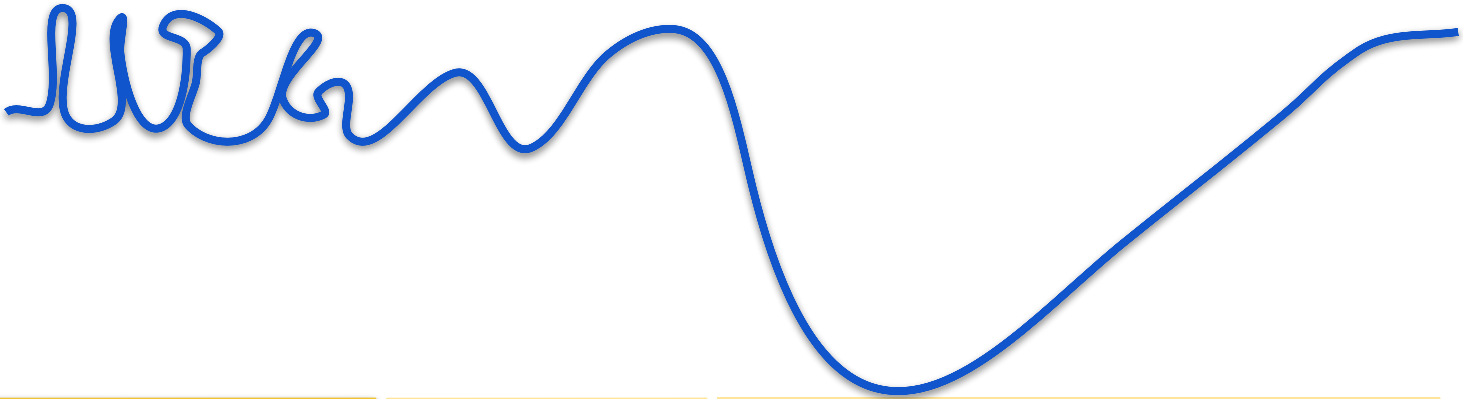
Computing with Signals



DA 623

Jan - May 2024

IIT Guwahati

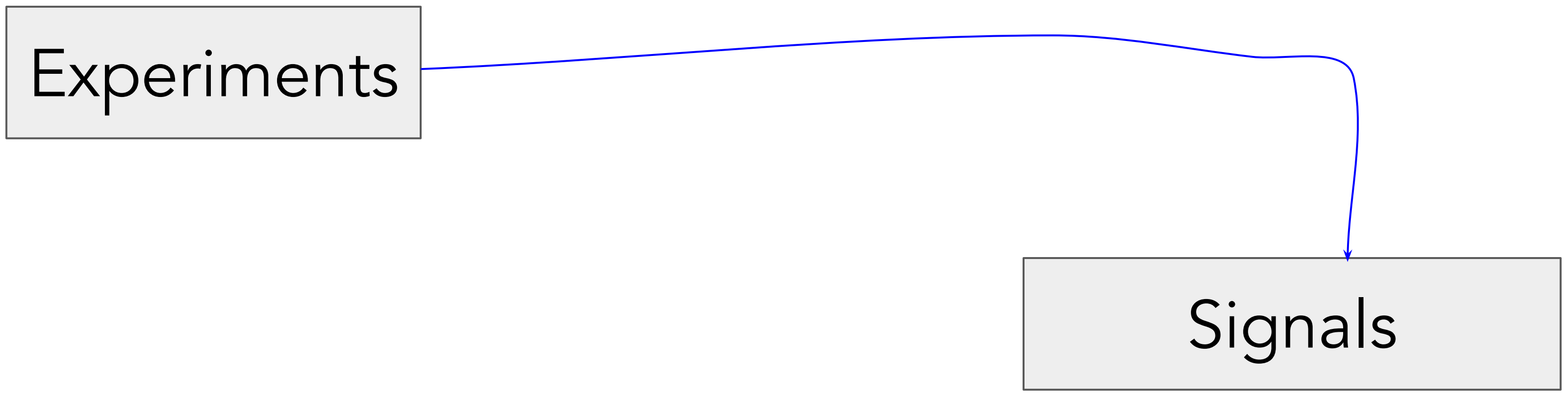
