Topic 3A
Choice Under Uncertainty: Part A

Einstein: God does not play dice.

Bohr: Einstein, stop telling God what to do

1 Introduction

In this section, I introduce certain examples of choice under uncertainty. In a way, these modules all relate to financial economics: choice of risky asset, problem of purchasing insurance and mean variance analysis of portfolio selection. Later, we will see some "non-financial" examples of choice under uncertainty. Together, these two chapters should demonstrate that the theoretical tools that we developed in chapter two provides an importanf framework to analyse real world phenomena in different contexts

2 Risky Asset

Let us make matters simple. Assume that I have an endowment y (of money). I can hold the money, in which case, irrespective of good or bad states of the world, I still have $\bar{y}$. Alternatively, I can invest 'a' Rs in a risky asset. The asset earns an interest rate of r_h in good state (prob p) of the world , and r_l in bad state (prob $1-p$), with $r_h > 0 > r_l > -1$. Thus, my random Income is $\tilde{y} = (y-a) + a(1+\tilde{r}) = y + a\tilde{r}$., where $\tilde{y} = y_l$ or y_h. Typically we think about bonds and shares as these assets, but think about anything of which the future price is uncertain: a piece of land, stock of gold, price of a house etc. The analysis applies everywhere.

So a consumer maximizes $EU = pv(y_h) + (1-p)v(y_l)$ with respect to a. The FOC is $pv'(y_h)r_h + (1-p)v'(y_l)r_l = 0$ (the fact that $r_l < 0$) guarantees a solution, and the SOC is $pv''(y_h)r_h^2 + (1-p)v''(y_l)r_l^2 < 0$. Thus, for SOC to be fulfilled, we need $v'' < 0 \rightarrow$ consumers must be risk averse. Solving the FOC, we get the optimal demand for risky asset: a^*.

Given the optimal demand, there are two questions which are to be asked (and answered). First, how do we know that there will be an interior optima, i.e. $a^* \in (0, \bar{y})$. Second, how does this a^* change when the parameters of the model (e.g. interest rate, income, nature of risk etc) change. Answers to the latter questons relate to the predictive power of our model and can be regarded as testable hypothesis (in a statistical sense), the validity of which can be checked using real world data.

2.1 Interior Solution for a^*

We have already seen that $r_l < 0$ guarantees a solution $a^* < \bar{y}$. The question remains, however, how do we know that $a^* > 0$. If we put $a = 0$ in the FOC, we get $pv'(\bar{y})\, r_h + (1-p)\, v'(\bar{y})\, r_l$. If the expression is positive, then the agent will never choose an $a^* = 0$. For te expression to be positive, all we need is $pr_h + (1-p)\, r_l > 0$, that is, the expected return from the risky asset must be greater than that of the safe asset.

2.2 Income and Risky Asset

Suppose income endowment, that is, y increases. The question that we would ask is whether a person will increase the risk of the portfolio,i.e whether $\frac{\partial a^*}{\partial y}$ is positive, negative or constant. Atiitudes towards risk provides a rough, intuitive guide. Suppose the agent exhibits DARA: as y increases, he becomes *less* risk averse. Therefore, such a person should invest more in risky asset.

To see it formally, we differentiate the first order condition with respect to y, noting that a depends on the parameters of the model. The differentiation yields

$$\frac{\partial a^*}{\partial y} = -\frac{pv''(y_h)\, r_h + (1-p)\, v''(y_l)\, r_l}{pv''(y_h)\, r_h^2 + (1-p)\, v''(y_l)\, r_l^2}$$

The denominator is negative (just the SOC): so

$$\begin{aligned} sign\left(\frac{\partial a^*}{\partial y}\right) &= sign\left[pv''(y_h)\,r_h + (1-p)\,v''(y_l)\,r_l\right] \\ &= sign\left[Ev''(\tilde{y}) * \tilde{r}\right] \end{aligned}$$

. The sign will depend on the agent's attitude towards risk.

Theorem 1 *The term inside the parenthasis is positive, negative OR constant as the agent is DARA/IARA/CARA.*

Proof. *See class notes.* ■

Therefore, if the agent exhibits DARA, then, with increasing income (wealth), he/she increases the amount of risky asset in the portfolio. We say that risk is a 'normal' good (amount of risk increases with income) for such agents.

2.3 Interest Rate and Risky Asset

Note that we have kept the interest rate as fixed. Interest rate can change in two ways: additive shift (each interest rate increases by a constant β, keeping the expected return to be negative. Thus, each $\tilde{r}' = \tilde{r} + \beta$, where β is small. Note that, there will be two effects on a^*. First, the cost of holding money in a safe asset increases, and hence there will be a shift towards purchasing more a^*. On the other hand, *had the agent kept his original portfolio unchanged,* he/she would have higher wealth. Such higher wealth will interact with a^* in terms of risk preference (as outlined in the above subsection). Hence the impact is ambiguous. If the agent exhibits DARA or CARA, $\frac{\partial a}{\partial \beta}$ in the vicinity of $\beta = 0$ is positive. For person with IARA, the net result will depend on the comparative strengths of such 'income' and 'substitution' effects.

On the other hand, the interest rate can change by a proportion. We can think of $\tilde{r}' = (1 + \beta_0)\,\tilde{r}$. If $\beta_0 = 0$, we get the original random r. The FOC of utility maximization becomes

$$pr_h v' \left(y + a\left(1+\beta_0\right) r_h\right) + \left(1-p\right) r_l v' \left(y + a\left(1+\beta_0\right) r_l\right) = 0$$

If $a\left(\beta_0\right)$ be the demand for a as a function of β_0, we have the following result

$$a\left(\beta_0\right) = \frac{a\left(0\right)}{1+\beta_0}$$

To see it, note that if we put the value of $a\left(\beta_0\right)$ in the FOC, the solution holds. Thus the consumer reduces his holdings proportionally by $\left(1+\beta_0\right)$.

