Binary Codes

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- Source:

Chapter 1 of Z. Kohavi and N. Jha, Switching and Finite Automata Theory, 3rd Ed., Cambridge University Press, 2010

Binary codes

- To simplify the problem of communication between human and machine, several codes have been devised in which decimal digits are represented by sequences of binary digits.

- Weighted Code
  - BCD
  - Self-complementing code
- Non-weighted code
  - Excess-3 code
  - Cycle code
    - Gray code
  - Reflected code

Weighted codes

- Each binary digit is assigned a decimal “weight,” and, for each group of four bits, the sum of the weights of those binary digits whose value is 1 is equal to the decimal digit which they represent.
- If w1, w2, w3, and w4 are the given weights of the binary digits and x_1, x_2, x_3, x_4 the corresponding digit values then the decimal digit
- $N = w_4 x_4 + w_3 x_3 + w_2 x_2 + w_1 x_1$ can be represented by the binary sequence $x_4x_3x_2x_1$.
- The sequence of binary digits that represents a decimal digit is called a code word.

Weighted codes

- The sequence $x_4x_3x_2x_1$ is the code word for N.
- The code word 8, 4, 2, 1 is known as Binary coded Decimal(BCD).

Decimal	$w_4 w_3 w_2 w_1$											
digit N	8	4	2	1	2	4	2	1	6	4	2	−3
0	0	0	0	0	0	0	0	0	0	0	0	0
1	0	0	0	1	0	0	0	1	0	1	0	1
2	0	0	1	0	0	0	1	0	0	0	1	0
3	0	0	1	1	0	0	1	1	1	0	0	1
4	0	1	0	0	0	1	0	0	0	1	0	0
5	0	1	0	1	1	0	1	1	1	0	1	1
6	0	1	1	0	1	1	0	0	0	1	1	0
7	0	1	1	1	1	1	0	1	1	1	0	1
8	1	0	0	0	1	1	1	0	1	0	1	0
9	1	0	0	1	1	1	1	1	1	1	1	1

Self-complementing codes

- It is apparent that the representations of some decimal numbers in the (2, 4, 2, 1) and (6, 4, 2, −3) codes are not unique.
  - For example, in the (2, 4, 2, 1) code, decimal 7 may be represented by 1101 as well as 0111.
- Adopting the representations shown in Table causes the codes to become self-complementing.
- A code is said to be *self-complementing* if the code word of the “9’s complement of *N*”, i.e., $9 - N$, can be obtained from the code word of *N* by interchanging all the 1’s and 0’s.
  - For example, in the (6, 4, 2, −3) code, decimal 3 is represented by 1001 while decimal 6 is represented by 0110.

Self-complementing codes

- BCD code (8, 4, 2, 1) is not self-complementing.
- A necessary condition for a weighted code to be self-complementing is that the sum of the weights must equal 9.
  - There exist only four positively weighted self-complementing codes, namely, (2, 4, 2, 1), (3, 3, 2, 1), (4, 3, 1, 1), and (5, 2, 1, 1).
  - there exist 13 self-complementing codes with positive and negative weights.

Nonweighted codes

- There are many nonweighted binary codes, such as Excess-3 Code, Cyclic Code, Gray Code etc..
- The Excess-3 code is formed by adding 0011 to each BCD code word.

Decimal digit	Excess-3				Cyclic			
0	0	0	1	1	0	0	0	0
1	0	1	0	0	0	0	0	1
2	0	1	0	1	0	0	1	1
3	0	1	1	0	0	0	1	0
4	0	1	1	1	0	1	1	0
5	1	0	0	0	1	1	1	0
6	1	0	0	1	1	0	1	0
7	1	0	1	0	1	0	0	0
8	1	0	1	1	1	1	0	0
9	1	1	0	0	0	1	0	0

Cyclic codes

- Code word having property of **successive decimal integers differ in only one digit** are referred to as cyclic codes.
- Gray code also have property of successive decimal integers differ by 1 bit, so it is also a cyclic code.
- Code useful is the simplicity of the procedure for converting from the binary number system into the Gray code

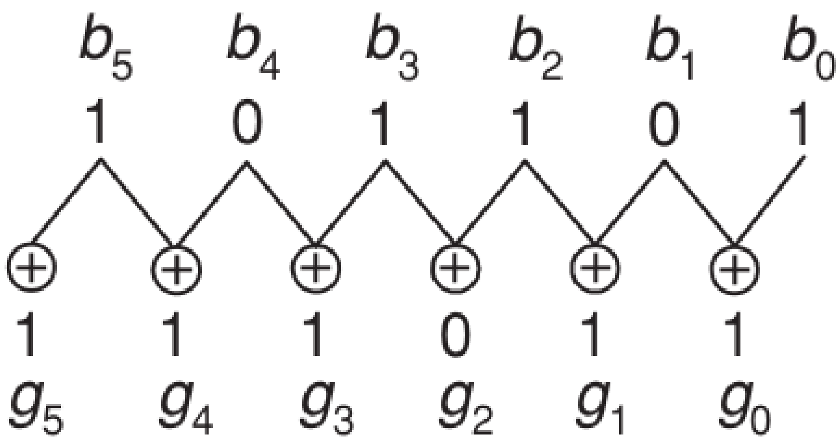
Gray Code

Table 1.5 Decimal numbers in the complete four-bit Gray code and in binary

Decimal number	Gray g_3	g_2	g_1	g_0	Binary b_3	b_2	b_1	b_0
0	0	0	0	0	0	0	0	0
1	0	0	0	1	0	0	0	1
2	0	0	1	1	0	0	1	0
3	0	0	1	0	0	0	1	1
4	0	1	1	0	0	1	0	0
5	0	1	1	1	0	1	0	1
6	0	1	0	1	0	1	1	0
7	0	1	0	0	0	1	1	1
8	1	1	0	0	1	0	0	0
9	1	1	0	1	1	0	0	1
10	1	1	1	1	1	0	1	0
11	1	1	1	0	1	0	1	1
12	1	0	1	0	1	1	0	0
13	1	0	1	1	1	1	0	1
14	1	0	0	1	1	1	1	0
15	1	0	0	0	1	1	1	1

Binary code to gray code conversion

- Let $g_n \cdots g_2\, g_1\, g_0$ denote a code word in the $(n + 1)^{th}$-bit Gray code, and let $b_n \cdots b_2 b_1\, b_0$ designate the corresponding binary number.



$$g_i = b_i \oplus b_{i+1}, \qquad 0 \le i \le n-1,$$
$$g_n = b_n,$$

Gray Code to Binary

- Start with the leftmost digit and proceed to the least significant digit, setting $b_i = g_i$ if the number of 1's preceding g_i is even and setting $b_i = g_i'$ if the number of 1's preceding g_i is odd
- Gray code word 1001011 represents the binary number 1110010

Reflected Code

- The term “reflected” is used to designate codes which have the property that the *n*-bit code can be generated by reflecting the (n − 1)th-bit code.
- Three-bit Gray code can be obtained by reflecting the two-bit code about an axis at the end of the code and assigning a most significant bit of 0 above the axis and 1 below the axis.

00	0	00	0	000
01	0	01	0	001
11	0	11	0	011
10	0	10	0	010
	1	10	0	110
	1	11	0	111
	1	01	0	101
	1	00	0	100
			1	100
			1	101
			1	111
			1	110
			1	010
			1	011
			1	001
			1	000