Instructors: Neeraj Sharma

Lecture-02

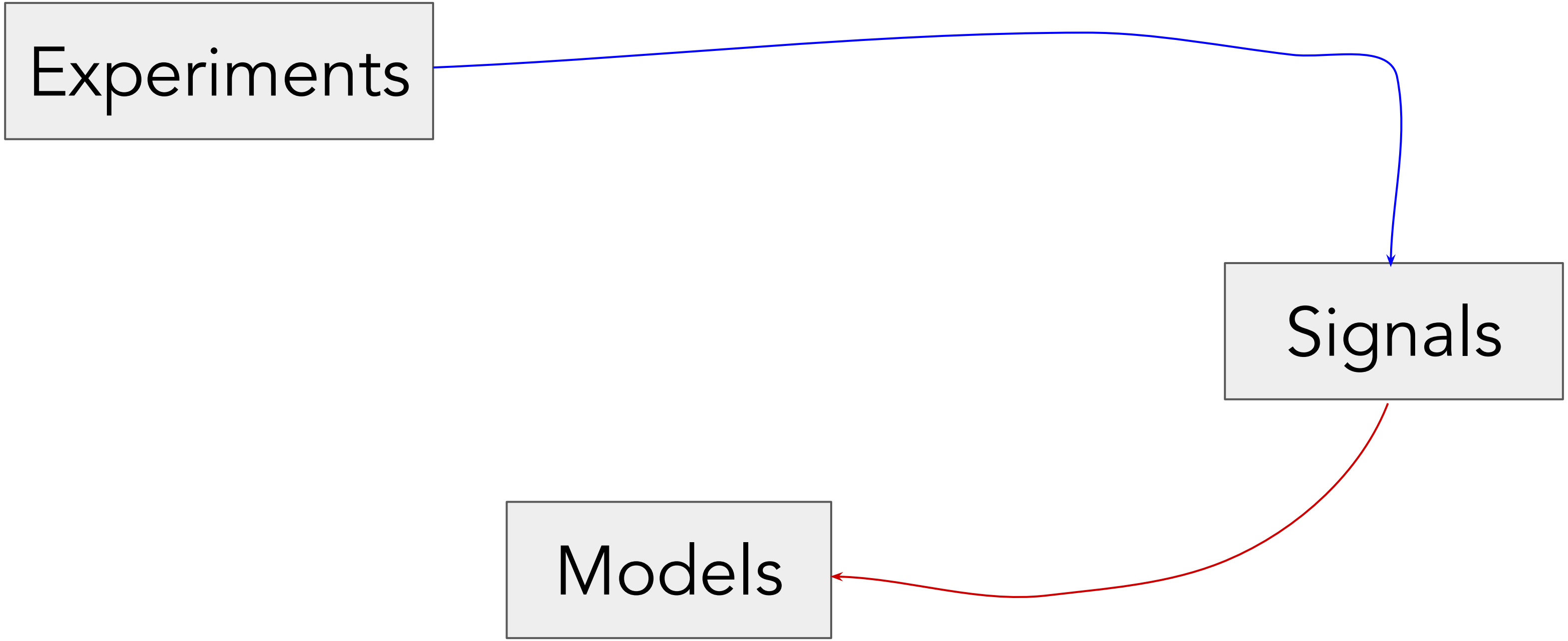
Real world

Experiments

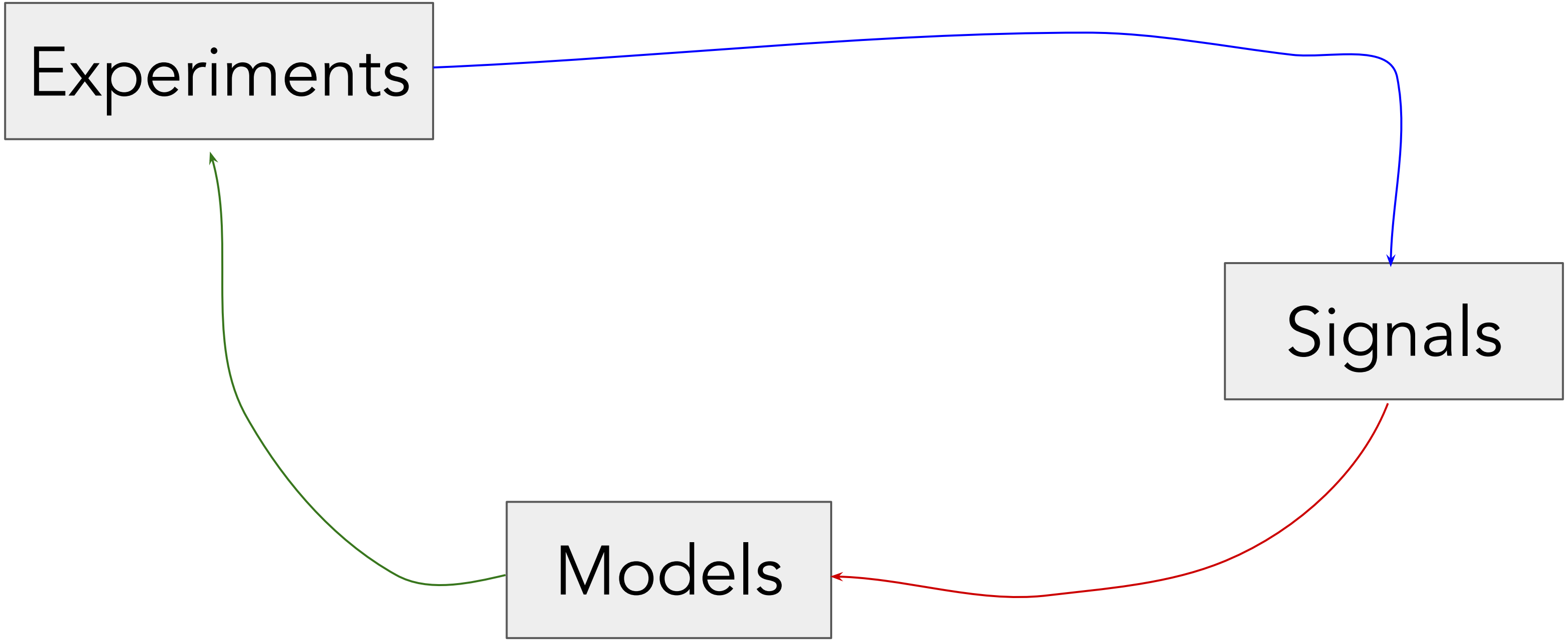
