PS-3 Sol^n outline.

a)

1) Bernoulli Utility f^n: $v = \ln c$.

initial wealth: c_0.

Invest c', remaining wealth $= c_0 - c'$.

c' can become $2c'$ or $\frac{c'}{2}$.

At the end, we have either $(c_0 - c' + 2c') = c_0 + c'$.

or $(c_0 - c' + \frac{c'}{2}) = c_0 - \frac{c'}{2}$.

Expc. Utility $= \frac{1}{2} \ln(c_0 + c') + \frac{1}{2} \ln\left(c_0 - \frac{c'}{2}\right)$.

Max^m with respect to c',

$$\frac{1}{2} \frac{1}{c_0 + c'} = \frac{1}{4} \frac{1}{c_0 - \frac{c'}{2}}$$

$$\Rightarrow 4\left(c_0 - \frac{c'}{2}\right) = 2(c_0 + c').$$

$$\Rightarrow 4c_0 - 2c' = 2c_0 + 2c'.$$

or. $4c' = 2c_0$

$$\Rightarrow c' = \frac{1}{2} c_0.$$

Note: Elasticity of X with respect to Y.

$$= \frac{\%\text{ Change in } X}{\%\text{ " " } Y} = \frac{dX/X}{dY/Y} = \frac{dX}{dY} \cdot \frac{Y}{X}.$$

Using this defn.

elasticity of c' w.r.t. c_0 $= \dfrac{dc'}{dc_0} \times \dfrac{c_0}{c'}$

$$= \frac{1}{2} \times 2 = 1.$$

<u>Note:</u> try with $u(c) = -e^{-\alpha c}$. (CARA)

2)

2) Markowitz's Bullet with −ve correlation:

The hyperbola is associated with the following eqn:

$$\mu = a\mu_1 + (1-a)\mu_2.$$

$$\sigma^2 = a^2\sigma_1^2 + (1-a)^2\sigma_2^2 + 2a(1-a)\rho\sigma_1\sigma_2$$

If $\rho = -1$, then

$$\frac{d\sigma^2}{da}\Big|_{a=1,\rho=-1} = 2(\sigma_1^2 - \rho\sigma_1\sigma_2)\Big|_{\rho=-1} = 2(\sigma_1^2+\sigma_1\sigma_2) > 0$$

~ly, $$\frac{d\sigma^2}{da}\Big|_{a=0,\rho=-1} = 2(-\sigma_2^2 + \rho\sigma_1\sigma_2)\Big|_{\rho=-1} = -2(\sigma_2^2+\sigma_1\sigma_2) < 0.$$

So, at the two extremes, the slopes are +ve and −ve, respectively.

Again, $\sigma^2 = (a\sigma_1 - (1-a)\sigma_2)^2.$

From here, if we put. $a = \frac{\sigma_1}{\sigma_1+\sigma_2} \Rightarrow \sigma^2 = 0$

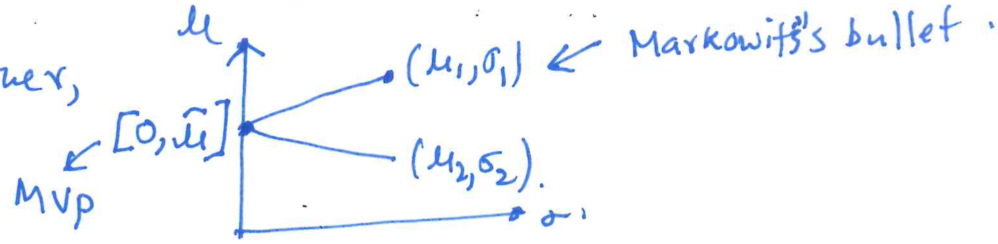
∴ it is possible to have a 0 risk portfolio.

In general; $\frac{d\sigma^2}{da} = 2.\sigma[\sigma_1+\sigma_2]$

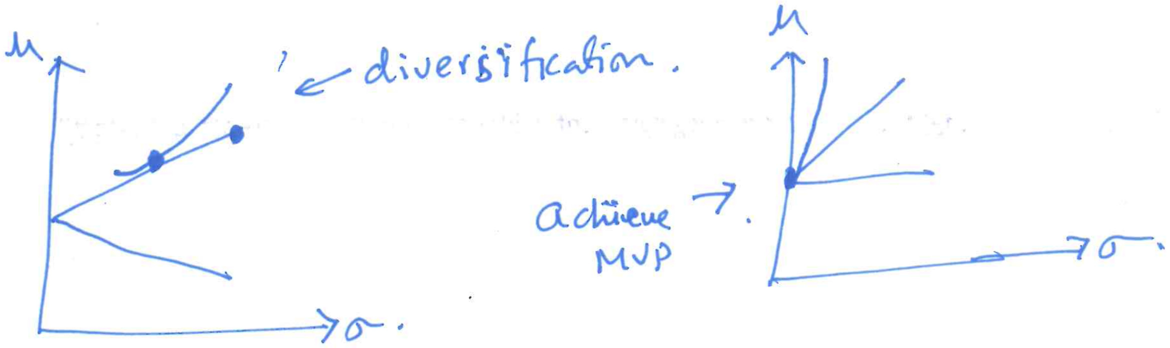
$\Rightarrow$ both μ and σ^2 varies monotonically with a.

