Mid Term Answer Key, 2024

Part A

1. If production function is $Y = K^{\alpha}L^{1-a}$, a being a positive fraction, the form of LR cost function must be (option a/Option b)

a) $C = \theta * Y$; b) $C = \theta Y^2$, where θ is a constant.

Ans: a.[**PS 1 Q6**]

2. Mr. X has Bernoulli utility function $v(x) = x^{.5}$. His income can be Rs 25 with probability 1/3 or Rs 36 otherwise. The risk premium is

____

ANS: His CE is a number x such that $\sqrt{x} = \frac{1}{3}\sqrt{25} + \frac{2}{3}\sqrt{36} = 5.67$. *So* $x = (5.67)^2 = 32.15$. The average value of the lottery is $\frac{1}{3} * 25.0 + \frac{2}{3} * 36.0 = 32.3\dot{4}$. Thus, $RP = 32.34 - 32.15 = 0.19$

3. The demand curve is $P = 20 - Q$. suppose market price is 10. Then consumer surplus must be ________

ANS: The consumer surplus will be the area of triangle bound by the following coordinates: $(20, 0), (10, 0)$ *and* $(10, 10)$. *The triangle has a height of* 10 *and base of* 10. *Hence* $CS = 0.5 * 10 * 10 = 50$

4. Logical explanation of diversification of risky assets is associated with the following economist (a) Markowitz, (b) Abrahamovitz, (c) von Neuman (tick)

ANS: (a), Markowitz

5. If quadratic formulation of Bernoulli utility function exhibit risk aversion, it must exhibit aversion to downward risk. (True/False).

ANS: False. Risk aversion requires $u'' < 0$, while downward risk aversion requires $u''' > 0$. For linear quadratic case, $u''' = 0$. Hence the agent is neutral to downward risk.

6. Under the status quo scenario, a company will go bankrupt with probability 0.9. In this case, the company's utility is zero. Current wealth of the company is Rs 900 and it has Bernoulli utility function $v(x) = x^{0.5}$. Investing some positive amount in R&D (from current wealth) will reduce

probability of bankruptcy to 0.5. The maximum amount that the company willing to invest is____________.

ANS: The amount of cash, say c, should be such that $.9*0+.1*\sqrt{900} \leq .5*0+.5*\sqrt{900-c}$*. solving which we get* $c \leq 864.0$.

7. Lottery L1 promises (2, 12) with probabilities (.2, .8). Now consider L2 which gives (8, 18) with (.8, .2). Which one will you prefer if you have mean variance utility? Ans________________

Mean of lottery $L1 = .2*2+.8*12 = 10.0$ *and variance* $.2*(2-10)^2 + .8*(12-10)^2 = 16.0$.

Mean of lottery $L2 = .8*8+.2*18 = 10.0$ *and variance* $.8*(8-10)^2 + .2*(18-10)^2 = 16.0$

Thus you will be indifferent.

8. Continue with Q7. Which lottery you will prefer if your Bernoulli utility function is $\ln(w)$? Ans__________

Expected utility from L1: $=.2*\ln(2)+.8*\ln(12) = 2.127$

Expected Utility from L2: $= .8*\ln(8)+.2*\ln(18) = 2.242$

So you will prefer L2.

[Both from PS3]

9. If Arrow Pratt measure of relative risk aversion is constant in wealth, the corresponding measure of absolute risk aversion must exhibit ________________ (decreasing/increasing/constant) risk aversion.

Note that $R_R(w) = w * R_A(w)$*. Thus,* $R_R' = R_A + wR_A'$*. If* $R_R' = 0$, $R_A' = -\frac{R_A}{w} < 0$*. So* R_A *is decreasing in* w*.*

10. A country like Ecuador undergoes Dollarization, that is, each price and income are now expressed in US$ instead of Ecuadorian Sucre, scaling up/down each item by appropriate exchange rate. Mr. Xavier, an Ecuadorian citizen, drank 2 cups of coffee every day before Dollarization. After the exercise, his demand for coffee will go up/ go down/remain same.

Same, because demand function is homogeneous of degree zero in prices and income.

[PS 1, Q4]

Part B

11 a) Given two lotteries L_1 and L_2 (consumptions are bound between c_0 and c_1), consumers with increasing Bernoulli utility function $v(c)$ prefer$L1$ over$L2$. Show how this condition can correspond to a comparison between probability density functions of L_1 and L_2(you can call them $f_1$and f_2). Consumptions, distribution functions etc. are continuous.

