# Topic 2

## Expected Utility and Extensions

There is a theory which states that if ever anyone discovers exactly what the Universe is for and why it is here, it will instantly disappear and be replaced by something even more bizarre and inexplicable.

There is another theory which states that this has already happened.

(Douglas Adams, The Hitchhikers Guide to Galaxy)

## 1 Introduction

Uncertainty is a basic feature of almost all economic actions. In topic 1, we considered actions of consumers and producers without any uncertainty. Now we will consider the state of nature to be unpredictable. For example, our action might be taking/not taking an umbrella. The state of nature can be sunny or rainy. Interaction between of our actions and states of nature will give rise different social or personal consequences (e.g. no umbrella/rain, umbrella/sunny) which occur with different probabilities. As it turns out, we need to take both consequences as well as the probabilities into account. The task is to figure out preferennces over consquences as well as associated probabilities.

A strategy is the typical "divide and conquer". First (in this chapter), we keep the consequences as fixed and consider preferences over probabilities. In a later chapter, we will keep the probabilities as fixed and consider preferences over different consequences (arising in different states of nature). Fortunately, as we will see, both strategies lead to the same sort of consistent outcome.

The rudiments of expected utility was first considered by Daniel and Nicholas (Daniel's brother) Bernoulli during the 18th century. However, a full fledged theory could only be developed during the 1940's by John von Neumann (no introduction is needed for Engineering students) and Oskar Morgenstern, a Princeton economist. It is that theory towards which we will turn to.

### 1.1 Expected Utility

Let us go back to history again. During the 18th century, the Mathematicians of Europe were hotly debating the so called Saint Petersburg Paradox, named by Daniel Bernoulli. Briefly the problem is as follows.

**Problem 1.1** *A fair coin is tossed. A gambler will gain* $2^t$ \$ *(or whatever) if the first head appears in the* $t-th$ *toss. How much will the gambler pay to enter the competition?*

**Solution 1.1** *If we go by expected value, then the gain to the gambler is* $G = \frac{1}{2} * 2 + \frac{1}{4} * 4 + \frac{1}{8} * 8 + .....$*, and the sum tends to infinity. Therefore, a gambler will pay infinite amount to enter this competition.*

The solution of course does not appeal to commonsense. Both Daniel and Nicholas provided a way out, stating that the return from money is not that money itself, but the "use of such money". They were, in fact, knocking at the door of (diminishing marginal) utility of money which make the sum $G$ bounded as $t \to \infty$. But the theory is more general than the solutions they proposed (Daniel thought logarithmic utility of money, while Nicholas focussed on square root)

#### 1.1.1 Lotteries

At the heart of Expected utility theory, lies the notion of lotteries, which is just the probability distribution over a set of consequences. For example, after the end semester exam, you can have party either at Lauriat (exam is good) OR at hostel mess (lousy exam). The consequence set is [L,M], and you can rank the consequences (e.g. L is better than M). A lottery is the probability distribution over good/bad exams, $\mathcal{L} = [p, 1-p; L, M]$. In general, if there are $n$ outcomes, a lottery is a probability distribution defined over $n$ outcomes $[p_1, p_2, ...p_n], p_i \geq 0$ and $\sum p_i = 1$. We will assume that these probabilities are *objectively known*. Note that a "sure outcome" is also a lottery: if you know that you are going to dine at mess for sure. The corresponding lottery is $[0, 1]$. Since we are focussing on probabilities, rather than outcomes, we will express a lottery only in terms of probabilities.

Sometimes, life offers you a lottery within a lottery which is known as a compound lottery. Put in a different way, for a compound lottery, another lottery is consequence. For example, suppose the four dining outcomes are [L, FC, Mess, Core I Canteen]. A simple lottery assigns 0.25 to each. But then, you could dine at [L, FC] if the exam is good with probability $[0.5, 0.5]$ (call this lottery A). Similarly for other (lottery B). But then your exam is good or bad with prob$[0.5; 0.5]$. So the exam is a (compound) lottery which gives you the two lotteries with prob $[0.5, 0.5]$.

