

CS528

Task Scheduling

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Scheduling Problems

**Ref: “Scheduling Algorithm” Book
by P. Brucker**

**Google “Scheduling Algorithm Brucker pdf” to get
a PDF copy of the Book
Soft copy uploaded to Course Website**

Parallel Machine Problems

- **P:** We have jobs j as before and m **identical machines** M_1, \dots, M_m .
- The processing time for j is the same on each machine.
- One has to assign the jobs to the machines and to schedule them on the assigned machines.
- This problem corresponds to an RCPSP with $r = 1$, $R_1 = m$, and $r_{j1} = 1$ for all jobs j .

Parallel Machine Problems

- **Q:** The machines are called **uniform** if $p_{jk} = p_j/r_k$.
- **R:** For **unrelated machines** the processing time p_{jk} depends on the machine M_k on which j is processed.
- ***MPM:** In a problem with **multi-purpose machines** a set of machines μ_j is associated with each job j indicating that j can be processed on one machine in μ_j only.*

Parallel Machines

Ti	P1	P2	P3	P4
T1	10	10	10	10
T2	12	12	12	12
T3	16	16	16	16
T4	20	20	20	20

P: Identical

Ti	P1	P2	P3	P4
T1	10	15	20	25
T2	12	18	24	30
T3	16	24	32	40
T4	20	30	40	50

**Q: Uniform : with
speed difference**
 $(S_1=1, S_2=2/3,$
 $S_3=1/2, S_4=2/5)$

Ti	P1	P2	P3	P4
T1	10	8	12	2
T2	12	28	25	13
T3	16	4	32	14
T4	20	38	42	22

**R: Unrelated :
heterogeneous**

Classification of Scheduling Problems

Classes of scheduling problems can be specified in terms of the three-field classification

$$\alpha \quad | \quad \beta \quad | \quad \gamma$$

where

- α specifies the **machine environment**,
- β specifies the **job characteristics**, and
- γ describes the **objective function(s)**.

Machine Environment : α

Symbol	Meaning
1	Single Machine
P	Parallel Identical Machine
Q	Uniform Machine
R	Unrelated Machine
<i>MPM</i>	<i>Multipurpose Machine</i>
<i>J</i>	<i>Job Shop</i>
<i>F</i>	<i>Flow Shop</i>

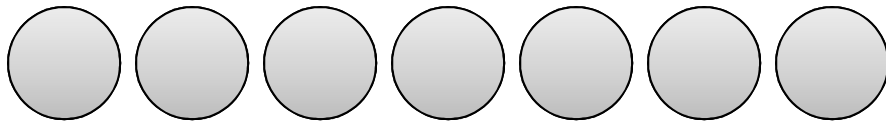
If the number of machines is fixed to m we write

$P_m, Q_m, R_m, MPM_m, J_m, F_m, O_m$.

Job Characteristics : β

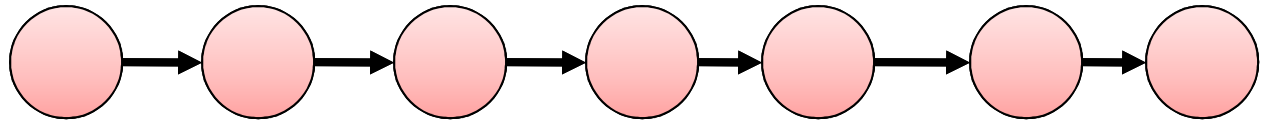
Symbol	meaning
pmtn	preemption
r_j	release times
d_j	deadlines
$p_j = 1$ or $p_j = p$ or $p_j \in \{1,2\}$	restricted processing times
prec	arbitrary precedence constraints
intree (outtree)	intree (or outtree) precedence
chains	chain precedence
<i>series-parallel</i>	<i>a series-parallel precedence graph</i>

Job Precedence Examples

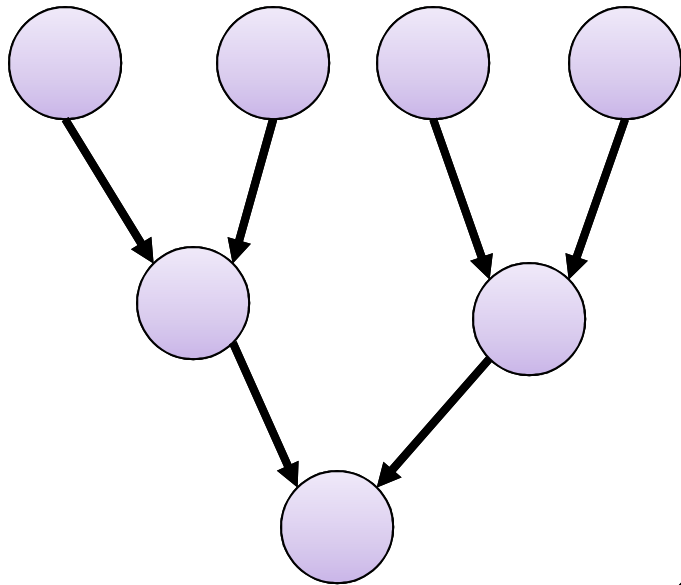


Independent Job (0s0p)