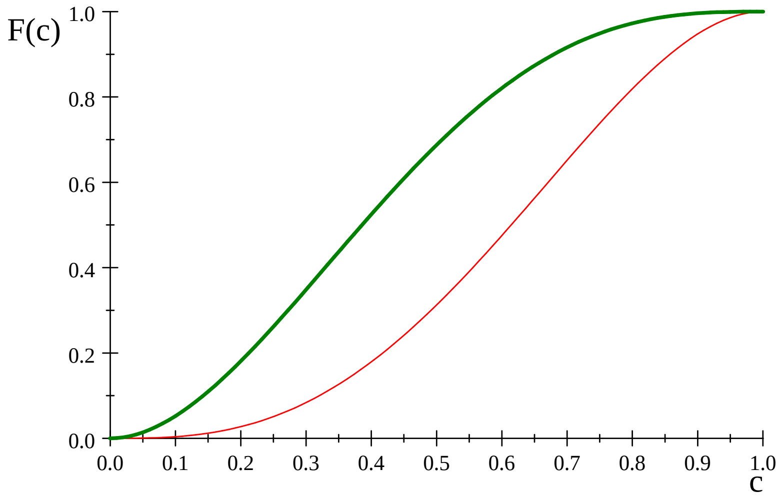
b) A lottery $L1$ is described as $L1 = [(0.2, 0.4, 0.4), (1, 2, 3)]$. In the probability simplex (probability triangle) done in class, indicate the set of lotteries that FOSD L1. In this question, you should answer focusing on the statistical definition of FOSD.

c) Consider the validity of the following statements (i) if $F(FOSD)G$, then $F(SOSD)G$. (ii) If $F(SOSD)G$ then $F(FOSD)G$. Focus on the statistical definition of FOSD and SOSD.

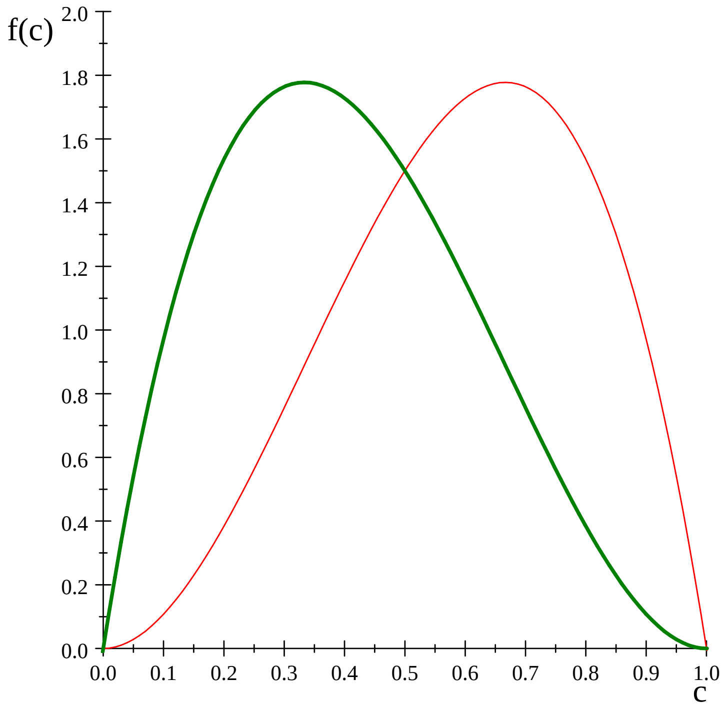
Ans: a) Suppose the lotteries have cumulative density functions $F_1(c)$ and $F_2(c)$. Then for part (a), you should show $F_1(c) < F_2(c) \rightarrow \int v(c)dF_1(c) > \int v(c)dF_2(c)$, that is, the probability of having less than $c \in (c_0, c_1)$ is higher under L_2 than L_1. (**done in class, lecture note 2, pp 12**)

The relation between F_2 and F_1 is shown here[1] (assuming $c_0 = 0$ and $c_1 = 1$). Green is F_2 and red is F_1.

[1] This is just an example, but the general case looks like the same.



Note that since F_1 must be below F_2 in the beginning and $F_1(c_0) = F_2(c_0) = 0$, the slope of F_1 must be less when c is low. Conversely, after a critical c^*, the slope under F_1 must be higher than that of F_2. The slope of the cumulative density function is the probability density function. Thus, the height of pdf under f_1 will be higher than f_2 for all $c > c^*$. Thus, L_1 provides more weight on higher values of consumption than L_2.



