3 - \beta x_4^3 \right) e^{\lambda_3 h}, \quad R_{24} = \left(\lambda_4 x_3^4 - \beta x_4^4 \right) e^{\lambda_4 h},$$

$$R_{31} = x_4^1, \quad R_{32} = x_4^2, \quad R_{33} = x_4^3, \quad R_{34} = x_4^4.$$

$$R_{41} = x_4^1 e^{\lambda_1 h}, \quad R_{42} = x_4^2 e^{\lambda_2 h}, \quad R_{43} = x_4^3 e^{\lambda_3 h}, \quad R_{44} = x_4^4 e^{\lambda_4 h}.$$

In the time domain t and space x, for the final solution of displacement, temperature, and stress and strain distributions we adopt a numerical inversion method based on the Stehfest method[40] [40]. In this method, the inverse $f(t)$ of the Laplace transform $f(s)$ is approximated by the relation

$$f(t) = \frac{\ln 2}{t} \sum_{j=1}^{N} V_j F\left(\frac{\ln 2}{t} j \right), \tag{38}$$

Where V_j is given by the following equation:

$$V_i = (-1)^{\left(\frac{N}{2}+1\right)} \sum_{k=\frac{i+1}{2}}^{\min\left(i,\frac{N}{2}\right)} \frac{k^{\left(\frac{N}{2}+1\right)}(2k)!}{\left(\frac{N}{2}-k\right)!k!(i-k)!(2k-1)!}. \tag{39}$$

The parameter N is the number of terms used in the summation in Eq. ~~equation~~ (36) and should be optimized by trial and error. Increasing N, increases the accuracy of the result up to a point, and then the accuracy declines because of increasing round-off errors. An optimal choice of $10 \le N \le 14$ has been reported by Lee *et al.* for some problem of their interest[41] ~~[41]~~. Thus, the solutions of all variables in physical space–time domain are given by

$$u(x,t) = \frac{\ln 2}{t} \sum_{i=1}^{N} V_i \bar{u}\left(x, \frac{\ln 2}{t}i\right),$$

(40)

$$T(x,t) = \frac{\ln 2}{t} \sum_{i=1}^{N} V_i \bar{T}\left(x, \frac{\ln 2}{t}i\right),$$

(41)

$$\sigma_{xx}(x,t) = \frac{\ln 2}{t} \sum_{i=1}^{N} V_i \bar{\sigma}_{xx}\left(x, \frac{\ln 2}{t}i\right). \tag{42}$$

8. NUMERICAL RESULTS AND DISCUSSION

In the present work, the generalized thermoelasticity theory under dual--phase--lag model is ~~analyzed~~analysed by considering a copper material. The material parameters are given as follows[13] ~~[13]~~

$$\lambda_o = 7.76 \times 10^{10} (\text{kg})(\text{m})^{-1}(\text{s})^{-2}, \quad \mu_o = 3.86 \times 10^{10} (\text{kg})(\text{m})^{-1}(\text{s})^{-2}, \quad T_0 = 293(\text{K}),$$

$$K_o = 3.68 \times 10^2 (\text{kg})(\text{m})(\text{K})^{-1}(\text{s})^{-3}, \quad c_e = 3.831 \times 10^2 (\text{m})^2 (\text{K})^{-1}(\text{s})^{-2}, T_1 = 1, \text{ h} = 3,$$

$$\alpha_t = 17.8 \times 10^{-6} (\text{K})^{-1}, \quad \rho_o = 8.954 \times 10^3 (\text{kg})(\text{m})^{-3}, \quad t = 0.5, \quad \tau_q = 0.15, \quad \tau_T = 0.1.$$

The temperature, ~~the~~ displacement, and ~~the~~ stress and ~~the~~ strain distribution are represented graphically. Numerical calculation is carried out for two groups. The first group is investigating how the non-dimensional temperature, displacement, and stress and strain vary with different values of the non-homogeneity parameter n under dual--phase--lag model with the magnetic field. The second group is investigating how the non-dimensional temperature, displacement, and stress and strain vary under the coupled theory CT at $\left(\tau_T = \tau_q = m = 0\right)$ and dual--phase--lag model DPL

at $\left(\tau_T = 0.1, \tau_q = 0.15, m = 1\right)$ with and without magnetic field. From Figures 1–8, it is observed ~~seen~~ that~~,~~ the temperature starts with its maximum value at the origin and decreases until zero is attained~~ing zero~~ beyond a wave front for the generalized theory, which agrees with the boundary conditions. The displacement takes negative values and gradually increases until it attains a peak value at a particular location in close proximity to the lower surface of the plate and then diminishes rapidly with increasing distance. The variation of stress with respect to distance, ~~which we observed that, the stress,~~ always starts from the zero value and terminates at the zero value to obey the boundary conditions. The strain starts with its maximum positive value and decreases slowly with the increase of the distance x until it attains a peak negative value and then increases until zero is attain~~attained~~~~ing zero~~.

