FinEngg - Group Assignment

Group Assignment members:

Akshat Jain B20AI054

Ishaan Shrivastava B20AI013

Navlika Singh B20AI025

Swara Patel B20AI044

Choosing 3 Risky Assets

We choose to analyze the stocks of various major stock market indices such as the S&P500, NASDAQ Composite, etc.

From these indices, we then picked **3 stocks of the larger companies -** `["AAPL", "AMZN", "TSLA"]`

Larger companies are usually less risky than smaller companies, which can help reflect more modern investment practices followed today.

Large companies often have more liquidity in their stock, which means that there are more buyers and sellers in the market and it is easier to buy or sell shares quickly.

This can be particularly important when trading options, which require a buyer and a seller to agree on a price and execute a transaction, especially in the case of self-financing strategies where the option writer is using the money generated by the self-financing strategy to buy portions of the security. i.e., the initial investment is fully funded by the cash flows generated by the options positions themselves (this will be discussed in more detail later)

Additionally, data about these companies is more readily available than other stocks. We chose these companies for our analysis as the **options pricing data for these was publicly available** on Yahoo Finance.

CRR Binomial model - American & European Put Options

💡 We calculate the American and European Put Option prices for **3 different stocks** for **3 different values of Time of Maturity.** We vary N, the number of time steps from `2` to `10` for all the values of strike prices available for that particular Time of Maturity.

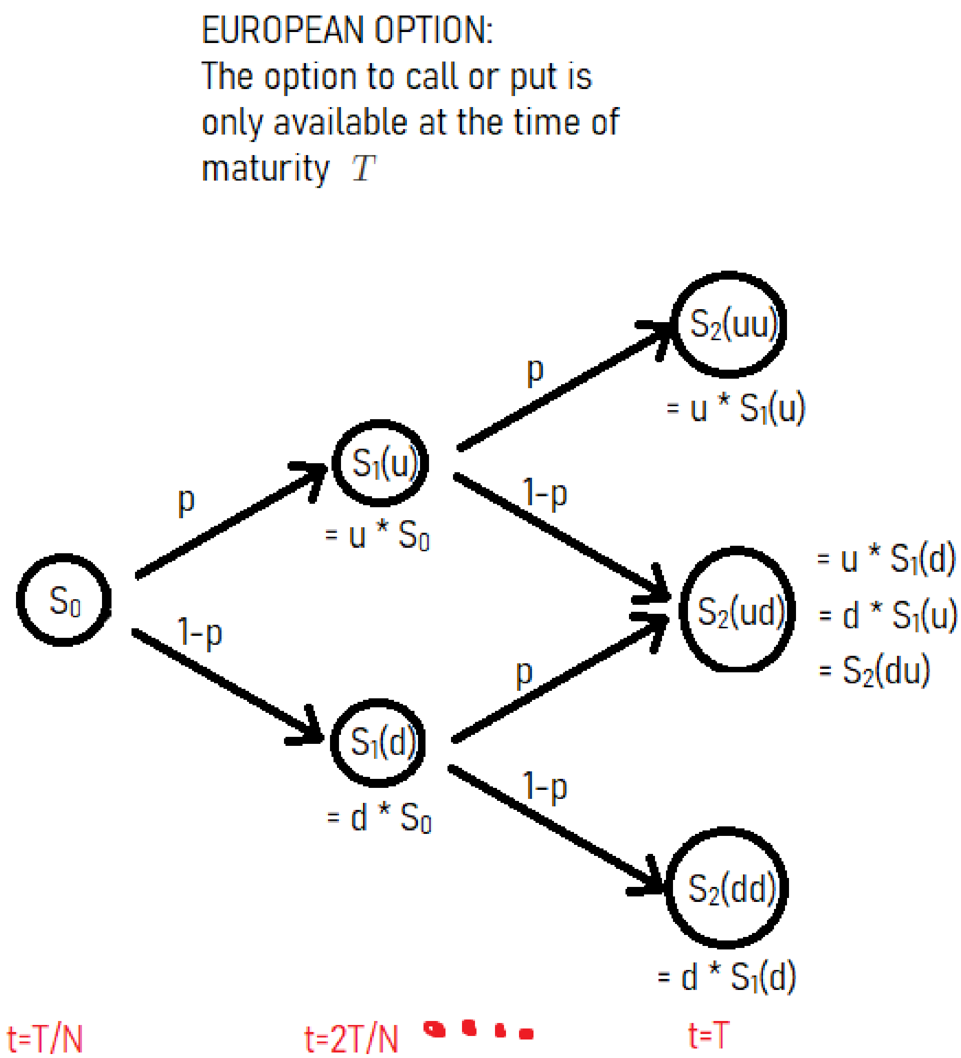
Using the Yahoo Finance API, we retrieve the Adjusted Close price for the past 1 year i.e. `27th April 2022` to `27th April 2023`, to calculate the volatility for each stock. Next we choose 3 different expiration dates for a Put option signed on `27th April 2023`, i.e. expiry dates = `[28th April 2023, 5th May 2023, 12th May 2023]`, resulting in 3 different times of expiration i.e., `1 day`, `8 days` & `15 days`.