Real world



Real world



Real world



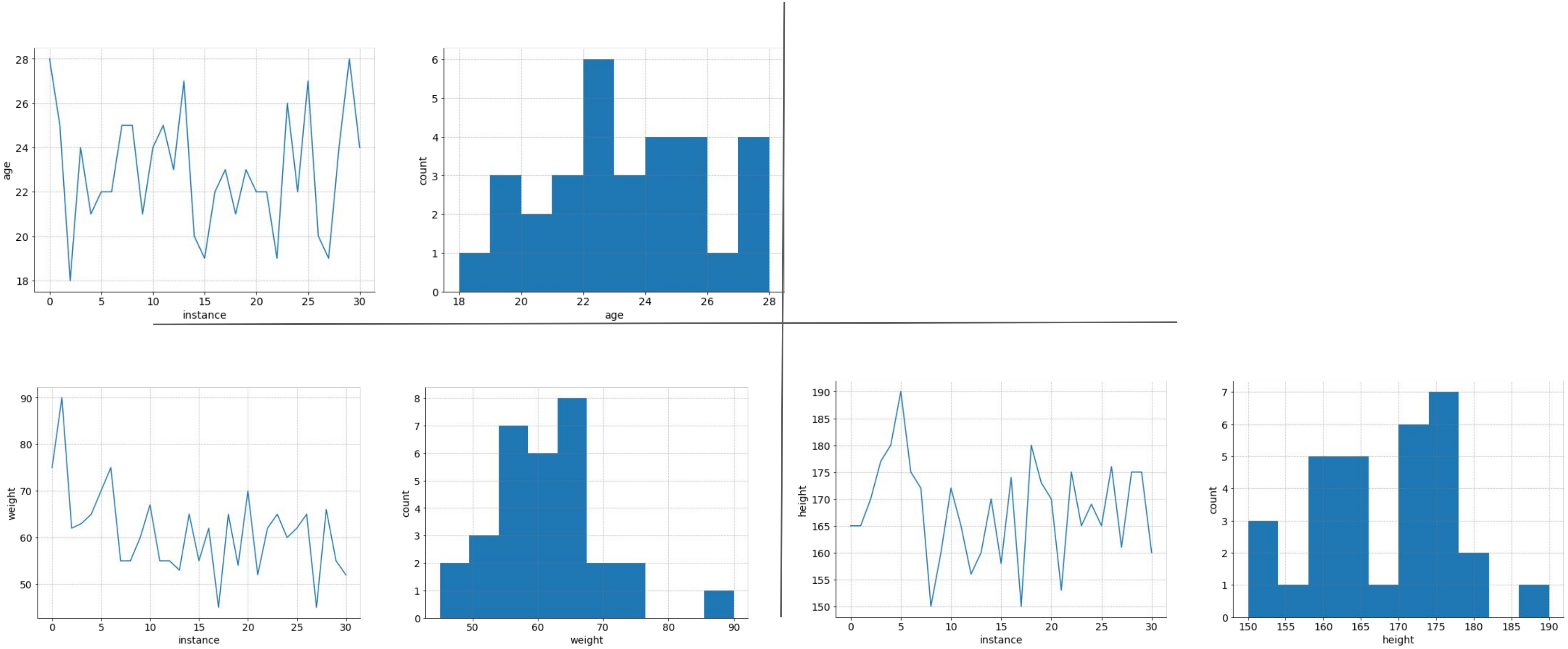
Experiment

Can you provide an estimate of this guy's?

