AEROSP 584: Homework 1

Due Date: September 18, 2024

Instructions

i) **Homework Preparation and Submission**:

- Refer to the homework preparation and uploading guidelines in the syllabus before starting your assignments.
- For Homeworks, you may choose to work in teams of up to 2 students, but each student must submit their homework individually.
- The goal of team collaboration is to enhance learning through idea exchange. Each team member should contribute equally to all problems and gain a deep understanding of the material.
- Submit all assignments via Canvas in a zip folder. The folder should contain:
  - A single PDF file that includes your solutions and, if applicable, a copy-paste of your MATLAB code.
  - The corresponding `.m` files of your MATLAB code.
- Name your zip file according to the following format:
  - If working individually: `MyName-584-HW-Number`.
  - If working in a team: `MyName-Team-TeamNumber-584-HW-Number`.
- For project submissions, name your zip file: `MyName-584-PR-Number`.
- Ensure that all submissions are neat and professional. Consider using LaTeX for typing, rulers for diagrams, and avoid crossouts. Highlight final answers with a box.
- Use the course's notation and terminology consistently in your work.

ii) **Matlab and Simulink Code**:

- Include all your MATLAB and Simulink code in the same zip file as your PDF.
- Ensure the PDF contains a copy-paste of your code.

iii) **MATLAB's `ode45`**:

- This homework requires the use of MATLAB's `ode45` function. An example file has been provided in the `Matlab material` folder on the course Canvas page.

iv) **Honor Code**:

- Review and adhere to the honor code guidelines outlined in the syllabus.

NOTE:

The dynamics notes refers to the "Geometry, kinematics, statics, and dynamics" book in the Canvas "Useful Books" folder

Problem 1 (10 points)

Let $\vec{x}(t)$ be a physical vector and let F_A be a frame. Show that, if $|\vec{x}(t)|$ is constant, then $\vec{x}(t)$ and $\overset{A\cdot}{\vec{x}}(t)$ are mutually orthogonal. (Note: This is in the notes somewhere, but try to do it on your own.)

Problem 2 (10 points)

Derive double transport by differentiating transport. (Note: This is in the notes somewhere, but try to do it on your own.)

Problem 3 (10 points)

Problem 4.17.11 in the dynamics notes.

Problem 4 (10 points)

Problem 4.18.1 in the dynamics notes.

Problem 5 (10 points)

Problem 7.12.1 in the dynamics notes.

Problem 6 (10 points)

Problem 7.12.6 in the dynamics notes.

Problem 7 (30 points)

This problem studies the relationship between the Euler-angle derivatives and the angular velocity vector. Let frame F_D be obtained by applying a 3-2-1 sequence of Euler-angle rotations to frame F_A, where ψ, θ, and ϕ denote the azimuth, elevation, and bank angles, respectively. The frames F_D and F_A are thus related by

$$F_A \xrightarrow[3]{\psi} F_B \xrightarrow[2]{\theta} F_C \xrightarrow[1]{\phi} F_D \tag{1}$$

Let the angular velocity of F_D relative to F_A evaluated at F_D be given by

$$\vec{\omega}_{D|A}\Big|_D = \begin{bmatrix} \cos 2t \\ \cos 2t \\ 0.025t \end{bmatrix}, \tag{2}$$

where t is the time in seconds. For all $t \in \{0, 0.01, 0.02, \ldots, 9.99, 10\}$ s, you will derive the corresponding Euler angles $\phi(t)$, $\theta(t)$, $\psi(t)$, and the components from the orientation matrix $\Theta_{\mathrm{D—A}}(t)$ in the case where $\phi(0) = \theta(0) = \psi(0) = 0$ rad. You'll need to use `ode45`. An example has been posted on the course Canvas page.

i) Integrate equation (4.10.10) from the dynamics notes to obtain $\phi(t)$, $\theta(t)$, $\psi(t)$, and use those angles to compute $\Theta_{D|A}(t)$ (see equations (2.13.32) and (2.15.3)). Plot the components of $\vec{\omega}_{\mathrm{D—A}}\Big|_{\mathrm{D}}$ versus time, and plot $\phi(t)$, $\theta(t)$, $\psi(t)$ versus time in another figure. Finally, plot all components from the orientation matrix $\Theta_{D|A}(t)$ versus time in a 3-by-3 figure grid using the `subplot` function in another figure.

ii) Integrate Poisson's equation (4.3.19) from the dynamics notes to obtain $\Theta_{D|A}(t)$. Plot all components from the orientation matrix $\Theta_{D|A}(t)$ obtained here (regular blue line) and the ones obtained in the a) (dashed orange line) versus time in a 3-by-3 figure grid using the `subplot` function. Do both coincide? **NOTE:** The example in the Canvas page shows how to integrate a matrix differential equation.

iii) Use the orientation matrices $\Theta_{D|A}(t)$ obtained in a) and b) to obtain two sets of Euler angles $\phi(t)$, $\theta(t)$, $\psi(t)$ by using `asin`, `acos`, and `atan2` functions. Plot $\phi(t)$, $\theta(t)$, $\psi(t)$ versus time. Use regular blue lines for the Euler angles obtained in the a), dashed orange lines for the angles obtained here using $\Theta_{D|A}(t)$ obtained in a), and dash-dot yellow lines for the angles obtained here using $\Theta_{D|A}(t)$ obtained in b). Do all Euler angles coincide?