Introduction to Financial Engineering
Project Assignment: Portfolio Optimization

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Defining the Market Index

The **Nifty 500** is a market index that represents the top 500 companies listed on the National Stock Exchange (NSE) in India. It includes large, mid, and small-cap stocks from a wide range of sectors, making it a well-diversified index that represents the overall market.

The Capital Asset Pricing Model (CAPM) is a widely used method for estimating the expected return on an investment by taking into account the risk-free rate, the expected market return, and the beta coefficient of the security. The beta coefficient measures the volatility of the security relative to the overall market.

The Nifty 500 is a good choice for the CAPM model because it provides a broad-based representation of the Indian stock market. As a result, it can serve as a suitable proxy for the market portfolio, which is a key assumption of the CAPM model.

Using the Nifty 500 as the market index in the CAPM model can help investors estimate the expected return on their investments more accurately. By calculating the beta coefficient of a security relative to the Nifty 500, investors can better assess the security's risk relative to the overall market.

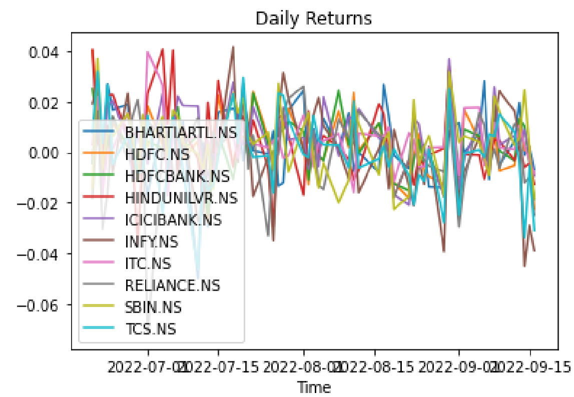
Overall, the Nifty 500 is a good choice for the CAPM model in the Indian market due to its diversification, large number of constituents, and broad-based representation of the market.

10 Risky Assets

We have chosen the following 10 risky assets from our above defined market index:

- RELIANCE
- TCS
- HDFCBANK
- INFY
- ICICIBANK
- HINDUNILVR
- ITC
- HDFC
- SBIN
- BHARTIARTL

As per the requirements of the assignment we have set the start date as **17th August 2022 and end date as 17th September 2022,** which gives us a net of 3 month's worth of data (as per the instructions of the project). It is also to be noted that in 3 month there are only **62 business days** which is the number of days the market is in play. Hence, real data that we are using is for 62 days.



Calculating daily returns, expected return and risk of the assets

Next, we calculate the daily simple returns, expected return and risk of the 10 risky assets mentioned above following the below mentioned formulas:

$$R_t = \frac{F - V}{V}$$

where;

R_t = Simple Daily Return

F = Final Price at time 't'

V = Initial Price at time '$t-1$'

$$E(R) = \mu = \text{mean}$$
$$= \bar{x} \rightarrow \text{mean}$$

$$Var(R) = \sigma^2 = \text{risk}$$
$$= \frac{\sum_{i=1}^{n} (x_i - \bar{x})^2}{N}$$

where:-

x_i = i^{th} observation/sample

$\bar{x}$ = mean of all observations

N = Number of samples total.

Calculating weights

Now, the next step in portfolio optimization is to calculate the weights assigned to each asset in the basket of assets. This is calculated following the below mentioned formula:

$$w_{min} = \frac{OC^{-1}}{OC^{-1}O^{T}}$$

where:-

$W_{min} = [w_{min\,1}, w_{min\,2}, \ldots w_{min\,n}]$ Weight matrix

$O = [1, 1, \ldots 1]_{1 \times n}$

$$C = \begin{bmatrix} \sigma_1^2 & \sigma_1\sigma_2\rho_{12} & \cdots & \sigma_1\sigma_n\rho_{1n} \\ \sigma_1\sigma_2\rho_{12} & \sigma_2^2 & \cdots & \vdots \\ \vdots & & \ddots & \\ \vdots & & \cdots & \sigma_n^2 \end{bmatrix}_{n \times n}$$ Covariance matrix