9. CONCLUSION~~s~~

This article investigates the generalized magneto-thermoelastic interaction in a functionally graded material under dual-phase-~~-~~lag model in the presence of a~~the~~ ~~-~~thermal shock. The material properties under consideration are assumed to vary exponentially with distance. The analysis of the results permits some concluding remarks:

- The ~~S~~significant effects of non-homogeneity parameter n are noticed on all quantities.
- By increas~~eing~~ of non-homogeneity parameter n, as FGM material becomes softer, all variable quantities are reduced.
- The presence of a magnetic field has a significant effect on the solution of the displacement, stress, and strain.
- The phase lags effect plays a significant role on all distributions.
- The difference between coupled theory (CT) and dual-phase-lag (DPL) model is very clear.

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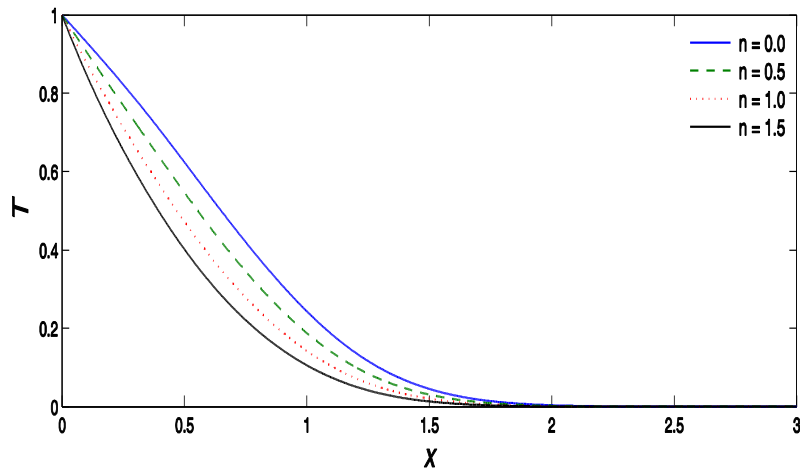
Fig.Figure. 1. The temperature distribution for different values of n .
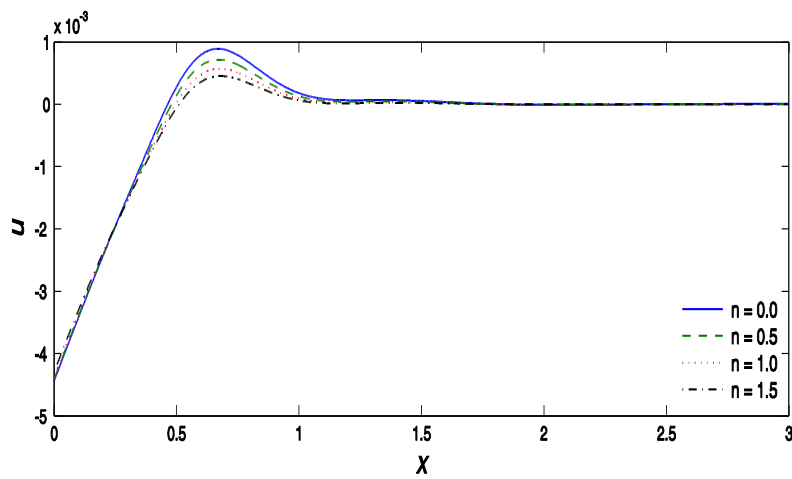
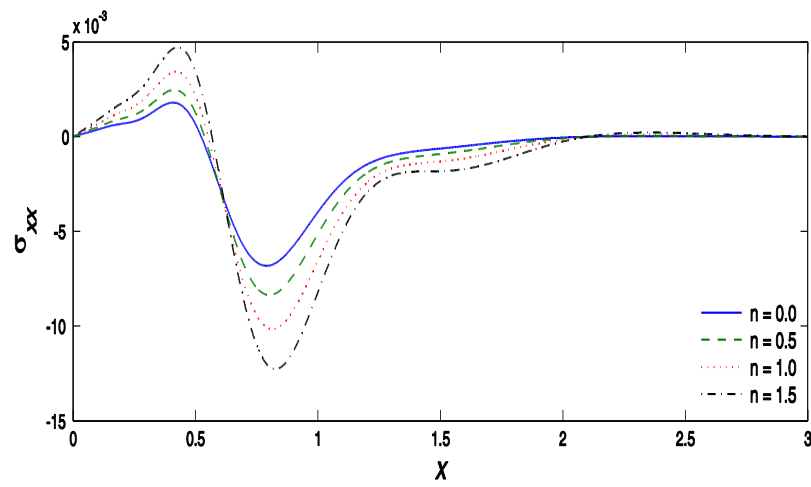**Fig.Figure. 2.** The dDisplacement distribution for different values of n .

