Soln to Problem 6

Preliminaries-1

- Production $Y_i = e+\varepsilon_i$.
- e is effort (e.g. 10 hours of work produces 10 kg of rice), but there is a random factor of ε_i.
- The random variable has zero mean and σ^2 variance.
- Tenant may pay (earn) a rent (wage) of R.
- Tenant's income $Y_T = sY-R$.
- s is the share.

Preliminaries-2

- $E(Y_T) = se-R$, $var(Y_T) = s^2 \sigma^2$.
- Similarly, $Y_L = (1-s)Y +R$.
- $E(Y_L) = (1-s)e+R$, $var(Y_L) = (1-s)^2\sigma^2$.
- Note that the cost of effort is borne only by the tenant
- Tenant is the agent, landlord is the principal.
- Landlord offers a contract (s,R).
- Cannot include e in the contract (*due to hidden action*)

ICC of The Tenant-1

- The tenant offers an "e" in response to "s" and "R" (the package).
- The landlord must figure out the optimal response of the tenant given any s and R.
- This, then, is the binding ICC.
- This is stage 1 of the landlord's optimization.

ICC of The Tenant-2

- Tenant's Utility (mean variance framework)

$$EU_T = E(Y_T) - \frac{\beta_T}{2} Var(Y_T) - \frac{1}{2} ce^2$$

$$= se - R - \frac{\beta_T}{2} s^2 \sigma^2 - \frac{1}{2} ce^2$$

- Here, β_T reflects risk aversion parameter for tenant.
- Tenant can choose only e.
- SOC is

$$s - ce = 0 \Rightarrow e = \frac{s}{c}$$

ICC of The Tenant-3

- By providing higher s, the landlord can elicit higher effort from tenant.
- This, then, becomes the *binding* ICC which must be obeyed by the tenant (because his/her utility is maximised).
- In the first best case, (effort can be monitored and enforced) e= 1/c.(Topic 3B)
- Here, to elicit the best effort, s=1.
- But that would reduce landlord's income.

Landlords' Problem

- Landlord maximises

$$EU_L = (1-s)e + R - \frac{\beta_L}{2}(1-s)^2\sigma^2$$

- Subject to two constraints
  - ICC of tenant, e=s/c.
  - Participation Constraint of the tenant, $EU_T = 0$

(assumption: outside option of the tenant is zero).

- PC will be binding because the landlord will increase R (or decrease – R) in order to extract as much as possible from tenant.

Maximization

- Put the ICC (e=s/c) in the landlord's problem.
- Then the landlord will be bothered only with the PC.
- From the objective function as well as PC, one can eliminate the expression for e.

Landlord's Problem

$$\max_{s,R} \ (1-s)\frac{s}{c} + R - \frac{\beta_L}{2}(1-s)^2\sigma^2$$

such that

$$\frac{s^2}{c} - R - \frac{\beta_T}{2}s^2\sigma^2 - \frac{s^2}{2c} = 0$$

$$\rightarrow \frac{s^2}{2c} - R - \frac{\beta_T}{2}s^2\sigma^2 = 0$$

Solution

- The Lagrangian is

$$L = (1-s)\frac{s}{c} + R - \frac{\beta_L}{2}(1-s)^2\sigma^2 + \lambda\left(\frac{s^2}{2c} - R - \frac{\beta_T}{2}s^2\sigma^2\right)$$

- First order conditions are

$$R: 1-\lambda = 0$$
$$s: \left(\frac{1}{c} - \frac{2s}{c}\right) + \beta_L(1-s)\sigma^2 + \lambda\left[\frac{s}{c} - s\beta_T\sigma^2\right] = 0$$

Optimal Share

- From the first equation, λ=1.
- From the second equation, (using λ=1), one gets (homework)

$$s^* = \frac{1 + c\sigma^2\beta_L}{c\sigma^2(\beta_L + \beta_T) + 1} < 1$$

- Thus, even if only the landlord is risk neutral (β_L =0), sharecropping emerges as a solution (because of pure moral hazard considerations).
- However, if the tenant is risk neutral, (β_T =0), the optimal solution is s=1, i.e. fixed rent.
- For moral hazard problem to have any bite, the agents must not be risk neutral.