GAME THEORY

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GAME THEORY

Game theory is the study of how and why individuals and entities (called players) make decisions about their situations. Game theory is used in a variety of fields to lay out various situations and predict their most likely outcomes. Businesses may use it, for example, to set prices, decide whether to acquire another firm, and determine how to handle a lawsuit.

NASH EQUILIBRIUM

The Nash equilibrium is a fundamental concept in game theory, named after the mathematician John Nash. It is an outcome reached that, once achieved, means no player can increase payoff by changing decisions unilaterally.In simple words occurs when no company is able to increase its profit by changing production level,OR a stable state of a game where each player's strategy is best response to the strategies of other players.

ECONOMIC MODELS

1.COURNOT DUOPOLYMODEL

2.BERTRAND DUOPOLY MODEL

COURNOT MODEL

- The Cournot model assumes an oligopoly, where a few firms dominate the market.
- Homogeneous Product: The firms produce identical or homogeneous products.
- Simultaneous Decision-Making: Firms choose their output levels simultaneously, without knowing the decisions of their rivals.
- No Price Competition: Firms compete on quantities, not prices. The market price is determined by the total quantity produced by all firms.

COURNOT NASH EQUILIBRIUM

Firm i simultaneously chooses a quantity of product q_i ; $q_i \geq 0$.

>>Prices set by market $q=q_1+q$.Output price=P

>>Marginal cost of production, $C_i \geq 0$

>>Maximising Profit $p(q)q_i-c_iq_i$

>>There are 3 possibilities

1)Firm1 knows the amount Firm2 is going to produce.

2)Firm2 knows the amount Firm1 is going to produce.

3)We find a pair of outputs that are mutual best responses.

We get $q_2=a-q_1-c_2/2$ $\quad$ $q_1=a-q_2-c_1/2$

Deriving the equilibrium we get

$q_1=a+c_2-2c_1/3$

BERTRAND DUOPOLY MODEL

The Bertrand model assumes that firms compete by setting prices for a homogeneous product, not by choosing quantities.The products offered by the firms are identical, meaning consumers will always buy from the firm offering the lower price.The model typically assumes that each firm can produce any quantity demanded at the price it sets, meaning there are no capacity constraints.

BERTRAND NASH EQUILIBRIUM

Firm i simultaneously chooses a quantity of product pi ;pi>=0.

>>FIRMS have symmetrical margin cost of production c>=0;

For equilibrium

The only equilibrium prices are p1=p2=c

Firm's 1 profit =a(p1-c);where a is probability that consumer buys from Firm1.

>>Po-c>a(p1-c)

GAMES CODED

1.ROULETT SPIN

RULES

Each player starts with the option to contribute a certain amount to the pot.
No.of players are predecided before starting of the game.
A random player is selected as the winner, with each player having an equal chance of winning.
The winner receives their calculated payout from the pot.

MATHEMATICS INVOLVED

The pot value is calculated using the formula:
Pot value=Base value/1+b*total contribution ; b is a constant that determines the rate of diminishing returns.
Probability of winning for all the players is equal.
Player’s Payout=(Player’s Contribution/Total contribution)*pot value
Diminishing Returns:
The pot value decreases as total ontributions increase, demonstrating diminishing returns. This reflects the idea that as more players contribute, the marginal value of each additional contribution decreases.

2.PRICE WAR SIMULATION

RULES

Each player in the game represents a firm. Multiple firms (players) compete against each other in a market.Each firm sets a price for their product. The price must be at least equal to the marginal cost of production (i.e., the minimum price a firm can charge without making a loss).

The firm inputs its price during the game.If a single firm sets the lowest price, it captures the entire market (100% market share).

If multiple firms set the same lowest price, the market share is divided equally among them.

MATHEMATICS INVOLVED

Profit=(Price−arginal Cost)×Market Share×Market Size.

$P_{min}=\min(P_1,P_2,\ldots,P_n)$

P_{min} is the lowest price set by any firm.

profit(Π_i)of each firm $\Pi_i=(P_i-C)\times S_i\times Q$

- P_i is the price set by the firm.
- C is the marginal cost of production.
- S_i is the market share of the firm.
- Q is the total market size, assumed to be 1000 units in this game.
- Winner=$\max(\Pi_1,\Pi_2,\ldots,\Pi_n)$

The firm with the highest Π_i is the winner.

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PRICE WAR SIMULATION OUTCOMES

PRICE CONVERGENCE TO MARGINAL COSTS-This outcome is a classic result of the Bertrand model. If firms have identical costs and compete by setting prices, the competition can drive prices down to the marginal cost. In this scenario, firms earn zero economic profit, as the price they charge just covers the cost of production.

PRICE ABOVE MARGINAL COST-Some firms set prices above the marginal cost, while others undercut them to capture the market.This outcome shows that firms may try to maintain some pricing power by setting higher prices, but they risk losing market share to competitors willing to undercut them.