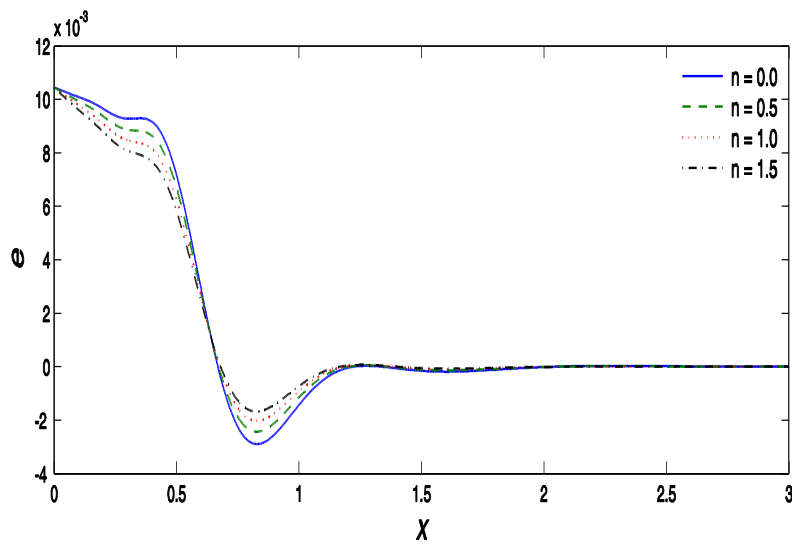**Fig.Figure. 3. SThe s**tress distribution for different values of n .

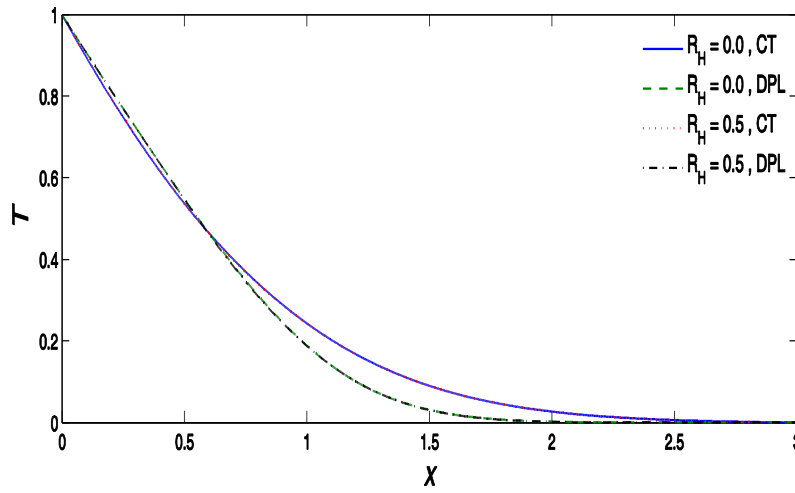**Fig.Figure. 4. SThe s**train distribution for different values of n .