**Definition 1.1** *Let* $p = [p_1, p_2, ..p_n]$ *and* $q = [q_1, q_2, ..., q_n]$ *be two simple lotteries. Then a compound lottery is defined by* $\mathcal{L} = \alpha p + (1 - \alpha) q$*, where* $\alpha \in (0, 1)$

The discussion also makes it clear that one can transform a compound lottery into a simple lottery and vice versa. So we can just focus on simple lotteries.

In case we have only three outcomes, a probability distribution can be represented in a probability simplex. Suppose the distribution is $[p_1, p_2, p_3]$ such that $p_1 + p_2 + p_3 = 1$. Then we can plot different values of $p_1$ and $p_3$ (bounded by $p_1 = 0$, $p_3 = 0$ and $p_1 + p_3 = 1$) and reading off $p_2$ as the vertical/horizontal distance from the line $p_1 + p_3 = 1$.

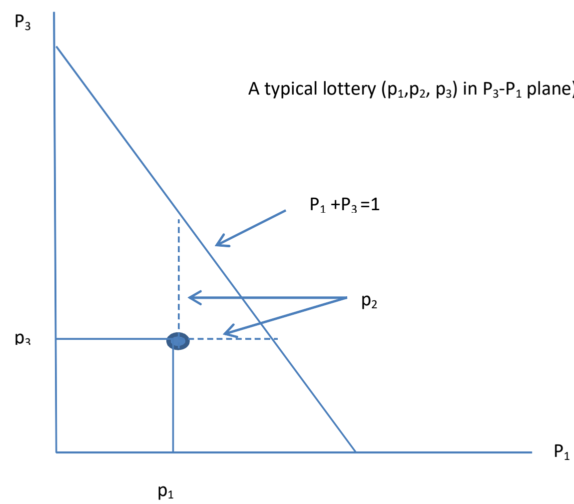


Figure 1: A Typical Lottery as a point in $P_1 - P_3$ Plane

Two points in the simplex give us two different lotteries, and a line segment joining these two points represent various compound lotteries which offer these two lotteries as linear combination of the others.

### 1.1.2 Expected Utility

However, the central question remains. How are we going to choose between different lotteries? We have already seen that expected values of consequences (e.g. in Saint Petersburg Paradox) is not satisfactory. Let $c_1, c_2, ..c_n$ be different consequences of an action with the corresponding lottery $\mathcal{L} = \{p_1, p_2, ..p_n\}$ Let there be an "utility of consequences function" $v : C \to \mathbb{R}$. Then, provided the set of lotteries obey certain regularity conditions, the utility from a lottery is represented by its expected utility: $U(\mathcal{L}) = \sum_i p_i v(c_i)$.[1] A lottery $L$ will be preferred to another lottery $L'$ if the expected utility from $L$ is more than expected utility from $L'$.

[1] The function $v$ is known as Bernoulli utility function, whereas $U(.)$ is known as von Neumann Morgenstern (vNM) utility function. In case of continuous probability distrubution, the summation is relpaced by an integration sign.

**Remark 1.1** *$v$, the Bernoulli utility functions have one major difference from traditional utility functions (which we have encountered in topic 1). In consumer theory, the utility functions are ordinal in nature, so as far as $U(X) > U(Y)$, we do not care about the particular form of the utility function. For example, both $U = \alpha \ln x + \beta \ln y$ and $V = f(U)$, such that $f' > 0$, represent same preferences. But here, the utility numbers matter, utility functions are both ordinal and cardinal in nature. The following example will make it clear.*

**Example 1.1** *Suppose the three choices are [FC, Canteen, Mess]. $v()$ assigns number 100, 36 and 1. $u()$ assigns number $10, 6, 1$, such that $u = \sqrt{v}$.Under ordinal utility, these numbers will not make any difference.; all it says that I like FC more than canteen and canteen more than the mess. Now consider lotteries $L_1 = [0.5, 0, 0.5]$ and $L_2 = [0, 1, 0]$.I like $L_1$.over $L_2$ if $v$ is the utility function, and other way round if $u$ is the utility function. So numbers matter: in a cardinal sense.*

The Bernoulli utility functions are only invariant to a linear (positive affine) transformation, i.e. $u$ and $v$ will maintain the preference ordering iff $v = a + bu$, where $b > 0$.

