CS 561 Artificial Intelligence
Lecture

SOLVING PROBLEMS BY SEARCHING
INFORMED SEARCH

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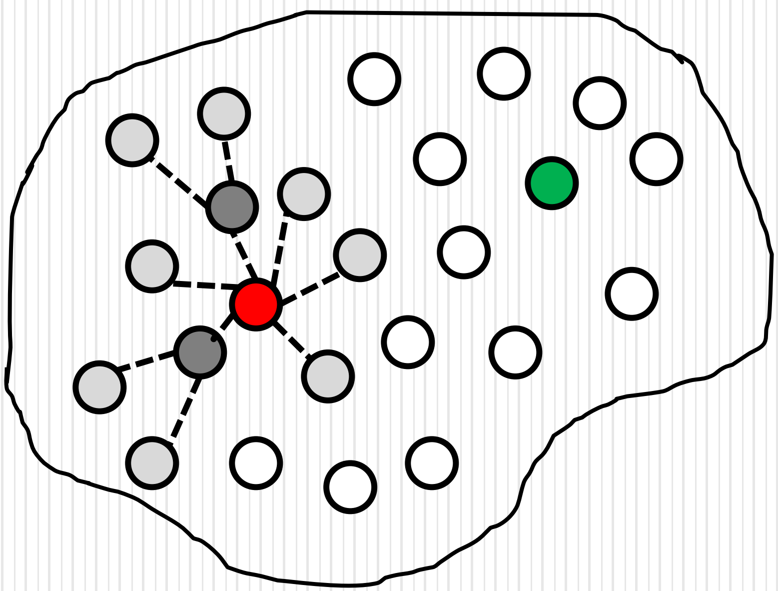
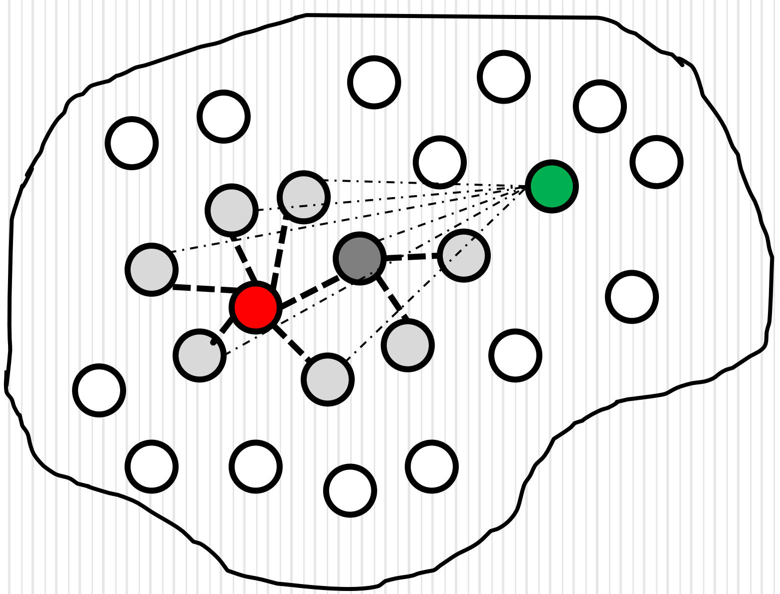
OUTLINE

- Informed (Heuristic) Search
  - Greedy Best-first Search
  - A* Search
  - Iterative Deepening A* (IDA*)
  - Recursive Best First Search (RBFS)
- More about heuristics
- Applications

INFORMED SEARCH STRATEGIES

Uses problem-specific knowledge beyond the definition of the problem itself





BEST-FIRST SEARCH

- Idea: use an evaluation function f(n) for each node
  - f(n) provides an estimate for the total cost.

  → Expand the node n with smallest f(n).

  → Choice of f determines the search strategy
- Implementation:
  - Order the nodes in frontier by f(n) (lowest f(n) first)
- Special cases:
  - greedy best-first search
  - A* search

Evaluation function is an estimate of node quality.

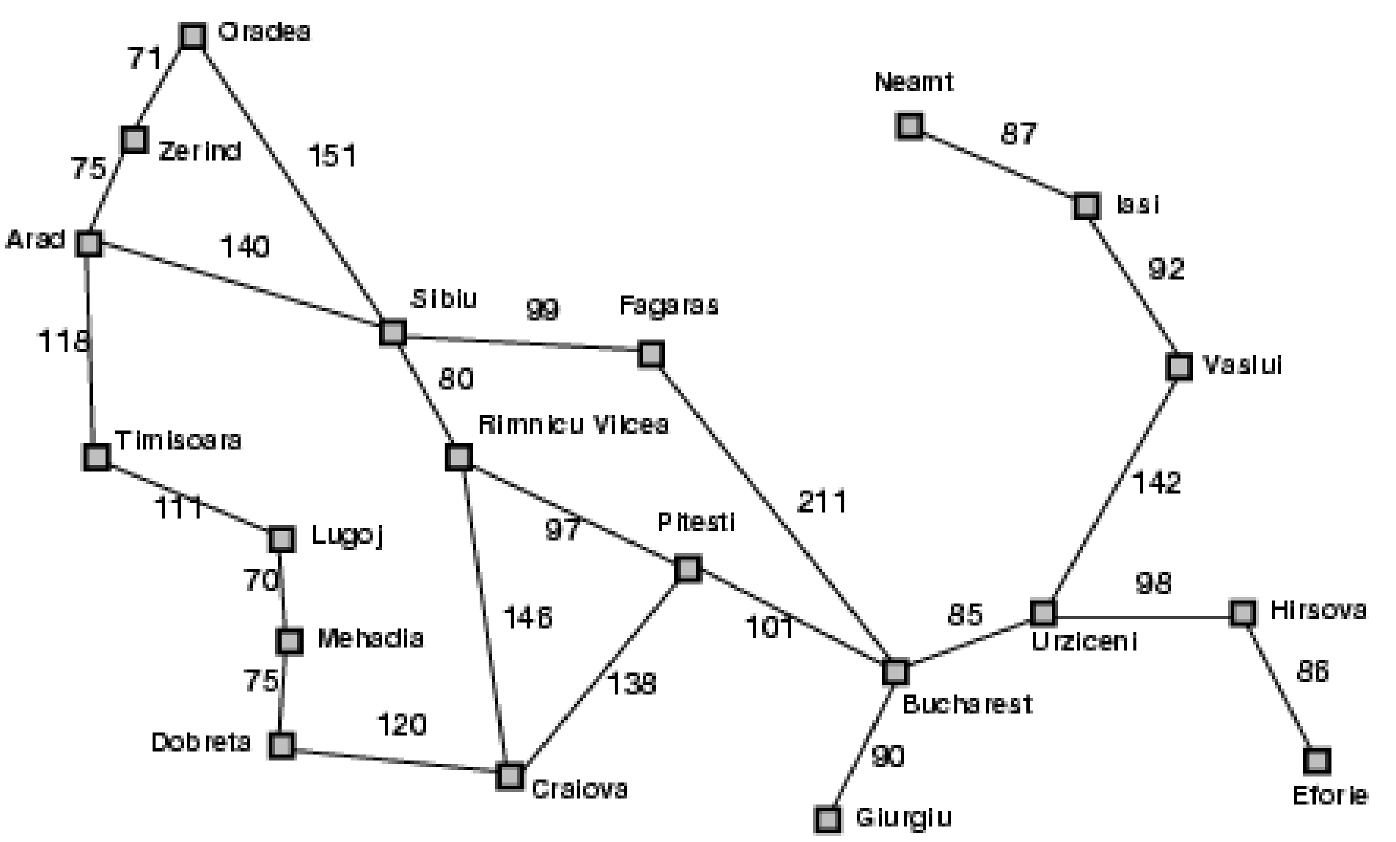
HEURISTIC FUNCTION

- f(n) consists of a heuristic function, denoted by h(n)
- Heuristic: "using rules of thumb to find answers"
- Heuristic function h(n)
  - Non-negative and problem-specific function
  - Estimate of (optimal) cost from n to goal
  - h(n) = 0 if n is a goal node

