Practice problems 8

1. For a balanced transportation problem of the form given below,

$$\begin{aligned} &\text{Minimize } \sum_i \sum_j c_{ij} x_{ij} \\ &\text{subject to } \sum_i x_{ij} = b_j, \quad j = 1, 2, ..., n, \\ &\text{subject to } \sum_j x_{ij} = a_i, \quad i = 1, 2, ..., m, \\ &\mathbf{x} \geq \mathbf{0}, \end{aligned}$$

with $\sum_i a_i = \sum_j b_j$, check the correctness of the following statements with proper justification.

(a) If $\mathbf{B}$ is a basis matrix of the above problem then the entries of $\mathbf{B}^{-1}$ are either $1, -1$ or 0.

(b) If the transportation problem (P) given above is feasible then it has an optimal solution.

(c) If $m = 4$ and $n = 4$, then there exists a $\theta-$loop with exactly $m + n - 1$ cells.

(d) In every row and column of the transportation array there must be at least one basic cell.

(e) If all the a_i and b_j are positive integers and x^* is an optimal solution satisfying the condition that $\Delta = \{(i, j) : i, j,$ such that x^*_{ij} is not an integer $\} \neq \phi$, then the cells corresponding to nonzero components of x^* contains a θ-loop.

(f) If $\mathfrak{B}$ is a collection of $m + n - 1$ basic cells and if the $\theta-$loop in $\mathfrak{B} \cup \{(p, q)\}$ is considered, then $\sum\{c_{ij} : (i, j) \text{ gets the allocation } +\theta\} - \sum\{c_{ij} : (i, j) \text{ gets the allocation } -\theta\} = c_{pq} - z_{pq}$ (where z_{ij} is as in simplex).

(g) If $m = 4$ and $n = 4$, then every column in the simplex table (leaving out the $B^{-1}b$) corresponding to any BFS will have atleast one zero entry.

(h) If in the 1st row of the array the cost c_{1j} is changed to $c_{1j} + 5$, for all $j = 1, 2, .., n$, then the optimal solution for the new problem is always equal to the optimal solution of the old problem.

(i) If all the a_i's and b_j's of the above transportation problem are integers then any basic feasible solution of the above problem will also take integer values.

(j) If $\bar{a_1}, \bar{a_2}, \bar{a_3}$ (where $\bar{a_i} = B^{-1}a_i$, a_i is the i th column of A), are three columns in any iteration (table) of the simplex algorithm applied to this problem, then $\{\bar{a_1}, \bar{a_2}, \bar{a_3}\}$ is linearly independent and the sum of the elements of the vector $u = \bar{a_1} + \bar{a_2} + \bar{a_3}$, is equal to 3.

(k) If $\mathcal{B}$ is a collection of basic cells of a transportation array and if $(p, q) \in \mathcal{B}$, then there exists at least one more cell from $\mathcal{B}$ in either the p-th row or q-th column of the array.

2. Consider the following transportation problem with c_{ij}'s, a_i's and b_j's as given below.

							a_i
	2	0	3	5	6	3	10
	1	3	4	0	9	7	24
	3	7	9	1	8	1	9
	4	8	7	3	6	8	36
b_j	24	4	1	19	23	8	

(a) Construct a basic feasible solution such that $\{(1,1),(1,3),(3,4)\} \subseteq \mathcal{B}$.

(b) Construct a $\theta-$loop which includes the cells $(1,1),(1,4),(3,4)$ and $(3,6)$.

(c) Is it true that given any collection of three cells in the above array there will be a BFS of the above problem containing those three cells?

(d) Solve the following transportation problem with, $\{(1,2),(1,5),(2,1),(2,4),(3,2),(3,3),(3,6),(4,4),(4,5)\}$ as the initial set of basic cells and by taking

   i. $u_3 = 0$ in every stage of solving (that is the third supply is removed).

   ii. $u_3 = 1$ initially (that is the third supply constraint is removed) and then in the next iteration (if any) by taking $v_3 = 0$ (that is the third destination constraint is removed).