**Axioms of Expected Utility** Here, we summarize the "regularity conditions". Let $L_1, L_2$ and $L_3$ be three non-degenerate lotteries. As before, let the preference ordering be represented by $\succeq$: ‘at least as good as’. Then we have

1. Axioms of Completeness: either $L_1 \succeq L_2$ OR $L_2 \succeq L_1$ or both
2. Transitivity: If $L_1 \succeq L_2$ and $L_2 \succeq L_3$ then $L_1 \succeq L_3$.
3. Independence: if $L_1 \succeq L_2$, then $\alpha L_1 + (1 - \alpha) L_3 \succeq \alpha L_2 + (1 - \alpha) L_3$
4. Continuity: Suppose $L_1 \succeq L_2$ and $L_2 \succeq L_3$. Then there exists an $\alpha$ and a $\beta$ such that $\alpha L_1 + (1 - \alpha) L_3 \succ L_2$ and $L_2 \succ \beta L_1 + (1 - \beta) L_3$

Axioms 1,2 can be easily defended: these are counterparts of what we have in consumer behavior. Axiom 4 is required to guarantee continuity of the expected utility function $U(L)$. It simply means that any lottery can be "beaten" by a mix of other lotteries and any lottery can "beat" a mix of other lotteries: there is no principle in life which is sacred enough! However, the important assumption is that of independence, which says that if I prefer one lottery to the other, then a mix of either with a third lottery will preserve the preference ordering. Many real life examples can be cited where this assumption cannot be defended.

**Indifference Curves** Armed with the knowledge of expected utility, we can draw indifference curves for various lotteries in $p_1 - p_3$ plane. Suppose there are only three consequences $c_1, c_2$ and $c_3$ and WLOG, let us assume $v(c_1) < v(c_2) < v(c_3)$. Then, the indifference curve is defined as $p_1 v_1 + p_2 v_2 + p_3 v_3 = \bar{k}$, or, $(v_3 - v_2) p_3 - (v_2 - v_1) p_1 = k - v_2$. Given our assumptions, this is a positively sloped straight line: if one increases weight on $p_1$ (worst outcome), one needs to increase weight on $p_3$ (best outcome) in order to keep the agent indifferent. The direction of increasing preferences is towards the north west: given $p_1$, any lottery which places higher weight on the best outcome (thus squeezing $p_2$) are to be preferred.

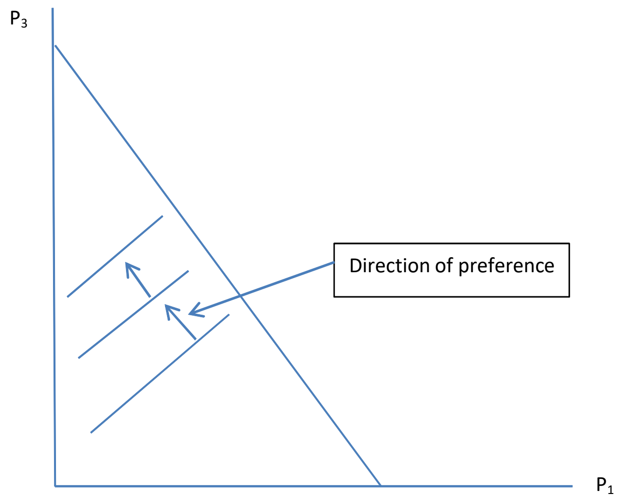


Figure 2: Indifference Curve for Lotteries

It is possible to restate the axioms in terms of the indifference curve representation. Notice that the space bounded by the three lines ($p_1 + p_2 = 1; p_1 + p_3 = 1$ and $p_2 + p_3 = 1$) is full of lotteries. The first axiom merely states that any two lotteries are either on the same indifference line or above/below to each other. The second axiom says, given three lotteries, if $L_1$ on a higher (same) indifference curve than (as) $L_2$ and $L_2$ is on a higher (same) indifference curve than (as) $L_3$, then $L_1$ must be on a higher (same) indifference curve than (as) $L_3$. The third axiom is explained in the following diagram