GREEDY BEST-FIRST SEARCH

- Special case of best-first search
  - Uses h(n) = heuristic function as its evaluation function
    - Example: straight line distance from n to Bucharest.
- Expand the node that appears closest to goal

ROMANIA EXAMPLE



Straight−line distance to Bucharest

Arad	366
Bucharest	0
Craiova	160
Dobreta	242
Eforie	161
Fagaras	176
Giurgiu	77
Hirsova	151
Iasi	226
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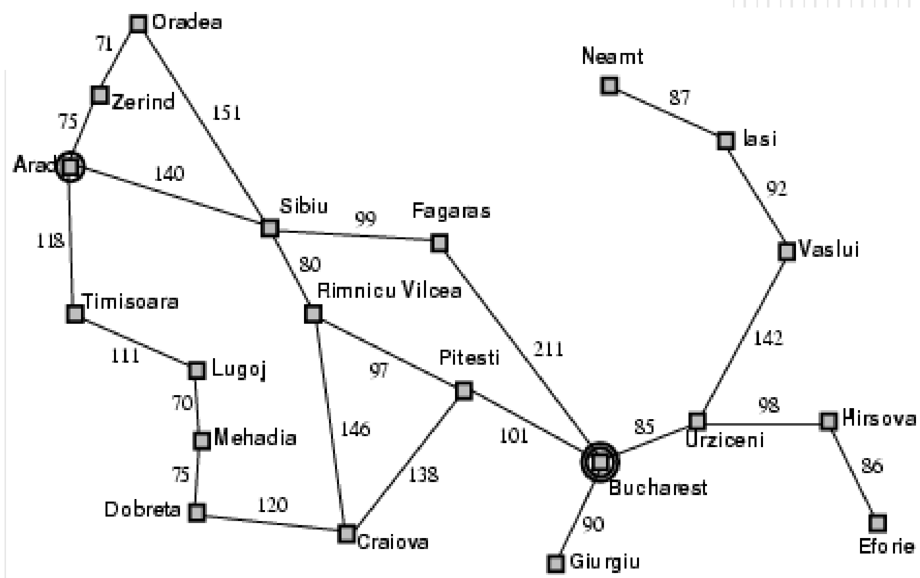
GREEDY BEST-FIRST SEARCH EXAMPLE



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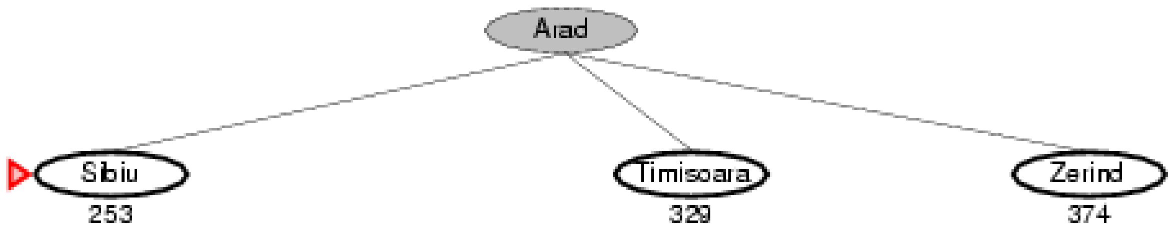

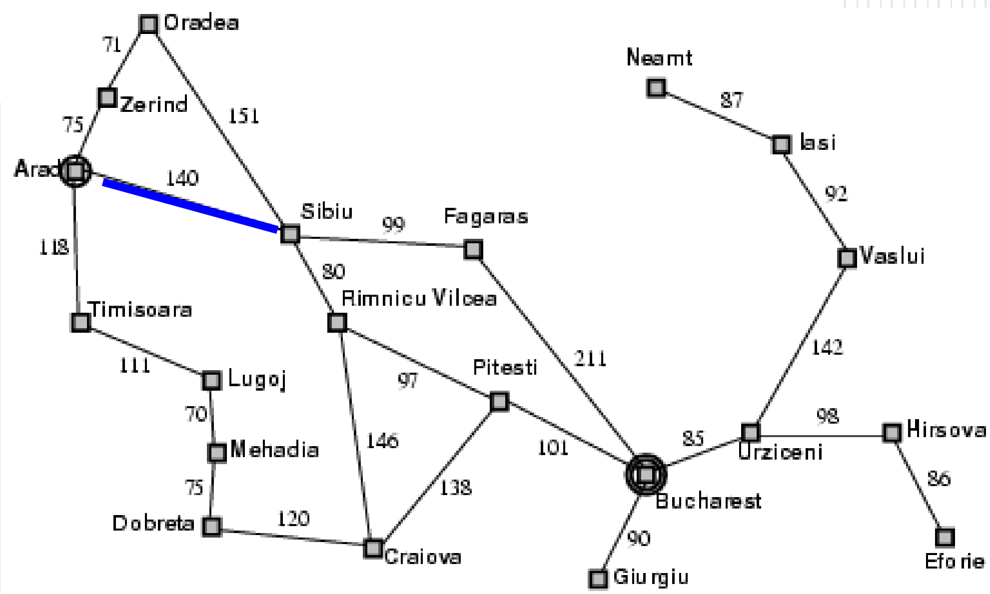

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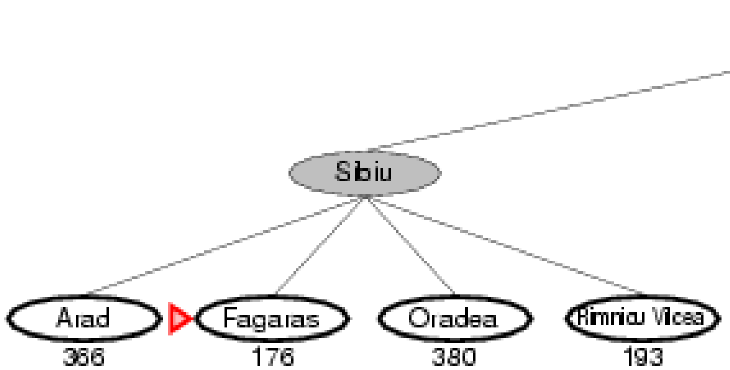

