CS501 Parallel Algorithms
Mid Semester Examination
Do rough work on extra sheets. Here, be brief and neat.

IIT Guwahati
Dept. of CSE
Sep 21, 2023

Roll No: **Name:**

Sheet 1 of 2

Q1. A cyclic linked list has the the following colours: 123, 155, 216, 323, 147, 389, 420 on consecutive vertices A, B, C, D, E, F, G respectively. After three steps of symmetry breaking, what are the colours of the vertices? [3 marks]

111 1011 — 1001 1011 — 1101 1000 — 1 0100 0011 — 1001 0011 — 1 1000 0101 — 1 1010 0100
1011 — 01 — 00 — 1000 — 11 — 01 — 00 (11-1-0-8-3-1-0)
11 — 01 — 110 — 00 — 11 — 01 — 00 (3-1-6-0-3-1-0)
11 — 01 — 11 — 00 — 11 — 01 — 00 (3-1-3-0-3-1-0)

Q2. Sort the following 4 × 4 mesh using the parallel Snake-like sorting algorithm discussed in the class. Show the steps. [4 marks]

13	07	06	02
04	11	14	12
10	01	08	15
09	05	16	03

13 7 6 2
4 11 14 12
10 1 8 15
9 5 16 3

→

2 6 7 13
14 12 11 4
1 8 10 15
16 9 5 3

→

1 6 5 3
2 8 7 4
14 9 10 13
16 12 11 15

1 3 5 6
8 7 4 2
9 10 13 14
16 15 12 11

→

1 3 4 2
8 7 5 6
9 10 12 11
16 15 13 14

→

1 2 3 4
8 7 6 5
9 10 11 12
16 15 14 13

Q3. Two arrays s and r are given, where $s[i]$ and $r[i]$ respectively represent the successor and rank of node i.

	00	01	02	03	04	05	06	07	08	09	10	11	12	13	14	15
s	06	03	08	03	15	00	10	11	11	04	01	01	12	04	15	12
r	1	1	1	0	1	1	1	1	1	1	1	1	0	1	1	1

Show how the s and r arrays will look, after two steps of pointer jumping on these nodes? [3 marks]

0	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15
10	3	11	3	12	6	1	1	1	15	3	3	12	15	12	12
2	1	2	0	2	2	2	2	2	2	2	2	0	2	2	1
3	3	3	3	12	1	3	3	3	12	3	3	12	12	12	12
4	1	4	0	2	4	3	3	3	3	2	2	0	3	2	1

Q4. Consider the optimal algorithm for 3-colouring of linked lists. After the sorting of the nodes on colours (which range from 4 to $\log^{(k)} n + 3$), the colours in a certain column are $4, 4, 5, 5, 6, 6, 7, 7, \ldots$ as you go down. Find the step at which the processor of that column would finish recolouring the nodes of that column. [4 marks]

Let $l = \log^{(k)} n$.
A node of colour C and row R is processed in step $C+R-4$.
Here, the last node is in row l. If its colour is x, it is processed in step $x+l-4$. If l is even, $x = 3+\frac{l}{2}$. Else, $x = 3+\lceil\frac{l}{2}\rceil$. Either way $x = 3+\lceil\frac{l}{2}\rceil$.
ie, the last node is processed in step $3+\lceil\frac{l}{2}\rceil+l-4 = l+\lceil\frac{l}{2}\rceil-1$

Q5(a). A rooted tree is an acyclic graph in which every vertex v has a parent pointer $p(v)$. We want to identify (A) all nodes with a vertex degree of one and (B) all nodes with a vertex degree of two. Consider the following code:

```
for v ∈ V pardo
    v.r = 0;
for v ∈ V pardo
    p(v).r = 1;
for v ∈ V pardo
    if (v.r == 0) then v.d = A
    else if (v.r == 1) then v.d = B;
```

Here every vertex has a parent.
So, $\deg(v) = 1$ if v has no child
$= 2$ if v has one child, $\forall v \in V$.
If v has >1 children, v faces a CW.

For each of the models state whether the code works correctly, incorrectly or crashes. EREW, CREW, CRCW Priority, CRCW Arbitrary, CRCW Common, CRCW Collision, CRCW Tolerant. [3 marks]

Crashes on CREW, EREW. On Priority, Arbitrary & Common, the code fails to distinguish 1 child vs >1 children. On Tolerant fails no child vs >1 children. On Collision, $0 \to \deg 1$ $1 \to \deg 2$ $\$ \to \deg > 2$.
So, works on Collision.

Q5(b). Suppose we use the code of 5(a) when we want to identify (A) all nodes with a vertex degree of one and (B) all nodes with a vertex degree greater than one. For each of the models state whether the code works correctly, incorrectly or crashes. EREW, CREW, CRCW Priority, CRCW Arbitrary, CRCW Common, CRCW Collision, CRCW Tolerant. [3 marks]

Crashes on CREW, EREW. Fails on Collision (see 5a) and Tolerant (See 5a). Works on Priority, Arbitrary & Common as $v.r == 1 \to \deg(v) \geq 2$.