- Height
- Weight
- Age

Experiment

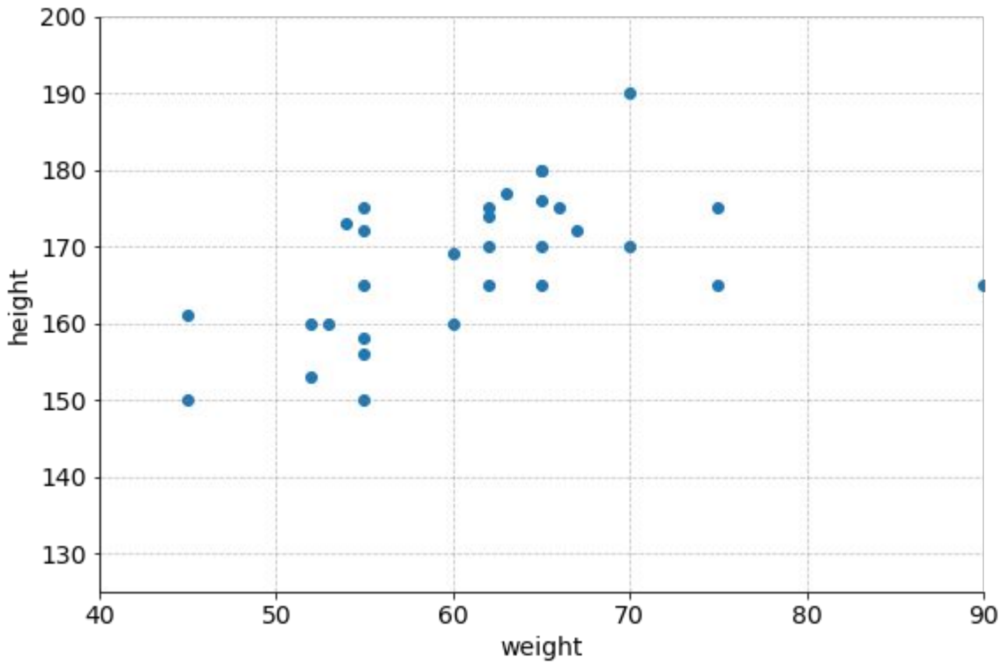
Cool! let's visualize the data we collected in 5 mins