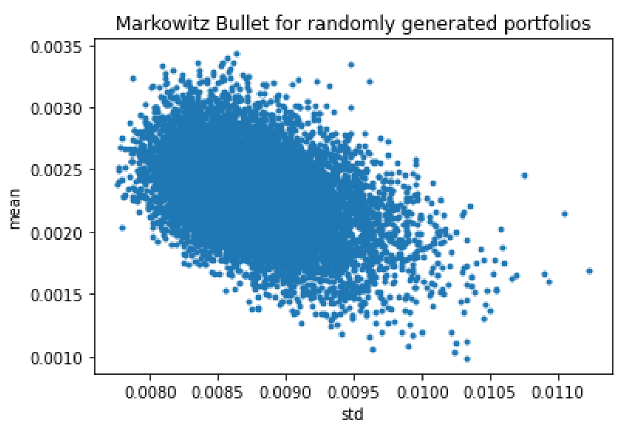
Markowitz Theory

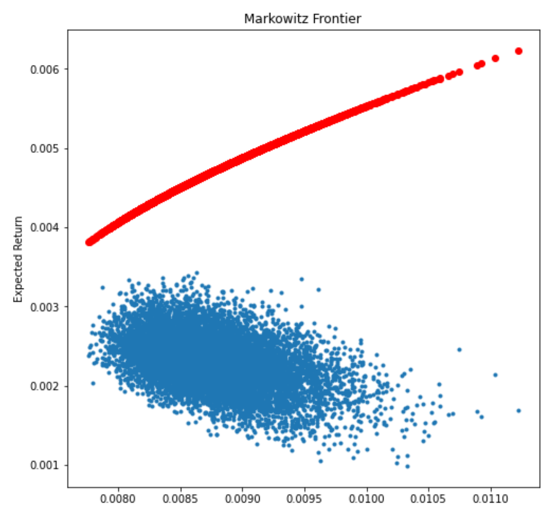
Following the Markowitz Theory, we get the following mean-standard deviation plot and the Markowitz efficient frontier. Now from the plots we can draw the following observations from the perspective of an investor. **Every rational investor will choose an efficient portfolio**, always preferring a **dominating portfolio to a dominated one**. However, different investors may select different portfolios on the efficient frontiers, drawn below, depending on their individual preferences. Give two efficient portfolios with mu1 <= mu2 and sigma1 <= sigma2, a cautious person may prefer that with lower risk sigma1 and lower expected retur mu1, while others may choose a portfolio with higher risk simga2, regarding the higher expected return mu2 as compensation for increased risk.

Getting the efficient Frontier:

We first get 1000 random portfolios with random weights and corresponding risks and returns, now we try to find an efficient frontier for each of these risks. Which is, if we pick any portfolio on efficient frontier, there is no portfolio with less risk and same return or same risk and more return.

For this we make an **optimization problem** to **maximize the returns** corresponding to each sigma (risk) with constraint that **sum of weights is 1**. This gives us **markowitz efficient frontier**.





CAP M Model

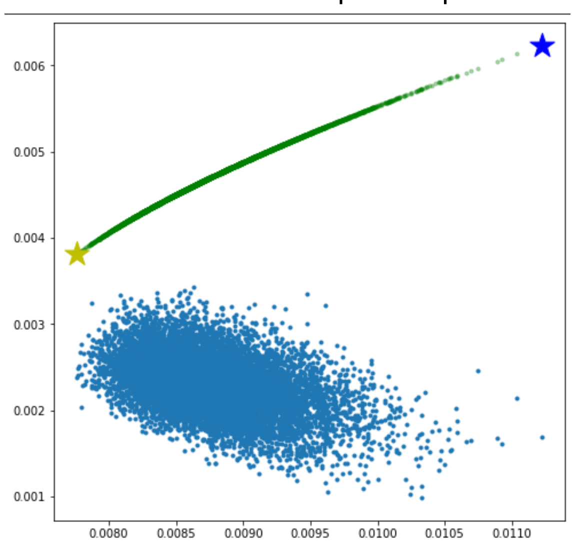
We have calculated the CM line following the below mentioned formula. We have taken the return for the **risk-free asset to be 0.074%**. This is following the real world risk-free return of six month bonds of Indian government. Furthermore, note that for the slope of the CML to be positive rate of risk free return must be less than the return of the market, which is indeed the case here.