As a final consideration, consider mean preserving spread of r. Suppose r shifted in such a way that $\bar{r} = E(r)$ is constant, but the variance $\left(\sigma_r^2\right)$ increases. Let $\tilde{r}' = \tilde{r} + \beta_1\left(\tilde{r} - \bar{r}\right)$.Here, $E\left(\tilde{r}'\right) = \bar{r}$ and $var\left(\tilde{r}'\right) = \left(1+\beta\right)^2 \sigma_r^2$. We can write new interest rate as $r' = \left(1+\beta\right) r - \beta\bar{r}$. That is, there is a proportional increase in r (in both states of the world), and a sure reduction in interest rate (in both states of the world).

Proportional increase in r implies a goes down, while absolute decrease in interest rate will reduce a if v exhibits CARA or DARA.

3 Financial Markets: An Introduction

Historically speaking, the framework of mean variance utility was the first attempt in portfolio selection model. It was pioneered by Harry Markowitz in 1952 (Nobel Prize in 1978), who was a graduate student at the University of Chicago at that time. Markowitzs' enduring legacy is the first formal treatment of the risk-return tradeoff.[1]

[1] As Bernstein notes in his highly readable (although probably not highly reccommendable just before the mid sem exam) *Against the Gods: The Remarkable Story of Risk*, "..That revolution touched off the intellectual movement that revolutionized Wall Street, corporate finance and business decisons around the world; its effects are still being felt today".

The theory simply says that we have preference over both mean (μ) and variance (i.e. standard deviation σ) of a portfolio. The preference is captured in a utility function $V(\mu, \sigma)$ .Here, $V_\mu > 0$, but V_σ can be positive/negative/zero according to attitudes towards risk.

Given the utility function, we can define an indifference curve along which $V(\mu, \sigma) = \bar{V}$. The slope is given by $\frac{d\mu}{d\sigma} = -\frac{V_\sigma}{V_\mu}$ The expression is negative if $V_\sigma < 0$ (risk aversion) and $V_\mu > 0$ (everybody likes higher return). Thes, the indifference curves are positively sloped. For the MRS to be positive (slope increases as we move to right), we must have $\frac{d(MRS)}{d\sigma} = -\frac{V_{\sigma\sigma}V_\mu^2 + V_{\mu\mu}V_\sigma^2 - 2V_{\mu\sigma}V_\mu V_\sigma}{(V_\mu)^3} > 0$. For this, our assumptions are $V_{\sigma\sigma} < 0$, $V_{\mu\mu} < 0$ and $V_{\mu\sigma} < 0$. Finally, to figure out the direction of increasing preferences, higher utility must be associated with points towards north-west. This can be summarized in the following indifference map.

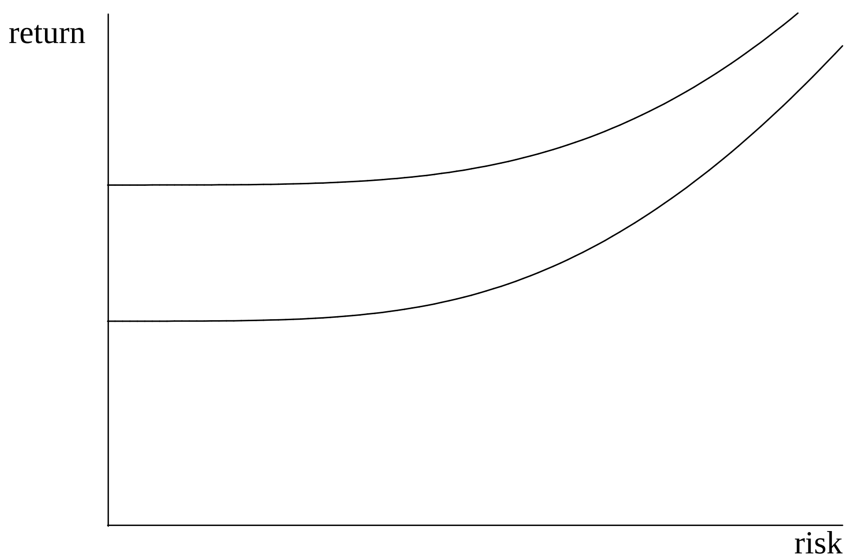

Figure 1: Risk-Return Indifference Map

Now we will turn to some applications of this model.

Now we would like to come back to the original question: with minimal knowledge of the utility function v, can we say something about the preference (over lotteries) of an EU maximizer? Note that, the theory neither mention anything about v and nor consider the whole distribution of income (only the first two moments suffice). Therefore, it offers considerable simplification over the expected utility paradigm or even the concepts of stochastic dominance. However, like every shortcuts, it also has its own pitfalls: the theory is consistent with EU hypothesis only under some restrictive conditions.

1. The Bernoulli utility function is quadratic in nature: $v(w) = \alpha w - \dfrac{\beta}{2}w^2 \implies E(v) = \alpha\mu - \dfrac{\beta}{2}\mu^2 - \frac{\beta}{2}\sigma^2 = V(\mu, \sigma)$. If $\beta > 0$, then one exhibits risk aversion. However, v exhibits IARA, which is somewhat counterintuitive.

2. The Bernoulli utility exhibits constant absolute risk aversion $v(w) = -\exp(-kw)$ and the return from the risky asset is normally distributed $f(w) = \frac{1}{\sqrt{2\pi\sigma^2}} \exp^{-\frac{1}{2}\left(\dfrac{w-\mu}{\sigma}\right)^2}$, then

$$E(v) = -\frac{1}{\exp\left[k\left(\mu - \dfrac{k}{2}\sigma^2\right)\right]} = V(\mu, \sigma)$$

But $\mu - \sigma$ approach is immensely popular with non-economists (i.e. Finance people), because it gives a huge leverage (pun unintended).

3.1 Portfolio Selection: One Risk Free Asset

Assume, as in section 1, that there is one risky and one risk free asset. Consumer has a fund Y. (s)he buys one riskless asset (e.g. puts money in bank) and buys another assert with risk (e.g. a piece of land).Let a be the risky asset quantity (assume 1 \$ buys one unit). Risk free asset (money in bank) gives a gross return of R..Risky asset (gross interest r) gives a mean return of μ_0 with a variance σ_0^2. Thus the budget contraint can be written as $B + a = Y$, where B is the riskless asset and a is the risky asset. The "trick" here is to express the equation in terms of μ, σ.