Murali Sreeshankar
Indian Athlete (Long Jump),
National record of 8.36 mts (2022)
Height = 180 cms

Modelling

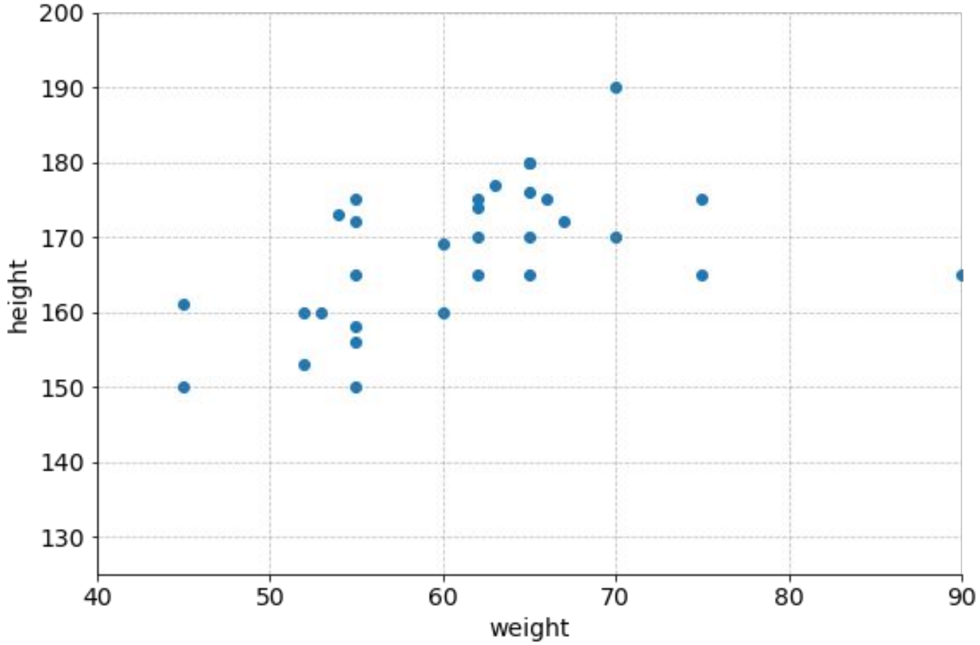
Can we fit a model between "weight" and "height"?



Modelling

Can we propose a model for relationship between “weight” and “height”?

- Some relationship in the scatter!
- What model will be good to start with?



Modelling

Can we propose a model for relationship between "weight" and "height"?

- Some relationship in the scatter!
- What model will be good to start with?