Fig.Figure. 5. T The temperature distribution for different theories and different values of R_H.
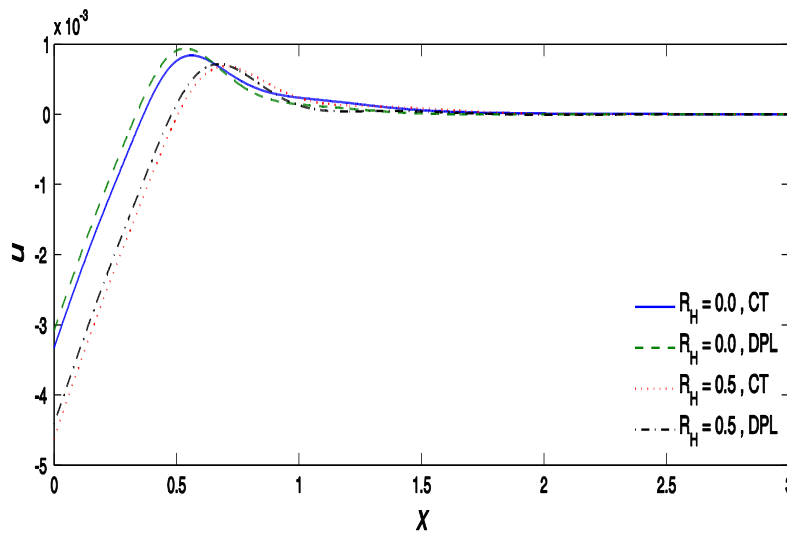

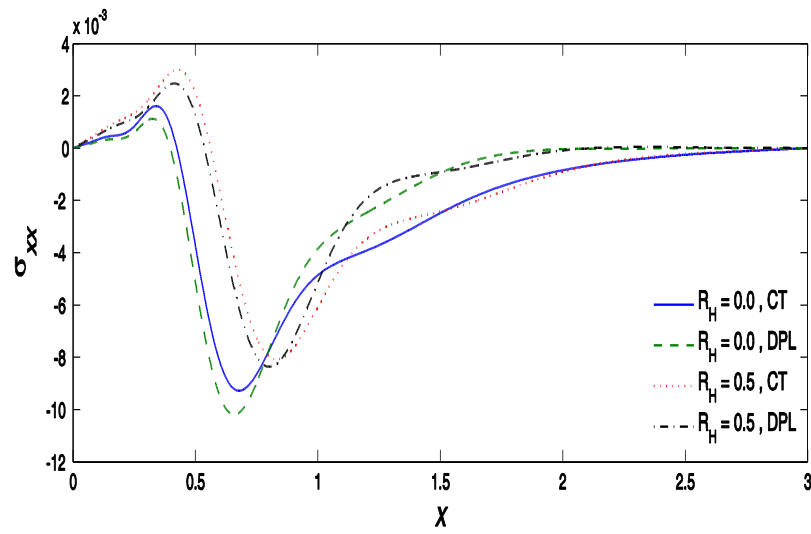**Fig.Figure. 6. D**The displacement distribution for different theories and different values of R_H.

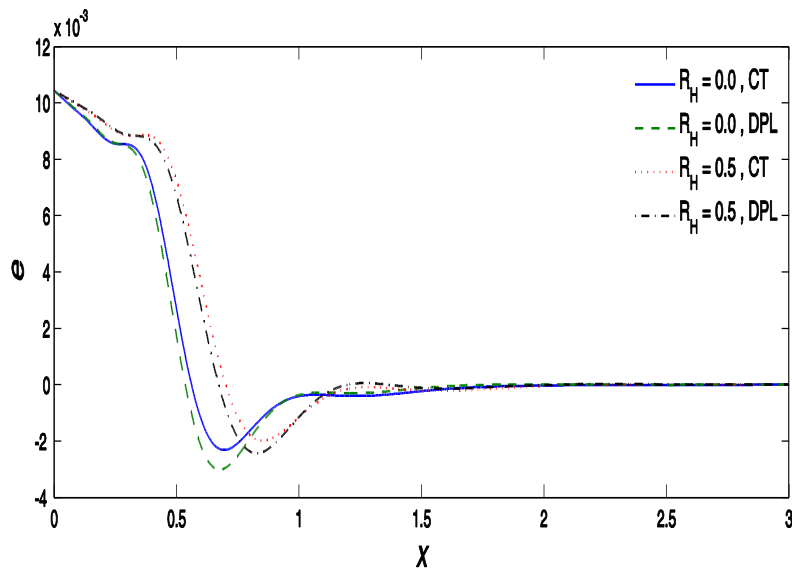**Fig.Figure. 7. SThe s**tress distribution for different theories and different value**s** of R_H .

Fig.Figure. 8. SThe strain distribution for different theories and different value**s** of R_H .