Topic 1

Introductory Tools of Economics

"(Economics is) a dreary, desolate and, indeed, quite abject and distressing one; what we might call, by way of eminence, the *dismal science*".

Thomas Carlyle (1795-1881), A Scottish philosopher and historian.

Hey Jude, don't make it bad./ Take a sad song and make it better.

.... The Beatles

1 Preliminaries

In this topic, we introduce some tools of economics without any reference to uncertainty or imperfect information. The reason is twofold. First, the topics and methodology will be used in later parts of the course. Second, one needs to know the particular vocabulary of economics. For example, a technical jargon used in Economics may have completely different meaning in everyday life (example: Mathematicians' use of the term '*well ordered*'). This is also a good place to know about the jargons.

Students with Economics background (or with enough confidence or both) may provide a cursory glance. Others need to go a little bit more thoroughly.

Economics is mainly concerned with three questions:[1] who consumers (this is the problem of consumer choice), who produces (this is the problem of producer behavior) and how these two groups interact (this is the problem of market structure).

The way we will proceed is the following. In each case, we will assume a highly simplistic scenario (which the economists call *a model*), and derive some logical conclusion from it. Then we relax some of the simplifying assumptions and see how far the logical conclusions can carry over.

[1] Other economists (e.g. Marxists) might give you different answer. As far as this course is concerned, this working definition will suit us perfectly.

1.1 Consumer Theory

Suppose a consumer cannot alter his/her money income (fixed) as well as prices ((s)he is a very small buyer in the market, so that his/her buying decision cannot alter the market price). His/her choice problem is to consume goods/services in such a way that (s)he gets the maximum benefit out of his/her consumption plan.

1.1.1 Elements of Choice

We will assume that the consumer has only two goods to consume (a generalization to n goods is straightforward, but we cannot quite draw the graphs!). Let the goods be apples and bananas. $X_1 = (a_1, b_1)$, $X_2 = (a_2, b_2)$ etc. are possible bundles (i.e. quantities of) apples and bananas. We will also make this (quite unrealistic) assumption that all a_i and b_i are continuous variables, that is, it is possible to have e apples and π bananas. The assumption will allow us to draw the graphs in a neat manner and facilitate use of calculus (and make our life easy).

We introduce a symbol: $\succeq$. It is to be read as (psychologically) 'at least as good as'.

Here we list our assumptions of choice. Given any two bundles, X_1 and X_2

1. The consumer can always compare two bundles. That is, either $X_1 \succeq X_2$, OR $X_2 \succeq X_1$ OR both. This is called the *axiom of completeness.* Note that, it is not very innocuous: it assumes that we have perfect knowledge over all possible bundles.

2. If $X_1 \succeq X_2$ and $X_2 \succeq X_3$, then $X_1 \succeq X_3$. This is called the *axiom of transitivity.*

If a consumers' choice obeys these two axioms, we say that the choice is *rational.*

Given $\succeq$, we can express two more relations.

- Indifference: If $X_1 \succeq X_2$ and $X_2 \succeq X_1$, then $X_1 \sim X_2$.
- Strict preference: If $X_1 \succeq X_2$ and $X_2 \not\succeq X_1$, then $X_1 \succ X_2$.

Our psychology dictates when we are indifferent or have strict preference. In a very crude way, we will suggest that the bundles containing more of one (or both goods) will be strictly preferred. That is,

- If $X_1 = (a_1, b_1)$ and $X_2 = (a_1 + \delta, b_1)$ or $X_2 = (a_1, b_1 + \delta)$, then $X_2 \succ X_1$. This is the assumption of monotonicity, otherwise known as "More Is Better",[2] or the *American Assumption*.