This part was also **done in class**.

b) Given L_1, the cumulative distribution function is given by

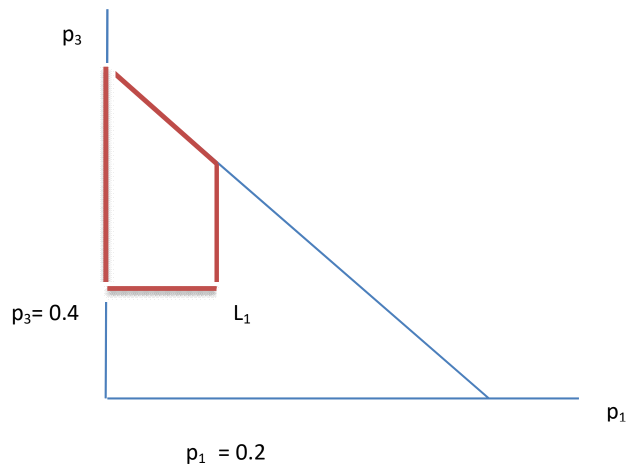
$$\begin{aligned} F(L_1) &= .02 \text{ if } c = 1 \\ &= .02 + .04 = .06 \text{ if } c = 1 \text{or } 2 \\ &= 1 \text{ if } c = 1 \text{ or } 2 \text{ or } 3 \end{aligned}$$

Thus, a lottery (p_1, p_2, p_3) will 'beat' L_1 if (omitting the consumptions)

$$\begin{aligned} p_1 &< .02 \\ p_1 + p_2 &< .06 \\ p_1 + p_2 + p_3 &= 1 \end{aligned}$$

From (3), $p_1 + p_2 = 1 - p_3$. Hence the second inequality requires $1 - p_3 < .6$ or $p_3 > .4$.

Since in the probability triangle diagram, we just focus on p_1 and p_3, we are done.



c) (i) If $F(FOSD)G$, then the cimulative distribution functions, (call them F and G) are such that $(F - G) < 0$ for all $c \rightarrow \int (F - G)\, dc < 0 \rightarrow F(SOSD)G$.

(ii) If $F(SOSD)G$ then not for all cases $F(FOSD)G$. SOSD allows for multiple crossings of F and G, but FOSD does not. So the statement is not true.

12. Mr Y's income in the good state of the world is 100\$ (occurs with probability $\frac{2}{3}$) and 30\$ in bad state (occurs with complementary probability). In each state, Mr. Y has a Bernoulli utility function $v(w) = \ln(w)$. An insurance company offers Mr Y a fair insurance contract with premium α and indemnity I.

a) In equilibrium, what should be the values of α and I?

b) After Mr Y have purchased the insurance, what is his wealth in good state (W_1) and in bad state (W_2)?

c) Is Mr.Y better off purchasing the insurance than a situation of autarky?

d) Now suppose, due to market imperfections, the insurance company offers contract with proportional loading. The loading factor is 1.5. Redo parts (a),(b),(c).

e) *If companies offer a loaded contract and consumer still purchases insurance, he/she never buys full insurance.* Comment on the validity of the statement

Ans:

a) Here, $w_0 = 100, d = 70$. If the contract is fair, then $\alpha = (1-p) I = \frac{1}{3} I$. Then

$$\begin{aligned} W_1 &= 100 - \alpha = 100 - \frac{1}{3} I \\ W_2 &= 100 - d - \alpha + I \\ &= 30 - \frac{1}{3} I + I \\ &= 30 + \frac{2}{3} I \end{aligned}$$

Mr Y maximizes

$$\frac{2}{3} \ln \left(100 - \frac{1}{3} I\right) + \frac{1}{3} \ln \left(30 + \frac{2}{3} I\right)$$

FOC is

$$\frac{2}{3} * \frac{1}{3} \frac{1}{100 - \frac{1}{3} I} = \frac{1}{3} * \frac{2}{3} * \frac{1}{30 + \frac{2}{3} I}$$

Solving this, $I = 70$. Hence $\alpha = \dfrac{70.0}{3} = 23.33$

(**Note:** you can directly apply the result that $I = d$. Then each step has to be explained).