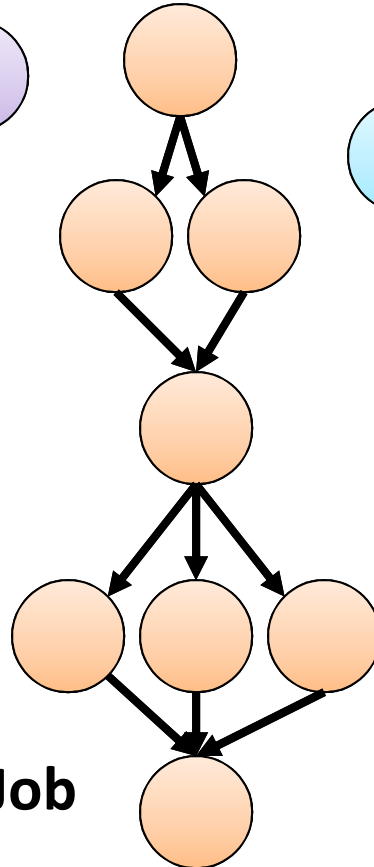
0 successor 0 predecessor



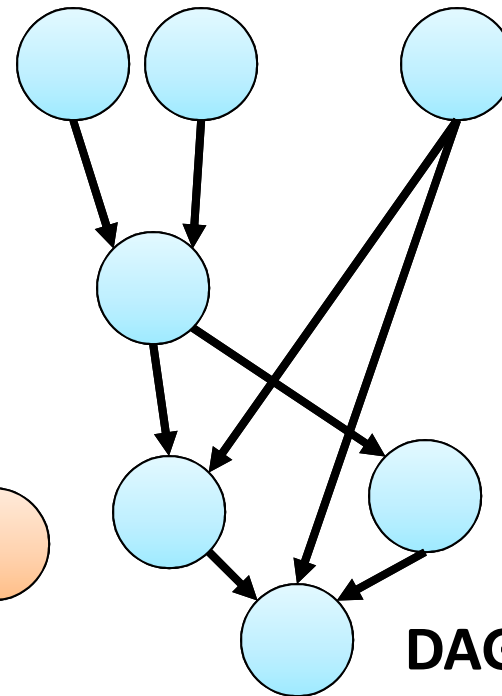
chain of Job (1s1p)



In/out Tree of Job
1pms or ms1p



SP of Job



DAG/prec of Job

Objective Functions : γ

Two types of objective functions are most common:

- **bottleneck objective functions**
 $\max \{f_j(C_j) \mid j= 1, \dots, n\}$, and
- **sum objective functions** $\sum f_j(C_j) = f_1(C_1) + f_2(C_2) + \dots + f_n(C_n)$.

C_j is completion time of task j

Objective Functions : γ

- C_{\max} and L_{\max} symbolize the bottleneck objective
 - C_{\max} objective functions with $f_j(C_j) = C_j$ (makespan)
 - L_{\max} objective functions $f_j(C_j) = C_j - d_j$ (maximum Lateness)
- Common sum objective functions are:
 - $\sum C_j$ (mean flow-time)
 - $\sum \omega_j C_j$ (weighted flow-time)

Objective Functions : γ

- ΣU_j (number of late jobs) and $\Sigma \omega_j U_j$ (weighted number of late jobs) where $U_j = 1$ if $C_j > d_j$ and $U_j = 0$ otherwise.
- ΣT_j (sum of tardiness) and $\Sigma \omega_j T_j$ (weighted sum of tardiness/lateness) where the tardiness of job j is given by

$$T_j = \max \{ 0, C_j - d_j \}.$$

Examples of Scheduling Problem

- $1 \mid \textit{prec}; p_j = 1 \mid \Sigma \omega_j C_j$
- $P2 \mid \mid C_{\max}$
- $P \mid p_j = 1; r_j \mid \Sigma \omega_j U_j$
- $R2 \mid \textit{chains; pmtn} \mid C_{\max}$
- $R \mid n = 3 \mid C_{\max}$
- $P \mid p_{ij} = 1; \textit{outtree}; r_j \mid \Sigma C_j$
- $Q \mid p_j = 1 \mid \Sigma T_j$

Scheduling of Independent Tasks

Polynomial algorithms

- A problem is called polynomially solvable if it can be solved by a polynomial algorithm.