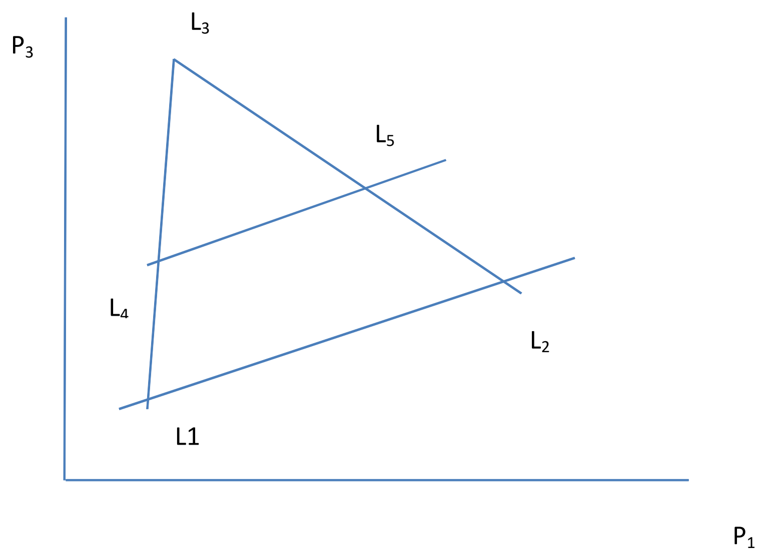


Figure 3: Independence Axiom

Here, $L_4 = \alpha L_1 + (1-\alpha) L_3$ and $L_5 = \alpha L_2 + (1-\alpha) L_3$. Notice that $L_1 \sim L_2$. Thus, $L_4$ and $L_5$ will also be on the same indifference curve (properties of similar triangles), which is parallel to the line segment $L_1 L_2$.

Armed with the knowledge of $v$ as well as the the probability distribution $L$, we can compare lotteries through the expected utility theorem. Note that, this is a tall order, we need to know the $v$ function as well as the exact probabilities on all possible events. The search is, therefore, for a mode of comparison (between lotteries) which is not as information heavy: some minimal information about $v$ or the probability distribution is all that is required. In order to do that, however, we need to investigate the properties of the function $v$ more closely. The 'by-product' of this investigation will come back to us again and again.

## 2 Attitudes Towards Risk

Briefly, suppose an agent is given a lottery, which promises \$50 with some probability $p$ and \$10 with the complementary probability. A person (with the expected \$ amount in pocket) is given either the choice of the lottery (entry fee equals the expected amount) OR a sure amount of money (more than \$10) instead of the lottery. If I am risk averse, not only I will part away with the sure money, but I will be willing to part away with a \$ amount which is less than the expected amount of money. In other words, I will pay to the lottery organizers a bribe in order to avoid the gamble, even if it gives me, *ex ante*, the exact amount which I have just now with me.

But what is the sure amount of \$? It is an amount which will give me in both states a constant utility. The sure amount is termed as *certainty equivalent* of the lottery $CE$ and is defined as $v(CE) = p * v(50) + (1 - p) * v(10)$. Suppose the expected value of the lottery is $Ec = p * 50 + (1 - p) * 10$. For the risk averse person, then $CE < Ec$. The difference between $CE$ and $Ec$ is known as *Risk Premium*,[2] $RP = Ec - CE$.

Given $v' > 0$ (people prefer more money than less), this is guaranteed if $v'' < 0$, i.e. the shape of $v$ is concave.

**Figure 3 Here: Risk Averse Person**

Similarly, a risk lover will have more welfare from the gamble than holding a sure amount. He/she will be willing to pay more in order to have the risk, i.e. $CE > Ec$. The utility function is convex: if $v''$ exists, the condition is guaranteed by $v'' > 0$.

**Figure 4 Here: Risk Lover Person**

For a risk neutral agent, $CE = Ec$, i.e. he is indifferent between the gamble and the sure amount of money. I leave the diagram as a homework.