These axioms of choice can be represented in a *utility function*[3] $u : \mathbb{R}^2 \to \mathbb{R}$ i.e. $u(a, b)$ which takes, as its argument, various quantities of a and b and returns a number, such that, iff $(a_1, b_1) \succeq (a_2, b_2)$ then $u(a_1, b_1) \geq u(a_2, b_2)$. Derivative of the utility function with respect to one of the arguments is called the *marginal utility* of the commodity,[4] e.g. $u_a = \dfrac{\partial u}{\partial a}$ is the marginal utility of a. The assumption of 'more is better' is embodied in positive marginal utility: $\frac{\partial u}{\partial a} > 0$. Another common assumption is that of *diminishing marginal utility*: $\frac{\partial^2 u}{\partial a^2} < 0$. As we consume more and more, the benefit increases at a decreasing rate.

Graphical Representation Given our preferences, we can always define an indifference set, $u(a, b) = \bar{u}$, i.e. the combinations of (a, b) such that

[2] As a counterpoint, read https://maggiemcneill.files.wordpress.com/2013/03/how-much-land-does-a-man-need1.pdf

[3] Existence of such a continuous function requires that preferences are complete, transitive and continuous. Continuity of preference means that $\succeq$ is preserved under limits. That is, suppose we have as sequence of bundles X^n and Y^n such that $X^n \succeq Y^n$. Then, if $\lim X^n = X$ and $\lim Y^n = Y$, we must have $X \succeq Y$. Essentially, this assumption rules out sudden "jumps" in preferences.

Note that, the proof is more technical (and so is the assumption of continuity). So, for sake of HS 239, we omit this.

[4] Anything 'marginal' means a small, infinitesimal increase.

the utility number is constant (and consequently, the consumer is indifferent between various bundles). Note that, the assumption of monotonicity prevents this set to be "thick". It has to be a line. Here we can make some educated guess about the shape of the line.

- If a is increased, utility goes up. To make utility constant, one must reduce b. In other words, the locus of a, b such that $u(a, b) = \bar{u}$, must be negatively sloped. Assuming differentiability, and taking the total derivative, one can show that $\frac{db}{da} = -\frac{u_a}{u_b}$. The slope of the indifference curve is called the 'marginal rate of substitution' or MRS. If I decrease consumption of a, by, say, 1 unit, the MRS tells me by how much I must increase consumption of b to achieve the same satisfaction. Such substitutions are determined by psychology.

- Suppose we draw a in horizontal axis. Then

$$\frac{d}{da}\left(\frac{db}{da}\right) = -\frac{u_{aa}\left(u_b\right)^2 + u_{bb}\left(u_a\right)^2 - 2u_a u_b u_{ab}}{u_b^3} > 0$$

  (assuming $u_{ab} > 0$) i.e. the negative value decreases as one moves to right.

- Keeping one of the arguments fixed, if the amount of other good increases, we move up to a higher indifference curve. Thus the direction of increasing utility is to the north-east. The consumer must prefer those bundles.

Combining these observations, we get an indifference map. The consumer seeks to obtain the highest indifference curve possible.

