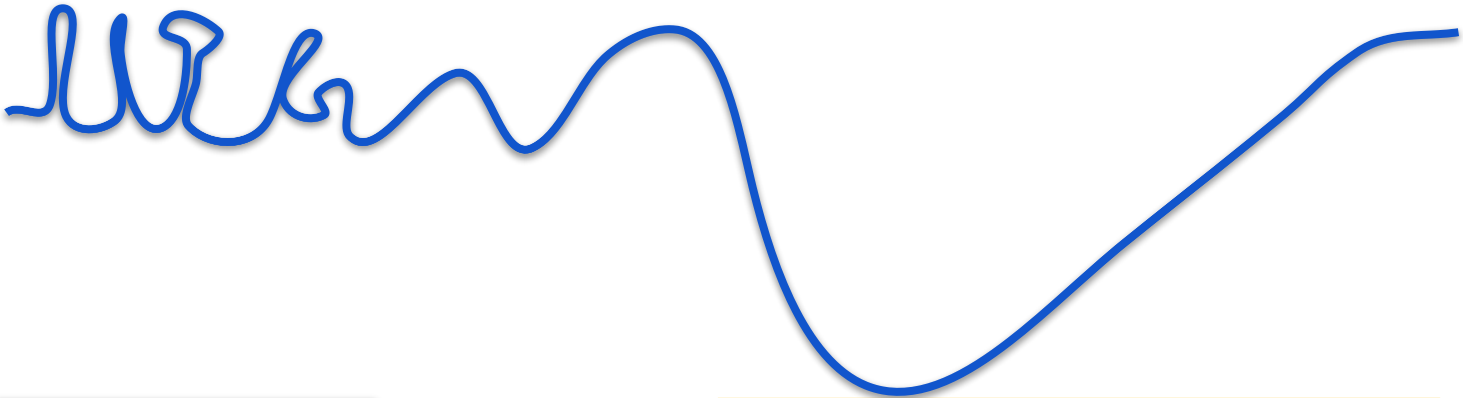
Computing with Signals



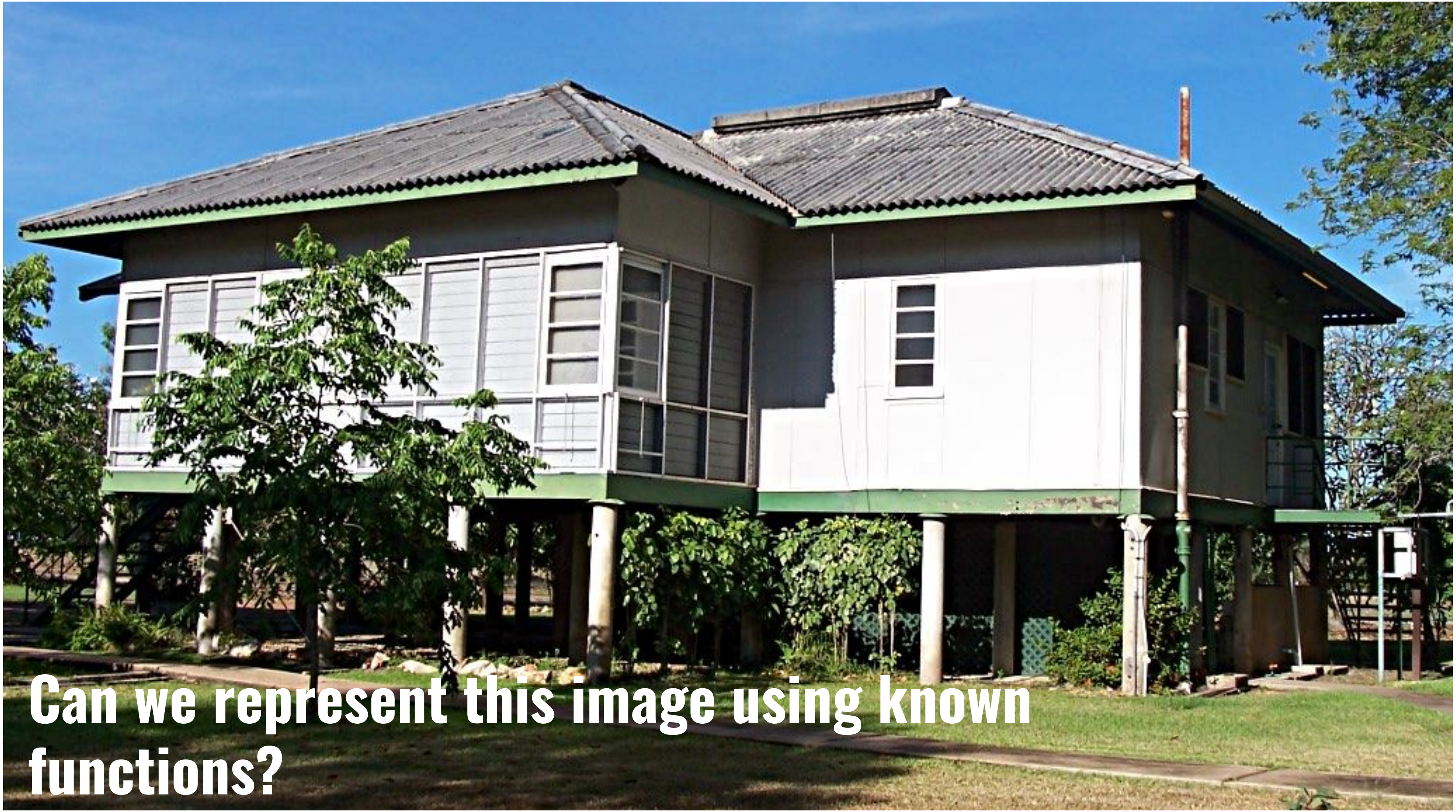
DA 623

Jan - May 2024

IIT Guwahati

Instructors: Neeraj Sharma

Lecture-14
(and more)



Can we represent this image using known functions?

