CS525: Formal Methods for System Verification, Quiz 1, Marks: 40, Time: 30 Mins

Name:

Roll Number:

1. Convert the following formula into Negation Normal Form (NNF). (10)
   $\neg(a \oplus (b \wedge (c \vee d)))$

* convert the following formula into Negative Normal Form:

$\sim(a \oplus (b \wedge (c \vee d)))$

$\sim(a \oplus b) = a \odot b$

$a \odot b = ab + \bar{a}\bar{b}$

$\sim(a \wedge b) = \sim a \vee \sim b$

$\sim(a \vee b) = \bar{a} \wedge \bar{b}$

Solution $a \odot (b \wedge (c \vee d))$

$= a \wedge (b \wedge (c \vee d)) \vee (\sim a \wedge \sim(b \wedge (c \vee d)))$

$= a \wedge (b \wedge (c \vee d)) \vee (\sim a \wedge (\sim b \vee \sim(c \vee d)))$

$= a \wedge (b \wedge (c \vee d)) \vee (\sim a \wedge (\sim b \vee (\sim c \wedge \sim d)))$

further simplification.

$= (a \wedge b \wedge c) \vee (a \wedge b \wedge d) \vee (\sim a \wedge [(\sim b \vee \sim c) \wedge (\sim b \vee \sim d)])$

or

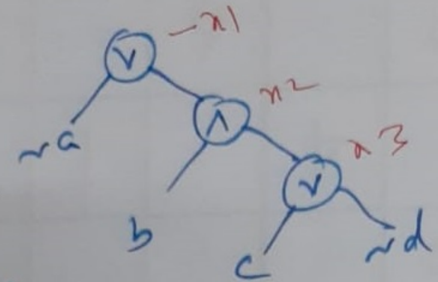
$\Rightarrow \boxed{(a \wedge b \wedge c) \vee (a \wedge b \wedge d) \vee (\sim a \wedge \sim b) \vee (\sim a \wedge \sim c \wedge \sim d)}$

2. Convert the following formula into Conjuctive Normal Form (CNF) using Tseitin transformation. (10)

   $(\neg a \vee (b \wedge (c \vee \neg d)))$

Q2:

$\sim a \vee (b \wedge (c \vee \sim d)$



$x_1 \wedge$

$(x_1 \rightarrow (\sim a \vee x_2)) \wedge$

$(x_2 \rightarrow (b \wedge x_3)) \wedge$

$(x_3 \rightarrow (c \vee \sim d))$

$\equiv x_1 \wedge (\sim x_1 \vee \sim a \vee x_2) \wedge (\sim x_2 \vee b) \wedge (\sim x_2 \vee x_3)$

$\wedge (\sim x_3 \vee c \vee \sim d)$

3. Consider the following clauses in a CNF formula. (10)
$c_1 = (\neg x_1 \lor x_2)$, $c_2 = (\neg x_1 \lor x_3 \lor x_5)$, $c_3 = (\neg x_2 \lor x_4)$, $c_4 = (\neg x_3 \lor \neg x_4)$, $c_5 = (x_1 \lor x_5 \lor \neg x_2)$, $c_6 = (x_2 \lor x_3)$, $c_7 = (x_2 \lor \neg x_3)$, $c_8 = (x_6 \lor \neg x_5)$.
We want to check the satisfiability of the formula using DPLL algorithm. Assume we have to select literal from $\neg x_1$, x_1, x_2, $\neg x_3$ and x_5 at decision level 1. Which literal will be selected by Jeroslow-Wang heuristic? Show complete calculation.

Q3. (1) x_1, $\sim x_1$, x_2, $\sim x_3$, x_5

$J(x_1) = 2^{-3}$ (occurs in c5)

$J(\sim x_1) = 2^{-2} + 2^{-3}$ (c1, c2)

$J(x_2) = 2^{-2} + 2^{-2} + 2^{-2}$ (c1, c6, c7)

$J(\sim x_3) = 2^{-2} + 2^{-2}$ (c4, c7)

$J(x_5) = 2^{-3} + 2^{-3}$ (c2, c5)

Max J value for x_2. So x_2 will be selected.

4. Consider the following clauses in a CNF formula. (10)
$c_1 = (\neg x_1 \vee x_2)$, $c_2 = (\neg x_1 \vee x_3 \vee x_5)$, $c_3 = (\neg x_2 \vee x_4)$, $c_4 = (\neg x_3 \vee \neg x_4)$, $c_5 = (x_1 \vee x_5 \vee \neg x_2)$, $c_6 = (x_2 \vee x_3)$, $c_7 = (x_2 \vee \neg x_3)$, $c_8 = (x_6 \vee \neg x_5)$.
Assume that we have selected x_1 at decision level 1 by some heuristic in DPLL algorithm. Show a partial implication graph that find a satisfiable solution of this formula. Clearly specify the BCP rules applied at each transition.