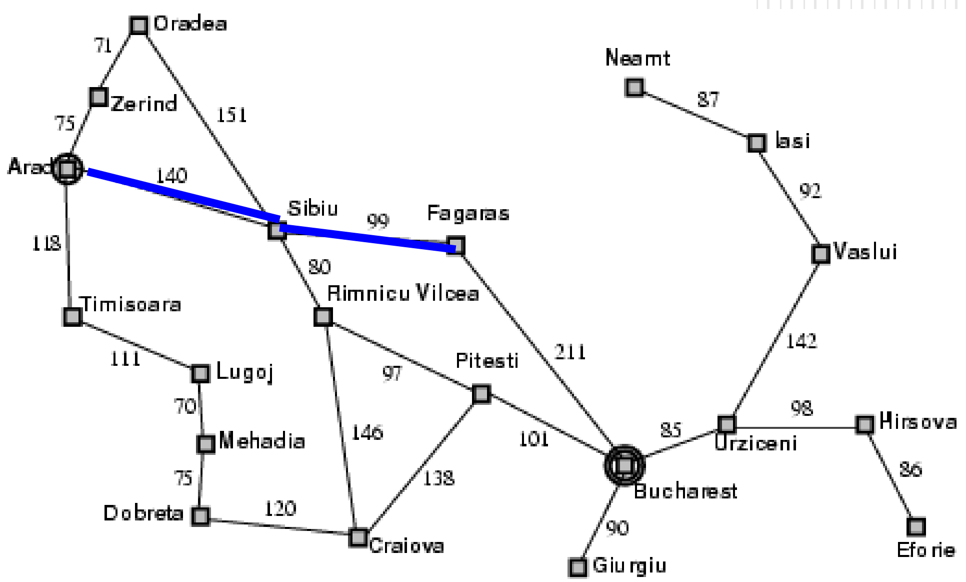

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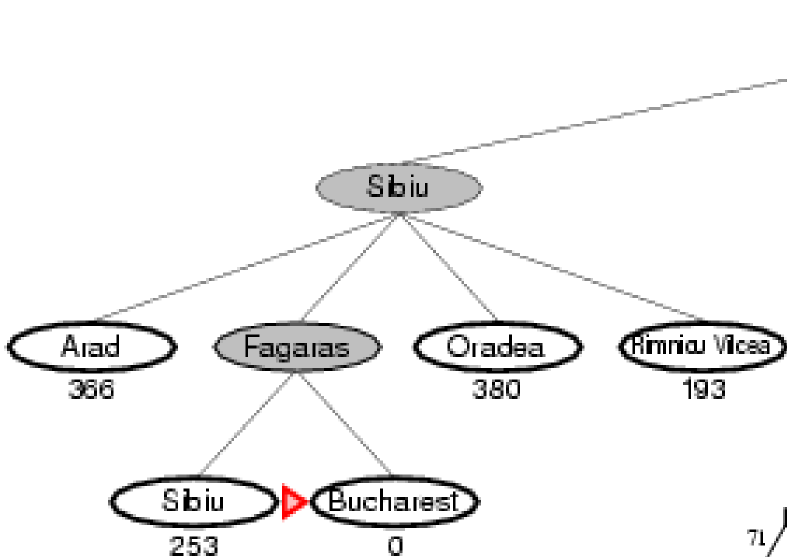
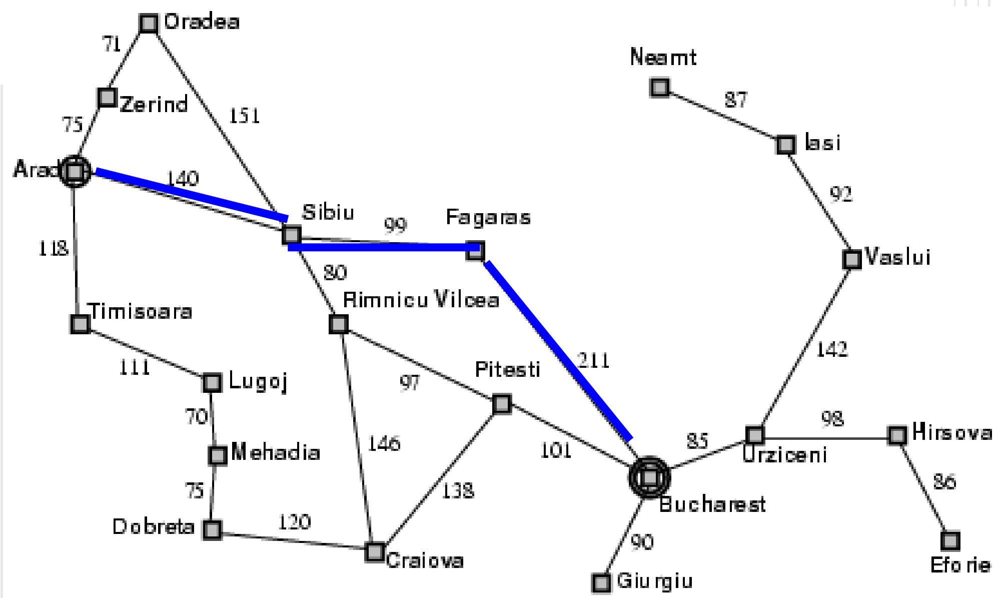
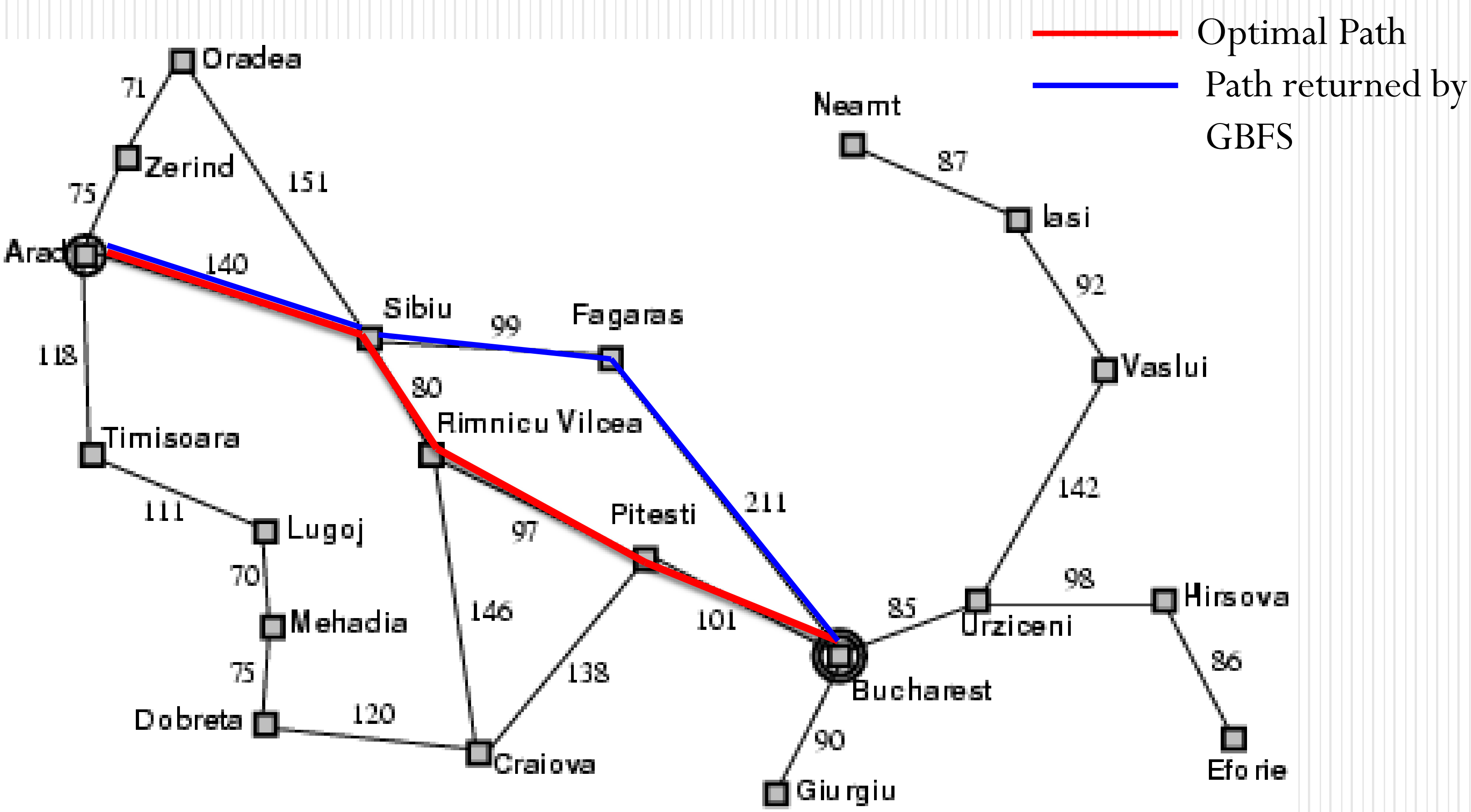


