AEROSP 584: Homework 4

Due Date: November 15, 2024

Instructions

i) **Homework Preparation and Submission**:

- Refer to the homework preparation and uploading guidelines in the syllabus before starting your assignments.
- For Homeworks, you may choose to work in teams of up to 2 students, but each student must submit their homework individually.
- The goal of team collaboration is to enhance learning through idea exchange. Each team member should contribute equally to all problems and gain a deep understanding of the material.
- Submit all assignments via Canvas in a zip folder. The folder should contain:
  - A single PDF file that includes your solutions and, if applicable, a copy-paste of your MATLAB code.
  - The corresponding `.m` files of your MATLAB code.
- In addition to the zip file mentioned above, please also upload the single PDF file separately, which is already included in the zip file. Hence your submission will be 1) a zip file and 2) a pdf file.
- Name your zip file according to the following format:
  - If working individually: `MyName-584-HW-Number`.
  - If working in a team: `MyName-Team-TeamNumber-584-HW-Number`.
- For project submissions, name your zip file: `MyName-584-PR-Number`.
- Ensure that all submissions are neat and professional. Consider using LaTeX for typing, rulers for diagrams, and avoid crossouts. Highlight final answers with a box.
- Use the course's notation and terminology consistently in your work.

ii) **Matlab and Simulink Code**:

- Include all your MATLAB and Simulink code in the same zip file as your PDF.
- Ensure the PDF contains a copy-paste of your code.

iii) **Honor Code**:

- Review and adhere to the honor code guidelines outlined in the syllabus.

Problem 1 (25 points). Position fixing using the Kalman Filter Let L_1, L_2, L_3 be three beacons and let P be your position. Let $\mathrm{F_A}$ be a frame and w be a point such that $\vec{r}_{P/w}\big|_{\mathrm{A}} = \begin{bmatrix}0.7212 & 2.4080 & 0\end{bmatrix}^{\mathrm{T}}$ and, for all $i \in \{1, 2, 3\}$,

$$\vec{r}_{L_i/w}\big|_{\mathrm{A}} = \begin{bmatrix} x_i \\ y_i \\ 0 \end{bmatrix} \mathrm{m}, \tag{1}$$

where $x_i, y_i \in \mathbb{R}$. Furthermore, for all $i \in \{1, 2, 3\}$, let the 2D distance from the beacons L_i to P be r_i, such that $r_i = \sqrt{(0.7212 - x_i)^2 + (2.4080 - y_i)^2}$. Suppose that $(x_1, y_1) = (0, 0)$ m, $(x_2, y_2) = (5, 5)$ m, and $(x_3, y_3) = (2.5, 0)$ m, Thus, it follows that $r_1 = 2.5$ m, $r_2 = 5$ m, $r_3 = 3$ m approximately. Furthermore, suppose that, for all $i \in \{1, 2, 3\}$, noisy 2D distance measurements from beacon L_i are available every one second. That is, for all $k > 0$,

$$Y_k = \begin{bmatrix} r_1 \\ r_2 \\ r_3 \end{bmatrix} + D w_k, \tag{2}$$

where $D = \mathrm{diag}(0.1, 0.1, 0.1)$ and $w_k \sim \mathcal{N}(0, I_3)$.

We will use the Kalman Filter to obtain an estimate of $\vec{r}_{P/w}\big|_{\mathrm{A}}$, which we will refer to as $\vec{r}_{\hat{P}/w}\big|_{\mathrm{A}}$ In order to do so, define, for all $k \geq 0$, $\hat{X}_k = \begin{bmatrix}\hat{x}_k & \hat{y}_k\end{bmatrix}^{\mathrm{T}}$, such that $\vec{r}_{\hat{P}/w,k}\big|_{\mathrm{A}} = \begin{bmatrix}\hat{x}_k & \hat{y}_k & 0\end{bmatrix}^{\mathrm{T}}$. For all $k \geq 0$, the dynamics model that will be used in the Kalman Filter is given by $\hat{X}_{k+1} = \hat{X}_k$. Furthermore, for all $k \geq 0$, the measurement model is given by

$$g(\hat{X}_k) = \begin{bmatrix} \sqrt{(\hat{x}_k - x_1)^2 + (\hat{y}_k - y_1)^2} \\ \sqrt{(\hat{x}_k - x_2)^2 + (\hat{y}_k - y_2)^2} \\ \sqrt{(\hat{x}_k - x_3)^2 + (\hat{y}_k - y_3)^2} \end{bmatrix}. \tag{3}$$

a) Write the Kalman filter equations to obtain an estimate of x_k.

b) Let $Q = 0$ and $R = 0.1$*eye(3), and suppose that $\hat{X}_0 = \begin{bmatrix}4 & 4\end{bmatrix}^{\mathrm{T}}$ m. For all $p_0 \in \{0.01, 0.1, 1, 10\}$ such that $P_{0|0} = p_0$*eye(2), obtain the estimates $\hat{X}_k$ for all $k \in \{0, 1, \ldots, 50\}$. In one figure, plot the 2D trajectory $\hat{X}_k$ for all $k \in \{0, 1, \ldots, 50\}$, for each value of p_0 and place a red point on the position of P. In another figure, use semilogy to plot the frobenius norm of $P_{k|k}$ versus k for all $k \in \{0, 1, \ldots, 50\}$.