Example

1 | | $\sum \omega_j C_j$ can be solved by

Scheduling the jobs in an ordering of non-increasing ω_j/p_j - values.

Complexity: $O(n \log n)$

Polynomial algorithms for $1 \mid \mid \Sigma C_j$

Example

$1 \mid \mid \Sigma C_j$ can be solved by

Scheduling the jobs in an ordering of non-increasing $1/p_j$ - values. \Rightarrow SJF

$C_i = Q_i + P_i$: Waiting time + Processing time
(SJF is optimal)

Complexity: $O(n \log n)$

Polynomial algorithms : $P \mid p_i=1 \mid C_{\max}$

- A problem is called polynomially solvable if it can be solved by a polynomial algorithm.

Example

$P \mid p_i=1 \mid C_{\max}$ can be solved by

Scheduling the jobs in phase wise, P jobs in one phase, require $\text{ceil}(n/P)$ phases.

Complexity: $O(n)$

P2 || C_{max}

- n tasks, 2 processors
- ET: $t_1, t_2, t_3, \dots, t_n$
- **Subset Sum problem : 1+e APPROX**
 - Ref: CLR Book Chapter 37 Section 4
- Divide the tasks in two sets such that
 - Difference of Sum of ETs of both the set is minimized
 - $\text{Min} (\text{Max}(\text{Sum}(\text{Set}_1), \text{Sum}(\text{Set}_2)))$

$$P_m ||| C_{\max}$$

- n tasks, m processors
- ET: $t_1, t_2, t_3, \dots, t_n$
- **m-Subset Sum problem**
- **INDEP(m) Problem: NPC in strong sense**
- Divide the tasks in m sets such that
 - Difference of Sum of ETs of all the set is minimized: **does not exceed a value K**
 - $\text{Min} (\text{Max}(\text{Sum}(\text{Set}_1), \text{Sum}(\text{Set}_2), \dots, \text{Sum}(\text{Set}_m)))$

$P_m | pmtn | C_{max}$

- n tasks, m processors, infinite pre-emption allowed
- ET: $t_1, t_2, t_3, \dots, t_n$
- Divide all the work among all the cores equally
 - $Avg = (\sum t_i) / m$ work to each cores
 - If $\max(t_i) > Avg$, $C_{max} = \max(t_i)$, Task need to execute serially
- Handle the boundary cases

$Q_m | \text{pmtn} | C_{\max}$

- n tasks, m uniform processors, infinite pre-emption allowed
- Longer task executed on high speed processor till its execution is long enough as compared to others
 - Sort the tasks based on LPT
 - Allocate long task to higher speed processors one by one
 - When execution time of longer task is no longer long as compared to other then co-execute

$Q_m \mid \text{pmtn} \mid C_{\max}$

Algorithm level

```
t := 0;  
WHILE there exist jobs with positive level {  
    Assign(t);  
     $t_1 := \min\{s > t \mid \text{a job completes at time } s\};$   
     $t_2 := \min\{s > t \mid \text{there are jobs } i, j \text{ with } p_i(t) > p_j(t)$   
         $\text{and } p_i(s) = p_j(s) \text{ at time } s\};$   
     $t := \min\{t_1, t_2\}$   
}
```

$Q_m \mid \text{pmtn} \mid C_{\max}$

Assign (t)

$J := \{i \mid p_i(t) > 0\};$

$M := \{M_1, \dots, M_m\};$

WHILE $J \neq \emptyset$ and $M \neq \emptyset$ {

 Find the set $I \subseteq J$ of jobs with highest level;

