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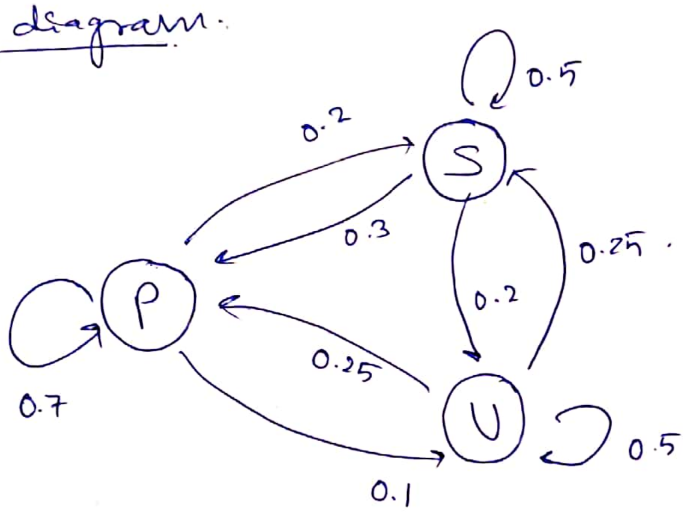
2018114008.

Assignment 5

Q1

let state P = professional, S = skilled, U = unskilled.

State diagram.



derived from

Transition matrix.

$P_{xy} \Rightarrow$

	P	S	U
P	0.7	0.2	0.1
S	0.3	0.5	0.2
U	0.25	0.25	0.5

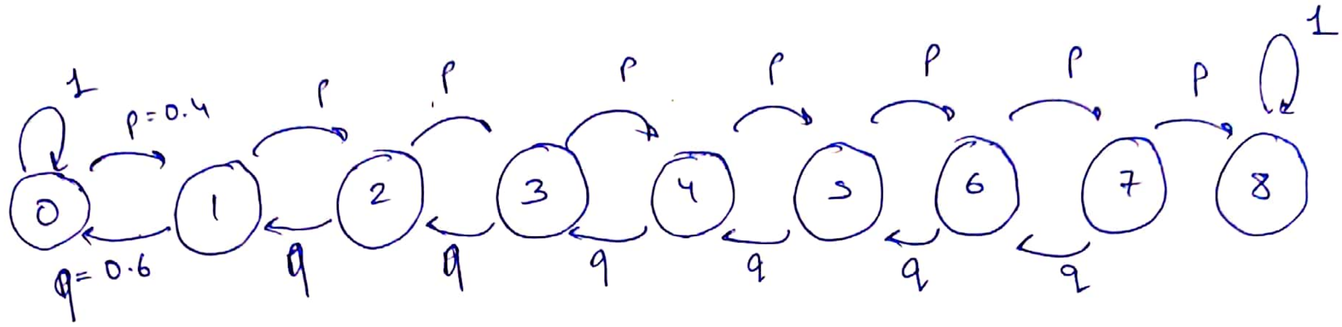
$$r_{U\to P}(2\text{ steps}) = r_{US}(1)\cdot r_{SP}(1) + r_{UU}(1) * r_{UP}(1) + r_{UP}(1) \times r_{PP}(1)$$

$$= 0.25\times 0.3 + 0.5\times 025 + 0.25 \times 0.7$$

$$= 0.375.$$

Q2

let state i represent money i x 100 (① represents Rs.100).



a_i represents absorption probability of state i.

To win, $a_8 = 1$, $a_0 = 0$.

For all states,

$$a_x = a_{x-\cancel{100}}\, 0.6 + a_{x+\cancel{100}}\, 0.4.$$

$$\Rightarrow (a_x - a_{x-1})\, 0.6 = 0.4\,(a_{x+1} - a_x)$$

let $a_x - a_{x-1} = S_x$.

$$\because S_x = \frac{0.4}{0.6} S_{x+1} \Rightarrow S_k = \left(\frac{0.4}{0.6}\right)^{-k} S_0. \text{ AND } \underbrace{\sum_{i=1}^{8} S_i = 1}_{\text{basic total probability}}$$

$$\Rightarrow \sum_{k=1}^{8} \left(\frac{0.4}{0.6}\right)^{-k} S_0 = 1 \Rightarrow S_0 = \frac{1}{\sum_{k=1}^{8} \left(\frac{0.4}{0.6}\right)^{-k}}$$

