HS239: Economics of Uncertainty and Information

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1. In a society, there are two types of people. If Q amount of good is consumed, type I people derive utility $U_1 = 4\ln(Q) - T$, while type II people derive utility $U_2 = 2\ln(Q) - T$. The cost of providing Q amount is $C = 0.5 * Q$. If they do not buy anything, they get a utility of zero. It is known that $\frac{1}{3}$ part of the society are type I people.

a) Suppose the monopolist has identified the types and wish to extract all consumer surplus from them. What should be the quantity and per unit price charged to each group?

b) If the monopolist cannot identify the types, and yet continue to offer the menu in (a), calculate monopolists' profit. Who benefits, and how?

c) Suppose the monopolist devises a screening mechanism: (T, Q) based on types. Write down the ICC and PCC for both types.

d) Determine which constraints bind at the optimum.

e) Using the information in 1(d), figure out the optimal contracts.

2. A firm hires two kinds of workers: alphas and betas. One cannot distinguish between them a priori. Alpha will produce \$3000 worth of output per month and a beta produces \$2500 worth of output per month. The firm proposes an entrance test to separate alphas from betas. For each question that they get right, alphas spend half an hour to study and betas spend an hour. For either type, an hour of study is as bad as giving up 20\$. A worker will be paid \$3000 if the get at least 40 answers correct and \$2500 otherwise. What is the equilibrium of the scheme?

3. An employer has hired someone to undertake a project. If the project fails, it will lose 20000. If it succeeds, the project would gain \$100000.The employee can work (effort =1) or shirk (effort =0). If he works, the project

will succeed only half the time. If he shirks, the project will fail for sure. The agent's utility is \$10000 less if he works. The reservation utility of the agent is \$5000. The employer is choosing between whether to pay the employee a fixed wage of \$20000 or a performance related pay in which the wage is 0\$ if the project fails and 40000\$ if the project succeeds. If both parties are risk neutral, what compensation scheme should the employer use?

4. It is known that some fraction d of all new cars are defective. Defective cars cannot be identified as such except by sellers. Each consumer is risk neutral and values a non defective car at \$16000. New cars sell for \$14000 and used cars sell for \$2000. If cars do not depreciate physically with use (consumers value an defective new car at per with an used car) , what is the proportion d of defective new cars?

5. Professor P hires a teaching assistant , Mr. A. Payoff function of P is $x - s$, where x is the number of hours taught by A and s is the total wage to A. If Mr.A teaches for x hours, his utility is $\left(s - \frac{x^2}{2}\right)$,and the reservation utility is zero.

a) If Professor P chooses x and s to maximize his utility subject to the constraint that Mr. A is willing to work for him, how many hours will Mr. A teach and what will be the payment?

b) Suppose the wage schedule is linear: $s = ax + b$. P chooses a, b, but A chooses x. What values of a and b should Professor choose, given that he cannot directly monitor x?

6. Consider the agrarian institution of sharecropping. Suppose a landlord is considering a tenant to offer certain contract. Both landlord and tenant are *risk averse* with the utility function $U_i = E(y_i) - \frac{\beta_i}{2} var(y_i)$, here, y_i is the (stochastic) income of person i $(i = LL, T)$. β_i measures the degree of risk aversion. Income of the tenant is $y_T = sY - R$, where s is tenant's share in output and R is rent (if $R > 0$) or wage (if $R < 0$). In addition, tenant bears a cost $\frac{ce^2}{2}$ for the effort level e. Income of landlord is $y_{LL} = (1 - s) Y + R$. However, output and effort are stochastically linked,

$Y = e + \varepsilon$, where $E(\varepsilon) = 0$, but $var(\varepsilon) = \sigma^2 > 0$. If the tenant does not work for the landlord, he can go out and earn a reservation utility of $\bar{U}$.

a) Suppose the landlord cannot enforce e. What will be the choice of s, R? Will $s < 1$ (so that sharecropping is justified)?

b) Show that,in case a, one of the parties can be risk neutral ($\beta_i = 0$ for $i = LL$ OR T)