Problem 1 (5.5 points):

In this problem, we will come up with the least squares formulation for determining the parameters of the general transfer function

$$G(s) = \left(\frac{b_0 + b_1 s + b_2 s^2 + \cdots + b_m s^m}{a_0 + a_1 s + a_2 s^2 + \cdots + s^n} \right) \tag{1}$$

from the frequency response $G(j\omega)$ collected from an experiment.

(a) Let $G(j\omega) = (G_R + jG_{IM})$. By cross-multiplying in (1), we have:

$$\left(a_0 + a_1(j\omega) + a_2(j\omega)^2 + \cdots + (j\omega)^n\right)(G_R + jG_{IM}) = \left(b_0 + b_1 j\omega + b_2(j\omega)^2 + \cdots + b_m(j\omega)^m\right) \tag{2}$$

Noting that

$$(j\omega)^p = \begin{cases} (-1)^{p/2}\omega^p & \text{if } p \text{ is even} \\ j(-1)^{(p-1)/2}\omega^p & \text{if } p \text{ is odd} \end{cases}$$

expand (2) to separate out its real and imaginary parts. Write the real and imaginary parts in (two) separate equations.

(b) Stack up these equations at various frequencies from ω_1 to ω_k to express as a least squares problem and write out the expression for the solution.

(c) Write a MATLAB script that can be used for solving the above problem (the least squares problem).

(d) Now, given the magnitude and phase data of a system at different frequencies (in rad/s) in the file `freq_resp_data.mat`; find the model $G(s)$ of the system using your MATLAB code. Plot the data on the corresponding bode plot of the system approximated by the least squares solution.

Problem 2 (4.5 points):

(a) Consider Problem 1 in Homework 4. Identify the system embedded in the file `piezoNozzle.p`. Write out your model. Verify the performance of your model using the input `V_example.mat`. Attempt this problem *using* and *without using* MATLAB System ID Toolbox and compare results. (Hint: The actual system order lies between 5^{th} and 10^{th} order).

(b) *Based on your model*, find what voltage signal $V^*(t)$ we should send in to generate the desired pressure wave P_{ref} given in the file `P_ref.mat`. Plot $V^*(t)$ against time. Also plot the signal $\mathcal{G}(V^*(t))$, where $\mathcal{G}$ denotes the *actual system* represented by the file `piezoNozzle.p` superimposed on a plot of P_{ref}. Briefly explain your process.