[2] Unfortunately, the specific term can have different meaning/definitions. Remember the caveat. This particular definition is due to Pratt.

## 2.1 Concavity and Attitude Towards Risk

It seems that the geometric property of concavity (function above the chord) is closely related to the idea of risk aversion. The geometric property is neatly summarized in so called Jensen's inequality for concave functions: $Ev(c) < v(Ec)$. Note that, this characterization does not need existence of $v''$ or its sign.

This also helps us to quantify risk aversion, or compare interpersonal attitudes towards risk. If concavity signifies risk aversion, then the degree of concavity (that is, the rate at which $v'$ falls) can be used as a measure of risk aversion. To wit, one may use $-v''$ as a measure of concavity. However, there is one problem: we know that Bernoulli utility functions are unique to positive linear transformation, that is, somebody with $v$ and another person with $u$ exhibits same preference towards lotteries if $u = a + b * v$. Yet, as $u'' = bv''$, the second agent will display higher (less) risk aversion if $b >$ or $< 1$. To get rid of such a situation, one divides $v''$ by the first derivative (proposed by K J Arrow). As a result, we have the Arrow-Pratt measure of absolute risk aversion

$$r_A(c) = -\frac{v''}{v'}$$

If the risk is in terms of percentage of initial consumption, e.g. $c(1 \pm \Delta c)$ with probabilities $p$ and $1-p$, then the measure is slightly modified, and we get the Arrow-Pratt measure of relative risk aversion

$$r_R(c) = -\frac{v''}{v'}c = c * r_A(c)$$

It is clear that both $r_A$ and $r_R$ depend on $c$. A natural question that we can ask is the following: how does attitude towards risk change as $c$ changes? One can distinguish between three behavior

1. Constant Risk Aversion: CRRA or CARA occurs if $r_R$ or $r_A$ does not change with consumption.

2. Increasing Risk aversion: IRRA or IARA

3. Decreasing risk aversion: DRRA or DARA.

At this point, one may ask whether constant $r_A$ or $r_R$ are special cases of a general attitude towards risk. The answer is yes. If we define$-\frac{v'}{v''}$ as risk tolerance ($r_T$), then the class of utility functions which exhibit linear risk tolerance, i.e. $r_T = \alpha + \beta c_T$ are said to be HARA (*hyperbolic absolute risk aversion*) utility functions. For different values of $\alpha, \beta$ we can recover CARA or CRRA utility functions.

An important theorem is due to Pratt, which neatly ties up different definitions of risk aversion. It states that the following three statements are equivalent for two concave and differentiable functions $u$ and $v$.. Suppose these functions belong to two gentlemen called Mr. U and Mr. V.

1. $-\dfrac{v''}{v'} > -\dfrac{u''}{u'}$

2. $v = G(u)$ where $G$ is a strictly increasing concave function.

3. If $\varepsilon$ is a mean preserving spread, then $RP_V > RP_U$ when both are given $\varepsilon$ increase in wealth with $E(\varepsilon) = 0$.

We will not prove the theorem here (the proof goes like $1 \Rightarrow 2 \Rightarrow 3 \Rightarrow 1$), but provide an intuition why this theorem makes sense. First proposition says $v$ exhibits more risk aversion than $u$, the second one says $v$ is more concave than $u$ (e.g. if $u$ is $c^{\cdot 5}$, $v$ is $(u)^{0.5}$) and the third statement says the person with $v$ will have a higher risk premium than the person with $u$. The first two are technical relations, while the third one is behavioral. What we show here is how the mathematical and behavioral definitions match up.

There is a reason why we focus on mean preserving spreads. For mean preserving spreads, the mean outcome $(c - \varepsilon, c + \varepsilon)$ remains same, and the risk increases. Since our focus is attitude towards risk (instead of return), we employ such lotteries.

# 3 Stochastic Dominance

Now we would like to come back to the original question: with minimal knowledge of the utility function $v$, can we say something about the preference (over lotteries) of an *EU* maximizer? Two things come to mind (a) a lottery which gives me higher return *at all states* will be preferred (if I get higher utility from consumption or wealth), (b) if two lotteries give same return, then the one with lower variance (risk) will be preferred by a risk averse person.

