AE 618: Finite Element Method for Fluid Dynamics
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# Assignment - 2

A. Consider the boundary value problem discussed in the class:

$$u_{,xx}(x) + f(x) = 0, \qquad on \qquad ]0.1[, \tag{1}$$
$$u(1) = g, \tag{2}$$
$$-u_{,x}(0) = t \qquad . \tag{3}$$

Assume $f(x) = qx$ where $q$ is a constant and $g = t = 0$.

1. Find the exact solution.
2. Employing the linear finite element space with equally spaced nodes, set up and solve the Galerkin-Finite Element equations for $n = 1, 2, 3, 4$, i.e. $h = 1, 1/2, 1/3, 1/4$. $n$ is the number of elements and $h$ is the element length. We are considering the case of all elements being of same length. The number of nodes is $n + 1$. In each case, you will need to calculate the stiffness matrix ($\mathbf{K}$) and vector ($\mathbf{F}$), and then carry out a solve for $\mathbf{Kd} = \mathbf{F}$.
3. Is the stiffness matrix banded? What is the consequence of boundary terms ($g$ and $h$) on the bandedness?
4. Let $re_{,x} = |u_{,x}^h - u_{,x}|/(q/2)$ denote the relative error in $u_{,x}$. Compute $re_{,x}$ at the midpoints of the 4 elements. They should be all equal.
5. Employing the data for $h = 1$, $h = 1/2$, $h = 1/3$ and $h = 1/4$ plot $ln(re_{,x})$ versus $ln(h)$.
6. What is the significance of the slope and the the y-intercept?

B. For the same problem, as described in part A, write a computer program to assemble the element level stiffness matrices and force vectors. Solve the matrix equation system using a linear equation solver (you can use a library or any other available program, for solving **A x=b**). Plot the solution and the slope. Compute the relative error for $n = 10$, $n = 50$ and $n = 100$. Calculate the slope as was done in the previous part. Comment on the results.