Problem 1 (7 points):

The function `aerialVehSim.p` simulates the dynamics of a small aerial vehicle. As illustrated in the attached sample code `aerialVehSim_Example.m`, given an initial state $\xi(k) \in \mathbb{R}^n$, input $u(k) \in \mathbb{R}^m$ and time interval T_s, the function outputs the state of the vehicle after time interval T_s, that is, $\xi(k+1)$. The state and input vectors are given as

$$\xi = \begin{bmatrix} x & y & z & \dot{x} & \dot{y} & \dot{z} & \phi & \theta & \psi & \dot{\phi} & \dot{\theta} & \dot{\psi} \end{bmatrix}^T, \quad u = \begin{bmatrix} u_1 & u_2 & u_3 & u_4 \end{bmatrix}^T$$

where x, y and z represent the vehicles position along the $x-$, $y-$ and $z-$axis respectively; ϕ, θ and ψ are the roll, pitch and yaw angles respectively; and u_1, u_2, u_3 and u_4 are the upward force, pitch torque, roll torque and yaw torque respectively.

(a) Using $T_s = 0.1s$, learn a discrete-time quadratic regulator (controller) for the system that satisfies the typical cost

$$\mathcal{J} := \sum_{k=0}^{\infty} \left(\xi(k)^T Q \xi(k) + u(k)^T R u(k) \right) \tag{1}$$

where $Q = 10I_n$ and $R = I_m$, I_n and I_m being identity matrices of appropriate dimensions. For initial conditions $\xi(0) = \begin{bmatrix} 1 & -1 & 0 & 0.5 & -1 & 1 & 0 & 0 & 0 & 0 & 0 & 0 \end{bmatrix}^T$, verify that your controller is indeed a regulator (that is, it drives the system state to zero). Use a simulation time of 10s. Using the `subplot` on MATLAB or otherwise, plot the positions x, y, and z against time on one figure, and the velocities $\dot{x}$, $\dot{y}$, and $\dot{z}$ against time on a separate figure. Include your codes in your submission.

(b) Learn a quadratic regulator for the system where only the positions x, y, and z are weighted in the cost of Eq. (1). The weights on the inputs remain a tenth of that on the positions. Deploy this controller to allow the vehicle execute a spiral maneuver where

$$x = 0.1 \sin \frac{t}{2}$$

$$y = 0.1 \cos \frac{t}{2}$$

$$z = 0.1t$$

for a total simulation time of 15s. Using MATLAB's `plot3` or otherwise, plot the desired maneuver trajectory and the actual trajectory executed by the vehicle in 3D (plot z against x and y).

Problem 2 (3 points):

You and your colleague are trying to identify a stable system by collecting *step response* data. Without having any means to visually examine the system output, your colleague has assumed that $30s$ worth of output data is sufficient. The collected data is given in `System_step_response.mat`. Examine (plot) the data, identify the system (and its model parameters), and superimpose a plot of the model's step response on the data. Justify your process.