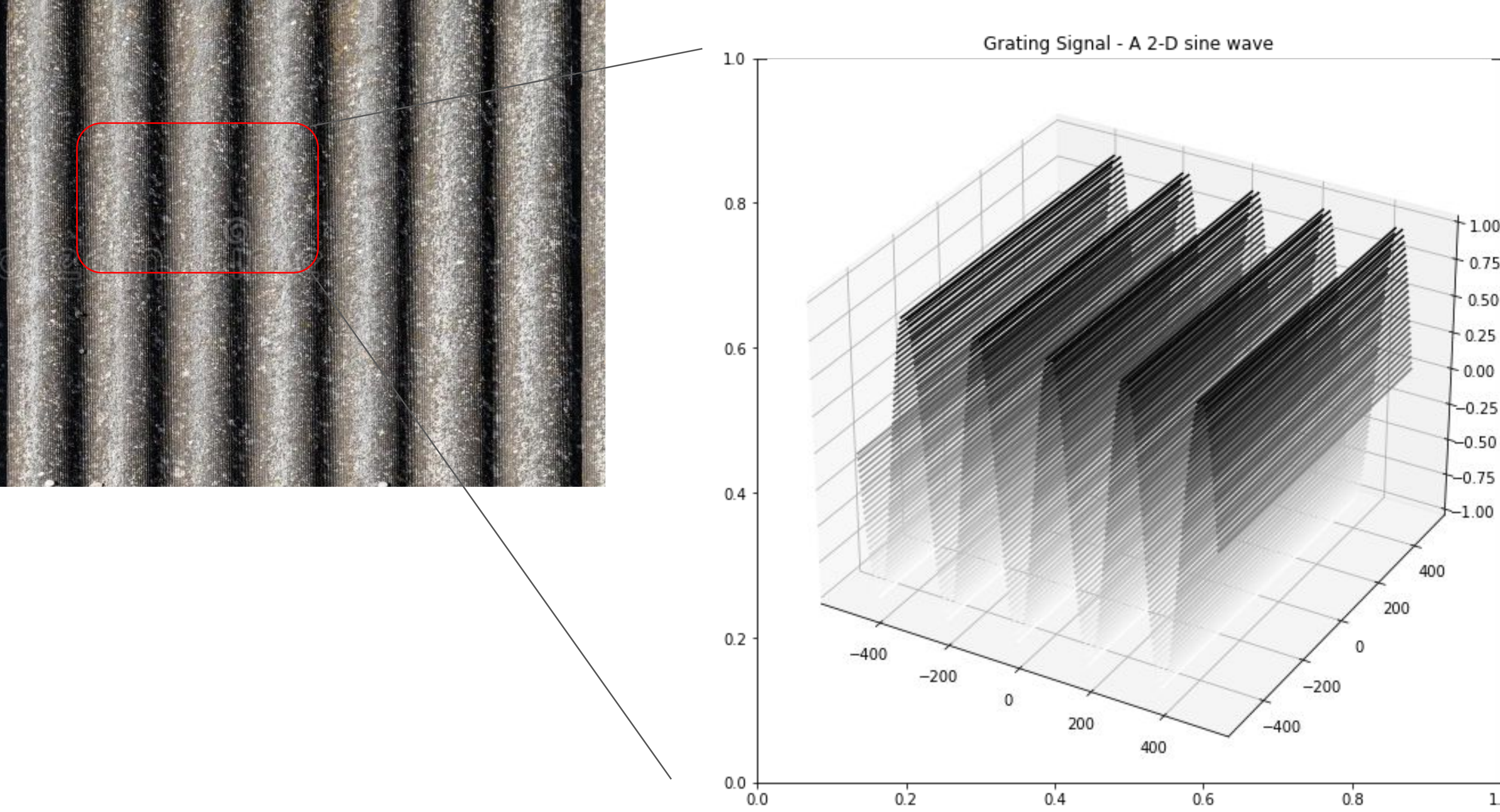
Modeling the patterns inside the image

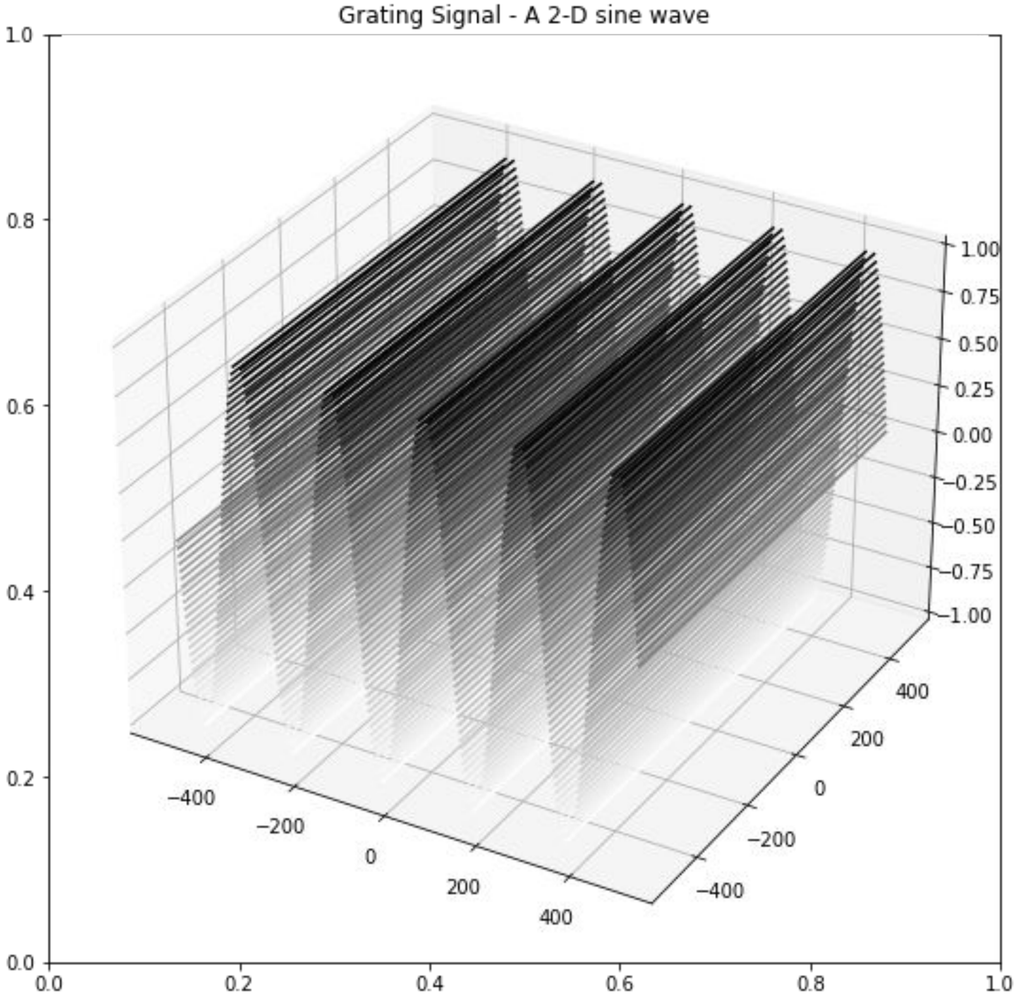
Do you spot a 1-D sine wave here

Do you spot a 1-D sine wave here

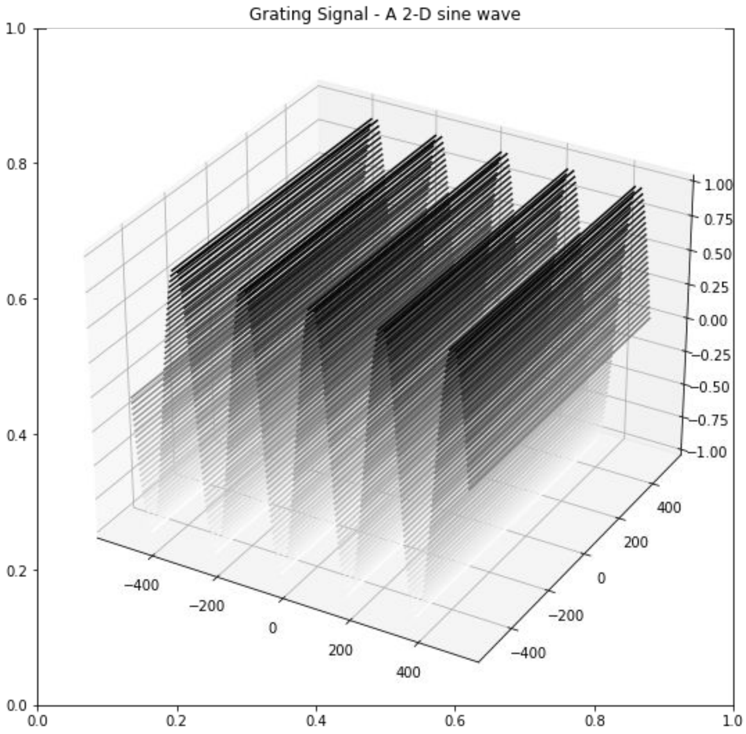
Do you spot a 1-D sine wave here

Do you spot a 2-D sine wave here

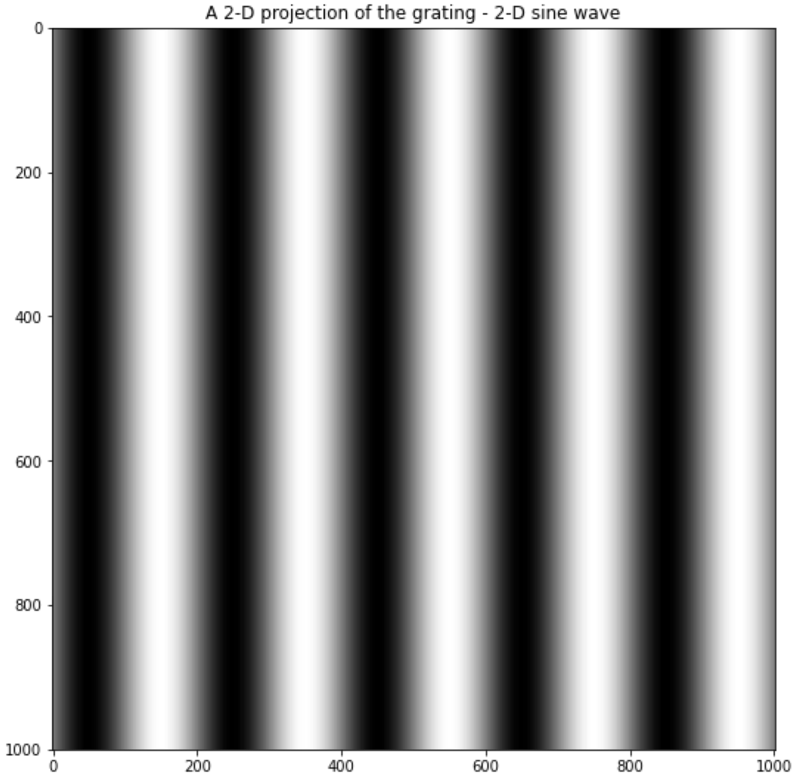