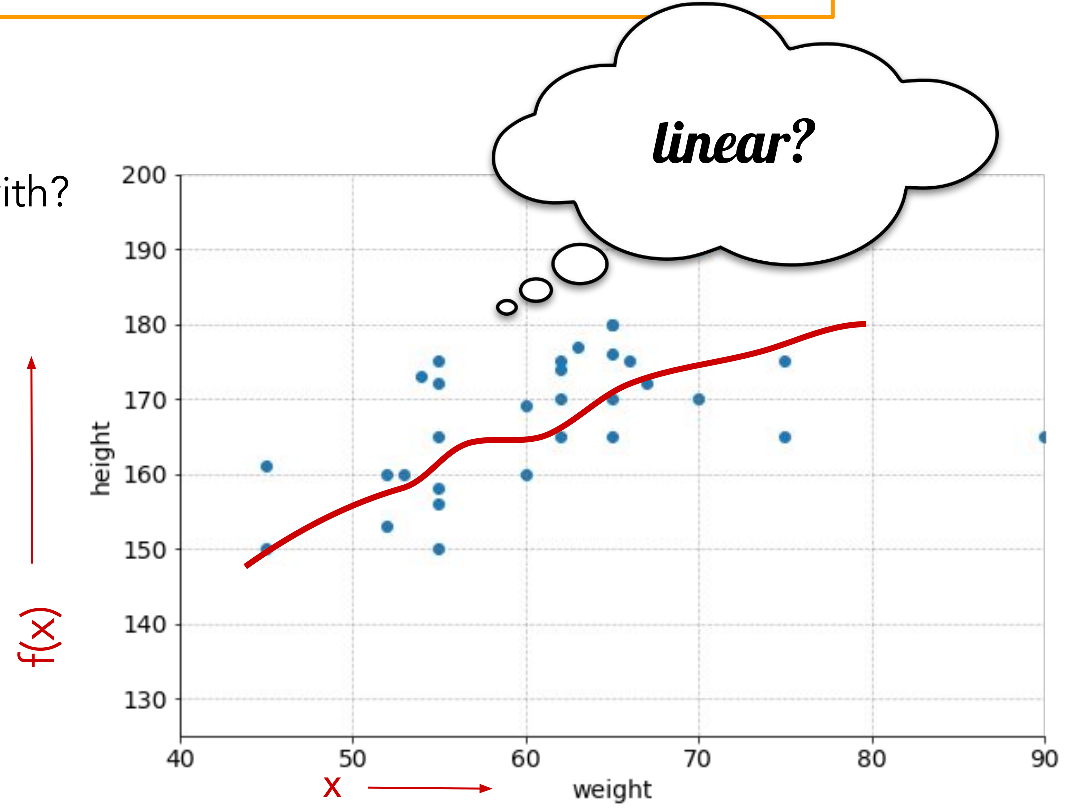
$$f(x) = ?$$

where,

x := weight
y := height
f(x):= model

Linear model:

$$f(x) = a\,x$$



Modelling

How do we estimate the model parameter here?

Linear model:

- $f(x) = a\,x$

  where,
  x := weight
  y := height
  f(x):= model

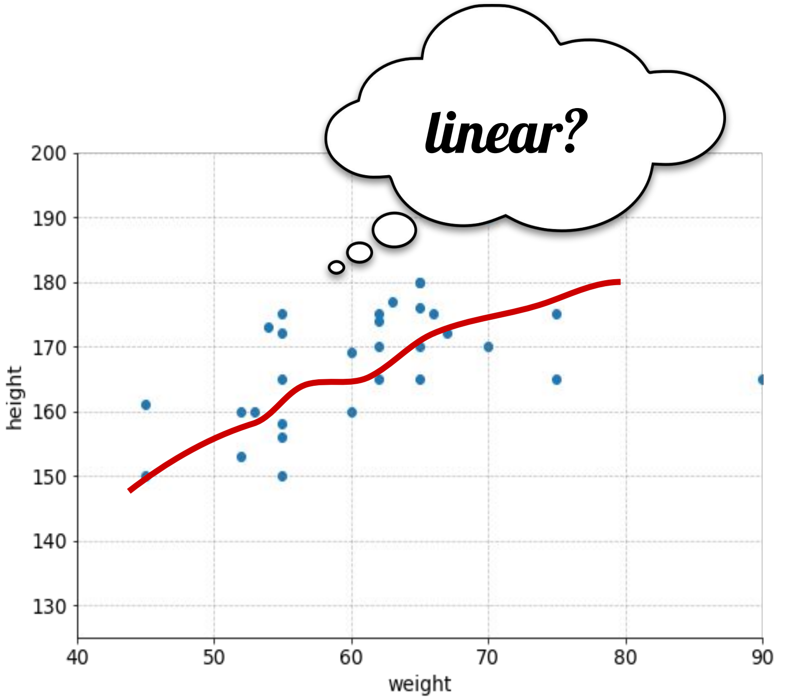
- $e_i = f(x_i) - y_i = ax_i - y_i$

  where,
  e := error

- Minimize error or some function of error over all data points

Data points

x	y
x_1	y_1
x_2	y_2
x_3	y_3
.	.
.	.
.	.
x_N	y_N

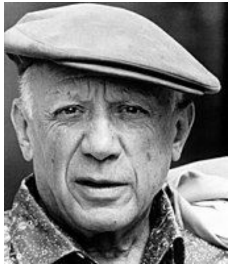


What is a model?

Art …

" We all know that art is not truth. Art is a lie that makes us realize truth, at least the truth that is given us to understand. The artist must know the manner whereby to convince others of the truthfulness of his lies. "

Pablo Picasso (1927)

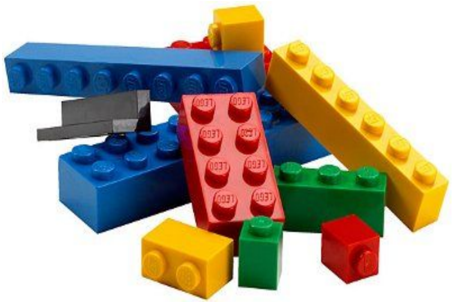


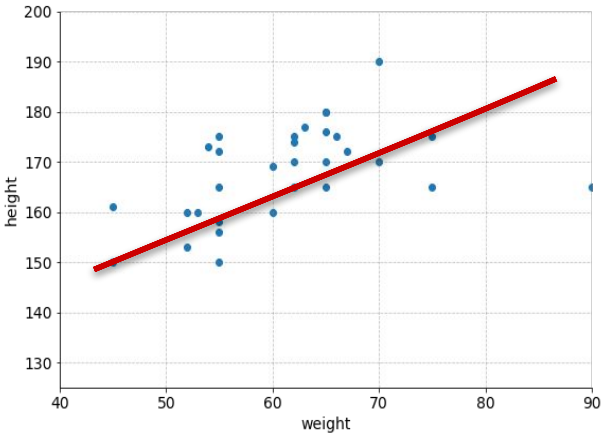
Model ...

"We all know that ~~art~~ model is not truth. ~~Art~~ Model is a lie that makes us realize truth, at least the truth that is given us to understand. The modeller artist must know the manner whereby to convince others of the truthfulness of his lies."

Models - useful?

- Knowledge synthesis
- Test hypotheses and discover unknowns
- Model predictions can serve as as interventions
- Guide newer, useful experiments
- Develop new technologies / applications





Signal ⇒ Model

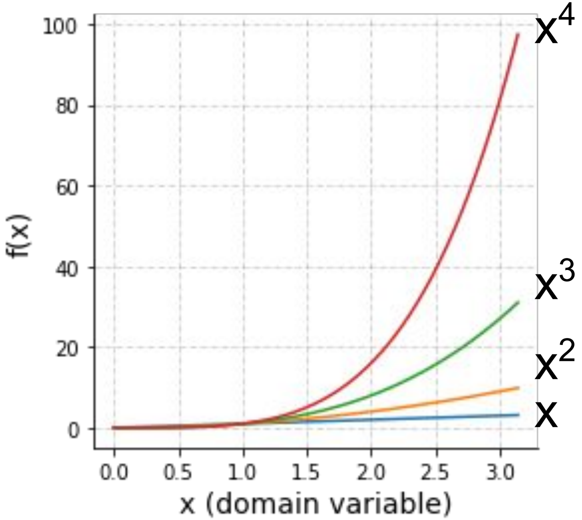
or

Representation ⇒ Compute

Signal → Model → Compute

or

Representation



Model

Taylor Series:

$$f(x) = f(a) + \frac{f'(a)}{1!}(x-a) + \frac{f''(a)}{2!}(x-a)^2 + \frac{f'''(a)}{3!}(x-a)^3 + \cdots$$