2.1 Probability of winning when start = 300 ruppees (a_3).

$$= P(a_3) = S_3 + S_2 + S_1$$

$$= \frac{\sum_{i=1}^{3} \left(\frac{0.6}{0.4}\right)^{i}}{\sum_{i=1}^{8} \left(\frac{0.6}{0.4}\right)^{i}} \Rightarrow \frac{(1.5)^3 - 1}{(1.5)^8 - 1} \simeq \cancel{0.0}\ \frac{2.375}{24.62890} = 0.0964.$$

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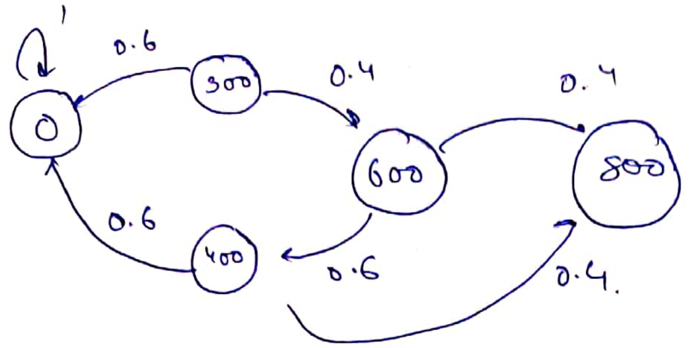
2.2 Let each state represent amount of money.



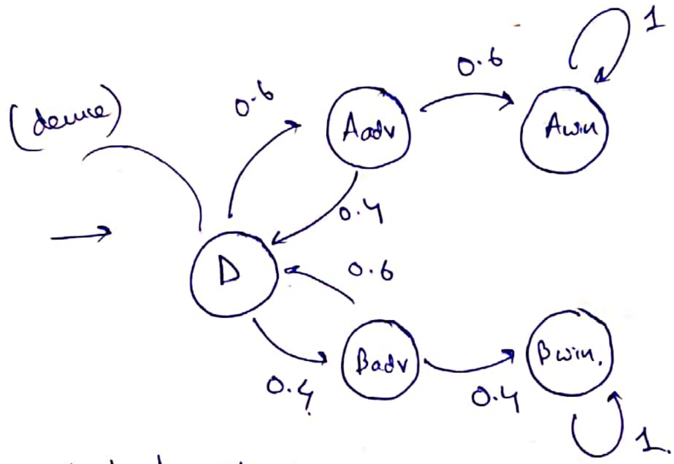
Answer = P(reaching 800 | initial state = 300)

$= P_{300,600}(1) * P_{600,800}(1) + P_{300,600}(1) \times P_{600,400}(1) \times P_{400,800}(1)$.

$= 0.4 \times 0.4 + 0.4 \times 0.6 \times 0.4 = \cancel{2.4}\ 0.16 + 0.096 = 0.256$.

Q3

State diagram →



a_i = absorption probab of state i.

$a_D = 0.6 A_a + 0.4 a_{Ba}$ — ①.

$a_{Aa} = 0.6 a_{Aw} + 0.4 D$ — ②.

$a_{Ba} = 0.4 a_{Bw} + 0.6 D$ — ③

from ①,②,③ → $a_D = 0.48 a_D + 0.36 a_{Aw} + 0.16 a_{Bw}$.

Hence, $= \frac{0.36 a_{Aw} + 0.16 a_{Bw}}{0.52 a_D}$.

P(A wins from D) = when $a_{Aw} = 1, a_{Bw} = 0$ $= \frac{0.36}{0.52} = 9/13$

P(B wins from D) = 1 − P(A wins for D) = 1 − 9/13 = 4/13, Q3.2

Let U_i be the expected length of game at state i.

$\because$ Game ends when $U_{Awin} = 0$, $U_{Bwin} = 0$.

To find $\rightarrow$ U_{deuce}.

$$U_D = 0.6(1 + U_{Aa}) + 0.4(1 + U_{Ba}).$$

$$U_{Aa} \downarrow 0.6(U_{Aw} + 1) + 0.4(U_D + 1) \qquad U_{Ba} \downarrow 0.4(U_{Bw} + 1) + 0.6(U_D + 1).$$

$$\therefore U_D = 0.6(1 + 0.6 + 0.4(U_D + 1)) + 0.4(1 + 0.4 + 0.6(U_D + 1)).$$

$$\Rightarrow U_D = 2 + 0.48 U_D.$$

$$\Rightarrow U_D = \frac{2}{0.52} = 3.84.$$