The binomial model is a mathematical model used to calculate the theoretical price of an option, given a set of assumptions about the behavior of the underlying asset. It is based on the idea that the price of the underlying asset can move up or down by a certain amount over each time interval, with the probability of an up move and a down move being known. Here are the six steps of the binomial model for option pricing:

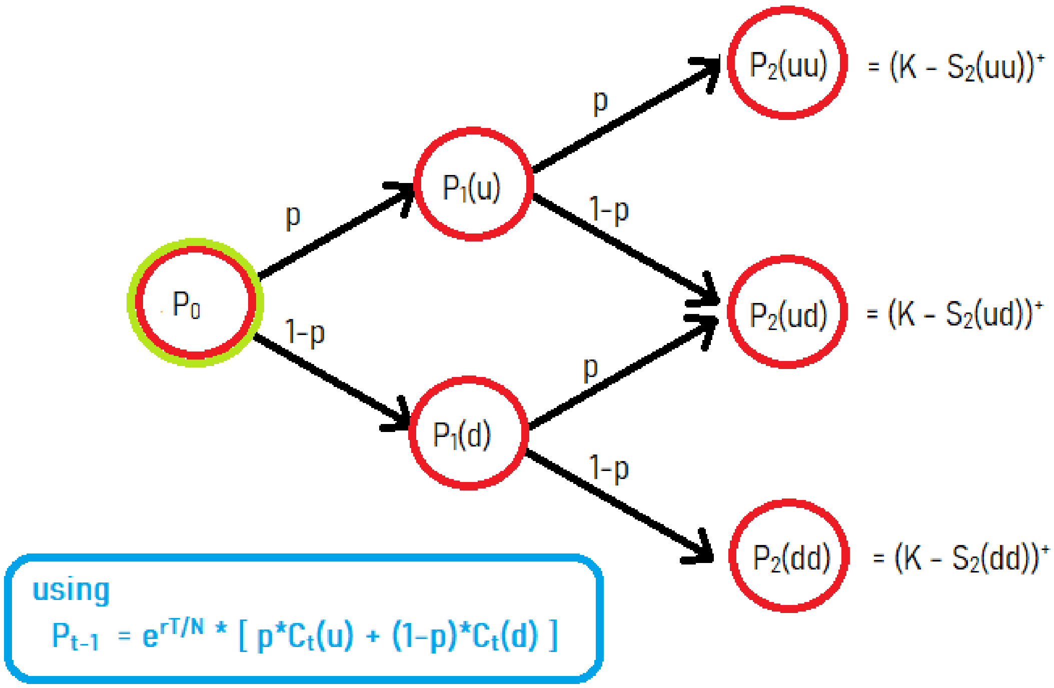
1. Divide the time to expiration into a number of equal intervals (or "steps").
2. Calculate the up and down factors for each step, based on the assumed volatility of the underlying asset.
3. Calculate the probability of an up move and a down move for each step, based on the up and down factors and the assumed risk-free rate.
4. Construct a binomial tree, representing the possible price paths of the underlying asset over time.
5. Starting at the final step of the tree, calculate the option payoff at each node (based on the difference between the strike price and the underlying asset price), and work backwards through the tree to calculate the option value at each node.
6. Finally, calculate the present value of the option by discounting the option value at time zero using the risk-free rate.

We have drawn a diagram to illustrate the various steps involved in calculating the values of the European put-options using the binomial model stock and put option price trees.

EUROPEAN STOCK PRICE TREE



PUT TREE



In the Cox-Ross-Rubinstein (CRR) model, the up and down factors for each time step in the binomial tree are calculated based on the assumed volatility of the underlying asset. Specifically, the up factor u is given by

$$u = e^{\sigma\sqrt{\Delta t}}, d = e^{-\sigma\sqrt{\Delta t}}$$

- where σ is the volatility of the underlying asset
- Δt is the length of the time step (i.e. the time to expiration divided by the number of steps). This equals $\dfrac{T}{N}$ in our implementation, where T is the time of maturity and N is the number of time steps we divide our model into.

This formula says that the up factor is equal to $e^{\sigma\sqrt{\Delta t}}$, which can be interpreted as the factor by which the underlying asset is expected to increase in price at each time step. The term $\sqrt{\Delta t}$appears because we are assuming that the volatility is an annualized value, and we need to adjust it for the length of the time step.

In the CRR model, it is **assumed that the volatility remains constant over time**, which is a simplifying assumption that allows for easier calculation of the up and down factors. However, in reality, volatility may change over time, which can affect the accuracy of the option pricing calculation.

Our code calculates the value of σ using past data, i.e. the standard deviation of returns. It takes as input the following parameters:

- `S`: the current stock price S_0
- `K`: the strike price K
- `T`: the time to maturity (in years) T
- `r`: the risk-free rate μ_{rf}
- `sigma`: the volatility σ
- `N`: the number of time steps N
- `option_type`: `"European"` or `"American"`

If the option is European, the value at the final time step is calculated based on the difference between the strike price and the stock price at that time step. If the option is American, the option value is calculated based on the maximum of the exercise value and the expected value of the option at the next time step.