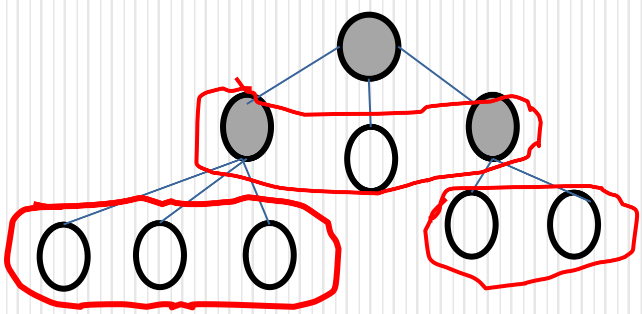


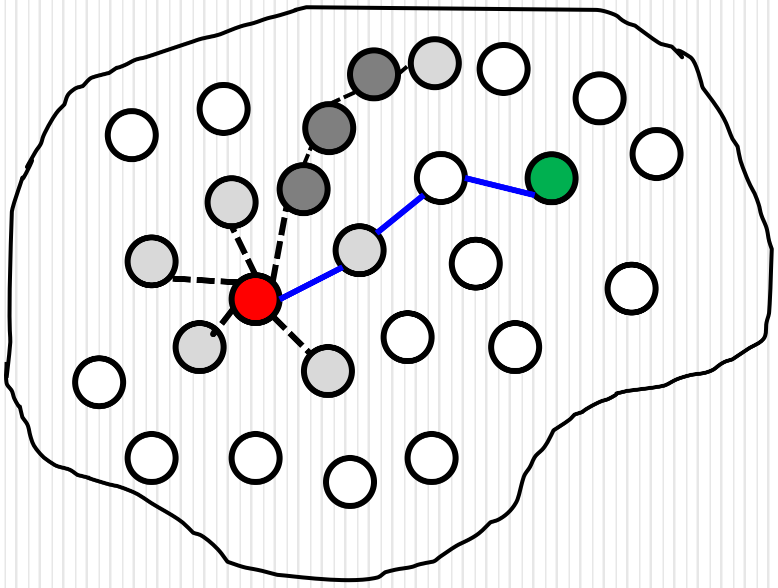
Optimal Path



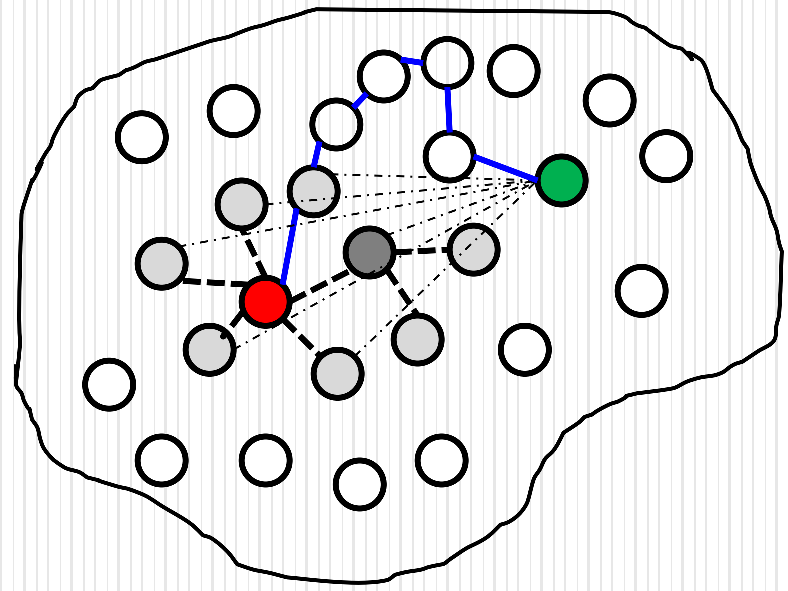
PROPERTIES OF GREEDY BEST-FIRST SEARCH

- Complete?
  - Not unless it keeps track of all states visited
- Optimal?
  - No – we just saw an example of a shorter path
- Time?
  - $O(b^m)$, can generate all nodes at depth m before finding solution
  - m = maximum depth of search space
- Space?
  - $O(b^m)$ – keeps all nodes in memory





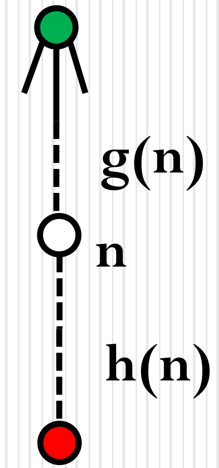
Uniform cost search $f(n) = g(n)$



Greedy Best First Search $f(n) = h(n)$

A* SEARCH

- Expand node based on estimate of total path cost through node
- Evaluation function $f(n) = g(n) + h(n)$
  - $g(n)$ = cost so far to reach n
  - $h(n)$ = estimated cost from n to goal
  - $f(n)$ = estimated total cost of path through n to goal
- Greedy Best First search has $f(n)=h(n)$
- Uniform Cost search has $f(n)=g(n)$



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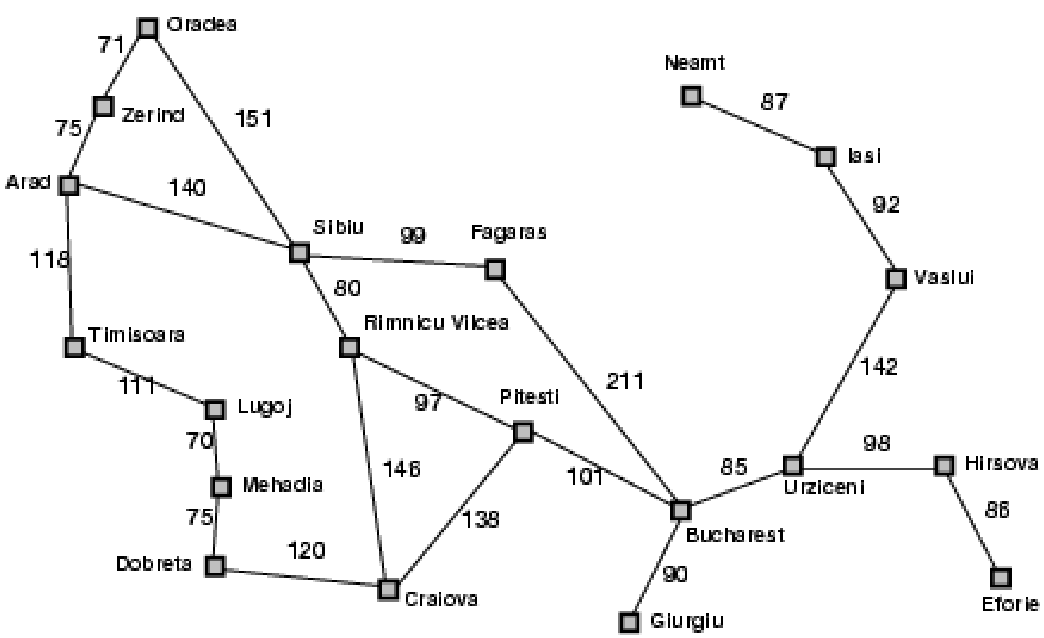