Model

Taylor Series:

$$f(x) = f(a) + \frac{f'(a)}{1!}(x-a) + \frac{f''(a)}{2!}(x-a)^2 + \frac{f'''(a)}{3!}(x-a)^3 + \cdots$$

Examples:

$$\sin(x) \approx x - \frac{x^3}{3!} + \frac{x^5}{5!} - \frac{x^7}{7!}$$

Model

Taylor Series:

$$f(x) = f(a) + \frac{f'(a)}{1!}(x-a) + \frac{f''(a)}{2!}(x-a)^2 + \frac{f'''(a)}{3!}(x-a)^3 + \cdots$$

Examples:

$$e^x = \sum_{n=0}^{\infty} \frac{x^n}{n!} = 1 + x + \frac{x^2}{2!} + \frac{x^3}{3!} + \cdots$$

More examples:

$$\sin x = \sum_{n=0}^{\infty} \frac{(-1)^n}{(2n+1)!} x^{2n+1} = x - \frac{x^3}{3!} + \frac{x^5}{5!} - \cdots \quad \text{for all } x$$

$$\cos x = \sum_{n=0}^{\infty} \frac{(-1)^n}{(2n)!} x^{2n} = 1 - \frac{x^2}{2!} + \frac{x^4}{4!} - \cdots \quad \text{for all } x$$

$$\tan x = \sum_{n=1}^{\infty} \frac{B_{2n}(-4)^n\left(1-4^n\right)}{(2n)!} x^{2n-1} = x + \frac{x^3}{3} + \frac{2x^5}{15} + \cdots \quad \text{for } |x| < \frac{\pi}{2}$$

$$\sec x = \sum_{n=0}^{\infty} \frac{(-1)^n E_{2n}}{(2n)!} x^{2n} = 1 + \frac{x^2}{2} + \frac{5x^4}{24} + \cdots \quad \text{for } |x| < \frac{\pi}{2}$$

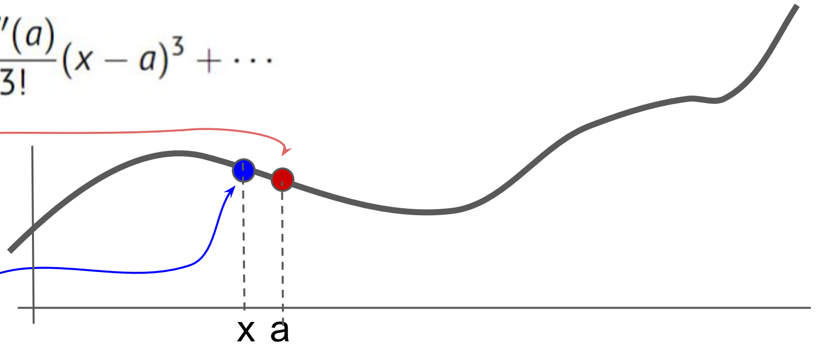
$$\arcsin x = \sum_{n=0}^{\infty} \frac{(2n)!}{4^n (n!)^2 (2n+1)} x^{2n+1} = x + \frac{x^3}{6} + \frac{3x^5}{40} + \cdots \quad \text{for } |x| \le 1$$

Taylor Series

- Assumes the function is differentiable
- Works like a charm if you know the function (in closed form) apriori
- Approximation only in the neighborhood of the sampled point

Using in practice requires derivative information of the signal.

$$f(x) = f(a) + \frac{f'(a)}{1!}(x-a) + \frac{f''(a)}{2!}(x-a)^2 + \frac{f'''(a)}{3!}(x-a)^3 + \cdots$$



Can we use ideas from Taylor?

Signal $\Rightarrow$ Model $\Rightarrow$ Processing

or

Representation

Next lecture we will continue …
modeling