American Put Option

The pricing formula for an American put option using the binomial option pricing model is:

$P = max(K - V(T), 0)$ where

- P is the price of the American put option
- V(T) is the value of the option at expiration
- K is the strike price

The value of the option at expiration, $V(T)$, can be calculated using the following recursive formula:

$V(i,j) = \max\left([K - S(i,j), (p \cdot V(i+1,j+1) + (1-p) \cdot V(i+1,j)) * e^{-r\Delta t}]\right)$ where

- $V(i,j)$ is the value of the option at the i-th time period and j-th price level of the underlying asset

- $S(i, j)$ is the price of the underlying asset at the i-th time period and j-th price level
- p is the probability of the asset price moving up in each time period
- r is the risk-free interest rate
- Δt is the length of each time period

The recursive formula calculates the expected value of the option at each time period and price level, taking into account the probability of the asset price moving up or down, and the discounted value of the option at the next time period.

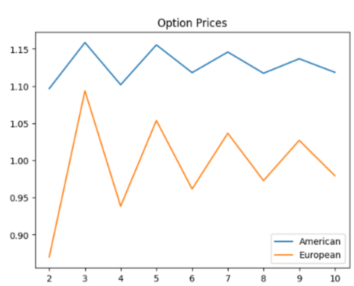
The value of the option at the current time period and price level can then be calculated as the maximum of the value at expiration and the expected value of the option at the next time period, minus the difference between the strike price and the current asset price.

American vs European Options

The value of an American call option can never be less than the value of a European call option, since the holder of the American option has the added flexibility to exercise the option early if it is profitable to do so.

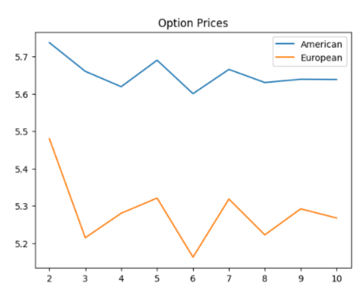
The early exercise feature gives American put options more value than European put options. This is because the early exercise feature allows the holder of an American put option to lock in a profit if the price of the underlying asset falls below the strike price before expiration. With a European put option, the holder must wait until expiration to exercise the option and realize any profit from the option.

AAPL



AMZN

TSLA



Black Scholes Equation

$$C = N(d_1)S_t - N(d_2)Ke^{-rt}$$

$$\text{where } d_1 = \frac{\ln \frac{S_t}{K} + (r + \frac{\sigma^2}{2})t}{\sigma\sqrt{t}}$$

$$\text{and } d_2 = d_1 - \sigma\sqrt{t}$$

C = call option price

N = CDF of the normal distribution

S_t = spot price of an asset

K = strike price

r = risk-free interest rate

t = time to maturity

σ = volatility of the asset

The Black-Scholes model is a mathematical model for pricing options contracts. It makes several assumptions, which include:

1. The underlying asset follows a lognormal distribution - this means that the asset price follows a pattern of random fluctuations with a certain degree of randomness.
2. The stock price has constant volatility - this means that the volatility of the stock price is assumed to be constant over the life of the option.
3. The risk-free rate is constant and known - this means that the interest rate used in the calculation of option prices is assumed to be constant and known.
4. The options are European - this means that the options can only be exercised on their expiration date.

Our code calculates the sigma using past data, i.e. the standard deviation of returns. It takes as input the following parameters:

- `S`: the current stock price S_0
- `K`: the strike price K

- `T`: the time to maturity (in years) T
- `r`: the risk-free rate μ_{rf}
- `sigma`: the volatility σ
- `option_type`: `"Call"` or `"Put"`

If `option_type` is `"call"`, the function will calculate the price of a call option using:

$$S * N(d_1) - Ke^{-rt} * N(d_2)$$

If `option_type` is `"put"`, the function will calculate the price of a put option using:

$$Ke^{-rt} * N(-d_2) - S * N(-d_1)$$

Black Scholes as a limiting case of CRR Binomial Model

The Binomial model assumes that the underlying asset can only move up or down by a fixed percentage over a fixed period. This means that the price of the asset at any given time can only take on one of two possible values, which makes it a discrete-time model.

On the other hand, the Black-Scholes model assumes that the underlying asset follows a continuous stochastic process such as a geometric Brownian motion. This means that the price of the asset can take on any value at any given time, making it a continuous-time model.

The Binomial model converges to the Black-Scholes model when the number of time steps increases to infinity and the step size decreases to zero. This is similar to the continuous-time assumption of the Black-Scholes model, where the price of the underlying asset can take on any value at any given time.

In order to show the limiting behaviour of the binomial model as it converges to the Black-Scholes model, we plotted the variation of predicted using the binomial CRR model for varying values of N from values in the range $[2, 300]$.