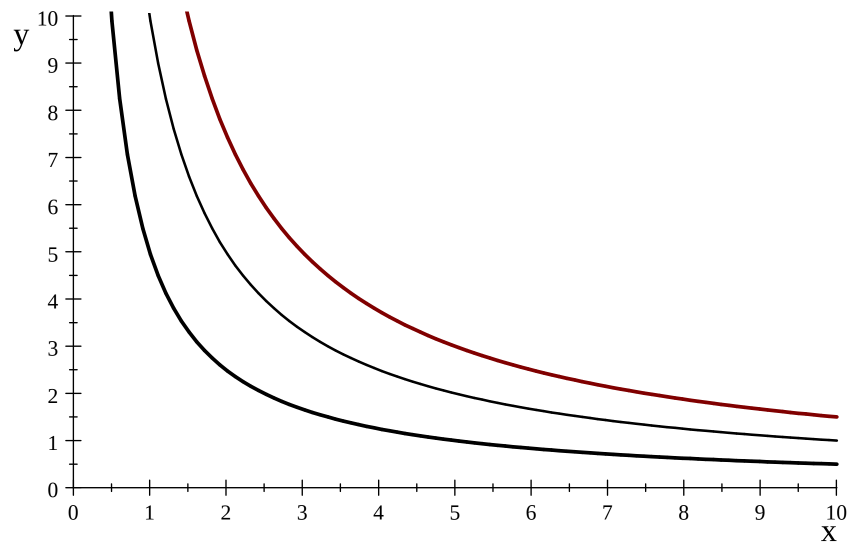

Figure 1:Indifference map of $u = \sqrt{xy}$

Constraint of Consumer Let the quality of goods (or services) be x_i (rather than apples and bananas). The consumer operates under the following constraint: given his money income M, and prices p_1, p_2, such that $p_1x_1 + p_2x_2 \leq M$. The value of x_1, x_2 which satisfy the above property are called feasible. The line where $p_1x_1 + p_2x_2 = M$ is called the budget line. On the budget line, notice that $\frac{dx_2}{dx_1} = -\frac{p_1}{p_2}$. If I reduce consumption of x_1 by 1 unit, the slope of the line tells me how much x_2 I can actually buy from the market. The slope of the budget line represents the market determined substitution rate between the two commodities.

1.1.2 Consumers' Problem

Mathematically, the problem is to

$$\begin{aligned} & \max_{x_1,x_2} u\left(x_1, x_2\right) \\ \text{such that } p_1x_1 + p_2x_2 & \leq M \end{aligned}$$

Given the 'more is better' assumption, the constraint will hold with equality, i.e. will be a binding constraint at the equilibrium.

The FOC of the consumer is[5]

$$\frac{MU_1}{MU_2} = \frac{p_1}{p_2}$$

That is, the 'internal' rate of substitution (MRS) matched with the market determined, 'external' substitution rate (price ratio). Graphically, this is achieved at the tangency point between the budget line and indifference curves. Solving the above equation and the budget constraint simultaneously, one gets the optimal demand x_i^*.

There are various techniques for solving for the optimal demand. A popular method is to form a Lagrangian $L = u + \lambda \left(M - \sum p_i x_i\right)$ and solving through the FOC, namely

$$\begin{aligned} MU_i &= p_i \lambda \\ M - \sum p_i x_i &= 0 \end{aligned}$$

For two goods, there will be three equations and three unknowns (λ, x_1, x_2) to solve for.

Demand Curve and Demand Function The optimal demand x_i^* is a function of own price, other prices and income, i.e. $x_i^* = x_i\left(p_1, p_2, M\right)$. This is known as the demand function for good x_i. If $\frac{\partial x_i}{\partial M} > 0$, then the good is called "normal". If the term is negative, then the good is "inferior". Similarly, consider a change in other price, p_j. If $\frac{\partial x_i}{\partial p_j} > 0$, then commodities i and j are "substitutes" (e.g. coffee and tea). If $\frac{\partial x_i}{\partial p_j} < 0$, then the commodities are "complements" (e.g. dosa and sambar). On the other hand, keeping p_j and M fixed, the relation between x_i and p_i ('own price') is known as *demand curve* for good i. As own price changes, there is a movement along

[5] Note that, the FOC does not guarantee "maximization" and we need to impose structure on $u\left(\right)$ to get the result (concavity etc.). We do not pursue the case here.

the curve (*negatively sloped*). As other variables change, demand curve either shifts to right or left.[6] If we horizontally add individual demand curves, we get the market demand for good i.

1.1.3 Change in Own Price: Further Considerations

If price of a good (say good i) falls, there are two separate impacts. First, one tends to consume more of the good (even if there is no change in your income) which has become relatively cheaper. This is called "substitution effect" of price change. On the other hand, had you kept your original consumption basket fixed, you would end up with higher income in the wallet (or bank), which you can use for buying more of good i This is called "income effect" of good i. The total effect is sum of income and substitution effect.