Grating Signal - A 2-D sine wave
1.0
0.8
0.6
0.4
0.2
0.0
0.0
0.2
0.4
0.6
0.8
1.00
0.75
0.50
0.25
0.00
−0.25
−0.50
−0.75
−1.00
−400
−200
0
200
400
400
200
0
−200
−400



Synthesizing 2-D Gratings



Grating Signal - A 2-D sine wave



A 2-D projection of the grating - 2-D sine wave

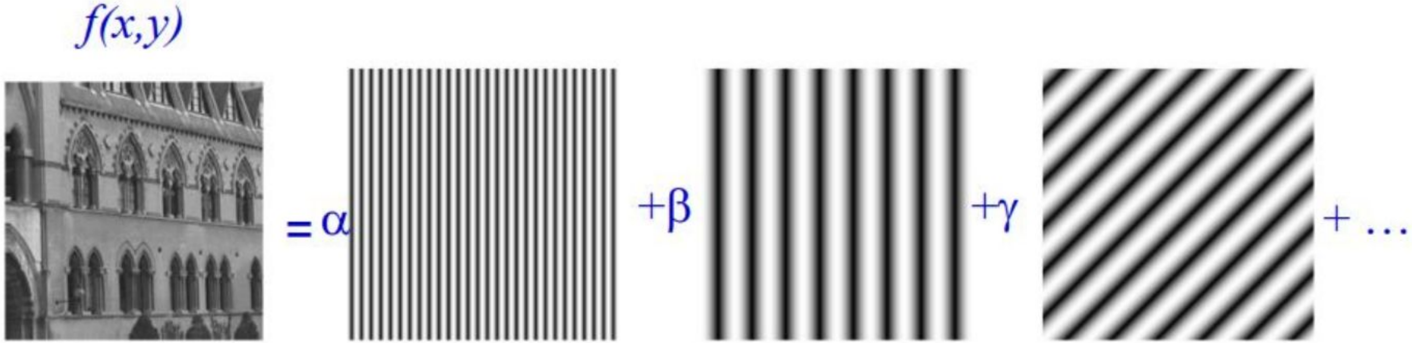
Joseph Fourier

Joseph Fourier's
Fourier Transform

The spatial function $f(x, y)$

$$f(x,y) = \int_{-\infty}^{\infty} \int_{-\infty}^{\infty} F(u,v) e^{j2\pi(ux+vy)} \, du \, dv$$

is decomposed into a weighted sum of 2D orthogonal basis functions in a similar manner to decomposing a vector onto a basis using scalar products.