In what follows, we will assume that realizations of $c$ as well as the probability distribution defined over it are continuous. Let $c$ be bounded by $c_1$ and $c_2$. We consider two different probability distributions (lotteries) $f$ and $g$ as well as the associated cumulative probability distributions $F(c)$ and $G(c)$. Thus, we must have $F(c_1) = G(c_1) = 0$ and $F(c_2) = G(c_2) = 1$. With slight abuse of notations, we will term these lotteries as lottery $F$ and $G$, respectively.

## 3.1 First Order Stochastic Dominance

We begin with a definition.

**Definition 3.1** *The distribution (lottery) $F$ first order stochastically dominates $G$ if $F(c) < G(c)$ for all $c$.*

In other words, the probability of getting less than a fixed $c^*$ is higher in case of lottery $G$ than lottery $F$.

**Theorem 3.1** *Suppose an expected utility maximizer has an increasing Bernoulli utility function $v$ (that is $v' > 0$). Then he would prefer $F$ to $G$ (the expected utility from $F$ will be higher than that from $G$) iff $F(FOSD)G$.*

**Proof.** (Only if part) Using integration by parts,[3] show that $EU_F =$

[3] Of course, the theorem/definitions are applicable for discrete probability distributions

$v(c_2) - \int v'(c)\, F(c)dc.$. Hence $EU_F - EU_G = \int v'(c) * (G - F)\, dc$. Thus, if $F(FOSD)G$, $G - F > 0$ for all $c \to EU_F - EU_G > 0$. ■

The if part is omitted (that is $EU_F - EU_G > 0 \to F < G$). The way it is done is the following. Suppose for some $c$, $F > G$. Then one shows that one cannot have $EU_F - EU_G > 0$).

Here is an intuition for this. $F$ assign higher weight (probabilities) to higher values of $c$, and therefore will be preferred. Note that, *at the beginning* (close to $c_1$), The slope of $G$, that is, the pdf $g$ must be higher than that of $F$ (i.e. the pdf $f$). However, since $F(c_2) = G(c_2)$, the slope of $F$ has to remain higher than that of $G$ after a certain $\bar{c}$. Thus the lottery $F$ provides higher weight to the "better" outcomes of the world.

For example, in the next figure, a possible relationship between $F$ (green) and $G$ (black) is plotted

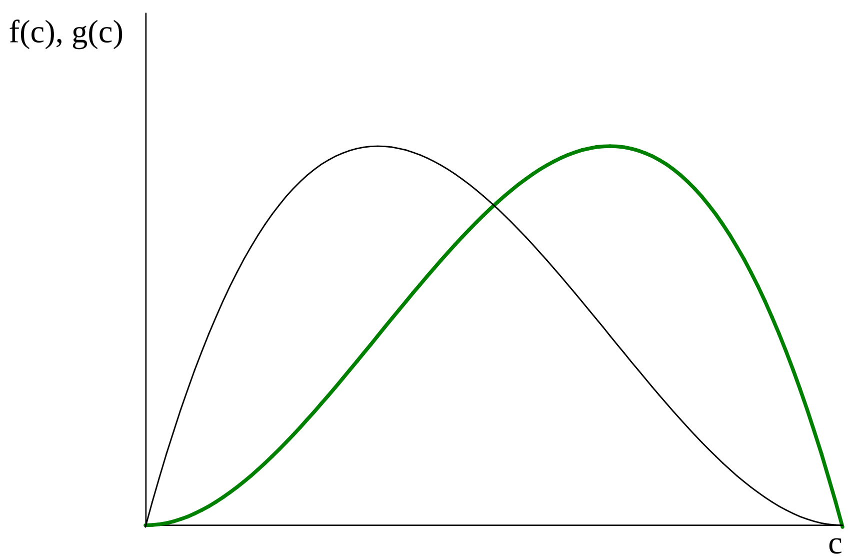


Figure 5: First order Stochastic Dominance

$(p_1, p_2, ..p_n)$ as well. To prove the above theorem in case of discrete distributions, we need a result called Abel's lemma (which parallels integration by parts in a discrete case). We do not wish to indulge in more maths than necessary.