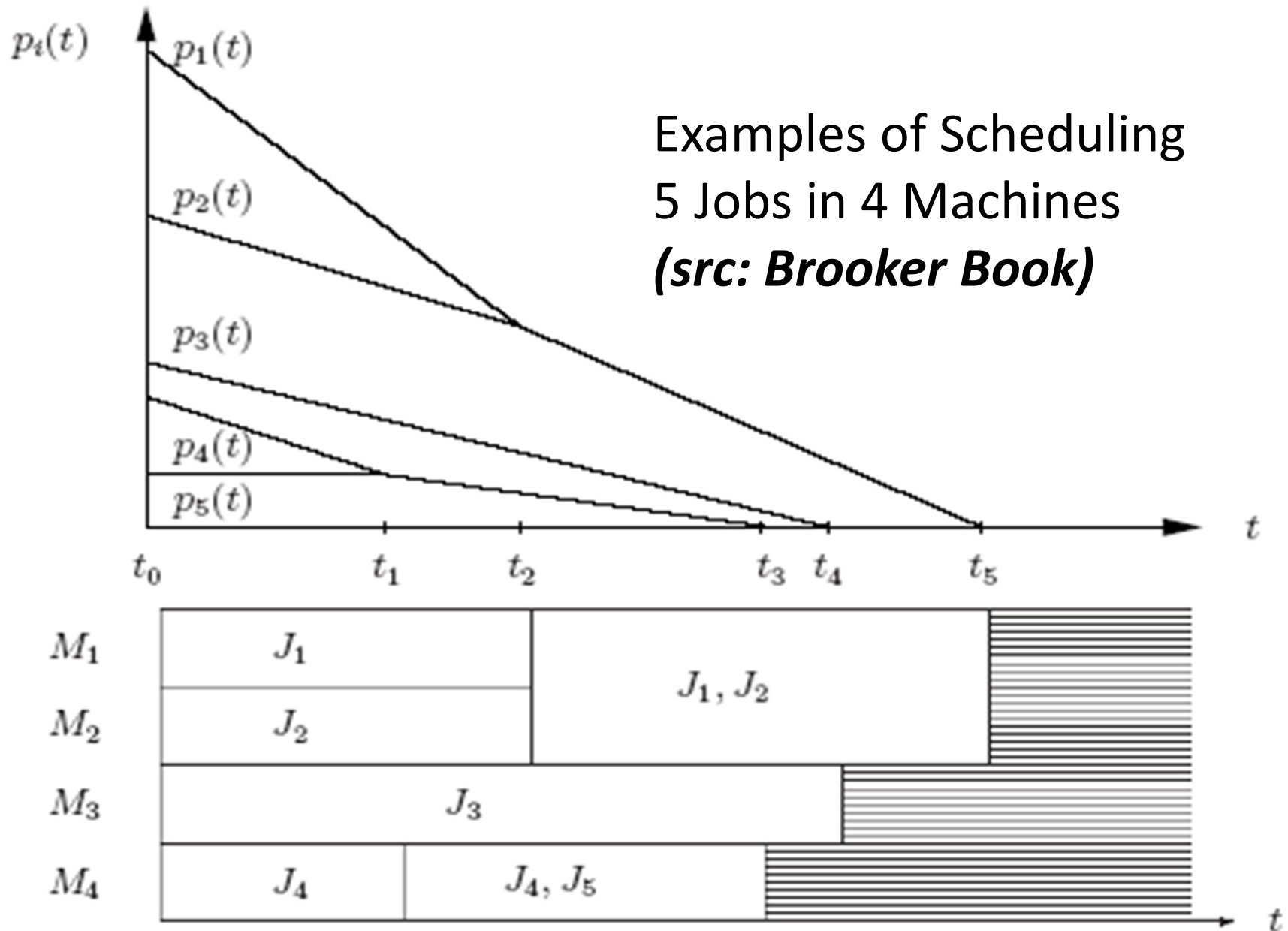
$r := \min\{|M|, |I|\};$

 Assign jobs in I to be processed jointly on the r
 fastest machines in M ;

$J := J \setminus I;$

 Eliminate the r fastest machines in M from M

$Q_m | pmtn | C_{max}$



$Q | p_{tmn} | \sum C_j$

- LPT on High speed is good to optimize $\sum e_j$ the sum of task execution time but not $\sum C_j$
- Modified version of SPT (shortest remaining time) rule. As $\sum C_j$ include waiting time of all the tasks
- Order the tasks according to non-decreasing processing time.
- Schedule task 1 on available highest speed machine up to time $t_1 = p_1/s_1$.
- Schedule 2nd task on M2 for t_1 time and then on M₁ from time t_1 to time $t_2 \geq t_1$ until it is completed and same process continues

$Q | p t m n | \Sigma C_j$

- Example $m=3$, $s_1=3$, $s_2=2$, $s_3=1$ and $n=4$,
 $p_1=10$, $p_2=8$, $p_3=8$, $p_4=3$
- SRT Job J_4 get scheduled on M_1 with speed s_1 for 1 time unit. Job 3 get scheduled on M_2 upto time 1 and then shifted to M_1 . Gant chat is given bellow with $\Sigma C_i = 14$