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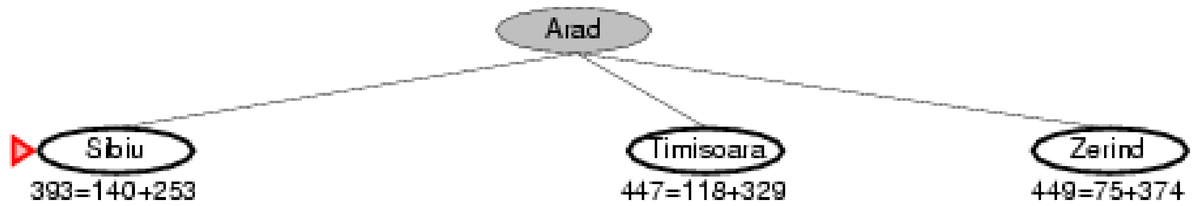

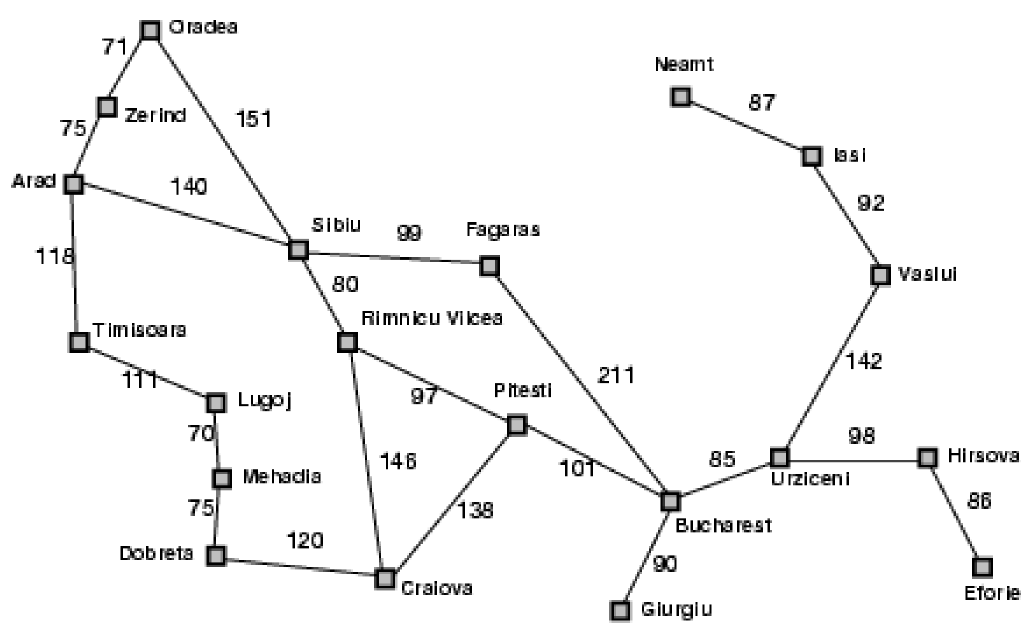

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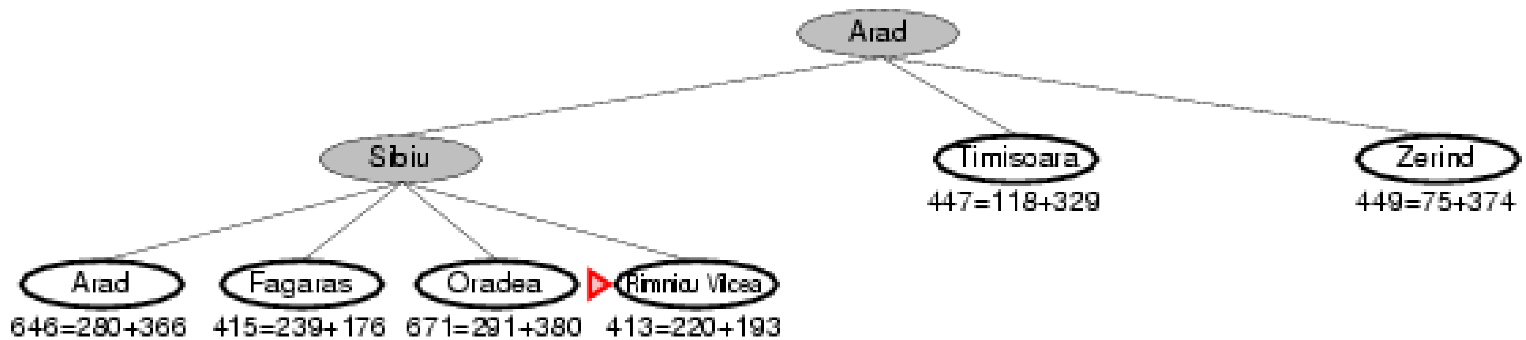