The intuition is the following. At each point on the budget line, the portfolio is some mixture of the riskless and risk free asset. Thus, each point on the budget line has a particular risk-return combination. Therefore, each point on the budget line can be treated as (linear) combination of two extreme cases: if the individual holds no risk free asset OR if the individuals' portfolio consists of only risky assets.

The portfolio income is $c = (Y - a) R + ar$. Thus, the expected return is $\mu = Ec = (Y - a) R + a\mu_0 = YR + (\mu_0 - R) a$ and portfolio risk is $\sigma^2 = var(c) = a^2\sigma_0^2$. From this, substituting the value of a, one can write

$$\mu = YR + (\mu_0 - R) \frac{\sigma}{\sigma_0}$$

This is our required constraint.[2]

Note that this is a straight line in $\mu - \sigma$ plane. If one puts everything in riskless asset ($a = 0$), the risk-return configuration is $(0, YR)$. If one puts everything in the risky asset ($a = Y$), then the configuration is $\left(Y^2\sigma^2, Y\mu_0\right)$The slope of the line, known as *Sharpe ratio* is given by $\frac{d\mu}{d\sigma} = \frac{\mu_0 - R}{\sigma_0} > 0$ if $\mu_0 > R$.[3] The ratio expresses the "price" of risk: to increase risk by 1 unit, how much the market offers as return?

Given this, we have all the diagrammatic tools to figure out the optimal portfolio. It is shown below in the following figure.

[2] What happens if $a = 0$? What happens if $a = Y$?

[3] In finance literature, the difference $E(r) - R$ is called the *risk premium*. Notice that the concept is differrent from Pratt's definition which we encountered in topic 2.

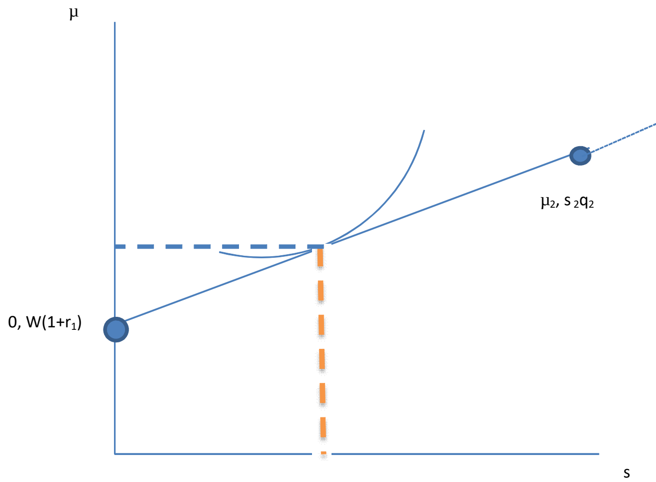

Figure 2: Optimal Portfolio

3.2 Two Risky Assets

Instread of one risk free and one risky asset, let us now assume two risky assets. Individually, each asset i has return-risk structure (μ_i, σ_i). Let $\mu_1 > \mu_2$. To make things interesting, let $0 < \sigma_2 < \sigma_1$

Let the total endowment Y be normalized to 1, so that we are dealing with shares in portfolio. The investor puts a\$ in asset 1 and $(1-a)$ \$ in asset 2. Return of the portfolio is $E(c) = \mu = a\mu_1 + (1-a)\,\mu_2 = \mu_2 + a(\mu_1 - \mu_2)$ and the associated risk is $Var(c) = E\,(c - E(c))^2 = a^2\sigma_1^2 + (1-a)^2\,\sigma_2^2 + 2a\,(1-a)\,\rho\sigma_1\sigma_2$. Here, ρ is the correlation between return between the two assets. The nature of preferences remain the same. However, the "budget line" is different.

To see this, notice that slope of the budget line (i.e. (the Sharpe ratio) can be written as $\frac{d\mu}{d\sigma} = \frac{d\mu}{da} \div \frac{d\sigma}{da}$. Here, $\frac{d\mu}{da} = \mu_1 - \mu_2 > 0$, so $sign\left(\frac{d\mu}{d\sigma}\right) = sign\left(\frac{d\sigma}{da}\right) = sign\left(\frac{d\sigma^2}{da}\right)$. So we can write $\frac{d\sigma^2}{da} = 2a\sigma_1^2 - 2(1-a)\sigma_2^2 + 2\,(1-2a)\,\rho\sigma_1\sigma_2$. If $a \approx 0$ (all money in asset 2), the expression is negative if $\rho\sigma_1 < \sigma_2$. If $a \approx 1$ (all money in asset 1), the expression is positive if

$\sigma_1 > \rho\sigma_2$. Both these conditions are satisfied if $\rho < \min\left[\frac{\sigma_1}{\sigma_2}, \frac{\sigma_2}{\sigma_1}\right]$. In other words, if the assets are not "strongly correlated" (such that the performance of one asset cannot predict the performance of nother asset), the "budget line" is likely to be a curve (actually a hyperbola). The curve is known as "Markowitz's bullet". The left extreme of the curve is known as minimum variance portfolio (MVP). Thus, it is never the case that investing in one asset only is likely to eliminate all risk. Rather, if one aims for minimizing risk, one should opt for diversication of assets.

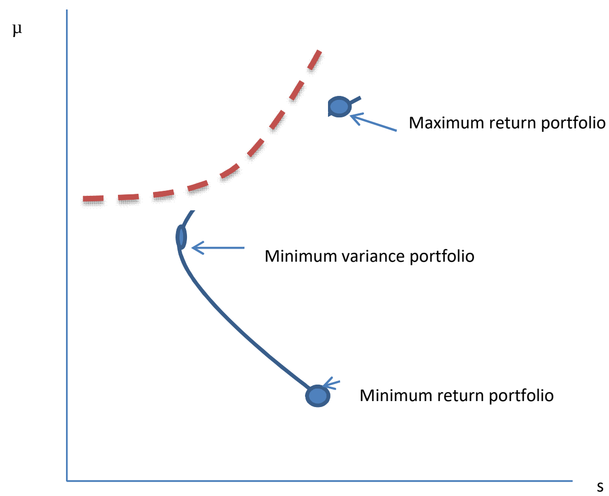

Figure 3

Given the indifference curve, the consumer attains *maximum utility* on the upward portion of the budget line.