2D Fourier Transform

Fourier Transform:

$$\boxed{F(u,v) = \iint_{-\infty}^{\infty} f(x,y)e^{-i2\pi(ux+vy)}dxdy}$$

u and v are frequencies along x and y, respectively

Inverse Fourier Transform:

$$\boxed{f(x,y) = \iint_{-\infty}^{\infty} F(u,v)e^{i2\pi(xu+yv)}dudv}$$

Reference:
https://fpcv.cs.columbia.edu/Monographs

2D Fourier Transform: Discrete Images

Discrete Fourier Transform (DFT):

$$\boxed{F[p,q] = \sum_{m=0}^{M-1} \sum_{n=0}^{N-1} f[m,n] e^{-i2\pi pm/M} e^{-i2\pi qn/N}}$$

$p = 0 \ldots M-1$

$q = 0 \ldots N-1$

p and q are frequencies along m and n, respectively

Inverse Discrete Fourier Transform (IDFT):

$$\boxed{f[m,n] = \frac{1}{MN} \sum_{p=0}^{M-1} \sum_{q=0}^{N-1} F[p,q] e^{i2\pi pm/M} e^{i2\pi qn/N}}$$

$m = 0 \ldots M-1$

$n = 0 \ldots N-1$

Sinusoidal Waves

In 1D the Fourier transform is based on a decompostion into functions $e^{j2\pi ux} = \cos 2\pi ux + j \sin 2\pi ux$ which form an orthogonal basis. Similarly in 2D

$$e^{j2\pi(ux+vy)} = \cos 2\pi(ux + vy) + j \sin 2\pi(ux + vy)$$

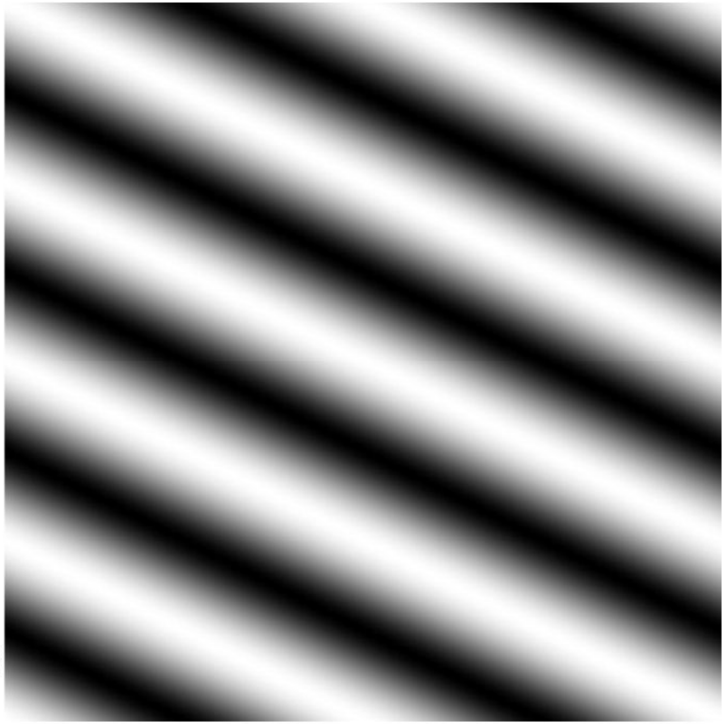
The real and imaginary terms are sinusoids on the x, y plane. The maxima and minima of $\cos 2\pi(ux + vy)$ occur when

$$2\pi(ux + vy) = n\pi$$

write $ux + vy$ using vector notation with $\mathbf{u} = (u, v)^\top, \mathbf{x} = (x, y)^\top$ then

$$2\pi(ux + vy) = 2\pi \mathbf{u}.\mathbf{x} = n\pi$$

are sets of equally spaced parallel lines with normal $\mathbf{u}$ and wavelength $1/\sqrt{u^2 + v^2}$.



