1. Can the feasible region of a linear programming problem in two variables have three distinct extreme directions?
   Ans=**No.**
   Proof: Suppose there are three distinct extreme directions, $\mathbf{d}_1, \mathbf{d}_2, \mathbf{d}_3$. Since any three vectors in $\mathbb{R}^2$ is LD, let us assume WLOG
   $\mathbf{d}_3 = \alpha_1\mathbf{d}_1 + \alpha_2\mathbf{d}_2 \quad (**)$.
   Since each of the vectors $\mathbf{d}_1, \mathbf{d}_2, \mathbf{d}_3$ are non negative, so both α_1, α_2 cannot be less than or equal to 0, or atleast one of α_1, α_2 is strictly positive.

   Case 1: If exactly **one** of α_1, α_2 is equal to 0.
   WLOG say α_1 is equal to 0.
   Then $\alpha_2 > 0$ and it **contradicts** the assumption that $\mathbf{d}_2, \mathbf{d}_3$ are **distinct** as **directions**.

   Case 2: If **both** α_1, α_2 are strictly greater than zero.
   Then it **contradicts** the assumption that $\mathbf{d}_3$ is an **extreme direction**.

   Case 3: If exactly **one** of α_1, α_2 is strictly less than 0.
   WLOG say $\alpha_1 < 0$. Then $\alpha_2 > 0$.
   Then $\mathbf{d}_2 = \frac{1}{\alpha_2}\mathbf{d}_3 + (\frac{-\alpha_1}{\alpha_2})\mathbf{d}_1$, which again **contradicts** the assumption that $\mathbf{d}_2$ is an **extreme direction**.

   Hence the conclusion.

2. Are the **distinct** extreme directions of $Fea(LPP)$ always LI?
   Solution: No.
   Consider the feasible region of a LPP in $\mathbb{R}^3$ as:
   $-x_1 + x_2 - x_3 \leq 1$
   $-x_1 - x_2 + x_3 \leq -2$
   $-x_1 - x_2 - x_3 \leq -5$
   $x_1 \geq 0$, $x_2 \geq 0$,$x_3 \geq 0$.
   Check that the above feasible region has more than three **distinct** extreme directions hence the set of all **distinct** extreme directions for this feasible region are LD.