3.3 Mutual Fund Theorem

Let us get a bit realistic now. Assume our total portfolio has one safe asset and $n = 2$ risky assets. How do the results of previous section gets modified? The answer is not much. Note that, each point of the Markowitz bullet gives one combination of two risky assets. The return from the safe asset (again, assume Y is normalized to 1), on the other hand, is on the x (μ) axis.

Hence, any line joining the safe asset point with any point of the Markowitz bullet will give you combinations of safe asset with a particular combination of risky assets. Beyond point A, the portfolio line coincides with the bullet.

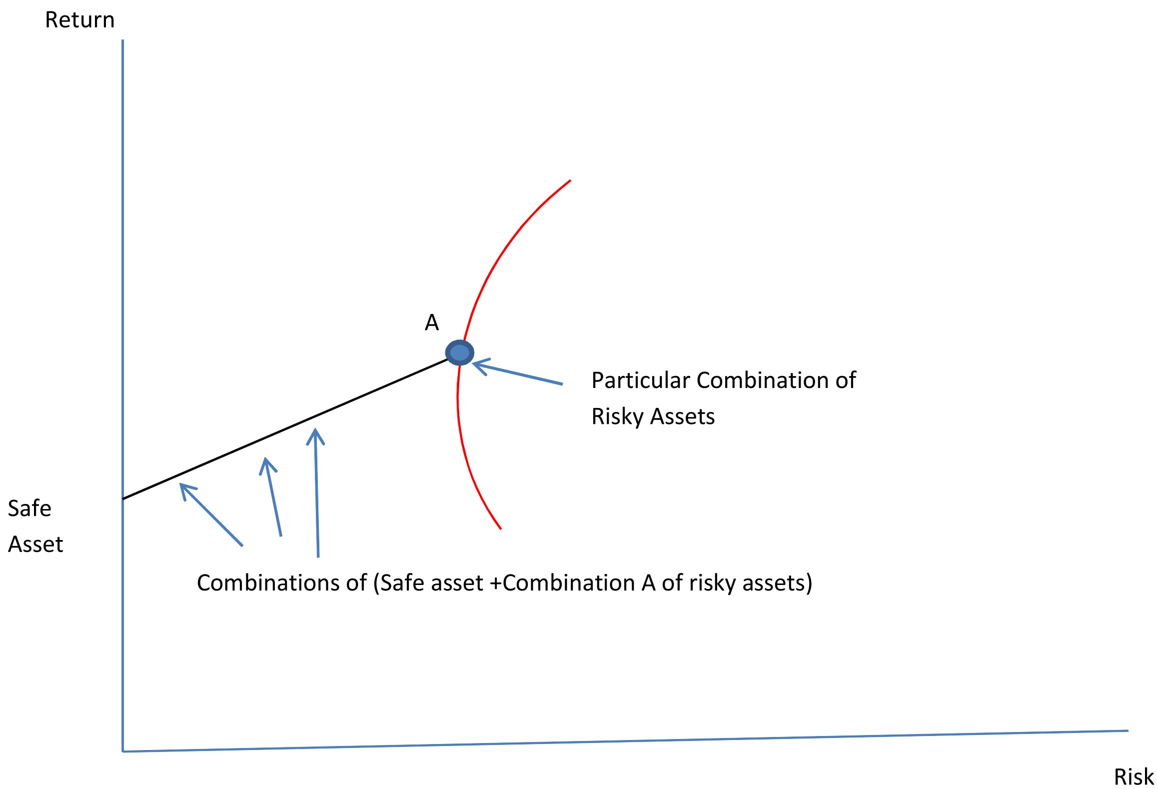

Figure 4: One Safe and Two Risky Assets

Given various mutual funds , it is clear that the constraint which is higher up will be preferred by the consumer. To see this, think about two mutual fund companies. One is offering A, and another B. All points on B portfolio is favored to A.

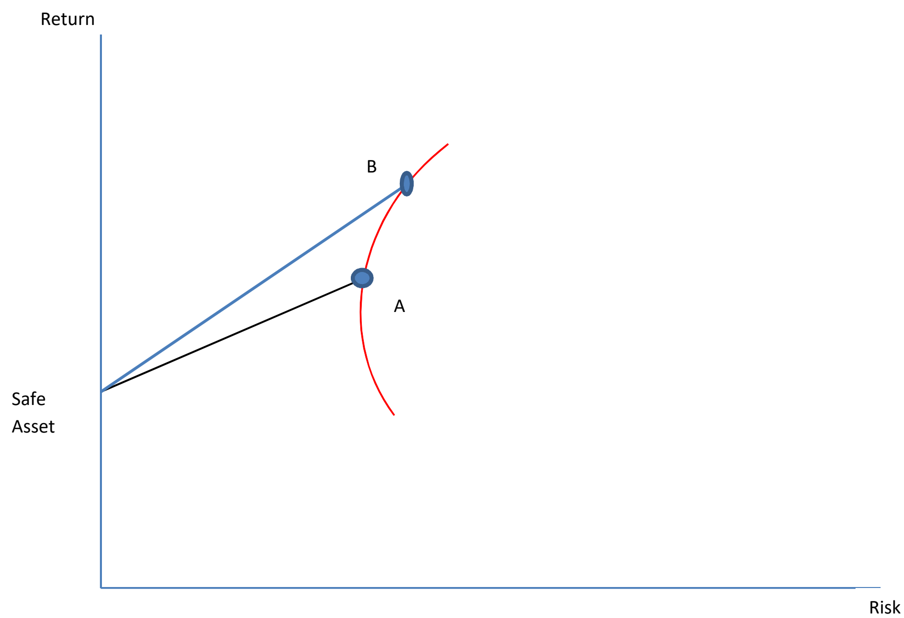

Figure 4a: B dominates A

If we proceed in the same way, the market constraint facing the consumer will be such that the constraint line is tangent to Markowitz bullet, and then it follows the same.