b) Here

$$\begin{aligned} W_1 &= 100 - 23.33 = 76.67 \\ W_2 &= 30 + \frac{2}{3} * 70.0 = 76.67 \end{aligned}$$

c) In autarky, Mr Y's expected utility is

$$\frac{2}{3} \ln (100.0) + \frac{1}{3} \ln (30) = 4.2038$$

With insurance, his expected utility is

$$\frac{2}{3}\ln(76.67)+\frac{1}{3}\ln(76.67)=4.3395$$

clearly, he is better off buying the insurance.

d) With a loading factor of $\frac{3}{2}$, $\alpha = m*(1-p)*I = \dfrac{I}{2}$. Then,

$$\begin{aligned} W_1 &= 100-\alpha = 100-\frac{1}{2}I \\ w_2 &= 100-d-\alpha+I \\ &= 30-\frac{1}{2}I+I \\ &= 30+\frac{1}{2}I \end{aligned}$$

Mr Y maximizes

$$\frac{2}{3}\ln\left(100-\frac{1}{2}I\right)+\frac{1}{3}\ln\left(30+\frac{1}{2}I\right)$$

the FOC is

$$\frac{2}{3}*\frac{1}{2}*\frac{1}{100-\frac{I}{2}}=\frac{1}{3}*\frac{1}{2}*\frac{1}{30.0+\frac{I}{2}}$$

$\rightarrow I = 26.667$. Hence $\alpha = \frac{26.667}{2} = 13.334$. Equilibrium insurance contract is $(13.334, 26.667)$

Income in good and bad states are, respectively,

$$\begin{aligned} W_1 &= 100-13.334 = 86.666 \\ W_2 &= 30+13.334 = 43.334 \end{aligned}$$

His expected utility is $\frac{2}{3}\ln(86.666)+\frac{1}{3}\ln(43.334) = 4.231 > 4.2038$. Therefore, he is still better off buying the insurance.

e) If it is proportional loading, then people purchase less than full insurance. If, however, the loading is fixed, the budget line shifts inwards in a parallel fashion. Hence consumers end up purchasing full insurance (income in both states are equal)

[Variant of PS 3, as well as lecture notes]

13. Mr A consumes two goods, x and y. His utility function is $u = x^{0.25}y^{0.75}$. Mr. A's income is Rs100. Price of X is 1 Rupee and that of Y is 1 Rupee.

a) What must be his level of utility given income and prices?

b) Now Mr. A is transferred to another town where prices are Rs 1 and 2, respectively. Mr A complains that this amounts to a fall in lifestyle measured in terms of utility. To counter this, his boss proposes that the salary be raised to Rs M. What must be the minimum value of M?

ANS:

Maximizing $u = x^{0.25}y^{0.75}$ with respect to the budget constraint $p_x x + p_y y + M$ gives us the following demand functions

$$\begin{aligned} x &= \frac{1}{4}\frac{M}{p_x} \\ y &= \frac{3}{4}\frac{M}{p_y} \end{aligned}$$

Plugging back the expression in the utility function gives us maximum utility given prices and income

$$\begin{aligned} v(p_x, p_y, M) &= \left(\frac{1}{3}\right)^{0.25}\left(\frac{3}{4}\right)^{.75}\frac{M}{(p_x)^{0.25}(p_y)^{0.75}} \\ &= 0.61\frac{M}{(p_x)^{0.25}(p_y)^{0.75}} \end{aligned}$$

Thus, we have

$$\begin{aligned} v(1, 1, 100) &= .62 * \frac{(100)}{(1)^{0.25}(1)^{0.75}} \\ &= 62.0 \end{aligned}$$

b) Moving to the new town, let the compensation be m. Thus it must be

$$
\begin{aligned}
.62 * \frac{(100+m)}{(1)^{0.25}\,(2)^{0.75}} &= 62 \\
&\rightarrow m = 68.18
\end{aligned}
$$

Part C

14. a) Mr. A has income c_0. A lottery promises him another x_i, where x_i $(i = 1, 2, 3, \ldots, n)$ are different discrete realizations of a random variable X with zero mean and variance σ^2. Show that his risk premium (in Pratt's sense) is approximately proportional to the Arrow-Pratt measure of absolute risk aversion at c_0.

Ans: Problem Set 2, done in class.

15. Consider the model of investment in risky asset as done in the beginning of topic 3 (maximizing expected utility). Show that if my Bernoulli utility function is DARA, my optimal purchase of risky asset will increase with my initial endowment Y. Safe asset does not earn any interest rate. The risky asset earns r_h with probability p and r_l otherwise, with $r_h > 0 > r_l > -1$.

Ans: Done in Class. Also in the Varian book.