PRICE WAR-Firms continuously lower their prices in response to competitors, potentially driving prices to the marginal cost level.

UNDERSTANDING DYNAMICS-The experiment shows how competitive pricing strategies impact market outcomes. Firms need to carefully consider their pricing strategies in competitive environments.

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ROULETT SPIN OUTCOMES

PLAYER BETS -Each player places a bet on a specific outcome (e.g., even, odd, red, or green).
Significance: Players choose their bets based on their risk preferences and potential payouts. Betting on even or odd numbers might have a lower payout but higher probability, while betting on a specific color column could have a higher payout with a lower probability.

Random Number Generation (Spin)-Random number is generated, simulating the spin of a roulette wheel.
The number determines the winning outcome for that round. This step introduces randomness and ensures that each possible outcome (even, odd, red, green) has a certain probability of occurring.

PAYOUTS BASED ON BETS-Players who bet correctly win a payout based on the risk of their bet.
Even/Odd Bet: If a player bets on even or odd and wins, the payout is lower but the probability of winning is higher.

The game determines which player has won based on the random spin and their bet .If multiple players bet on the same winning outcome, they all receive their respective payouts.
If no player bet on the winning outcome, no payouts are made, and the game proceeds to the next round.
The randomness of the outcome ensures that all players have an equal chance of winning, regardless of their contribution or bet amount. This random element is what makes the game unpredictable and engaging.

BEST STRATEGY TO PLAY

ROULETT SPIN

If All Players Play Optimally

When players are playing optimally, they focus on maximizing their expected value while minimizing risk. In the context of a Roulette game:

Bet on Outcomes with Higher Probability: In a typical Roulette game, betting on outcomes with higher probabilities, such as even or odd numbers, red or black colors, can be considered optimal. These bets generally cover a large portion of the possible outcomes and have nearly a 50% chance of winning (ignoring the house edge like the green zero in traditional roulette).

Avoid Risky Bets with Low Probability: Betting on specific numbers or combinations, which have a much lower probability of winning (e.g., 1 in 37 for a single number in European Roulette), is less optimal from a purely mathematical standpoint. The high payout (35:1) does not fully compensate for the low probability of winning.

If All Players Don't Play Optimally

Chasing Losses (Martingale Strategy): A common but risky strategy is the Martingale system, where a player doubles their bet after every loss to recover losses with a single win.

High Risk of Ruin: The Martingale strategy can lead to massive losses if a losing streak occurs, especially since many casinos have table limits that prevent endless doubling.

Betting on Specific Numbers: Some players may bet on specific numbers or combinations due to personal beliefs, lucky numbers, or perceived patterns. These bets have very low probabilities of winning and are not optimal from a mathematical standpoint.

High Variance: This approach results in high variance, where the player might occasionally win big but will more often lose, leading to an overall negative expected value.

BEST STRATEGY TO PLAY

PRICE WAR SIMULATION

If All Players Play Optimally

In an optimal scenario, rational firms will anticipate that competitors will lower their prices to capture the market. As a result, they will eventually set their prices at or just above their marginal cost of production.This is the Bertrand equilibrium, where no firm can profitably undercut another without incurring losses. At this point, firms earn zero economic profit, as the price equals the marginal cost

Mathematical Justification:

Price (P) = Marginal Cost (MC):

When P = MC, firms are indifferent between increasing or decreasing prices because doing so would either reduce their market share (if they increase prices) or lead to losses (if they decrease prices below MC).

Profit (π) = (P - MC) * Quantity Sold (Q) = 0 at equilibrium.

If All Players Don’t Play Optimally

If competitors are not playing optimally, they may set prices above their marginal cost, leaving room for profit. The best strategy is to undercut them slightly to capture a significant portion of the market while still earning a profit above the marginal cost.

Mathematical Justification:

Price (P) > Marginal Cost (MC):

Assume competitors set prices P1 > MC. By setting a price P2 just below P1 but still above MC, the firm can capture the market and earn positive profits.

Profit (π) = (P2 - MC) * Quantity Sold (Q).

The risk is that competitors may drop their prices in response, leading to a potential price war.

CONCLUSION

This project explored two fundamental models of oligopolistic competition in game theory: The Cournot model and the Bertrand model.The project demonstrated the importance of strategic decision-making in oligopolistic markets and how different models can predict varying outcomes based on the nature of competition. Understanding these models is crucial for firms aiming to navigate competitive environments, policymakers seeking to regulate markets, and economists studying market dynamics.

Cournot Model:

Summary: In the Cournot model, firms compete by choosing quantities of output, assuming their competitors' output levels remain fixed. The market price is determined by the total output produced by all firms.

Bertrand Model:

Summary: In the Bertrand model, firms compete by setting prices for identical products, assuming their competitors' prices remain fixed. The firm offering the lowest price captures the entire market, or the market is shared among firms with the same lowest price.