**Problem Set 2, HS 239**[1]

**In Class, 14/2/2024**

1. Consider a lottery with money rewards 100, 200 and 300. The probabilities are denoted by $p_1, p_2$ and $p_3$ respectively.

(a) In the probability triangle diagram, plot an indifference curve such that expected monetary reward is a constant. What is the slope of this curve?

(b) Ram is an expected utility maximizer, and his Bernoulli utilities are $v(100) = 2$, $v(200) = 8$ and $v(300) = 12$ . Sketch an indifference curve for Ram. What is the slope of this curve?

2. Mr. A has a utility function

$$u(x, y) = x^{0.25} y^{0.75}$$

We will assume that the utility function is *cardinal* in nature.

Price of good $x$ is 20 Rupees, that of $y$ is 10 rupees and his overall income is 100 rupees. Suppose Mr. A is offered the following lottery. He can invest all his income in a gamble. The lottery will give him Rs 150 with probability 0.5, and $Rs$ 50 with probability 0.5. His other option is not to participate in the game at all. What will he do?

3. Mr. B has income $C_0$. A lottery gives him income of $C_0 + x_i$ where $i = 1, 2, 3, ..n$. Here, $x_i$ are different realizations of a random income $x$ with the property that $E(x) = 0$. The risk-premium ($\Pi$) is a number such that

$$v(C_0 - \Pi) = \sum p_i v(C_0 + x_i)$$

Show that, if the risk is "small", then

$$\Pi \approx \frac{var(x)}{2} * \left( -\frac{v''}{v'} \right)$$

4. Suppose the lottery in Q3 gives an income $C_0(1 + x_i)$. The rest is same as Q3. In this case, show that

$$\Pi \approx \frac{var(x)}{2} * \left( -\frac{v''}{v'} c \right)$$

[1] These problems are indicative in nature. There is no guarantee that only these and/or similar problems will be asked in the examination or that the exam is "problems only".

5. A person has a Bernoulli utility function of the form $v = \sqrt{w}$. His initial wealth is Rs 4. He holds a lottery ticket, which will give him Rs 21 with probability 0.5 and 0 otherwise.

a) Find the expected utility.

b) What is the lowest price $p_{\min}$ at which he will sell the ticket?

c) If he does not hold the ticket, at what price he must be willing to pay to buy the ticket?

6. We have the following statements about two differentiable, concave and increasing functions $u$ and $v$

A) $v$ exhibits higher abolsute risk aversion than $u$.

B) There is a $G$ such that $v = G(u(w))$ where $G$ is concave and increasing.

Show that $A \Longrightarrow B$.

7. If an agent's utility function shows CARA, demonstrate that the utility function must be $v(x) = -e^{-\alpha x}$.