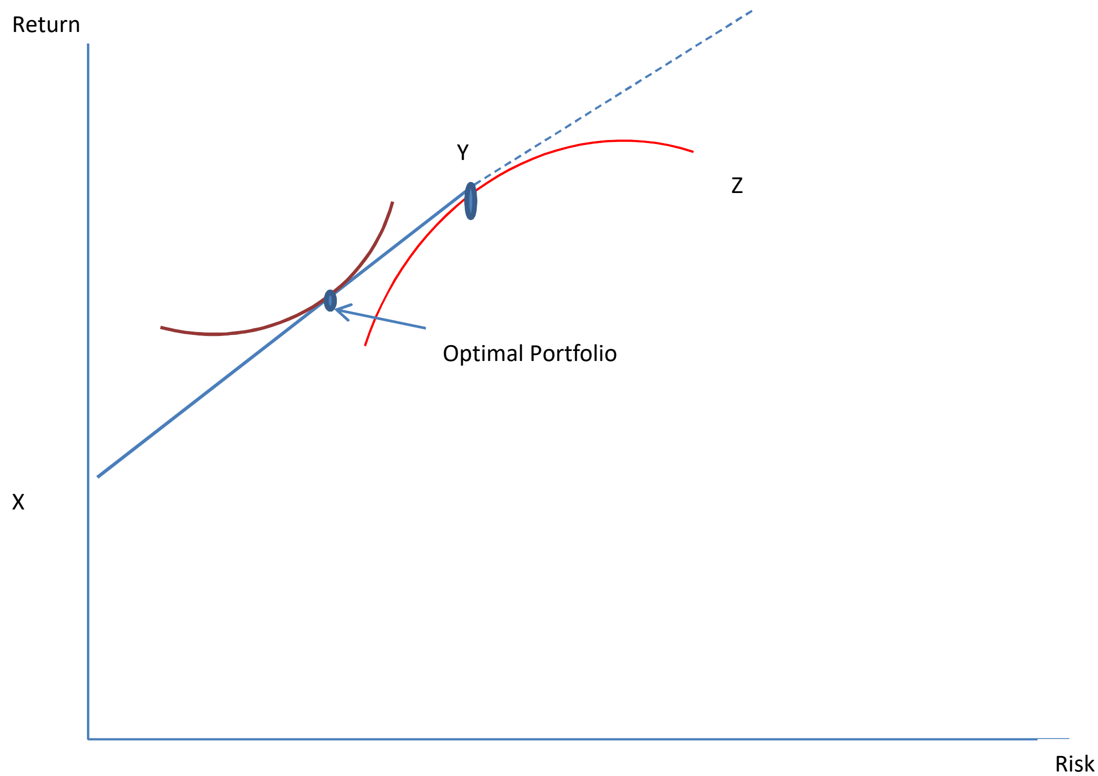

Figure 4b: Optimal Portfolio Choice

The market offers portfolio along XYZ. Given the choices, a rational consumer chooses the portfolio where the indifference curve is tangent to XYZ. Ruling out corner solutions (all safe asset/all risky asset), we see that the solution must occur at some points on XY.

But now suppose there are different consumers. All of them are presented with the opportinity line XYZ and all of them are situated at some points between XY (no corner solutions). But this means the allocation of risky portfolio for each of them are identical, i.e. what is given by point Y. I may be rich and show more apetite to risk, putting $800 in risky asset and

\$200 in safe asset(splitting my portfolio in a 4 : 1 ratio). You may be poor and less prone to risk, putting \$440 in riskless asset and \$60 in risky asset (splitting the portfolio in 3 : 22 ratio). If point Y implies that risky assets are being combined at 3 : 1 ratio, my investment in risky assets will be \$600 and \$200, respectively. For you, the corresponding split will be \$45 and \$15.

Thus, irrespective of asset allocation, each agent will hold risky assets *in the same proportion*. It is as if the risky assets are managed by a giant mutual fund firm, collecting the money from everyone, but investing it in the same proportion over different risky assets. This remarkable result is called the "*Mutual Fund Theorem*".

4 Insurance

Insurance is to give up something in good states of the life so as to minimize loss of livelihood in the bad states. Suppose the net wealth of a person is w_0. If the person experiences an accident (random event), the income is likely to be $(w_0 - d)$, where d is the damage cost. Thus the agents statewise income is $(w_0, w_0 - d)$. *Is it possible to have a better deal?*

Nowadays, insurance companies are ubiquitous (the first modern insurance contract was written in Italy during the 15th century), but the behavior is as old as human civilization.As we will see in later chapters, one does not need even an insurance company. In a way, insurance is opposite analysis of purchase of risky asset. When one buys risky asset, one purchases risk. If I buy insurance, I reduce my exposure to risk.[4]

As before, we will cover both formal and graphical analysis. To do so, we need to know how to represent the preferences and constraint in a state-space.

[4] Existence if insurance is often cited as in indirect proof for risk aversion.

4.1 State Space Approach to Uncertainty

Remember, in topic 2, we had represented expected utility of the lottery $\mathcal{L} = [p, 1-p; c_1, c_2]$ as $Ev(c) = pv(c_1) + (1-p)\,v(c_2)$, with the property that $v'(c_i) > 0$. We had fixed c_i and did change p to obtain an indifference curve in probability space. Here, we will take the complementary approach: we will "fix" p (i.e. treat p as parameter) and cahnge c. As we shall see, this will bring us closer to the traditional analysis which we have encountered in topic 1.

We will start with plotting the indifference curve in $c_1 - c_2$ space, also known as state space. The name arises because c_i are realization of c under different states of the world. Note that the indifference curve should the the locus of c_i such that $pv(c_1) + (1-p)\,v(c_2) = k$. Taking derivative we see that the slope is $\frac{dc_2}{dc_1} = -\frac{p}{1-p}\frac{v'(c_1)}{v'(c_2)} < 0$. The slope changes as we move to the right according to the sign of $\frac{d}{dc_1}\left(\frac{dc_2}{dc_1}\right)$. Note that the sign of the term is equal to the the sign of $\left(-\dfrac{[v'(c_1)]^2\,v''(c_2) + [v'(c_2)]^2\,v''(c_1)}{[v'(c_2)]^3}\right)$. This is negative if $v'' < 0$, that is, the person is risk averse. For such agents we can draw the indifference map as in the following figure. Note that the value of the expression is higher if v'' is higher. So, for more risk averse agents, the indifference map has higher curvature.