If we want to compute substitution effect, we need to be more specific on the definition of "constant income". Here is a procedure proposed by the economist Hicks.

1. Suppose price have changed from (p_1, p_2, M) to (p_1', p_2, M) .We can assume this is a price fall.

2. At the new price situation, take back enough income, say, reduce M to M', such that at (p_1', p_2, M'), the utility derived by the consumer is same as that of (p_1, p_2, M). Then the choice of x_1 reflects (Hicksian)[7] substitution effect.

3. Now give back the consumer the lost income. The choice will move from situation (p_1', p_2, M') to situation (p_1', p_2, M): a parallel movement of budget line. This is the income effect.

Figure 2: Income and Substitution Effect:

[6] For example, if the good is "normal" and income increases, then demand curve will shift to right.

[7] The name suggests that there are alternate notions of income and substitution effect.

Welfare Analysis: Consumer Surplus Typically, we read the demand curve with p_i on the vertical axis and x_i on the horizontal axis, this was the handiwork of Alfred Marshall, a Victorian Economist, who allegedly drew the first demand curve. Here is a justification. Note that, the Lagrangian says that marginal (psychological) benefit is equal to price, multiplied by a term λ (Lagrange multiplier). As it turns out, there is a nice economic interpretation of λ. If we reinterpret λ as marginal utility per dollar, the first order conditions merely show that at any level of x_i, price that the consumer is willing to pay is the marginal utility from consumption at this level, expressed in money terms. The demand curve, then, can be read up from the quantity axis: the height of the curve (price) is precisely the marginal benefit of consumption (in money terms).

Therefore for any consumption level up to a particular $\bar{x}_1$, the area under the demand curve is the total willingness to pay for consumption up to $\bar{x}_1$. However, as consumers pay a single price for consumption, the area of the rectangle bound by the price $\bar{p}_1$ and quantity $\bar{x}_1$ is less than the total WTP. As a result, the consumer always gets a surplus benefit while participating in market (in which he/she has to pay a single price for all units). The benefit is known as consumer surplus, CS.

{Figure 3: Consumer Surplus}

1.2 Producers

'Supply side' of the market(s) is characterized by, first, a technical relation and then an optimizing behavior. We will present the details of the technical relation first before turning into the optimizing decision of the producer.

1.2.1 Production Function

A production function is the technical specification between inputs and the *maximum* output which can be produced. Suppose we have only two inputs: labor and capital (machines). A production function (which is a description of the production process) says that the output Q, depends on K, L by the following relation $Q = F(K, L)$. The marginal product of any factor (that is input) of production is defined by $\frac{\partial F}{\partial K}$, say. We assume that marginal product is positive, i.e. $\frac{\partial F}{\partial K} > 0$ and decreasing, i.e. $\frac{\partial^2 F}{\partial K^2} < 0$.

The properties are very close to that of utility function. But there is one important difference. Utility functions are 'psychological' in nature, and the utility number does not convey any special meaning. If my utility number from consuming 2 apples and 5 bananas is 20, and yours is 40, it is meaningless to claim that your utility is twice that of me. On the other hand, output of any production is measured in terms of quantities. Therefore, the numbers matter.

Short Run and Long Run The concepts of '*short*' and '*long*' runs are important in all branches in Economics. Briefly, a short run occurs when we can change only a few factors of production given an external change (e.g. a sudden surge in demand). For example, higher production can be achieved by increasing the number of labor than buying more machines (being costly). We say that labor is 'variable' factor and capital is 'fixed' factor. After some time, say six months, we may adjust the number of machines.

Thus, in short run some (may be all) factors are fixed, while in LR, all factors are variable. Note that the length of the 'run' depends on production technology, availability etc.