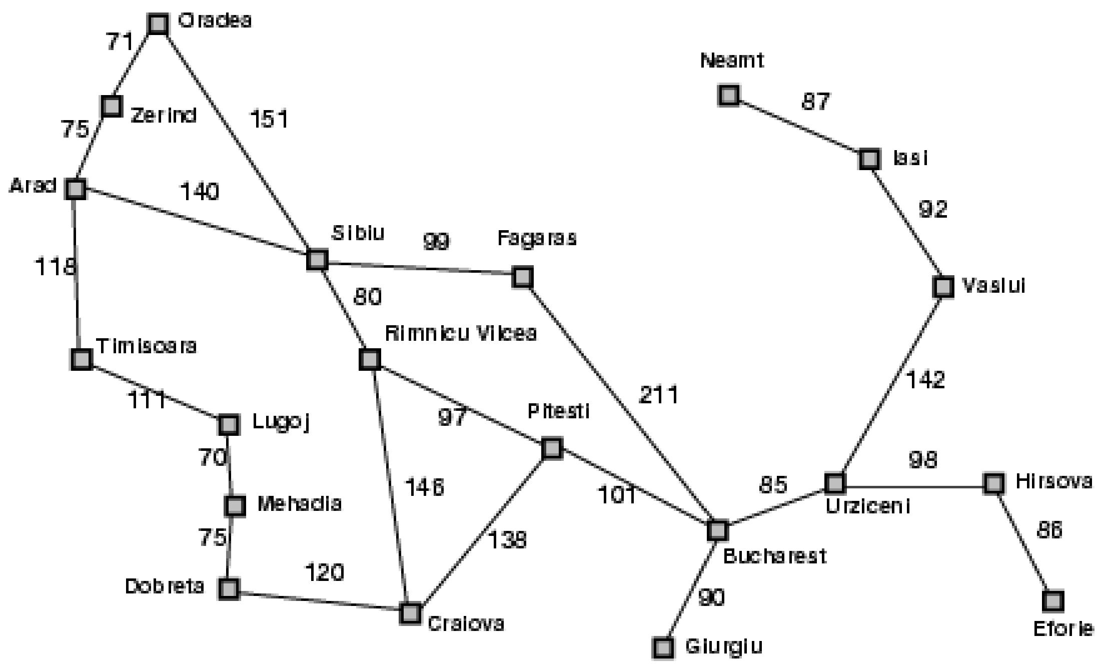

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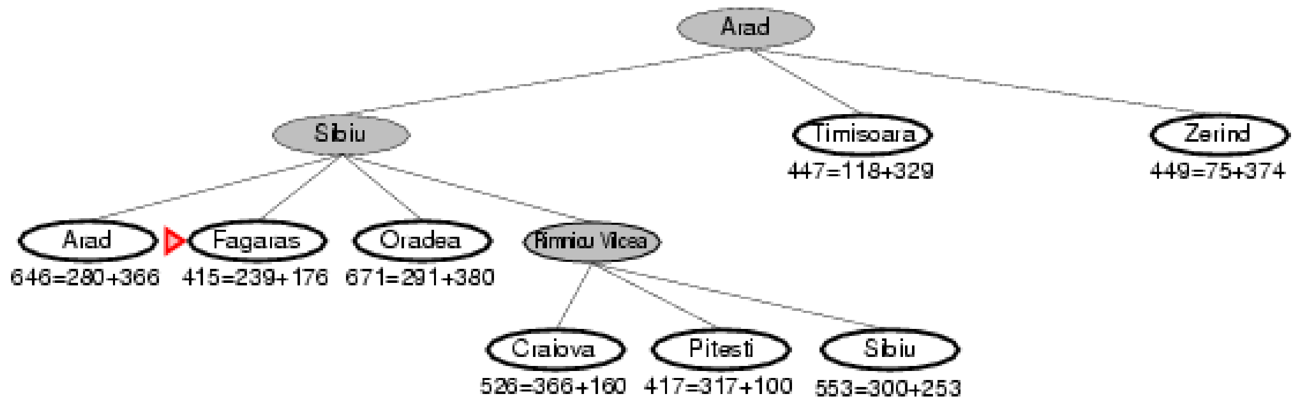

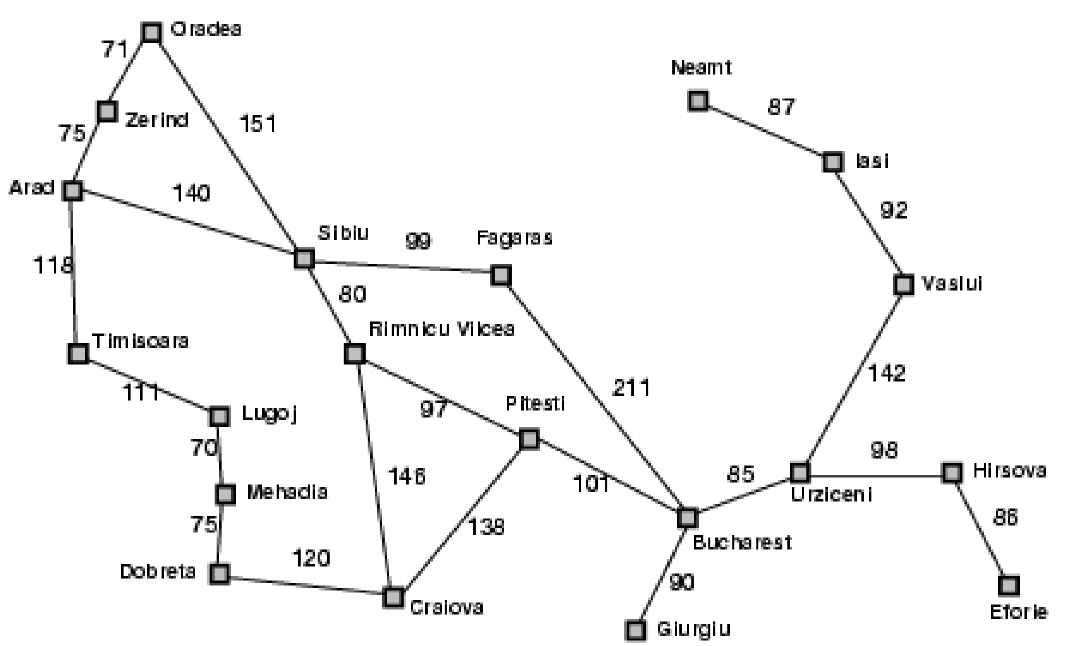

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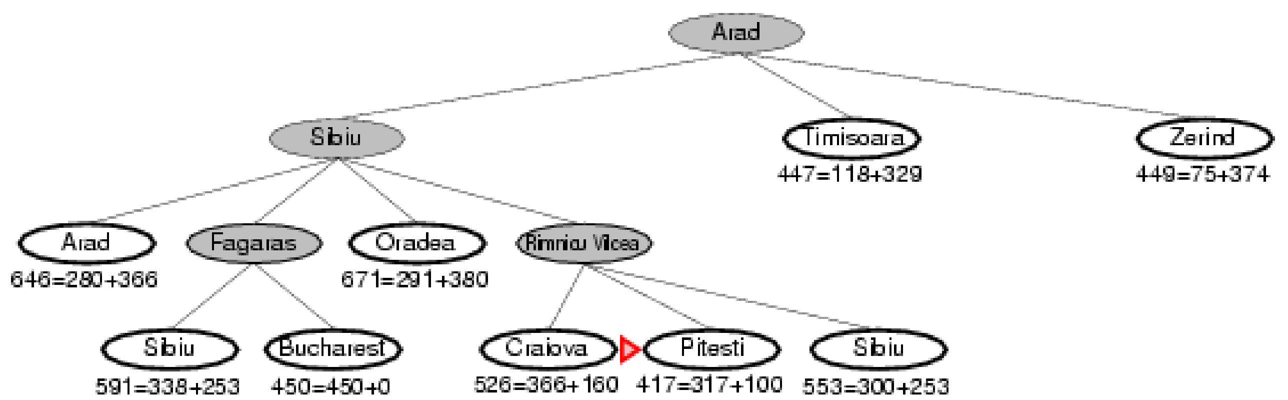