$\Rightarrow$ The 'bullet' is a straight line.

All facts put together,



$\Rightarrow$ You can have diversification with $\rho = -1$, or achieve MVP.



3). (i) Find μ and σ of L1 and L2.

$$\mu = E(x) = \cdot 2 \times 2 + \cdot 8 \times 12.$$

$$\sigma^2 = E(x-\mu)^2 = \cdot 2\,(2-\mu)^2 + \cdot 8\,(12-\mu)^2.$$

Similarly for the other lottery.

Then Compare EU of the above lotteries with the function

$u = \ln w$.

[Hint: you will see one lottery is dominating the other in mean - variance sense.

However, the EU ranking is opposite]

4). $W_0 = 100$

$W_0 - d = 70 \Rightarrow d = 30.$

$p = \frac{2}{3}$, $1-p = \frac{1}{3}$ (prob. of accident).

with Insurance $\alpha = \frac{I}{3}$ (fair insurance).

- with insurance, $W_1 = 100 - \alpha = 100 - \frac{I}{3}$

$$W_2 = 70 - \alpha + I = 70 - \frac{I}{3} + \frac{I}{3}$$
$$= 70 + \frac{2}{3} I$$

The agent maximises $\frac{2}{3}\ln\left(100 - \frac{I}{3}\right) + \frac{1}{3}\ln\left(70 + \frac{I}{3}\right)$.

- In the second case, the agent maximises

$$\frac{2}{3}\sqrt{100-\frac{I}{3}}+\frac{1}{3}\sqrt{70+\frac{2}{3}I}$$

- Do you see any diffrence in α^*?

5. Same as above. Now

$$\alpha = 1.25 * \frac{1}{3} * I$$

$\Rightarrow$ repeat Q^n 4. Do you see any difference?

6. India wins: utility $\ln(W)$.

AOT wins utility $\frac{1}{2}\ln(W)$: state dependent utility. (utility terms)

[The $\frac{1}{2}$ is the 'weight' on your wealth if other team wins.
If this is close to 1, you view Indian's win and AOT's win "at par"]

$W_0 = 1000$.

AOT wins $= 1000 - x$.

India wins $= 1000 - x + 2x = 1000 + x$.

Then the exercise is

$$\max_{x} \frac{1}{2} \ln(1000+x) + \frac{1}{2} \times \frac{1}{2} \ln(1000-x).$$

I leave it as a trivial exercise.

[Interesting: do not use specific values. Let the probabilities be $p, 1-p$, the bookie offering you mx $(m>1)$ if your team wins and let the state dep. utility be $\alpha \ln x$ $(\alpha<1)$ if your team loses.

Then you maximise.

$$\max_{x} \; p \ln(1000+(m-1)x) + (1-p) \cdot \alpha \ln(1000-x).$$

See how x^* depends on various parameters.

7. Let the ~~probabi~~ utility of death be "0".

If not treated, his utility is.

$$p_0 \times u(c) + (1-p_0) \times 0$$

$$= p_0 \, v(c).$$

If he spends on the Physician, his utility is

$$= p \, u(c-z) + (1-p) \times 0 = p \, u(c-z)$$

Thus he will spend as long as.

$$p\,v(c-z) \geq p_0\,v(c) \quad \Big|\; p > p_0.$$

$\Rightarrow$ This defines an indifference relation between p and Z.

[trade off : higher p $\Rightarrow$ ~~lower~~ higher Z. $\Rightarrow$ low $(c-z)$]

$U(p,z) = p\,v(c-z) = \bar{k} = p_0\,v(c)$ $\Rightarrow$ the max^m Z I am willing to spend.

$$MRS = \frac{dz}{dp} = \frac{v(c-z)}{p\,v'(c-z)} > 0 \quad \text{if } v' > 0.$$

$\Rightarrow$ Extra amt that i am willing to pay given a small Δ in probability.

- Then find ~~the~~ how the slope changes with p? That is

$$\frac{d}{dp}\left[\frac{dz}{dp}\right] > 0, \quad < 0 \quad \text{or} = 0 \,?$$

Implicitly differentiate MRS and find out.