### 3.2 Second Order Stochastic Dominance

The idea of FOSD requires that the cumulative probability curves never cross.. However, this may be unreasonable. So we propose an weaker idea of stochastic dominance: the lottery under $F(SOSD)G$ if, $\int_{c_1}^{c^*} F(c)dc < \int_{c_1}^{c^*} G(c)dc$, for ant $c^* \in (c_1, c_2)$ In other words, the curves may cross, but the sum of area (under $F$) should be less than that of $G$.

The basic theorem is stated below

**Theorem 3.2** *Suppose an expected utility maximizer is risk averse, i.e.$v'' < 0$. Then he would prefer $F$ to $G$ (the expected utility from $F$ will be higher than that from $G$) iff $F(SOSD)G$.*

The proof applies integration by parts twice. Here we provide an outline of the proof. Again, we do only the "only if" part.

1. Define $S_F(c^*) = \int_{c_1}^{c^*} F(c)dc$. Note that $S(c_1) = 0$. $S$ can be thought of as a 'super cumulative function'.

2. Take the expected utility under $F$. From the theorem of FOSD, we already know that $EU_F = v(c_2) - \int v'(c)Fdc$. Applying integration by parts once more using $F$ as the second function, and noting that $S_F(c_1) = 0$

$$EU_F = v(c_2) - v'(c_2)S_F(c_2) + \int_{c_1}^{c_2} v''(c)\, S_F(c)dc$$

3. Compare $EU_F$ and $EU_G$. Note that

$$EU_F - EU_G = v'(c_2) * (S_G(c_2) - S_F(c_2)) + \int_{c_1}^{c_2} v''(c)\,(S_F - S_G)\, dc$$

4. We wish to create a benchmark case: all risk averse individuals should prefer F over G. Similarly, all risk lover individuals ($v'' > 0$) should

prefer G over F. At the border line, a risk neutral person (with $v'' = 0$) is indifferent between these two lotteries. To ensure this, we must have $S_G(c_2) - S_F(c_2) = 0$ Note that $S_F(c_2) = \int_{c_1}^{c_2} F(c) * 1 * dc = \int_{c_1}^{c_2} [F(c) * c] - \int_{c_1}^{c_2} f(c) c dc = c_2 - E_F(c)$, where the last term is expected value of the lottery under $F$. Similarly, $S_G(c_2) = c_2 - E_G(c)$. That a risk neutral person will be indifferent between $F$ and $G$. is possible iff both lotteries provide same return, i.e. $E_G(c) = E_F(c)$.

5. So what should be the relation between $F$ and $G$? We say that $G$ is a mean preserving spread of $F$.

6. Note, if I am risk averse ($v'' < 0$), I will prefer $F$ to $G$ if $S_G(c) > S_F(c)$, i.e. for any $c^*$ between $c_2$ and $c_1$

$$S_G - S_F > 0 \rightarrow EU_F - EU_G$$

The intuition of the result is following. We can think of $G$ as a mean preserving spread of $F$. That is, both lotteries $G$ and $F$ have same mean return, but for $F$, all the $c's$ are relatively more clustered around $E(c)$, while $G$ assigns more probability weight to the extremes of the distribution (fat-tail distribution in finance jargon). It is evident, as in the figure below, that $G$ has higher variance than $F$.