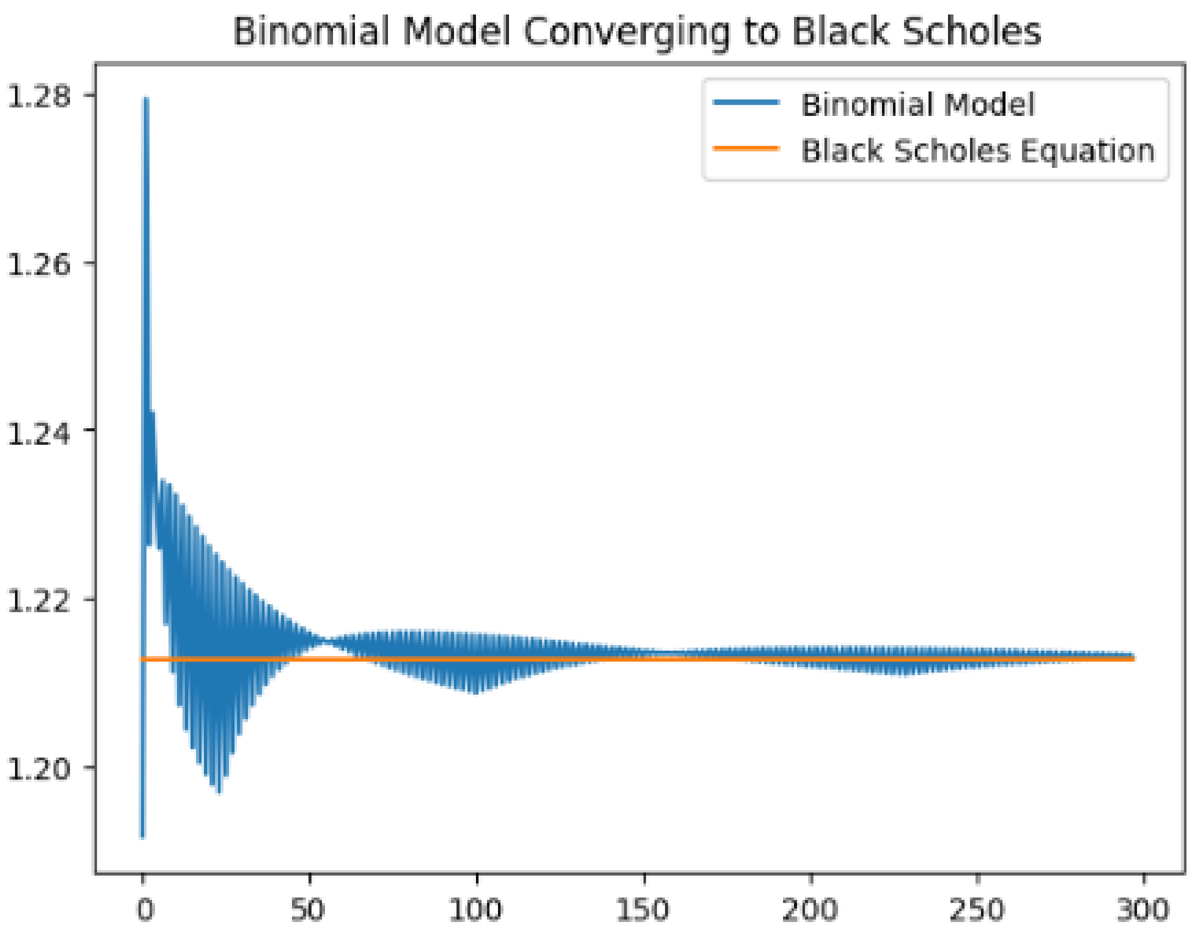
Below is the plot of `AAPL` predicted option price using CRR Binomial Model (with the Black-Scholes limiting value superimposed on top for reference) along the y-axis, against the number of steps the time period is divided into N along the x-axis.

`AAPL`

`S = 167.5`

`K = 166.789`

`T = 1`



Variation in Strike Price:

`AAPL`

`S = 167.3`

`K = 170.789`

`T = 1`

`AAPL`

`S = 167.3`

`K = 172.5`

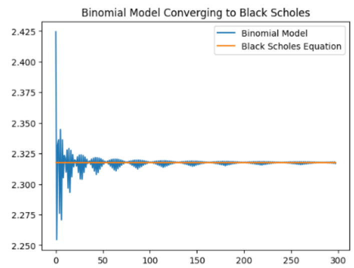
`T = 1`

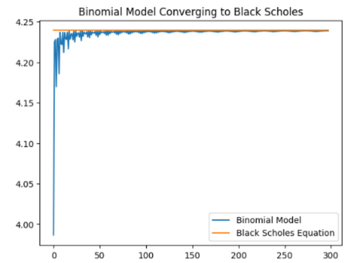
`AAPL`

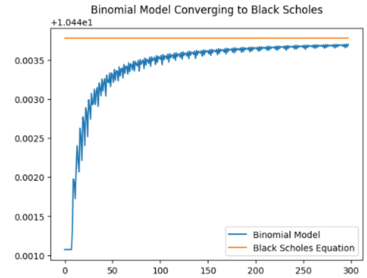
`S = 167.3`

`K = 179`

`T = 1`







Variation in Stocks:

`AAPL`

`S = 167.3`

`K = 170.789`

`T = 1`

`AMZN`

`S = 110.5`

`K = 106.5`

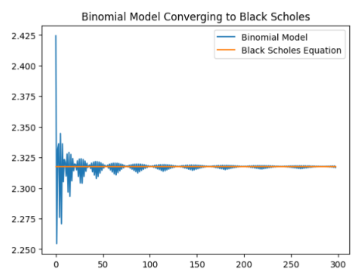
`T = 1`

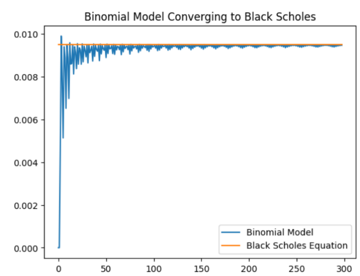
`TSLA`

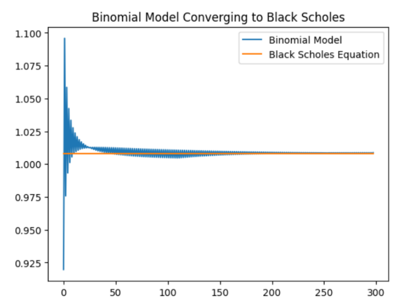
`S = 159.6`

`K = 160`

`T = 1`







Variation in Time Period:

`AAPL`

`S = 167.3`

`K = 170.789`

`T = 1`

`AAPL`

`S = 167.3`

`K = 172.5`

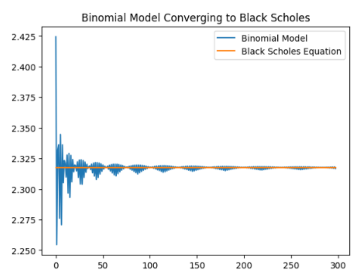
`T = 8`

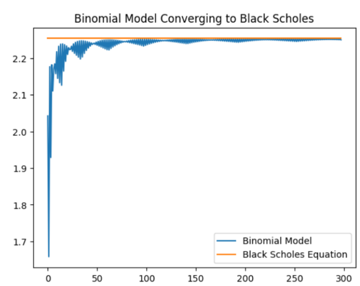
`AAPL`

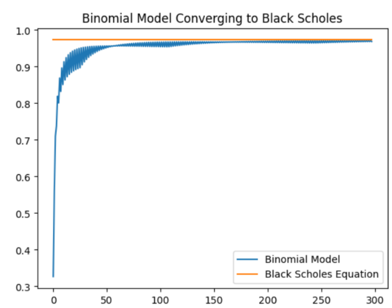
`S = 167.3`

`K = 170.0`

`T = 15`







Self-financing strategy (Delta Hedging)

The value of Δ essentially captures the ratio between how much the call/put option valuation changes with respect to the stock price in a binomial model. The value of Δ can then be used in order to compute amount of stock to be bought for a self-financing strategy to be able to finance itself regardless of whether the stock goes up or down.

At every time step, the value of Δ computed along the binomial tree gives us the fraction of stock to own in order to hedge against the fluctuations in the market to prevent losses in case the stock falls down.

```
graph LR
  B["S1(u)"]
  C["S1(d)"]
  S0 --p--> B
  S0 --1-p--> C
  D["P1(u) = (K - S1(u))+"]
  F["P1(d) = (K - S1(d))+"]
  P0 --p--> D
  P0 --1-p--> F
```

In order to hedge against variations in the market, we require the portfolio to be able to finance itself in both cases:

1. The value of the stock goes up - $S_1(u)$
2. The value of the stock goes down - $S_1(d)$

From this, we get two equations at time $t = T$:

$$1.\ P_1(u) = \Delta(0)S_1(u) + (C(0) - \Delta(0)S(0))e^{rT}$$
$$2.\ P_1(d) = \Delta(0)S_1(d) + (C(0) - \Delta(0)S(0))e^{rT}$$

For both requirements listed to hold, both the above equations must hold as well. On solving these equations, we get:

$$\Delta(0) = \frac{P_1(u) - P_1(d)}{S_1(u) - S_1(d)}$$

To calculate the weekly values of the implied volatility, we calculate the value of $\Delta(0)$ using the historical values of the stock price at the beginning of every week. ‘

Thus, we obtain the required values of Δ to invest in the stock at the beginning of every week until option maturity.

