HS239: Economics of Uncertainty and Information

PS 4: 2024

In Class, 22/3/2024

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1. This question is designated to make you go through Marshallian inefficiency of sharecropping, step-by-step. Assume that an agricultural production process has production function $y = \sqrt{e}$ and cost as $c(e) = ke$, where e is the intensity of effort.

a) If the landlord tills his/her own land, what must be (i) the level of effort and (ii) landlord's income?

b) If the landlord hires a tenant, and opts for a fixed rent $\bar{R}$, what must be the i) tenant's income, (ii) landlord's income?

c) Now suppose the landlord demands $s\%$ of output, leaving $(1-s)\%$ to the tenant. Re-do part (b).

d) Now assume that the landlord fixes $\bar{R}$ to be what he was getting in part (c) and offers a fixed rent contract to tenant ($s = 0$). Re-do part b.

e) Argue why (d) is superior to (c).

2. In the model of sharecropping with risk-sharing, suppose we have the following specifications

$$\begin{aligned} U_L &= E(y_L) - \frac{1}{3}Var(y_L) \\ U_T &= E(y_T) - \frac{4}{5}var(y_T) - c\,(e) \\ c(e) &= \frac{k}{2}e^2 \end{aligned}$$

Reservation utility of the agent= 10. The production function is $y = e + \varepsilon$, where ε is an rv with mean 0 and variance σ^2. The landlord offers a sharecropping contract to the tenant:$y_T = sy - R$.

Characterise the optimal contract.

3. Assume an agent has the utility function $v = -e^{-Y}$.Show that evaded income $E = Y - X$ is independent of Y.

4. Assume an agent has the utility function $v = \ln(Y)$.Show that the proportion of income not declared $\left(\frac{X}{Y}\right)$ is constant.

5. Consider the model of consumption over time. Assume that the utility functions are $\ln c_i$. Without uncertainty, what are the optimal savings?

6. Continue with the above example. Now suppose income of period 0 is $(y_1 + 3)$ with probability .5 and $(y_1 - 3)$ with the complementary probability. How does your answer change in the above problem?

In 5 and 6, you may assume $y_0 = y_1 = 10, r = 5\%$, $\beta(1 + r) = 1$

7. Consider the model of consumption over time. Show that precautionary savings will be higher for agents with higher coefficient of absolute prudence $A(w) = -\dfrac{u'''}{u''}$

8. This refers to the model of tax evasion and loss aversion. Assume that a loss averse agent has the following utility function (β is a fraction)

$$\begin{aligned} v(z) &= z^{\beta} \text{ if } z \geq 0 \\ &= -\gamma\,(-z)^{\beta} \text{ if } z < 0; \gamma > 1 \end{aligned}$$

Let us assume that staying 'honest' is the status quo.

a) What are the gains and losses if the taxpayer evades?

b) Show that the taxpayer either evades or stays honest (*in effect, a zero-one decision*).