Multi-level logic Minimization

Dr. Chandan Karfa

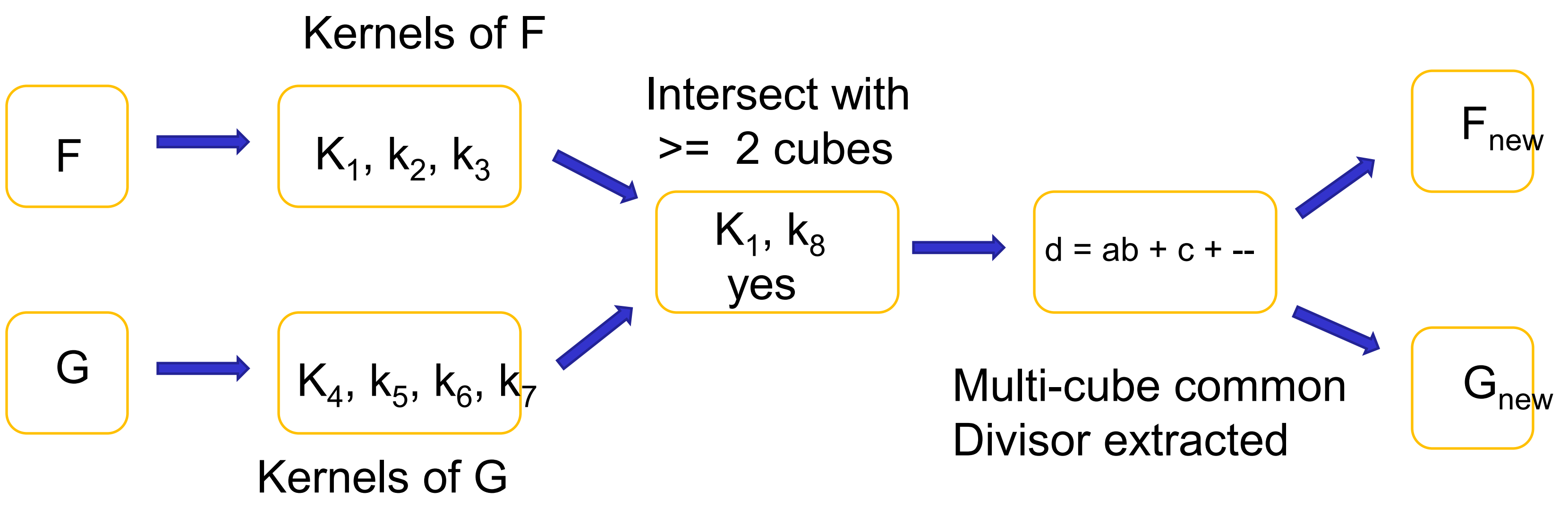
CSE IIT Guwahati

Text Book

- Chapter 6, Z. Kohavi and N. Jha, Switching and Finite Automata Theory, 3rd Ed., Cambridge University Press, 2010.

The Algebraic Method

1. Find kernels of F and G
2. Find kernels in intersections of K(F) and K(G)
3. Extract multi-cube common divisor D
4. Rewrite F and G using D



Rectangle covering – Identify all Kernels

- Identifying all Kernels of an Expression
- Consider a sum-of-products expression f with p cubes and q distinct literals.
- A $p \times q$ *cube–literal incidence matrix* can be defined for f in which element (i, j) is 1 if the j th literal is used in the ith cube, and 0 otherwise.
- *A rectangle* of this matrix denotes a set of rows and columns in which all entries are 1.
- Let (r, c) denote the row and column subsets of the rectangle.
- A rectangle $(r1, c1)$ is said to *contain* another rectangle $(r2, c2)$ if $r1 \supseteq r2$ and $c1 \supseteq c2$.
- A rectangle is called *prime* if it is not strictly contained in another rectangle.

Rectangle covering

- The co-rectangle of rectangle (r, c) is denoted as (r, $\bar{C}$) where $\bar{c}$ is the complement of the column subset c, i.e., it includes all columns of the matrix not in c.
- $f = uwz + uxz + yz + uv$.
  - 4 cubes and 6 distinct literals

Prime Rectangles:

1. ({*uwz, uxz*}, {*u, z*})
2. ({*uwz, uxz, uv*}, {*u*})
3. ({*uwz, uxz, yz*}, {*z*}).