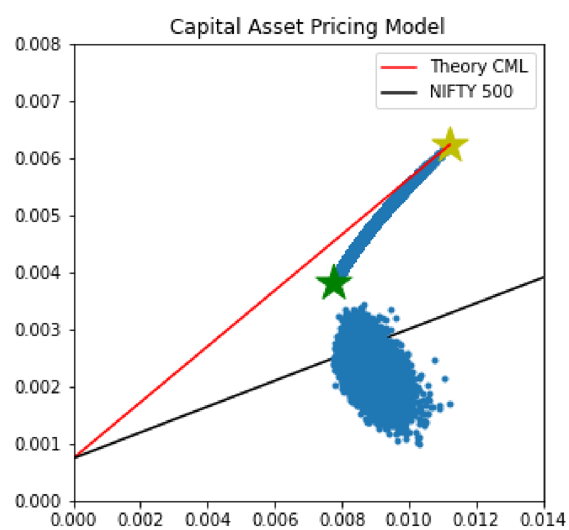
The respective line is also plotted below. It can be seen that the **CML is tangent to the Markwotiz efficient frontier** , which is indeed the expected behavior following the CAP M model. The tangency point with coordinates sigma_m, mu_m plays a special role. Every portfolio on the capital market line can be constructed from the risk-free security and the portfolio with standard deviation sigma_m and expected return mu_m.
Since every investor will select a portfolio on the capital market line, everyone will be holding a portfolio with the same relative proportions of risky securities. But this means that the portfolio with standard deviation sigma_m and expected return mu_m has to contain all risky securities with weights equal to their relative share in the whole market. Because of this property it is called the market portfolio.

$$\mu = \mu_{rf} + \left(\frac{\mu_M - \mu_{rf}}{\sigma_M}\right)\sigma$$

where:-
μ_{rf} = risk free return.
μ_M = market return.
$\sigma_m = \sqrt{\text{market risk}}$
$\sigma = \sqrt{\text{portfolio risk}}$
μ = expected return of portfolio

In practice, deducing this market portfolio is redundant and we use any market index in place of Market Risk and Market return. Here, as discussed above we make use of NIFTY500 index as the market index and compare its performance to actual market risk and market returns.





We see that, **NIFTY500 line is not a tangent** to the efficient frontier, however it still lies in the upper half of the bullet and is therefore a reasonable approximation. However, the **actual CML computed via the market weights is a tangent and this confirms the CAPM theory.**