Traditionally, textbooks assume that labor is more variable factor of production, while capital is more fixed (there is no basis of such assumption). The short run production function then can be represented only as function of labor, e.g. $Y = F(\bar{K}, L) = G(L)$, with $G' > 0, G'' < 0$

In long run, production functions are typified by their returns to scale property. If, for example, we increase all K, L by a factor λ, and output increases by more than the factor λ, the production function exhibits increasing returns to scale. Similarly, we can define constant and decreasing returns to scale.

1.2.2 Cost Minimization

The producer minimizes the cost of production to meet a target output Q. Suppose w is the wage rate of labor and r is the cost of machines (rental rate). If the producer could choose both K and L, his problem is

$$\begin{aligned} & \min_{K,L} wL + rK \\ \text{such that } F(K,L) &= \bar{Q} \end{aligned}$$

where, $\bar{Q}$. is the target rate of output.

This is long run cost minimization, when one can vary both K and L at will.

Long Run Cost Function Solving the minimization exercise, we get $L = L(w,r,Q)$ and $K = K(w,r,Q)$, that is the inputs depend on the input prices as well as the target level of output. The functional relationship is known as conditional factor demand function. Putting this into the objective function, we get the cost function $C(Q) = wL(w,r,Q) + rK(w,r,Q) = C(w,r,Q) = C(Q)$,assuming w, r are just parameters.

- Marginal cost of production is defined by $MC = \frac{dC}{dQ} = C'(Q)$
- Average cost of production is $AC = \frac{C(Q)}{Q}$

There is a close relation between marginal and average cost (or, all margin and average you can think of). If average cost is falling, marginal cost must be below average. If average cost is rising, marginal cost must be above average. If average cost is constant, marginal cost is equal to average cost.

There is also a close relation between (by this time, this should be evident) cost of production and technology. For this reason, economists pay close attention to cost structure, because cost data is more easily available (than technology data). For example, if long run production technology is such that $Y = K^a L^{1-\alpha}$, then average cost of production is constant: if we scale up the production by a factor λ, both total cost and production scales up by λ, leaving the average cost unchanged. Similarly, a production function is such that $Y = K^a L^\beta, \alpha + \beta > 1$ will be characterized by declining AC (*Economies of scale*). With $\alpha + \beta < 1$, AC will be increasing (*Diseconomies of scale*).[8]

Short Run Cost Function In short run, one (or more) of the factors is fixed. Hence the short run cost function has the following form $C_s(Q) = F + \phi(Q)$, where F is the fixed cost of production (arising out of fixed factor) and $\phi(Q)$ is the variable cost of production (arising out of fixed factor. In short run the average cost then consists of two factors: average fixed cost (AFC) and average variable cost (AVC). One can also define the short run marginal cost by $\phi'(Q)$.

If the production process exhibits diminishing marginal productivity (in variable factor), then short run marginal cost must be increasing in Q. Successively, to produce one unit more, one has to hire more and more units of variable factor, thus pushing up the marginal cost of production as Q increases.

Supply Decision: Competitive Firm In a competitive market, there are many firms producing and selling identical product. A single firm cannot change the market price OR input prices. Given the market price, the firm then maximizes its profit $\pi = pq - C(q)$ by choosing its output level q. The first order condition states $p = MC(q)$. Ths, given any price, the intersection between market price and MC will determine how much the firm is willing to produce, which implies that the marginal cost curve *is* the supply curve for the firm in the short run. Again, by horizontally adding the individual supply curves, one gets the market supply curve.

[8] Note

$$AC = \frac{wL + rK}{f(K, L)}$$

These properties are easily established.