`AAPL`

`T = 1`

```
Option Type: American
VOL:  0.2718777351749825
Steps: 2 | Current Stock Price : 156.7007293701172 | Delta : -3.309181576749097 | Strike Price : 235.0000 | Actual option price : 68.6500 | Option Price: 71.4848 | Future Stock Price : 164.9000
VOL:  0.2718777351749825
Steps: 3 | Current Stock Price : 156.7007293701172 | Delta : -3.309181576749097 | Strike Price : 235.0000 | Actual option price : 68.6500 | Option Price: 71.4848 | Future Stock Price : 164.9000
VOL:  0.2718777351749825
Steps: 4 | Current Stock Price : 156.7007293701172 | Delta : -3.309181576749097 | Strike Price : 235.0000 | Actual option price : 68.6500 | Option Price: 71.4848 | Future Stock Price : 164.9000
VOL:  0.2718777351749825
Steps: 5 | Current Stock Price : 156.7007293701172 | Delta : -3.309181576749097 | Strike Price : 235.0000 | Actual option price : 68.6500 | Option Price: 71.4848 | Future Stock Price : 164.9000
VOL:  0.2718777351749825
Steps: 6 | Current Stock Price : 156.7007293701172 | Delta : -3.309181576749097 | Strike Price : 235.0000 | Actual option price : 68.6500 | Option Price: 71.4848 | Future Stock Price : 164.9000
VOL:  0.2718777351749825
Steps: 7 | Current Stock Price : 156.7007293701172 | Delta : -3.309181576749097 | Strike Price : 235.0000 | Actual option price : 68.6500 | Option Price: 71.4848 | Future Stock Price : 164.9000
VOL:  0.2718777351749825
Steps: 8 | Current Stock Price : 156.7007293701172 | Delta : -3.309181576749097 | Strike Price : 235.0000 | Actual option price : 68.6500 | Option Price: 71.4848 | Future Stock Price : 164.9000
VOL:  0.2718777351749825
Steps: 9 | Current Stock Price : 156.7007293701172 | Delta : -3.309181576749097 | Strike Price : 235.0000 | Actual option price : 68.6500 | Option Price: 71.4848 | Future Stock Price : 164.9000
VOL:  0.2718777351749825
Steps: 10 | Current Stock Price : 156.7007293701172 | Delta : -3.309181576749097 | Strike Price : 235.0000 | Actual option price : 68.6500 | Option Price: 71.4848 | Future Stock Price : 164.9000
Option Type: European
VOL:  0.052150775775044074
Steps: 2 | Current Stock Price : 156.7007293701172 | Delta : -89.93864459226657 | Strike Price : 235.0000 | Actual option price : 68.6500 | Option Price: 71.4848 | Future Stock Price : 164.9000
VOL:  0.052150775775044074
Steps: 3 | Current Stock Price : 156.7007293701172 | Delta : -89.93864459226657 | Strike Price : 235.0000 | Actual option price : 68.6500 | Option Price: 71.4848 | Future Stock Price : 164.9000
VOL:  0.052150775775044074
Steps: 4 | Current Stock Price : 156.7007293701172 | Delta : -89.93864459226657 | Strike Price : 235.0000 | Actual option price : 68.6500 | Option Price: 71.4848 | Future Stock Price : 164.9000
Steps: 5 | Current Stock Price : 156.7007293701172 | Delta : NaN | Strike Price : 235.0000 | Actual option price : 68.6500 | Option Price: 71.4848 | Future Stock Price : 164.9000
Steps: 6 | Current Stock Price : 156.7007293701172 | Delta : NaN | Strike Price : 235.0000 | Actual option price : 68.6500 | Option Price: 71.4848 | Future Stock Price : 164.9000
VOL:  0.0
Steps: 7 | Current Stock Price : 156.7007293701172 | Delta : NaN | Strike Price : 235.0000 | Actual option price : 68.6500 | Option Price: 71.4848 | Future Stock Price : 164.9000
VOL:  0.052150775775044074
Steps: 8 | Current Stock Price : 156.7007293701172 | Delta : -89.93864459226657 | Strike Price : 235.0000 | Actual option price : 68.6500 | Option Price: 71.4848 | Future Stock Price : 164.9000
VOL:  0.052150775775044074
Steps: 9 | Current Stock Price : 156.7007293701172 | Delta : -89.93864459226657 | Strike Price : 235.0000 | Actual option price : 68.6500 | Option Price: 71.4848 | Future Stock Price : 164.9000
Steps: 10 | Current Stock Price : 156.7007293701172 | Delta : NaN | Strike Price : 235.0000 | Actual option price : 68.6500 | Option Price: 71.4848 | Future Stock Price : 164.9000
```

