End Semester Examination
CS 564

Full marks – 100 30th April, 2024

Instructions

1. *There are more questions than you can possibly answer in the time allotted to you. Attempt as many as you can while presenting cohesive, comprehensible, and legible proofs.*
2. *All questions carry equal marks.*
3. *State clearly any assumptions that you may have to make.*
4. *No clarifications will be provided during the examination.*
5. Don't Panic.

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1. (a) Prove that if the language L is in NP, then so is the language L^*.

   (b) Consider the following argument for proving $P \neq NP$.

      i. Suppose $P = NP$.
      ii. Then for some $k \geq 0$, we have $SAT \in DTIME(n^k)$.
      iii. As every language in NP is reducible to SAT, consequently $NP \subseteq DTIME(n^k)$.
      iv. As we have assumed that $P = NP$, we therefore have $P \subseteq DTIME(n^k)$.

v. Yet through deterministic time hierarchy we already know that $\mathrm{DTIME}(n^k) \subsetneq \mathrm{DTIME}(n^l) \subseteq \mathrm{P}$, for some large l.

vi. So, we arrive at a contradiction.

vii. Thus, it must be the case that P cannot be equal to NP.

2. There exists a language L such that $\mathrm{P}^L \neq \mathrm{NP}^L$.

3. Show that if $L \in \mathrm{NP} \cap \mathrm{co-NP}$, then $\mathrm{NP} = \mathrm{NP}^L$.

4. Prove that $\mathrm{NSPACE}(f(n)) \subseteq \mathrm{SPACE}(f(n)^2)$, for $f(n) \geq n$.

5. Prove that if $\Sigma_i = \Pi_i$ for some $i > 0$, then the polynomial hierarchy collapses at the i^{th} level.

6. Prove that if $\mathrm{NP} \subseteq \mathrm{P}_{/poly}$, then the polynomial hierarchy collapses.

7. Show that if $\mathrm{DTIME}(n) = \mathrm{NTIME}(n)$, then $\mathrm{DTIME}(n^2) = \mathrm{NTIME}(n^2)$.

8. Show that if every unary language that is in NP is also in P, then EXP = NEXP.

9. Prove that $\mathrm{P} \neq \mathrm{SPACE}(n)$.

10. Prove that if $L \in \mathrm{P}$, so is L^*.

Hint : Use dynamic programming.