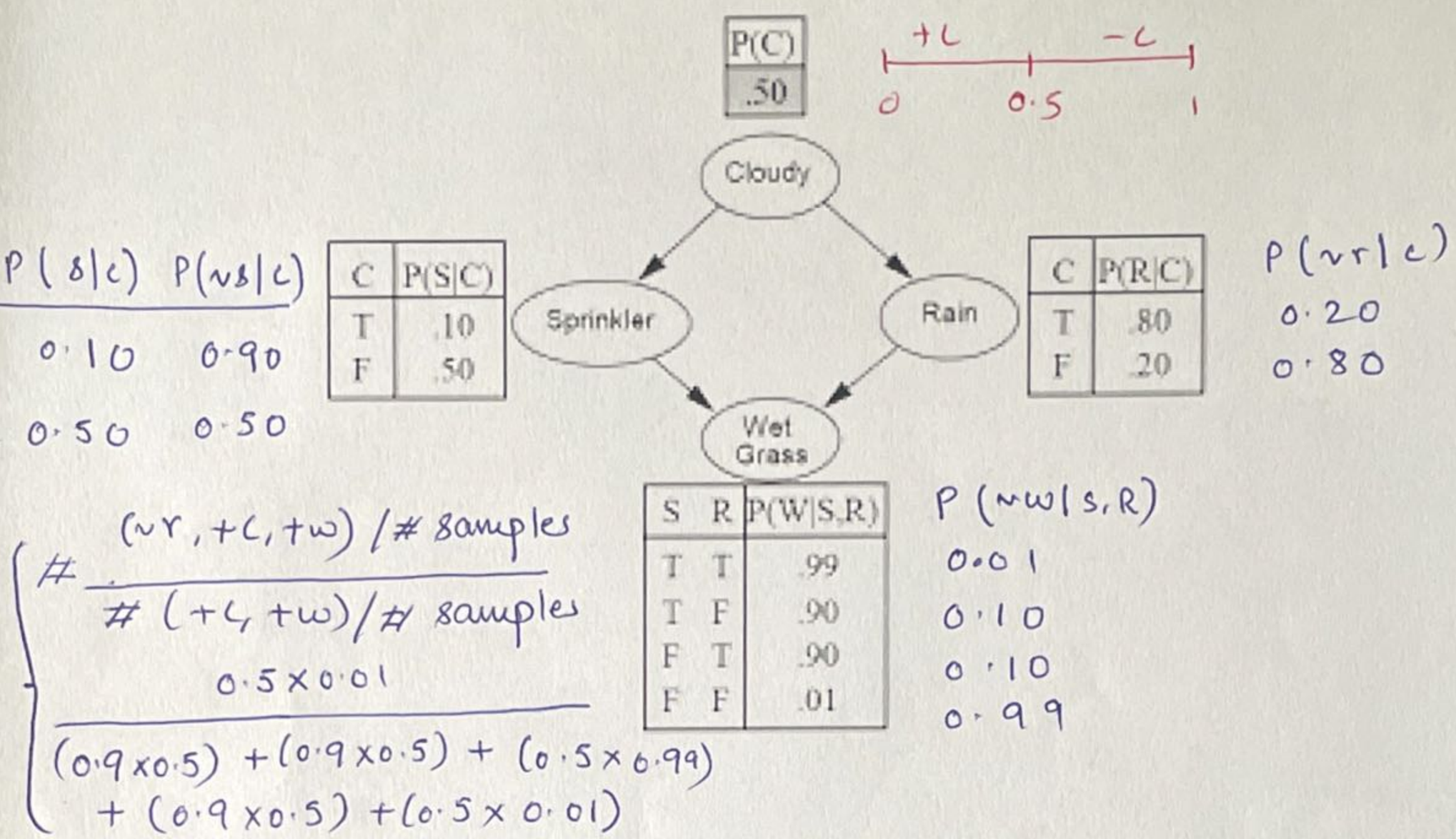
Date : 09-02-2024

Roll No.	(B.Tech./MTech./Ph.D.)	Name:



C	P(S|C)
T	.10
F	.50

C	P(R|C)
T	.80
F	.20

S	R	P(W|S,R)
T	T	.99
T	F	.90
F	T	.90
F	F	.01

C	P(S|C)	P(~S|C)
+C	0.10	0.90
~C	0.50	0.50

P(~r|C)
0.20
0.80

P(~W|S,R)
0.01
0.10
0.10
0.99

+L −L
0 0.5 1

$$\# \frac{(\sim r, +c, +w) / \# \text{samples}}{\# (+c, +w) / \# \text{samples}} = \frac{0.5 \times 0.01}{(0.9 \times 0.5) + (0.9 \times 0.5) + (0.5 \times 0.99) + (0.9 \times 0.5) + (0.5 \times 0.01)}$$