`AMZN`

`T = 1`

```
Option Type: American
VOL:  0.4389083395149661
Steps: 2 | Current Stock Price : 115.54000091552734 | Delta : -2.646742736656094 | Strike Price : 235.0000 | Actual option price : 68.6500 | Option Price: 71.4848 | Future Stock Price : 164.9000
VOL:  0.4389083395149661
Steps: 3 | Current Stock Price : 115.54000091552734 | Delta : -2.646742736656094 | Strike Price : 235.0000 | Actual option price : 68.6500 | Option Price: 71.4848 | Future Stock Price : 164.9000
VOL:  0.4389083395149661
Steps: 4 | Current Stock Price : 115.54000091552734 | Delta : -2.646742736656094 | Strike Price : 235.0000 | Actual option price : 68.6500 | Option Price: 71.4848 | Future Stock Price : 164.9000
VOL:  0.4389083395149661
Steps: 5 | Current Stock Price : 115.54000091552734 | Delta : -2.646742736656094 | Strike Price : 235.0000 | Actual option price : 68.6500 | Option Price: 71.4848 | Future Stock Price : 164.9000
VOL:  0.4389083395149661
Steps: 6 | Current Stock Price : 115.54000091552734 | Delta : -2.646742736656094 | Strike Price : 235.0000 | Actual option price : 68.6500 | Option Price: 71.4848 | Future Stock Price : 164.9000
VOL:  0.4389083395149661
Steps: 7 | Current Stock Price : 115.54000091552734 | Delta : -2.646742736656094 | Strike Price : 235.0000 | Actual option price : 68.6500 | Option Price: 71.4848 | Future Stock Price : 164.9000
VOL:  0.4389083395149661
Steps: 8 | Current Stock Price : 115.54000091552734 | Delta : -2.646742736656094 | Strike Price : 235.0000 | Actual option price : 68.6500 | Option Price: 71.4848 | Future Stock Price : 164.9000
VOL:  0.4389083395149661
Steps: 9 | Current Stock Price : 115.54000091552734 | Delta : -2.646742736656094 | Strike Price : 235.0000 | Actual option price : 68.6500 | Option Price: 71.4848 | Future Stock Price : 164.9000
VOL:  0.4389083395149661
Steps: 10 | Current Stock Price : 115.54000091552734 | Delta : -2.646742736656094 | Strike Price : 235.0000 | Actual option price : 68.6500 | Option Price: 71.4848 | Future Stock Price : 164.9000
Option Type: European
VOL:  0.0
Steps: 2 | Current Stock Price : 115.54000091552734 | Delta : NaN | Strike Price : 235.0000 | Actual option price : 68.6500 | Option Price: 71.4848 | Future Stock Price : 164.9000
VOL:  0.0
Steps: 3 | Current Stock Price : 115.54000091552734 | Delta : NaN | Strike Price : 235.0000 | Actual option price : 68.6500 | Option Price: 71.4848 | Future Stock Price : 164.9000
VOL:  0.09143598642303156
Steps: 4 | Current Stock Price : 115.54000091552734 | Delta : -60.98528904440175 | Strike Price : 235.0000 | Actual option price : 68.6500 | Option Price: 71.4848 | Future Stock Price : 164.9000
Steps: 5 | Current Stock Price : 115.54000091552734 | Delta : NaN | Strike Price : 235.0000 | Actual option price : 68.6500 | Option Price: 71.4848 | Future Stock Price : 164.9000
Steps: 6 | Current Stock Price : 115.54000091552734 | Delta : NaN | Strike Price : 235.0000 | Actual option price : 68.6500 | Option Price: 71.4848 | Future Stock Price : 164.9000
Steps: 7 | Current Stock Price : 115.54000091552734 | Delta : NaN | Strike Price : 235.0000 | Actual option price : 68.6500 | Option Price: 71.4848 | Future Stock Price : 164.9000
VOL:  0.09143598642303156
Steps: 8 | Current Stock Price : 115.54000091552734 | Delta : -60.98528904440175 | Strike Price : 235.0000 | Actual option price : 68.6500 | Option Price: 71.4848 | Future Stock Price : 164.9000
VOL:  0.0
Steps: 9 | Current Stock Price : 115.54000091552734 | Delta : NaN | Strike Price : 235.0000 | Actual option price : 68.6500 | Option Price: 71.4848 | Future Stock Price : 164.9000
Steps: 10 | Current Stock Price : 115.54000091552734 | Delta : NaN | Strike Price : 235.0000 | Actual option price : 68.6500 | Option Price: 71.4848 | Future Stock Price : 164.9000
```

`TSLA`

`T = 1`

```
Option Type: American
VOL:  0.02371869554988013
Steps: 2 | Current Stock Price : 240.06666564941406 | Delta : 24.82647222312742 | Strike Price : 235.0000 | Actual option price : 68.6500 | Option Price: 71.4848 | Future Stock Price : 164.9000
VOL:  0.02221506376943906
Steps: 3 | Current Stock Price : 240.06666564941406 | Delta : 28.30098065631681 | Strike Price : 235.0000 | Actual option price : 68.6500 | Option Price: 71.4848 | Future Stock Price : 164.9000
VOL:  0.02077595608244773
Steps: 4 | Current Stock Price : 240.06666564941406 | Delta : 32.35747138844973 | Strike Price : 235.0000 | Actual option price : 68.6500 | Option Price: 71.4848 | Future Stock Price : 164.9000
VOL:  0.022659436708691763
Steps: 5 | Current Stock Price : 240.06666564941406 | Delta : 27.20184702462115 | Strike Price : 235.0000 | Actual option price : 68.6500 | Option Price: 71.4848 | Future Stock Price : 164.9000
VOL:  0.021641746288889385
Steps: 6 | Current Stock Price : 240.06666564941406 | Delta : 29.820300133250132 | Strike Price : 235.0000 | Actual option price : 68.6500 | Option Price: 71.4848 | Future Stock Price : 164.9000
VOL:  0.02216488717863971
Steps: 7 | Current Stock Price : 240.06666564941406 | Delta : 28.429260494246773 | Strike Price : 235.0000 | Actual option price : 68.6500 | Option Price: 71.4848 | Future Stock Price : 164.9000
VOL:  0.022161920423520887
Steps: 8 | Current Stock Price : 240.06666564941406 | Delta : 28.436872496983316 | Strike Price : 235.0000 | Actual option price : 68.6500 | Option Price: 71.4848 | Future Stock Price : 164.9000
VOL:  0.021475904024267065
Steps: 9 | Current Stock Price : 240.06666564941406 | Delta : 30.282637938351847 | Strike Price : 235.0000 | Actual option price : 68.6500 | Option Price: 71.4848 | Future Stock Price : 164.9000
VOL:  0.022258348106833206
Steps: 10 | Current Stock Price : 240.06666564941406 | Delta : 28.191017588823527 | Strike Price : 235.0000 | Actual option price : 68.6500 | Option Price: 71.4848 | Future Stock Price : 164.9000
Option Type: European
VOL:  0.02371869554988013
Steps: 2 | Current Stock Price : 240.06666564941406 | Delta : 24.82647222312742 | Strike Price : 235.0000 | Actual option price : 68.6500 | Option Price: 71.4848 | Future Stock Price : 164.9000
VOL:  0.02221506376943906
Steps: 3 | Current Stock Price : 240.06666564941406 | Delta : 28.30098065631681 | Strike Price : 235.0000 | Actual option price : 68.6500 | Option Price: 71.4848 | Future Stock Price : 164.9000
VOL:  0.019922945442164736
Steps: 4 | Current Stock Price : 240.06666564941406 | Delta : 35.1875898908786 | Strike Price : 235.0000 | Actual option price : 68.6500 | Option Price: 71.4848 | Future Stock Price : 164.9000
VOL:  0.0223994562534723
Steps: 5 | Current Stock Price : 240.06666564941406 | Delta : 27.83695074869664 | Strike Price : 235.0000 | Actual option price : 68.6500 | Option Price: 71.4848 | Future Stock Price : 164.9000
VOL:  0.0214016934396936
Steps: 6 | Current Stock Price : 240.06666564941406 | Delta : 30.493012741102742 | Strike Price : 235.0000 | Actual option price : 68.6500 | Option Price: 71.4848 | Future Stock Price : 164.9000
VOL:  0.021711851372401857
Steps: 7 | Current Stock Price : 240.06666564941406 | Delta : 29.628038357451107 | Strike Price : 235.0000 | Actual option price : 68.6500 | Option Price: 71.4848 | Future Stock Price : 164.9000
VOL:  0.021902077321326117
Steps: 8 | Current Stock Price : 240.06666564941406 | Delta : 29.115616966520708 | Strike Price : 235.0000 | Actual option price : 68.6500 | Option Price: 71.4848 | Future Stock Price : 164.9000
VOL:  0.020841129441674575
Steps: 9 | Current Stock Price : 240.06666564941406 | Delta : 32.15541441511704 | Strike Price : 235.0000 | Actual option price : 68.6500 | Option Price: 71.4848 | Future Stock Price : 164.9000
VOL:  0.021941066421615304
Steps: 10 | Current Stock Price : 240.06666564941406 | Delta : 29.01223246598844 | Strike Price : 235.0000 | Actual option price : 68.6500 | Option Price: 71.4848 | Future Stock Price : 164.9000
```

