5. **Professor P hires a teaching assistant , Mr. A. Payoff function of P is $x - s$, where x is the number of hours taught by A and s is the total wage to A. If Mr.A teaches for x hours, his utility is $s - \frac{x^2}{2}$, and the reservation utility is zero.**

a) If Professor P chooses x and s to maximize his utility subject to the constraint that Mr. A is willing to work for him, how many hours will Mr. A teach and what will be the payment?

b) Suppose the wage schedule is linear: $s = ax + b$. P chooses a, b, but A chooses x. What values of a and b should Professor choose, given that he cannot directly monitor x?

Ans:

a) In this case, Mr P chooses x, s to maximise $(x - s)$ such that $s - \frac{x^2}{2} \geq 0$. Thus, P should choose an x such that $s - \frac{x^2}{2} = 0$. Thus, the principal chooses x in such a way that $x - \frac{x^2}{2}$ is maximised. Solving this, one gets $x^* = 1$.

b) Given that the wage schedule is linear, the agent chooses his x given a, b. Thus, the agent maximises $s - \dfrac{x^2}{2} = (ax + b) - \dfrac{x^2}{2}$. Maximising this, one gets $x = a$. This, then, becomes the ICC.

Now principal maximises $x - s = a - (ax + b) = a - a^2 - b$ such that $s - \frac{x^2}{2} = \frac{a^2}{2} + b = 0$.

I leave the exercise (optimal values of a and b) as a homework.