Table 6.2 Cube–literal incidence matrix for f

	Literal					
Cube	u	v	w	x	y	z
uwz	1	0	1	0	0	1
uxz	1	0	0	1	0	1
yz	0	0	0	0	1	1
uv	1	1	0	0	0	0

({*uwz, uxz*}, {*u, z*}) whose co-rectangle is ({*uwz, uxz*}, {*v, w, x, y*}}

Rectangle covering

- A co-kernel of an expression can be derived from a prime rectangle (r, c) that contains at least two rows.
- Its co-rectangle (r, $\bar{c}$) yields the corresponding kernel, which can be derived as the sum of the cubes in r restricted to the literals in $\bar{c}$.
- The prime rectangle ({uwz, uxz}, {u, z}) yields co-kernel uz.
- Its co-rectangle ({uwz, uxz}, {v, w, x, y}) yields the kernel $w + x$
- Kernel is obtained by restricting $uwz + uxz$ to literals in {v, w, x, y}.

Extraction

- If two or more expressions have common divisors, the divisors can be extracted.
- The rectangle-covering method can be extended to perform extraction as well.
- There are two types of extraction methods:
  1. **Cube extraction**: Cube extraction refers to the extraction of a cube.
  2. **Kernel extraction**: Extraction of kernel from two or more expressions.

Find kernels in Intersections

- To perform kernel extraction, a **kernel–cube incidence matrix** is defined analogously to the cube–literal incidence matrix.
  - The kernels of each expression are identified.
  - The set of kernels for expression f_i is denoted by $K(f_i)$.
- To derive such a matrix, we first represent each cube in a kernel with a new variable and the kernel by a set of such variables.
  - For each minterm/cube in kernels, a new variable is introduced
- The sets of kernels can now be represented in terms of these variables.
- Form an auxiliary function f_a as a sum of cubes, where a cube is the product of the new variables corresponding to a kernel for all the expressions under consideration.

Example

$f_1 = (uwz + uxz + yz)$

$f_2 = (vw + vx + vyz)$

$K(f_1) = \{(w + x), (uw + ux + y)\}$

$K(f_2) = \{(w + x + yz)\}$

$K(f_1) = \{\{a_w, a_x\},\{a_{uw}, a_{ux}, a_y\}\}$

$K(f_2) = \{\{a_w, a_x, a_{yz}\}\}.$

$f_a = (a_w a_x + a_{uw} a_{ux} a_y + a_w a_x a_{yz})$

Kernel–cube incidence matrix

- The row headings in the kernel–cube incidence matrix denote the cubes, representing the kernels.
- The columns headings denote the new variables.
- Element (i, j) of this matrix is 1 if the j^{th} new variable is used in the i^{th} cube, and 0 otherwise.
- A prime rectangle in such a matrix corresponds to a kernel in the intersection.
- If the rows of such a rectangle correspond to different expressions, then the kernel intersection corresponds to the subexpression that can be extracted from these expressions.

kernel–cube incidence matrix

$f_a = (a_w a_x + a_{uw} a_{ux} a_y + a_w a_x a_{yz})$

Table 6.4 Kernel-cube incidence matrix for f_a

			Literals corresponding to cubes					
Kernel	Representation	Id	a_w	a_x	a_y	a_{uw}	a_{ux}	a_{yz}
$w + x$	$a_w a_x$	f_1	1	1	0	0	0	0
$uw + ux + y$	$a_{uw} a_{ux} a_y$	f_1	0	0	1	1	1	0
$w + x + yz$	$a_w a_x a_{yz}$	f_2	1	1	0	0	0	1

- Prime rectangle is $(\{a_w a_x, a_w a_x a_{yz}\}, \{a_w, a_x\})$.
- This corresponds to the kernel intersection $w + x$.

$f_1 = (uwz + uxz + yz)$

$f_2 = (vw + vx + vyz)$

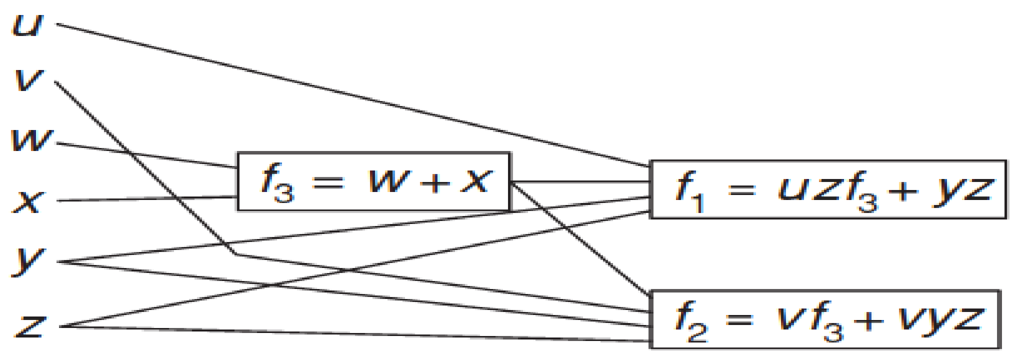


Fig. 6.6 Kernel extraction.