<u>Security Market Line</u>

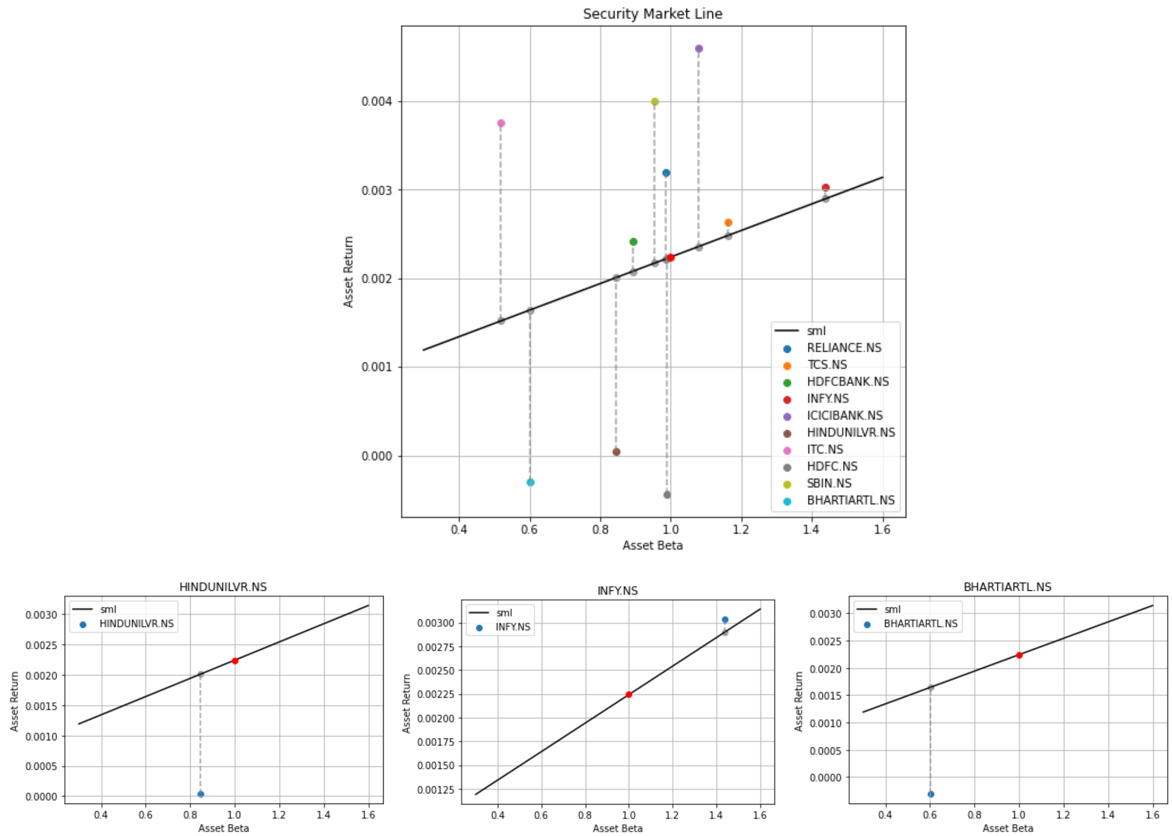
We have calculated the security market line using the following mentioned formulas. Plots of the same are drawn below.
Note that the **beta factor** is an indicator of the expected changes i the return on a particular portfolio or individual security in response to the behaviour of the market as a whole.
The CAPM describes a state of equilibrium in the market. Everyone is holding a portfolio of risky securities with the same weights as the market portfolio. Any trades that may be executed by investors will only affect their split of funds between the risk-free security ad the market portfolio. This will remain so as long as the estimates of expected returns and beta factors satisfy.
However, as soon as some new information about the market becomes available to investors, it may affect their estimates of expected returns ad beta factors. The new estimates values may no longer satisfy the below equation of SML.
Suppose, that mu_v > mu_rf + (mu_m - mu_rf)Beta for a particular security. In this case investors will want to increase their relative position in this security, which offers a higher expected return than required as compensation for systematic risk. Demand will exceed supply, the price of the security will begin to rise and the expected return will decline.
On the other hand, if the reverse inequality mu_v < mu_rf + (mu_m - mu_rf)Beta holds, investors will want to sell the security. In this case supply will exceed demand, the price will fall and the expected return will increase. This will continue until the prices and with them the expected returns of all securities settle at a new level, restoring equilibrium.
These inequalities are important in the sense that the send a clear signal to investors whether any particular security is underpriced, or respectectively overpriced, that is, whether it should be bought or sold.
Following the above theoretical concepts, it can be seen that many real world stocks such as ITC, SBIN, ICICIBANK, and RELIANCE have mu_v > mu_rf + (mu_m - mu_rf)Beta which implies that investors will want to hold onto the shares of these companies. On the other hand, for companies like BHARTIARTL, HINDUNILVR, and HDFC mu_v < mu_rf + (mu_m - mu_rf)Beta which implies that the investors will want to sell the security. Then there are companies like HDFCBANK, INFY, and TCS which lie on the SML and investors may end up being confused as to what to do with these shares.

$$\beta_v = \frac{Cov(A, M)}{\sigma_M^2}$$

where
$A \rightarrow$ Asset
$M \rightarrow$ Market.
$\sigma_M^2 \rightarrow$ risk of market

$$\mu_v = \mu_{rf} + (\mu_M - \mu_{rf})\beta_v$$

where;
μ_v = expected return of asset
μ_{rf} = risk free return
μ_M = market expected return.
β_v = beta value.



Conclusion

In this project, we have used real world NIFTY500 3 months worth of data and analysed it following financial models of ‘Markowitz Theory’, ‘CAP M’, and ‘Security Line’. We have observed that the data does indeed follow the above mentioned theories. Furthermore, from the values and plots obtained we are able to make intelligent decisions as to how to invest in the market based on previous data.