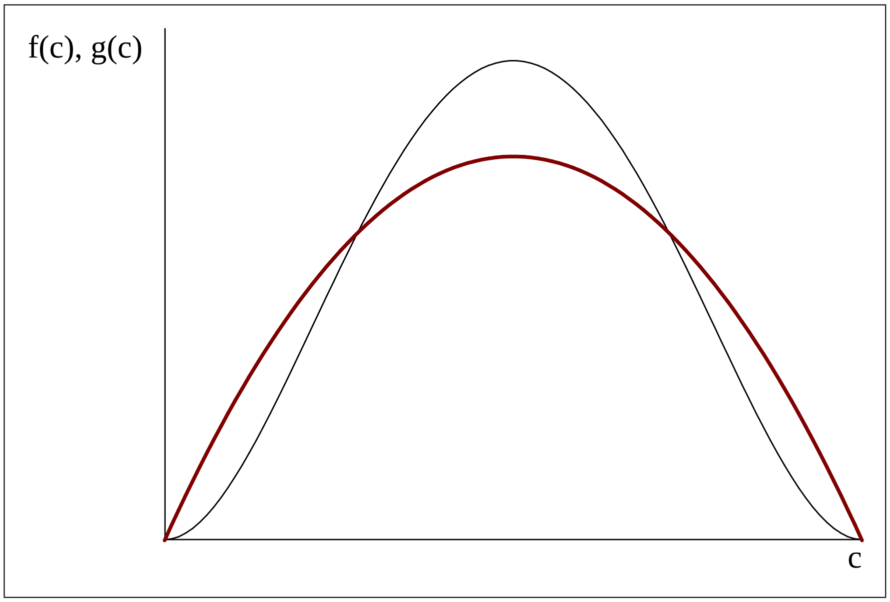


Figure 6: Second order Stochastic Dominance

In the above figure, the thick red line is pdf for lottery $G$, and the thin line is pdf for lottery $F$.

At this point, we should mention a third point: the relationship between preference and variance is subtler than what we wrote before. The lottery $G$ has higher variance than $F$, but the higher variance must come from a mean preserving spread of $F$. In fact, there is a nice theorem ($F(SOSD)G \Leftrightarrow G$ is a mean preserving spread of $F \Leftrightarrow$ every risk averse agent prefers $F$ over $G$). If these conditions are not met, we can have a situation where two lotteries have the same mean, $G$ might have higher variance, but a risk averse person (for a certain $v$) may prefer $G$ over $F$.

## 4 Downward Risk Aversion

So far, we have considered the importance of the second derivative ($v''$) of the Bernoulli utility function as a measure of risk aversion. Here, we provide an example (and there are others) where the third derivative ($v''$) matters.

An agent is downward risk averse if he/she prefers risk at the ‘upper end’ of a lottery than the ‘lower end’. To fix ideas, consider a fair gamble $[.5, .5; c_0, c_1]$ with $c_0 < c_1$. Let $\tilde{c}_0$ be a mean preserving spread of $c_0$, e.g. $\tilde{c}_0 = c_0 \pm \varepsilon$ with probabilities .5 and .5. We can write the resulting (compound) lottery as $L_1 = [.5, .5; \{.5, .5, c_0 - \varepsilon, c_0 + \varepsilon\}, c_1]$. In a similar fashion, define $L_2 = [.5, .5; c_0, \{.5, .5, c_1 - \varepsilon, c_1 + \varepsilon\}]$. A downward risk averse person will prefer $L_2$ over $L_1$.

For downward risk aversion, one may surmise that ‘concavity’ of $v$ falls (increass) as $c$ increases (falls), i.e. $d(v'') = v''' > 0$ (In other words, $v''$ becomes, say, $-1$ from $-2$ as $c$ increases). A positive third derivative implies downward risk aversion. There are examples of utility functions that exhibit risk aversion, but not downward risk aversion.

However, downward risk aversion and decreasing risk aversion are not same. The following problem will make it clear.

**Problem 4.1** *If a Bernoulli utility function $v$ exhibits DARA, show that it will exhibit downward risk aversion. Is the converse also true?*

## 5 Mean Variance Utility

Yet, one can further reduce the ‘dimensionality’. Remember that any lottery, along with the outcomes, are characterized by its mean and variance. In so far agents care only about the risk and return (and not any higher moments), their utility can be characterized by $V(\mu, \sigma)$, with $V_\mu > 0$, and $V_\sigma < 0$ if the agent is risk averse. Thus, one favors lottery $F$ over $G$ if $V(\mu_F, \sigma_F) > V(\mu_G, \sigma_G)$. Unfortunately, there is a problem with this shortcut: mean variance utility is consistent with expected utility theory only under some stringent conditions. We will turn to mean variance utility (and its application) in the next section.