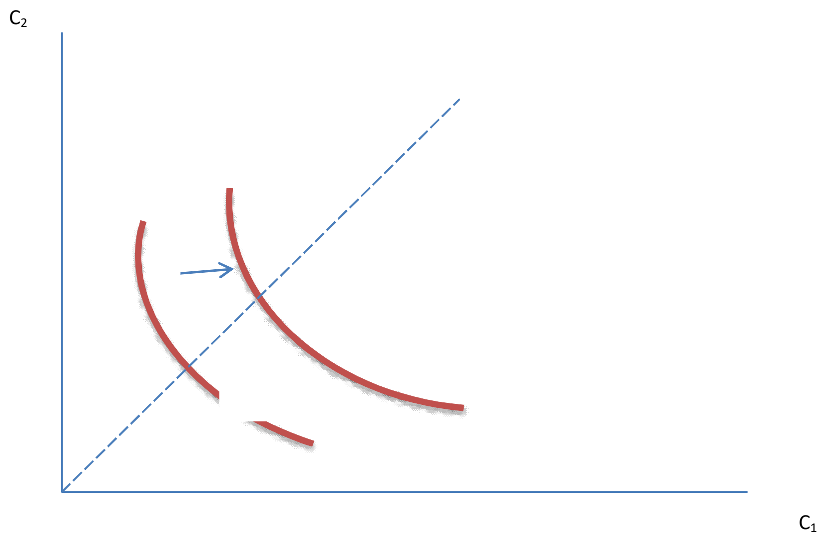

Figure 5: Indifference Curve in State Space

It is clear that higher indifference curves are preferred, i.e. points to the north-east represent better bundles.

Of special interest is the 45° line through the origin. Along the line, $c_1 = c_2$. Therefore, when the indifference curves cross the line, the slope is $\frac{p}{1-p}$

4.1.1 Certainty Equivalence and Risk Premium

Note that, on the 45° line, the consumer gets equal amount of c (call it $\tilde{c}$) in both states. Given any c_1^*, c_2^*, along the indifference curve we must have $v(\tilde{c}) = pv(c_1^*) + (1-p)v(c_2^*)$. But then by the definition that we have seen in topic 2, $\tilde{c}$ is nothing but the certainty equivalence of the lottery. Thus, given any combination of c_1, c_2, the corresponding certainty equivalence is obtained where the indifference curve passing through the point cuts the 45° line. Second, given any combination, let us draw a line with slope $\frac{p}{1-p}$ through it. The generic equation of the line is $pc_1 + (1-p)c_2 = k$, which happens to pass through (c_1^*, c_2^*). Where the line cuts the 45° line, the value of $\bar{c}$ is such that $p\bar{c} + (1-p)\bar{c} = \bar{c} = pc_1^* + (1-p)c_2^*$. Thus, $(\bar{c}, \bar{c})$ represents expected value of the lottery. The difference between CE and expected value is the risk premium (in Pratt's sense).

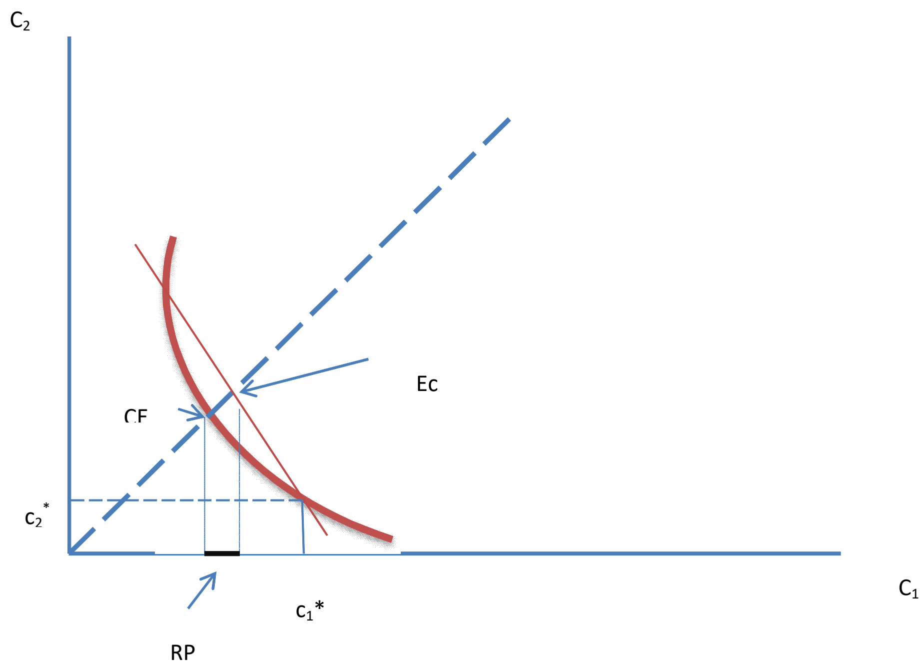

Figure 5a: Certainty Equivalence and Risk Premium in State Space

4.2 The Problem of Insurance

Now that we are done with the preliminaries, we can start talking about insurance proper. An insurance contract is a pair (α, I) where α is the the premium I pay to the insurance company (regardless of state) and I is the indemnity that the insurance companies pay me if there is an accident or damage. Thus, with an insurance, my income is $(w_1.w_2) = (w_0 - \alpha, w_0 - \alpha - d + I)$. We will assume that insurance companies are rich and therefore they are risk neutral. Their expected profit is $p(\alpha) + (1-p)(\alpha - I)$. Further, if we assume competition in the insurance market, the profit of a typical firm will be driven to zero. Therefore, $E\pi = 0 \rightarrow \alpha =$

$(1-p)\,I$. This means insurance is "*actuarially fair*", that is, the premium that I pay equals the expected value of indemnity.