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Q6. Give a lower bound for comparison based parallel sorting of n items with $n.2^{\sqrt{\log n}}$ processors [4 marks]

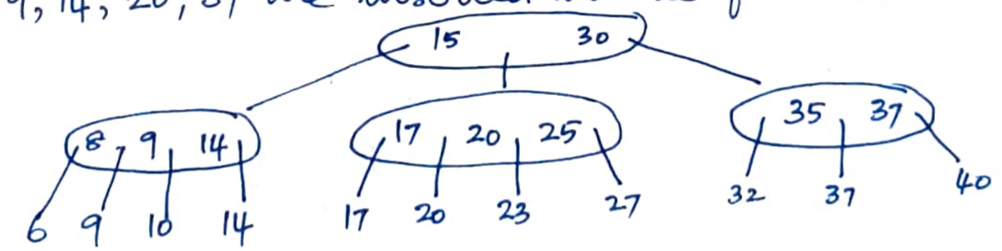
$p = n \cdot 2^{\sqrt{\log n}}$ $\quad p/n = 2^{\sqrt{\log n}}$ $\quad \log(p/n) = \sqrt{\log n}$

lower bound: $\Omega(\log n / \log(p/n)) = \Omega(\sqrt{\log n})$

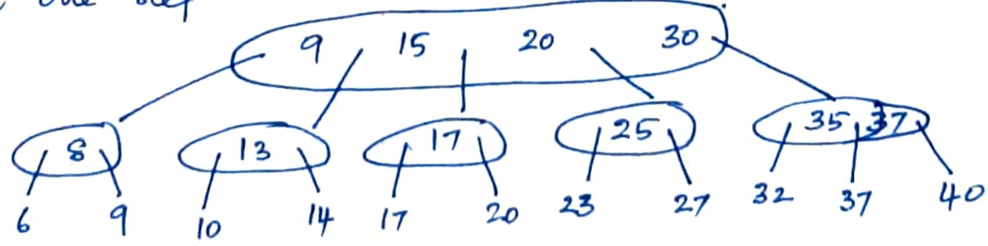
Q7. Consider a 2-3 tree with the following internal nodes: $A(15:30)$, $B(8)$, $C(20,25)$, $D(35)$, where A is the root and B, C, D are its children. The leaves of the tree are labelled $6, 10, 17, 23, 27, 32, 40$. Elements $9, 14, 18, 19, 20, 21, 22, 34, 36, 37, 38, 39$ are to be inserted in the tree. The CREW PRAM algorithm we discussed is to be used for a batch insertion. Sketch the tree as it would look when the first wave moves past the bottommost level of non-leaves. [4 marks]

$6 \ \boxed{9} \ 10 \ \boxed{14} \ 17 \ \boxed{18 \ 19 \ 20 \ 21 \ 22} \ 23 \ 27 \ 32 \ \boxed{34 \ 36 \ 37 \ 38 \ 39} \ 40$

9, 14, 20, 37 are inserted in the first wave



After one step.



Q8. Solve the following recurrence relation: $T(n, r^2) = 1 + T(\frac{n}{r}, r^3)$ when $n > r$; $T(r, r^2) = 1$. [4 marks]

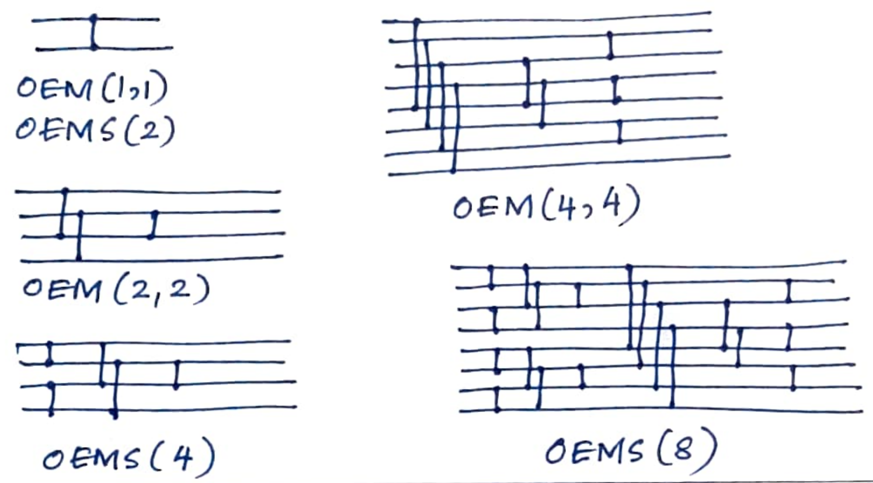
$$T(n,r^2) = 1 + T\left(\frac{n}{r}, r^3\right) = 2 + T\left(\frac{n}{r^{1+1.5}}, r^{2(1.5)^2}\right) = \cdots$$

$$= k + T\left(\frac{n}{r^{1+\epsilon+\epsilon^2+\cdots+\epsilon^{k-1}}}, r^{2\epsilon^k}\right) \quad \text{where } \epsilon = 1.5$$

when $n = r^{\epsilon^k} + 2\epsilon^k - 2$, ie, $k = O(\log_2 \log_r n)$

$$T(n,r^2) = k+1 = O(k) = O(\log_2 \log_r n)$$

Q9. Sketch the Odd Even Merge Sort network for 8 elements. [4 marks]



Q10. Sketch the Biotonic-Sort-Merge Sort network for 8 elements [4 marks]