Quiz 1: Answer Key
HS 239, 2024

1. *A person has the Cobb-Douglas utility* $v = \frac{1}{4}\ln(x_1) + \frac{3}{4}\ln(x_2)$ *. Price of good 1 is 1, that of good 2 is 2 and income is 100. Assume that price of good 2 falls to 1. Compute the change in consumer surplus.*

(10)

[**Straight from lectures and problem set 1, noting that** v **is equivalent to** $u = x_1^{.25}x_2^{.75}$]

ANS:

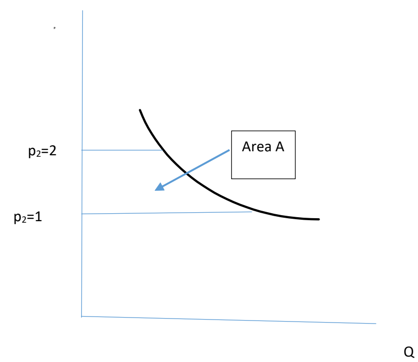
We need to maximize utility subject to budget constraint $p_1x_1 + p_2x_2 = M$. Use the Lagrangian (or any method of your choice) **to show that** the demand functions are $x_1 = \frac{1}{4} * \frac{M}{p_1}$ and $x_2 = \frac{3}{4} * \frac{M}{p_2}$.

(if you do this part correctly, you get 8 points)

Now $M = 100$ and p_2 falls to 1 from 2. Hence, the required change in CS is the area (on price axis) bounded by the demand curve and the two prices, i.e.

$$\begin{aligned}
\Delta CS &= \int_1^2 \frac{3}{4} * \frac{100.0}{p_2} dp_2 \\
&= 75.0 * [\ln 2.0 - \ln 1] \\
&= 75.0 * \ln(2.0) \\
&= 51.99
\end{aligned}$$

This is clear in the following figure.



Consumer Surplus: Change

2. *Explain why*

a) Indifference curves cannot cross.

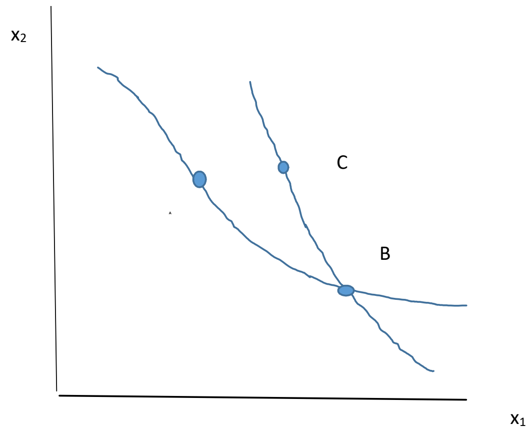
b) In perfect competition, the marginal cost curve is the supply curve of a firm.

(5+5=10)

[**Both of this were discussed in class and/or available in lecture notes**]

ANS

a) Suppose, Indifference curves cross (see the following figure).



Now $B \sim C$ (same indifference curve) and $B \sim A$. By transitivity of indifference relation, $C \sim A$. But, by construction, C contains more of both commodities compared to bundle A. By monotonicity, C is strictly preferred to A. This is a contradiction as the agent, at the same time, strictly prefers C over A and is indifferent between C and A.

b) Under perfect competition, a single producer maximizes his/her profit $pq - c(q)$ taking p as given. The profit maximization exercise leads to

$$p = c'(q) = MC(q)$$

Thus, against each price, the profit maximizing output supplied can be read off from the MC curve. As this uniquely relates price and quantity supplied, the marginal cost curve is the supply curve.

(Note: An "AA" answer should also supply the relevant diagram).

3. *If the production function is* $Q = \sqrt{KL}$, *price of labor is* 4 *and that of capital is* 1, *obtain the long run cost function.*

(Done in class, also in PS1)

(**10**)

ANS

Problem is

$$\begin{aligned} \min C &= 4L + K \\ \text{such that } \sqrt{LK} &= Q \end{aligned}$$

The relevant Lagrangian is

$$\mathcal{L} = 4L + K + \mu\left(Q - \sqrt{LK}\right)$$

FOC's are

$$\begin{aligned} 4 &= \frac{\mu}{2}\sqrt{\frac{K}{L}} \\ 1 &= \frac{\mu}{2}\sqrt{\frac{L}{K}} \\ \sqrt{LK} &= Q \end{aligned}$$

Dividing the first equation by the second

$$\frac{K}{L} = 4 \Longrightarrow K = 4L$$

Putting in constraint

$$\sqrt{L * 4L} = Q \Longrightarrow L = \frac{Q}{2}$$

Thus

$$K = 4L = 2Q$$

Potting in the constraint, the optimum minimized cost is

$$C = 4 * \frac{Q}{2} + 2Q = 4Q$$

This is the cost function.

(Hence MC and AC are equal to 4)