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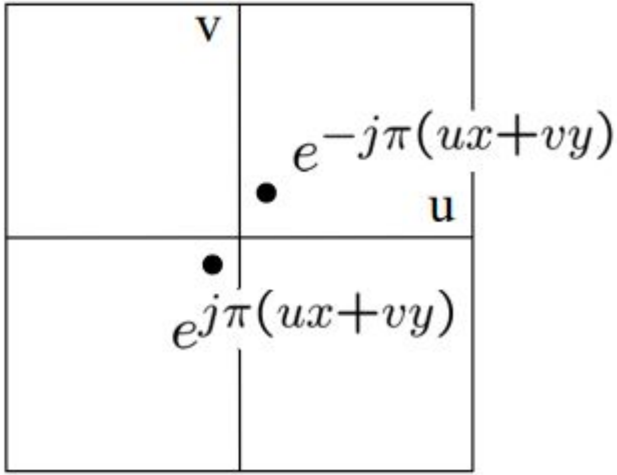
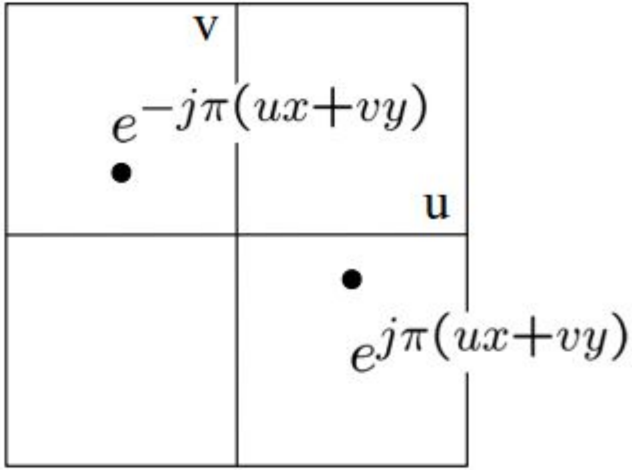
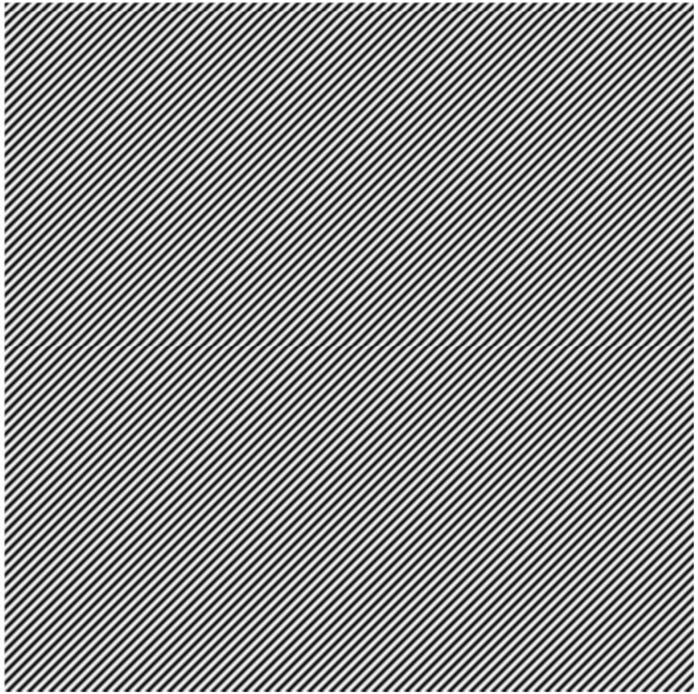
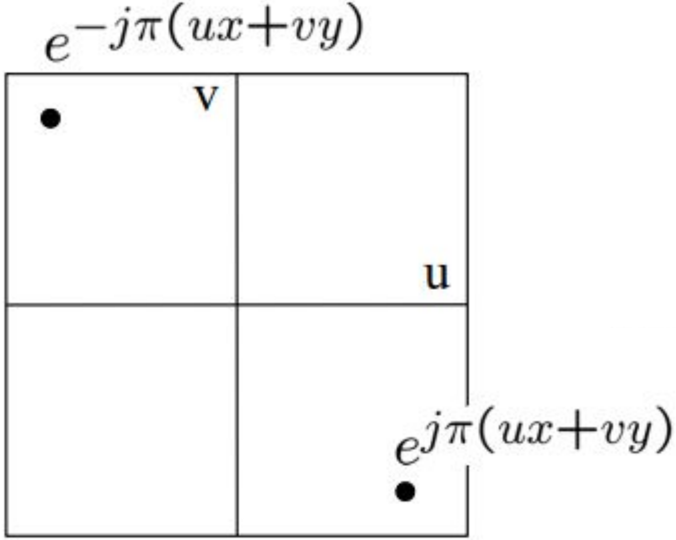
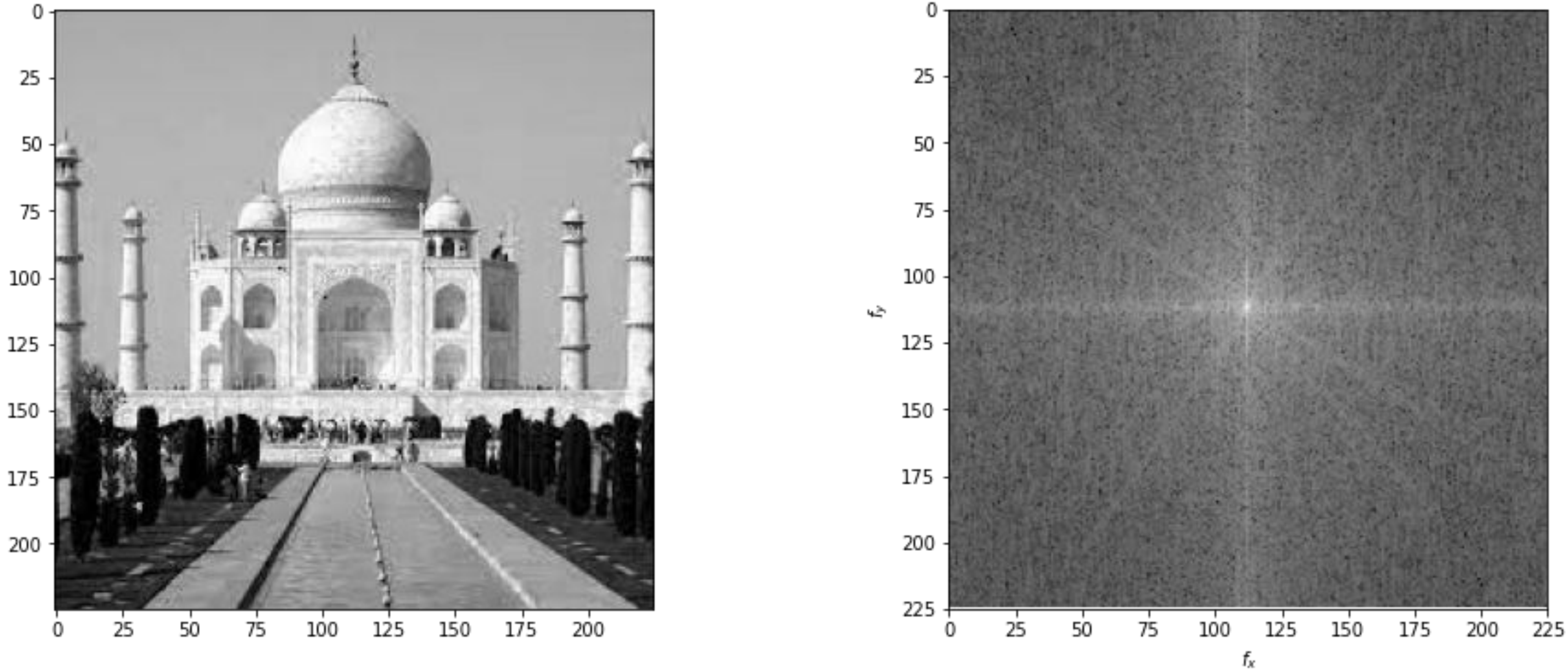

v
$e^{-j\pi(ux+vy)}$
u
$e^{j\pi(ux+vy)}$


v
$e^{-j\pi(ux+vy)}$
u
$e^{j\pi(ux+vy)}$




$e^{-j\pi(ux+vy)}$
v
u
$e^{j\pi(ux+vy)}$



0
25
50
75
100
125
150
175
200
0
25
50
75
100
125
150
175
200
0
25
50
75
100
125
150
175
200
225
f_y
0
25
50
75
100
125
150
175
200
225
f_x