Problem 2 (50 points). Let $g = 9.80665$ m/s^2 be the acceleration due to gravity, and let $\phi = \pi/6$ rad. Suppose that a 3-axis accelerometer and a 3-axis rate gyro are attached to a quadcopter following an inclined, circular trajectory. Let $\mathrm{F_A}$ be an inertial frame, let $\mathrm{F_B}$ be a frame fixed to the quadcopter, and suppose that the axes of both the rate gyro and the accelerometer are aligned with $\mathrm{F_B}$. Let c be the center of mass of the quadcopter, let w be a point with zero inertial acceleration, and let $\vec{r}_{c/w}(t)$ and $\mathcal{O}_{\mathrm{B/A}}(t)$ be the position vector of the quadcopter center of mass and the orientation matrix of $\mathrm{F_B}$ relative to $\mathrm{F_A}$ at time t, respectively. Furthermore, suppose that $g_A = \begin{bmatrix} 0 & 0 & -g \end{bmatrix}$ m/s^2 and, for all $k \in \{0, 1, \ldots, 2000\}$, the measurements from the sensors are given by

$$\omega_k = \begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix} + D_1 w_{1,k} \quad \text{rad}, \tag{4}$$

$$a_k = \begin{bmatrix} -1 - g \sin\phi \sin kT \\ -g \sin\phi \cos kT \\ -g \cos\phi \end{bmatrix} + D_2 w_{2,k} \quad \text{m/s}^2, \tag{5}$$

where $T = 0.01$ s is the sampling rate of these sensors, $D_1 = D_2 = \mathrm{diag}(0.1, 0.1, 0.1)$, $w_{1,k} \sim \mathcal{N}(0, I_3)$, and $w_{2,k} \sim \mathcal{N}(0, I_3)$. Furthermore, suppose that

$$\vec{r}_{c/w}\big|_{\mathrm{A}}(0) = \begin{bmatrix} 1 \\ 0 \\ 0 \end{bmatrix} \text{m}, \tag{6}$$

$$\overset{\mathrm{A}\bullet}{\vec{r}}_{c/w}\big|_{\mathrm{A}}(0) = \begin{bmatrix} 0 \\ \cos\phi \\ \sin\phi \end{bmatrix} \text{m/s}, \quad \mathcal{O}_{\mathrm{B/A}}(0) = \begin{bmatrix} 1 & 0 & 0 \\ 0 & \cos\phi & \sin\phi \\ 0 & -\sin\phi & \cos\phi \end{bmatrix}. \tag{7}$$

a) For all $k \in \{0, 1, \ldots, 2000\}$, obtain the estimates $\vec{r}_{c,w}\big|_{\mathrm{A}}(kT)$ via a Kalman filter, using only the rate-gyro and accelerometer measurements. Compare it against the trajectory in the rcwA.mat file by plotting both this reference trajectory and the estimated trajectory in a 3D plot.

b) For all $k \in \{0, 1, \ldots, 2000\}$, let $y_{\mathrm{meas},k} \in \mathbb{R}^3$ be the position vectors from the rcwA.mat file and suppose that these are sampled every T_{MOCAP} seconds to obtain noisy position measurements $y_k \in \mathbb{R}^3$, such that, for all $k \in \{0, 1, \ldots, 2000\}$ such that $\mathrm{mod}(kT, T_{\mathrm{MOCAP}}) = 0$,

$$y_k = y_{\mathrm{meas},k} + D_3 w_{3,k}, \tag{8}$$

where $D_3 = \mathrm{diag}(0.005, 0.005, 0.005)$, and $w_{3,k} \sim \mathcal{N}(0, I_3)$. Use the position measurements in your Kalman filter with $R = 0.001 I_3$, $Q = 10 I_6$, and $P_{0|0} = 10 I_6$. Do this for $T_{\mathrm{MOCAP}} = 1$ and $T_{\mathrm{MOCAP}} = 0.1$ s. In one figure, use a 3D plot to plot the trajectory in the rcwA.mat file and the estimated trajectories obtained for $T_{\mathrm{MOCAP}} = 1$ and $T_{\mathrm{MOCAP}} = 0.1$ s. In another figure, plot the 3 components of the trajectory in the rcwA.mat file and the estimated trajectories obtained for $T_{\mathrm{MOCAP}} = 1$ and $T_{\mathrm{MOCAP}} = 0.1$ s versus time in a 3-by-1 figure grid using the subplot function.

NOTE: You will find the rewA.mat file in the folder "Matlab material" posted on Canvas.

Problem 3 (15 points). Reading Assignment and Question Submission

Breanna Johnson, an expert in entry guidance and control systems working at NASA Johnson Space Center, will be our invited speaker on December 3rd, 2024. To prepare for her talk, please read the following three papers:

i) **Entry Guidance: A Unified Method**, Ping Lu.
https://arc.aiaa.org/doi/10.2514/1.62605

ii) **Mid-Lift-to-Drag Ratio Rigid Vehicle Control System Design and Simulation for Human Mars Entry**, Breanna Johnson et al.
https://arc.aiaa.org/doi/epdf/10.2514/6.2018-0615

iii) **Pterodactyl: Development and Performance of Guidance Algorithms for a Mechanically Deployed Entry Vehicle**, Breanna Johnson et al.
https://arc.aiaa.org/doi/10.2514/6.2020-1011

For each paper, write a brief summary (max half a page per paper) highlighting the main contributions, methodologies, and findings. Your summaries should demonstrate a clear understanding of the material.

Additionally, prepare at least one thoughtful question for Breanna Johnson related to the content of these papers. Your question should reflect critical thinking and may pertain to areas such as challenges faced, potential applications, or implications of the research.

Submission Instructions:

- Include your summaries and question(s) in your homework submission.
- Ensure that your work is original and written in your own words.
- Cite any additional references you use.