Given this restriction, an agent maximizes

$$\begin{aligned}
& pu\,(w_1) + (1-p)\,u(w_2) \\
= \quad & pu\,(w_0 - \alpha) + (1-p)\,u\,(w_0 - \alpha - d + I) \\
= \quad & pu\,(w_0 - (1-p)I) + (1-p)\,u(w_0 - (1-p)I - d + I) \\
= \quad & pu\,(w_0 - (1-p)\,I) + (1-p)\,u\,(w_0 + pI - d)
\end{aligned}$$

Note that the agent can choose either α or I because they are uniquely related.

Maximizing with respect to I, and setting the FOC to zero, we get $u'(w_1) = u'\,(w_2) \rightarrow w_1 = w_2$. That is, if I buy insurance, it must be the case that my wealth before the accident and after the accident must be equal: $w_0 - (1-p)\,I = w_0 + pI - d \rightarrow d = I$. In other words, the damage cost must be equal to the indemnity. Such an insurance is known as "full insurance".

Thus we get a stunning result: if the insurance is actuarially fair, everybody will fully insure themselves. However, what is to be demonstrated is the fact that consumer actually gains if he/she moves from $(w_0, w_0 - d)$ (that is "autarky") to $(w_0 - \alpha, w_0 - \alpha - d + I)$. The easier way is to represent this in a diagram.

Note that the isoprofit curve of the company is $\alpha + (1-p)\,I = C$. The way we have represented w_1, w_2, we can write $\alpha = w_0 - w_1$ and $I = w_2 - w_1 + d$. Thus, the isoprofit line in the $w_2 - w_1$ space is $(w_0 - w_1) - (1 - p)\,(w_2 - w_1 + d) = C$. Note that, given w_1 (that is the premium received), lower value of w_2 (lower I) will be preferred. Similarly, given w_2 (indemnity), lower values of w_1 (guaranteed by higher values of α) are preferred. So the preference of the firm is increasing to the left in $w_2 - w_1$ plane. Notice that, if insurance is actuarially fair, then we must have $C = 0$, or $w_2 = \frac{1}{1-p}\,(-d + w_0 - pw_1 + dp)$. Slope of the line is $-\frac{p}{1-p}$. Further, if $w_1 = w_0$, then $w_2 = w_0 - d$. That is, the isoprofit line, when it is actuarially fair, must pass through the autarky point. We can summarize the discussion in the following diagram.

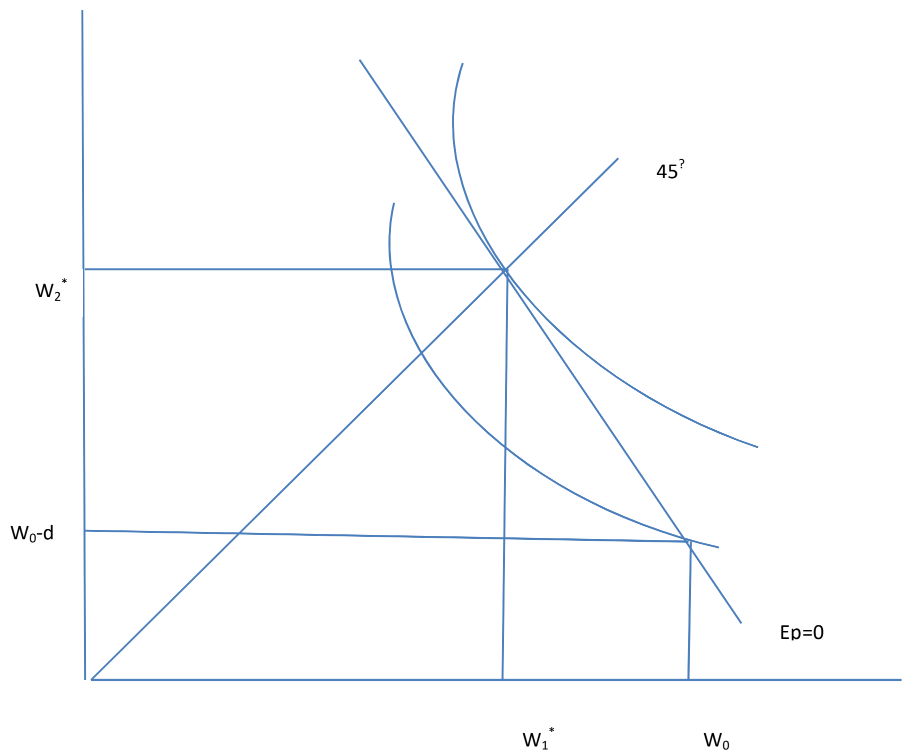

Figure 6: Purchase of Full Insurance

Some observations are in order. First, notice that the slope of the iso-profit line is $-\frac{p}{1-p}$, which is equal to the slope of indifference curve on the 45° line. The point $(w_0, w_0 - d)$ is below the 45° line, so that the slope of the indifference curve (IC1) is lower (in terms of magnitude) than $-\frac{p}{1-p}$ at that point, given our assumption of risk aversion. In other words, the actuarially fair line must cut the autarky point from above. Given the choice of availing insurance, the consumer moves off the isoprofit line and locates himself where the slope of indifference curve (on IC2) equals $-\frac{p}{1-p}$. Since IC2 is above IC1 (utility level at autarky), the consumer must have benefited from availing the insurance.

The result might seem interesting at first glance, but it lacks any predictive power. This arises because of the assumptions that we made regarding the nature of the firm. We will modify some of these assumptions and see where do these lead us to.

4.2.1 "Loading"

Loading is a situation where the individual firms no longer offer a fair insurance. In other words, there is a difference between expected indemnity and premium. It might be the case that the market structure is such that monopolistic elements allow the firms to reap up some profit.