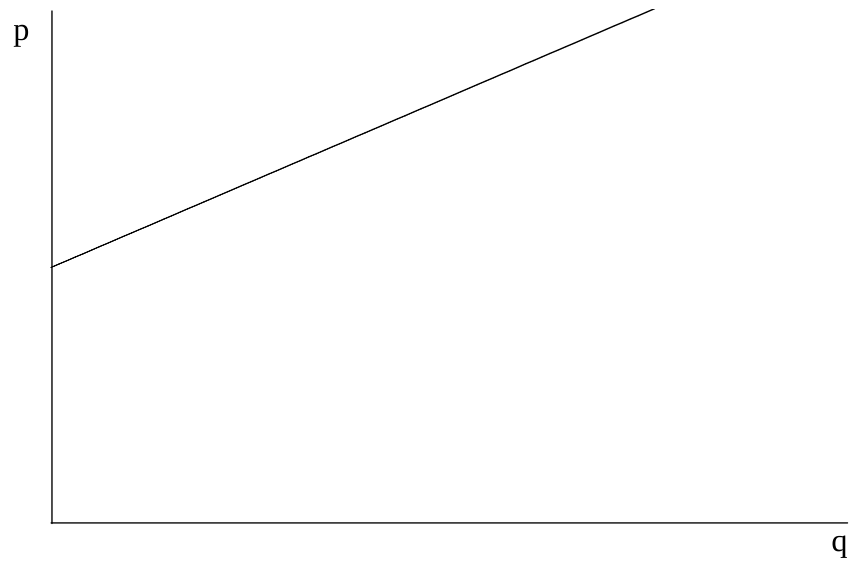

Figure 4: A Typical Supply Curve: $p = a + bq$

Shifts in individual supply curve (leftward/rightward) happen if marginal cost of production changes. A reduction in marginal cost will shift the curve to right (positive supply shock). Reduction in marginal cost happens if the technology improves (you can produce the same unit with lower quantity of inputs) OR if the prices of inputs fall (or both). At the market level, a positive supply shock is achieved if, in addition to the above factors, we also have new entrants coming in as well as effect from complementary markets (e.g. higher oil production will also increase supply of natural gas and vice versa).

At any price, the area under the supply curve is the total variable cost of production. However, by selling the quantity in the market, the firm gets a revenue which is more than TVC. The difference is known as producer surplus, a measure of benefit that the producers reap by participating in the market.[9]

[9]If the fixed cost is zero, there will be no difference between producer surplus and profit.

{Figure 5: Producer Surplus}

Market equilibrium occurs when the downward sloping demand curve (hopefully) intersects the upward sloping supply curve. The intersection of these two curves determine the ruling p^* and Q^* prevailing in the market.

{Figure 6: Market Equilibrium and Surplus}

Social welfare is measured as sum of consumer and producer surplus, $W = CS + PS$. Total surplus is maximized at $q = q^*$. That is, the surplus at any q, $\int_0^q (D(\tau) - s(\tau))\, d\tau$ is maximum at $q = q^*$. Note that social welfare is maximized through action of selfish consumers (interested in maximizing their own utility) and producers (interested in maximizing their own profit).[10] This is one enduring appeal of perfectly competitive market, and the associated policy of "no policy" by the free market economists. At any $q \neq q^*$, there is a mismatch between the buyers price (marginal utility of the good) and sellers' price (marginal cost of the good).

1.2.3 First Best Outcome

Here, we add a small subsection on what Economists say as first best. Suppose there are two individuals (or social groups), 1 and 2. If you can make one better off *without making the other worse off*, the situation is not first best, and can be improved upon. Suppose I have 10$ in my pocket which I must divide among two individuals. If I give 5$ to individual 1 and 2 $ to individual 2, then can improve on individual 2 by giving him/her more (drawing out

[10]This is not quite a class of History of Economic Thought, but the following quote from the pioneer of modern economics, Adam Smith (1723-1790) is worth mentioning. In a somewhat classical English, he wrote "..*Every individual... neither intends to promote the public interest, nor knows how much he is promoting it... he intends only his own security; and by directing that industry in such a manner as its produce may be of the greatest value, he intends only his own gain, and he is in this, as in many other cases, led by an* ***invisible hand*** *to promote an end which was no part of his intention*". (*Theory of Moral Sentiments*, part IV, Chapter 1)

of my reserve of 3$), without harming individual 1. Then the situation is not first best. However, if I have allocations like $(10,0),(9,1),..(7,3)....(0,10)$, to make one better off I have to take some money from the other person. All such situations are first best. Notice that first best does not speak anything about inequality aversion.