Steps for Kernel Extraction in Intersections

- Identify the kernels of each expression
- Introduce a new variable for each minterm/cube in the kernels
- Rewrite the kernels in terms of new variables.
- Form an auxiliary function f_a as a sum of cubes.
- Construct kernel–cube incidence matrix
- A prime rectangle in such a matrix corresponds to a kernel in the intersection.

Cube Extraction

- For Cube Extraction following steps needs to be performed:
  1. Auxiliary expressions f_a is formed as the sum of all the expressions in the logic network.
  2. Cube-literal incidence matrix is obtained for f_a.
  3. Each cube of each expression is tagged with an identifier for that expression.
  4. Rest of the approach is same as finding a prime rectangle.

f_1 = (uwz + uxz + yz + uv) and f_2 = (vz + wyz).

Cube Extraction

- $f_1 = (uwz + uxz + yz + uv)$ and $f_2 = (vz + wyz)$.
- The auxiliary function $f_a = f_1 + f_2 = (uwz + uxz + yz + uv + vz + wyz)$.

Table 6.3 Cube–literal incidence matrix for $f_a = f_1 + f_2$. "Id" identifies the expression to which a cube belongs

Cube	Id	Literal					
		u	v	w	x	y	z
uwz	f_1	1	0	1	0	0	1
uxz	f_1	1	0	0	1	0	1
yz	f_1	0	0	0	0	1	1
uv	f_1	1	1	0	0	0	0
vz	f_2	0	1	0	0	0	1
wyz	f_2	0	0	1	0	1	1

The prime rectangle ({yz, wyz},{y,z})

The corresponding cube yz.

Cube Extraction

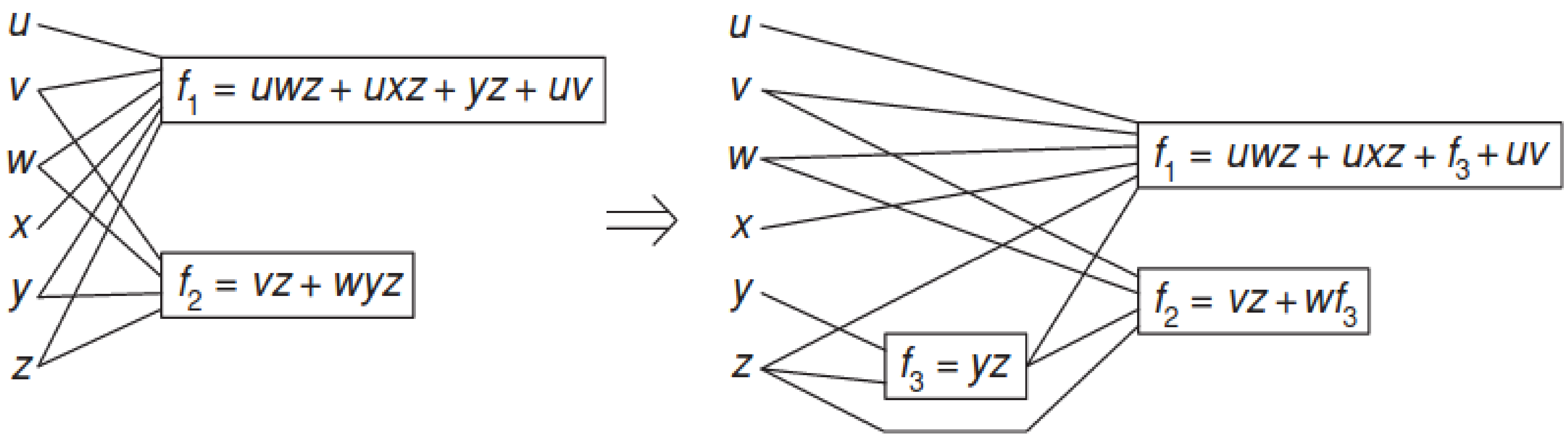


Fig. 6.5 Cube extraction.