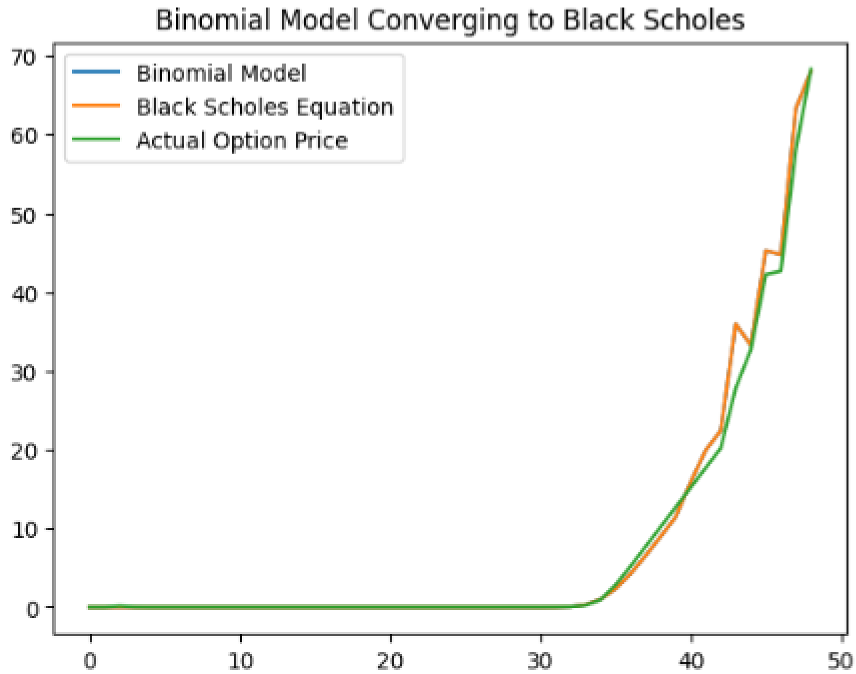
Analysis - Black Scholes vs Binomial vs Actual Option price

Black Scholes and Binomial Option pricing converge to roughly the same value which results in there not being much of a visual difference between the two, which can be seen in the graph as the orange curve `“Black Scholes Equation”` overlaps on top of the blue curve `"Binomial Model”`

However, there is a difference between the model prices we predicted with the actual market price. This is because of some of the fundamental assumptions that limit the Black-Scholes and the CRR model -

1. These models assume constant values for the risk-free rate of return and volatility over the option duration. In the real world, brokers adjust their option pricing to take into account the variable values of volatility which reflects in the difference between our models and the real world.

2. Option Prices obtained from Binomial Model and Black Scholes are same, essentially because we used a larger step size.


Binomial Model Converging to Black Scholes
Binomial Model
Black Scholes Equation
Actual Option Price
70
60
50
40
30
20
10
0
0
10
20
30
40
50