1. Given the order of variables to be [*Cloudy, Sprinkler, Rain, WetGrass*] and the sequence of random numbers: 0.90, 0.27, 0.97, 0.42, 0.03, 0.12, 0.54, 0.12, 0.5, 0.95, 0.91, 0.81, 0.15, 0.65, 0.63, 0.48, 0.39, 0.70, 0.74, 0.80, 0.55, 0.23, 0.11, 0.65, 0.17, 0.01, 0.57, 0.34

Generate 5 samples and answer the following queries:

1. P(+w) using Forward sampling. $P(+w) = 4/5$
2. P(+r|+s) using Rejection sampling. $P(+r|+s) = \frac{P(r,s)}{P(s)} = \frac{\#(r,s) / \# \text{samples}}{\#(s) / \# \text{samples}} = 2/5$
3. P(~r|+c, +w) using Likelihood weighting.

P(~r|+c,+w) = 0.0027

1) ~C +S ~r +W, +C ~S +r +W, +C ~S ~r ~W, +C ~S +r +W

+C ~S +r +W

2) ~C, +S ~r +W, ↑ +C, +S +r +W, ↑↑ ~C +S +r +W, ~C +S ~r +W

+C +S +r +W ↑ : indicates rejected samples

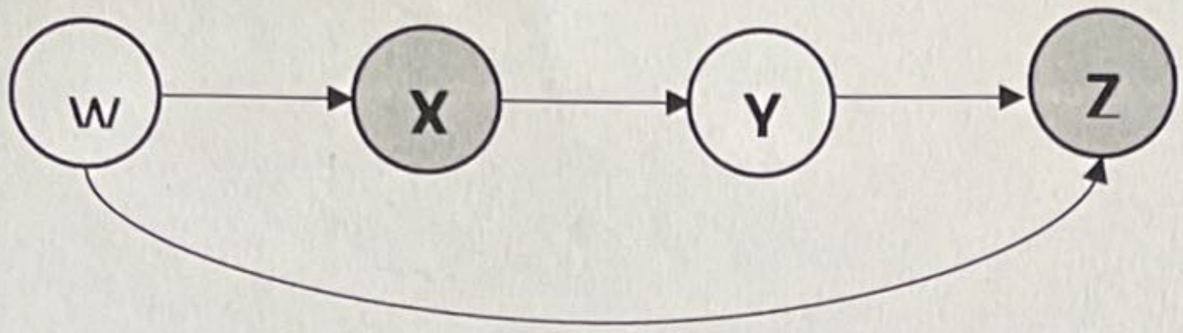
3) 0.90 0.27 (rand. num)

+C ~S +r +W → 0.9×0.5 +C ~S +r +W → 0.9×0.5

+C +S +r +W → 0.5×0.99 +C ~S +r +W → 0.9×0.5

+C ~S ~r +W → 0.5×0.01

2. Consider the following Bayes Net, where we have observed that X = +x and Z = +z.



P(W)	
+w	0.5
-w	0.5

P(X|W)		
+w	+x	0.8
+w	-x	0.2
-w	+x	0.4
-w	-x	0.6

P(Y|X)		
+x	+y	0.1
+x	-y	0.9
-x	+y	0.7
-x	-y	0.3

P (Z|Y, W)			
+w	+y	+z	0.6
+w	+y	-z	0.4
+w	-y	+z	0.1
+w	-y	-z	0.9
-w	+y	+z	0.2
-w	+y	-z	0.8
-w	-y	+z	0.5
-w	-y	-z	0.5

Consider doing Gibbs sampling for this example. Assume all variables are initialized to the values +w, +x, +y, +z. Consider Y to be the variable to be resampled. Calculate the probabilities for new values of Y at this stage of the Gibbs sampling procedure.

P (Y = +y at the next step of Gibbs sampling) = $\frac{0.1 \cdot 0.6}{0.1 \cdot 0.6 + 0.9 \cdot 0.1} = \frac{2}{5}$

P (Y = −y at the next step of Gibbs sampling) = $\frac{0.9 \cdot 0.1}{0.1 \cdot 0.6 + 0.9 \cdot 0.1} = \frac{3}{5}$

$$P(+y \mid +w, +x, +z) = P(y \mid w x z)$$

$$= \frac{P(w, x, y, z)}{P(w x z)}$$

$$= \frac{P(w)\,P(x \mid w)\,P(y \mid x)\,P(z \mid y w)}{\sum_y P(w, x, y, z)}$$

$$= \frac{P(w)\,P(x \mid w)\,P(y \mid x)\,P(z \mid y w)}{\sum_y P(y)\,P(x \mid w)\,P(y \mid x)\,P(z \mid y w)}$$