Practice problems 2

1. Consider the problem (P) given by :
   Max $\mathbf{c}^T\mathbf{x}$
   subject to $A_{5\times2}\mathbf{x} \leq \mathbf{b}$, $\mathbf{x} \geq \mathbf{0}$.

   (a) Check whether the above problem (P) with $\mathbf{c} = [-1, 0]^T$ and $\mathbf{b} = [0, 3, 1, 1, 0]^T$ has an optimal solution.

   (b) If the above problem is changed to a minimization problem, then find a $\mathbf{c} \neq \mathbf{0}$ and a $\mathbf{b} \neq \mathbf{0}$ such that the changed problem has an optimal solution (no matter what the matrix A is).

   (c) Given $\mathbf{b} = [-1, -2, 0, 0, 0]^T$ suggest entries of $\bar{\mathbf{a}}_1 \neq \mathbf{0}$ the first column of A, such that the feasible region of the above problem (P) is nonempty (no matter what the other columns of A are).

   (d) If $\bar{\mathbf{a}}_1$, the first column of A is given as $[-21, -1, 0, -1, -1]^T$, then is $Fea(P)$ nonempty? If no justify. If yes, then suggest an extreme direction of $Fea(P)$.

   (e) Suggest entries of $\bar{\mathbf{a}}_1$, the first column of A such that the feasible region of the above problem (P) is unbounded (no matter what the other columns of A are).

   (f) If the first row of A is given by $[1, 2]$ and if $Fea(P) \neq \phi$, then will the above problem always (no matter what the other rows of A are) have an optimal solution ?

   All the parts in the above question are independent.

2. Consider a transportation problem with 2 supply stations and 3 destinations such the maximum capacity of the two supply stations S_1 and S_2 are given by 20 units and 40 units respectively. The demands at each of the 3 destinations are 20 units each.

   (a) Find all the extreme points of the feasible region $\mathbf{S}$, of the above problem.

   (b) Find an optimal solution of the above problem.

   (c) Does the feasible region $\mathbf{S}$ of the above problem has an extreme direction?

   (d) Does there exist an $\mathbf{x} \in \mathbb{R}^6$ such that $\mathbf{x}$ lies at the point of intersection of 6 LI defining hyperplanes of $\mathbf{S}$ but $\mathbf{x}$ is not an extreme point of $\mathbf{S}$?

3. Can the feasible region of a linear programming problem in two variables have three distinct extreme directions?

4. Can the feasible region of a linear programming problem in three variables have four distinct extreme directions?