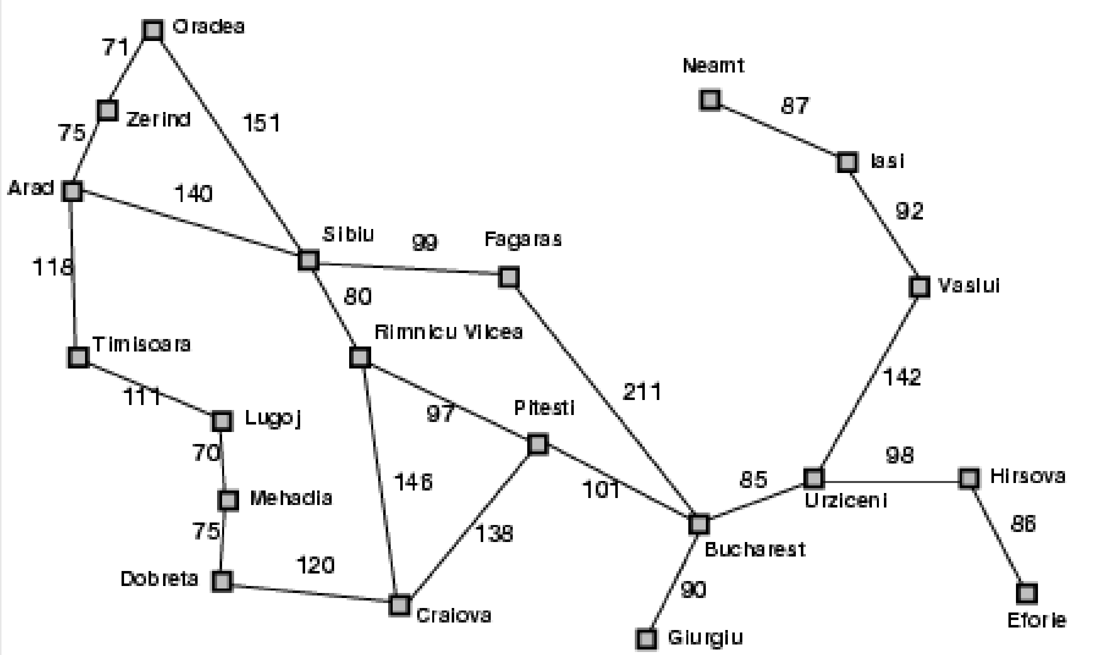

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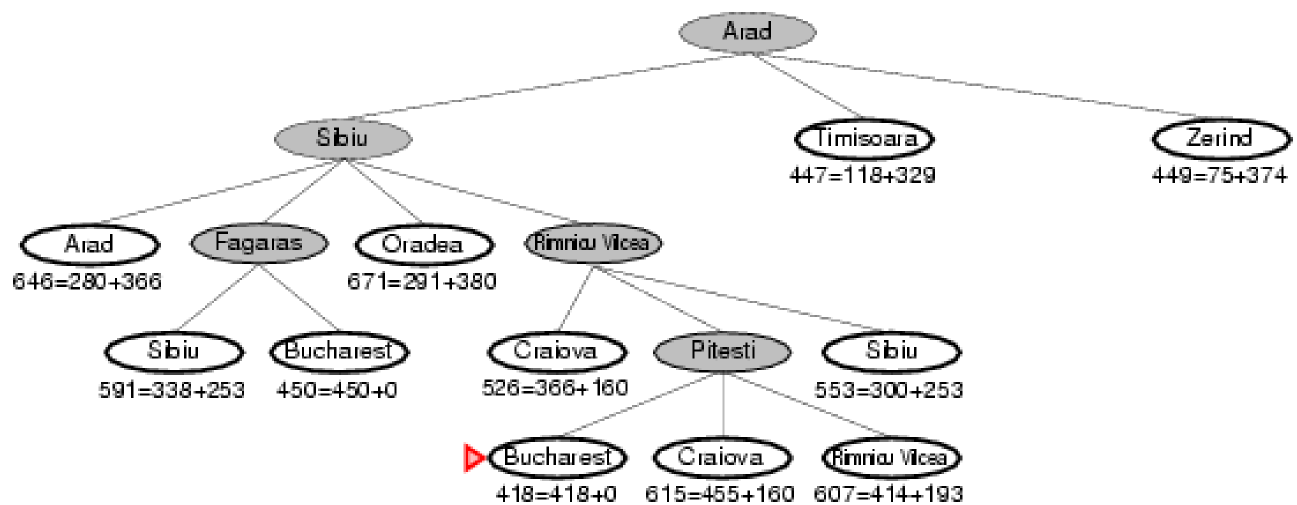

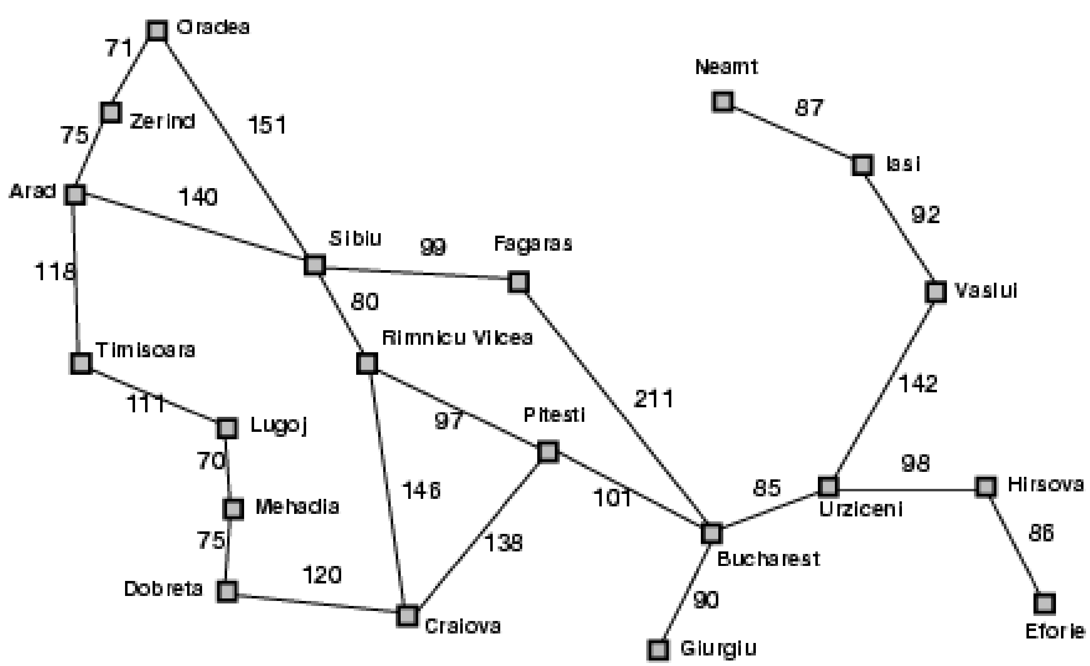

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A* SEARCH

- A* search is both complete (finds a solution if one exists) and optimal (finds the optimal path to a goal) if:
  - the branching factor is finite
  - Step cost are $> \epsilon > 0$
  - h(n) satisfies the conditions of
    - Admissibility
    - Consistency

ADMISSIBLE HEURISTICS

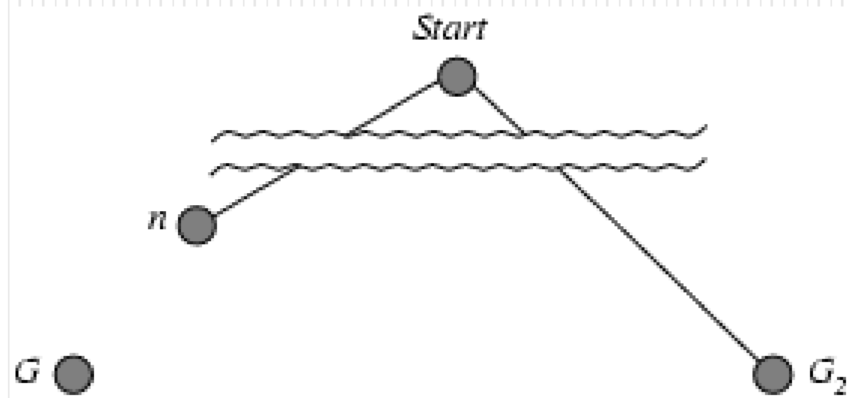
- A heuristic $h(n)$ is admissible if for every node n, $h(n) \leq h^*(n)$, where $h^*(n)$ is the true cost to reach the goal state from n.
- An admissible heuristic never overestimates the cost to reach the goal, i.e., it is optimistic
- Example: $h_{SLD}(n)$ (never overestimates the actual road distance)
- Theorem:

  If $h(n)$ is admissible, A^* using `TREE-SEARCH` is optimal

OPTIMALITY OF A* (PROOF)