There are two equivalent ways of getting into the first best. Suppose person i has utility $U^i(x_i)$ from allocation x_i. Then, either

$$\begin{aligned} & \max \sum U^i \\ \text{s t } x_1 + x_2 &= \bar{x} \end{aligned}$$

Or

$$\begin{aligned} & \max U^i(x_i) \\ \text{s.t. } U^j(x_j) &= m \text{ (a constant)} \\ x_1 + x_2 &= \bar{x} \end{aligned}$$

will yield the same outcome. They may seem trivial, but notice that the second formulation actually picks up the fact that, *first best*, in itself, will not guarantee equal distribution of resources. The constant m can be anything.

1.3 Elements of Statistics

Here we provide a very brief recapitulation of the statistical tools that we are going to require. The section is intentionally brief because most of the items have been taught in your main course work.

Random variables are those which occur with some probability (e.g. the faces of dice when one throws one). They are of two types: discrete (can take values line $0,1,2,3,..$) or continuous (e.g. temperature). If $x_1, x_2...x_n$ are different realizations of the discrete random event, and if $p_1, ..p_n$ be the associated probabilities we must have $p_i \geq 0$ and $\sum p_i = 1$. In case of continuous distributions, we have a probability density function $f(x)$ such that $f(x) \geq 0$ and $\int f(x) = 1$ when the integration is defined over the supports of the distribution.

Expected value of a r.v. (random variable) is

$$\begin{aligned} E(x) &= \mu = \sum p_i x_i \\ &= \int x f(x) dx \text{ for continuous distribution} \end{aligned}$$

The 'expected value' is like a long run forecast (e.g. the expected minimum/maximum of temperature, the data of which is collated for 30/40 years).

Given the expected value, variance is a measure of how the data will be dispersed around the expected value, again for a long term view. It can be defined as $\sigma^2 = E\left(x-\mu\right)^2$. Often, it is useful to work with its square root, the standard deviation (to get the units correct). Variance can also be written as $\sigma^2 = E\left(x^2\right) - \mu^2$

If we have data on two variables x and y, with mean μ_x and μ_y respectively, the covariance is defined as

$$cov\left(x,y\right) = E\left(x-\mu_x\right) * \left(y-\mu_y\right)$$

The expectation is defined over the *joint probability* of x and y. Covariance can be written as $cov\left(x,y\right) = E(xy) - \mu_x\mu_y$ If two variables x and y are random, the joint variance given by $var\left(x \pm y\right) = var\left(x\right) + var\left(y\right) \pm 2cov(x,y)$

(Linear) Correlation is defined as

$$\rho\left(x,y\right) = \frac{cov\left(x,y\right)}{\sigma_x\sigma_y}.$$

Apart from being a unit free measure, correlation has a great advantage: it lies between -1 and 1. But ρ has some problems: higher ρ can exist between variables which are totally unrelated (e.g. tea production in Assam and divorce rate in Papua New Guinea). Unless backed by theory, such events are known as spurious correlation. Also, ρ close to zero *may not be* interpreted as absence of association. It indicates that there is no linear association between these two (but there can be nonlinear association).

Terms to Remember

- Preferences: weak, strong, indifferent
- Marginal utility, indifference curve, MRS, budget line.
- Demand Function, Demand Curve
- Consumer Surplus
- Production Function
- Cost Minimization, cost function.
- Marginal cost, average cost.
- Short run and long run
- Supply curve, market equilibrium
- Producer surplus
- First Best
- Statistics: mean, variance, covariance, correlation