In case of *proportional loading*, the premium and the indemnity are linked through the following equation $\alpha = m\,(1-p)\,I$, where $m > 1$ is the mark up over expected payments. Using the same equations which we have done before, one can write the constraint as $w_2 = \frac{[w_0 - m(1-p)d]}{m(1-p)} + \frac{m(1-p)-1}{m(1-p)}w_1$. The slope of the line is $\frac{dw_2}{dw_1} = -\frac{1-m(1-p)}{m(1-p)}$. We assume that the mark up m is not so great such that the slope of the constraint is still negative. It can easily be shown that the value is less than the fair insurance line, i.e. $\frac{1-m(1-p)}{m(1-p)} < \frac{p}{1-p}$. Finally, if $w_1 = w_0$, then $w_2 = w_0 - d$. Thus, the loaded line must pass through the autarky point

By collecting all information, we have the following diagram, with "loading"

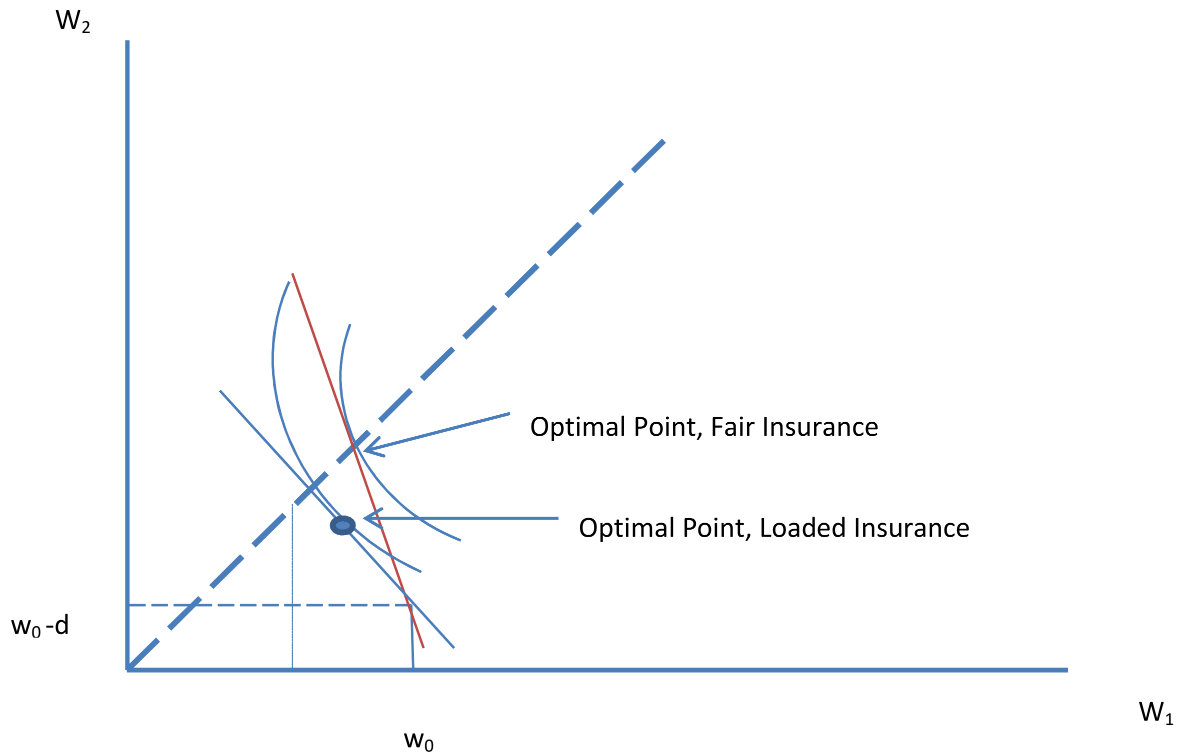

Figure 6a: Loading

We omit IC1 for clarity, but the point of autarky is still available. Consumers still can make things better by moving up through the loaded line. However, the tangency occurs below the 45° line. It must be so : the slope of the line is lower than $-\frac{p}{1-p}$. Thus, the tangency will be achieved, for any IC, below the line (in fact any IC will cut the new line from above at the 45°.line, hence cannot be a tangency).

One consequence is the following. Consumers no longer buy full insurance. In fact, they are willing to pay less when the going gets good, and willing to receive a partial refund when going gets bad. To see it, consider a consumer maximising $pu\left(w_0 - m\left(1-p\right)I\right)+(1-p)u\left(w_0 - m\left(1-p\right)I - D + I\right)$ with respect to I. The result is $(1-m(1-p))u'(w_2) = pmu'\left(w_1\right)$. Since

$pm > 1 - m(1-p)$, we have $u'(w_1) > u'(w_2) \to w_1 < w_2 \to d < I$.

Second, one can also readily decipher the role of risk aversion. Irrespective of the attitude towards risk, the slope of IC's on 45° line is $-\frac{p}{1-p}$. But for a risk averse person, the IC curve away rapidly, i.e. the slope falls off quickly. Thus, it would achieve a tangency with the loaded line at a point which is closer to the full insurance point than that of a person who is less risk averse. The result more or less conforms with our intuition.

In case of fixed loading, the premium and the endowment is related through the following equation: $\alpha = (1-p)\,I + \kappa$, where $\kappa > 0$. The isoprofit curve of the firm in $w_1 - w_2$ plane is $pw_1 + (1-p)\,w_2 = w_0 - \kappa - (1-p)\,d$. Here, if we put $w_1 = w_0 - \kappa$, then $w_2 = w_0 - \kappa - d$. Thus, having a fixed loading is identical to a reduction in original wealth to $w_0 - \kappa$ even at autarky. The slope of the isoprofit line however remains the same.

If κ is not "too high", the isoprofit line, parallel to the actuarially fair line, will make consumers better off compared to autarky (and the consumer will buy full insurance). However, if κ is high, isoprofit line shifts too far to left, and, as a result, the consumers are better off by not purchasing any insurance. I leave this as a homework.

4.2.2 Epilogue

These results do suggest that if the insurance market is imperfect, then many consumers will be left out of the *formal* insurance market. But people are smart: if formal insurance markets are not available (due to poverty, say), societies have still devised institutions that mimic insurance and minimize risk. We will turn to such an example in the next topic.