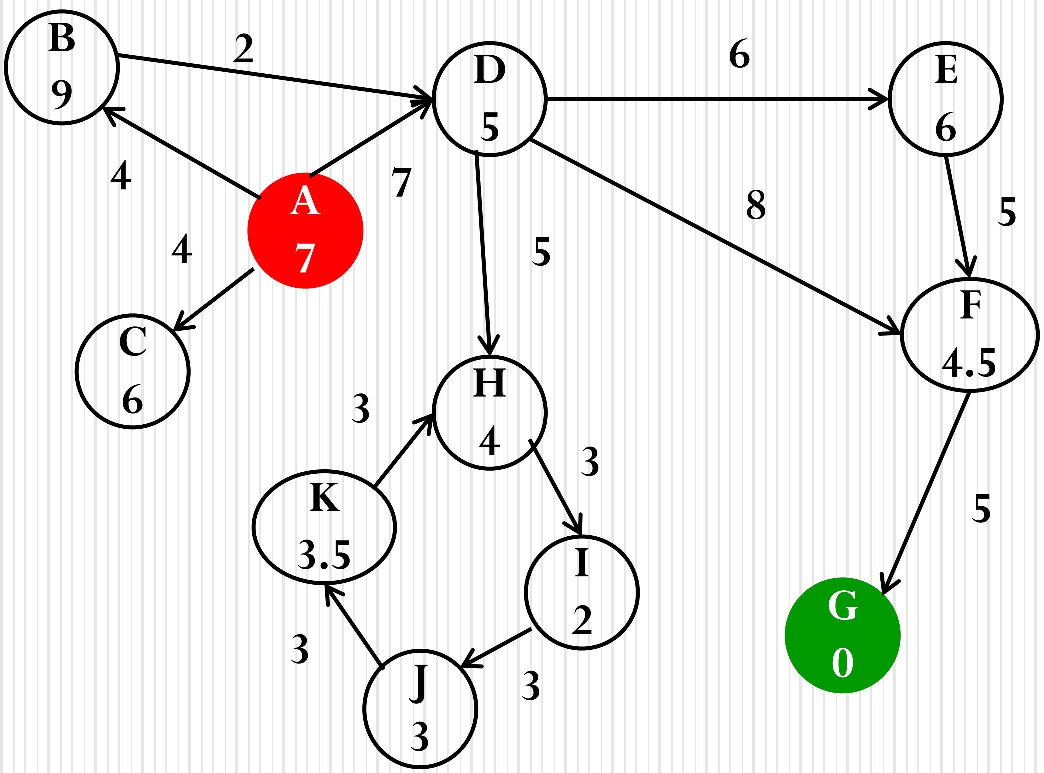
- Suppose some suboptimal goal G_2 has been generated and is in the frontier. Let n be an unexpanded node in the frontier such that n is on a shortest path to an optimal goal G.

We want to prove:
f(n) < f(G2)
(then A* will prefer n over G2)



- $f(G_2) = g(G_2)$ since $h(G_2) = 0$
- $f(G) = g(G)$ since $h(G) = 0$
- $g(G_2) > g(G)$ since G_2 is suboptimal
- $f(G_2) > f(G)$ from above
- $h(n) \leq h^*(n)$ since h is admissible (*under*-estimate)
- $g(n) + h(n) \leq g(n) + h^*(n)$ from above
- $f(n) \leq f(G)$ since $g(n)+h(n)=f(n)$ & $g(n)+h^*(n)=f(G)$
- $f(n) < f(G2)$ since $f(G) < f(G_2)$

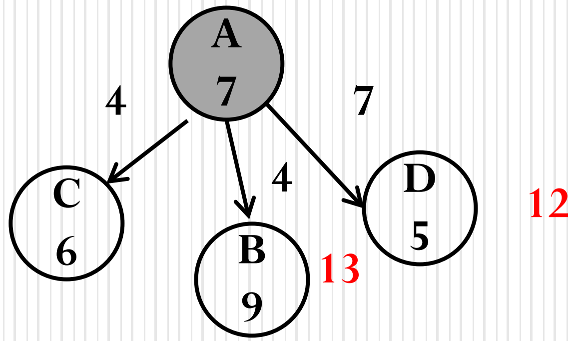
A * Example



A * Example

TREE-SEARCH VERSION

f(n) = 4+6=10



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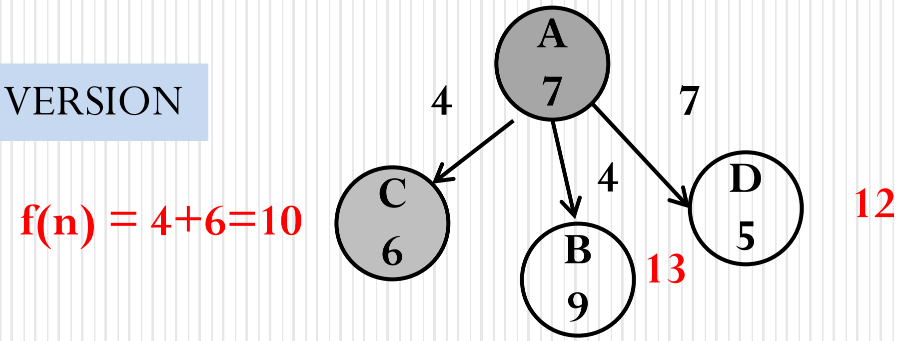

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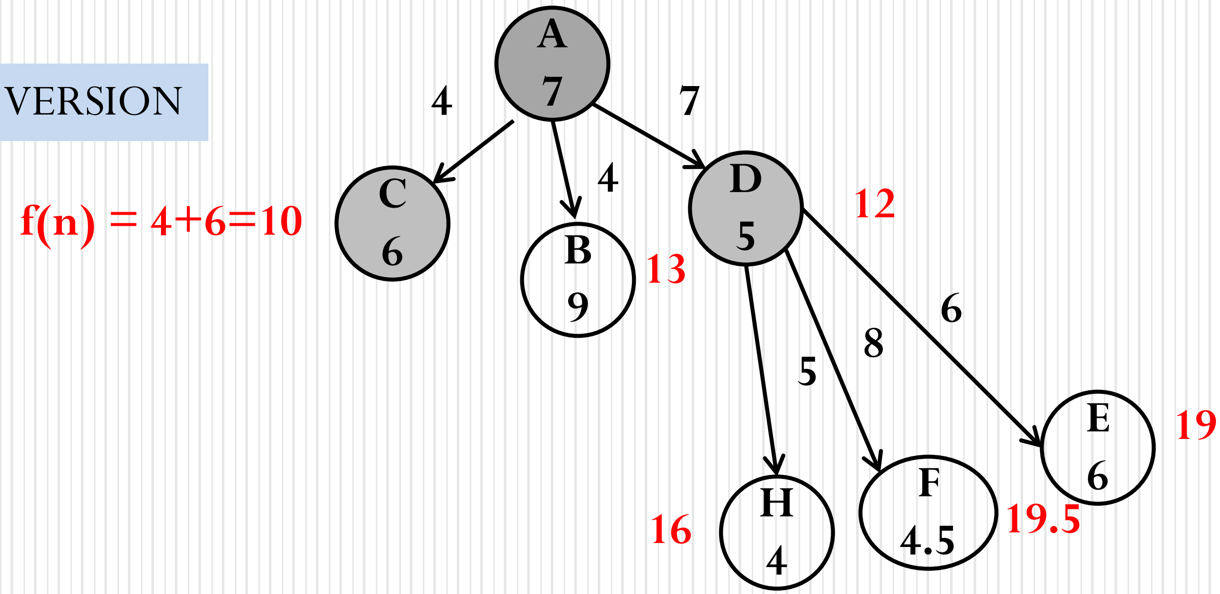

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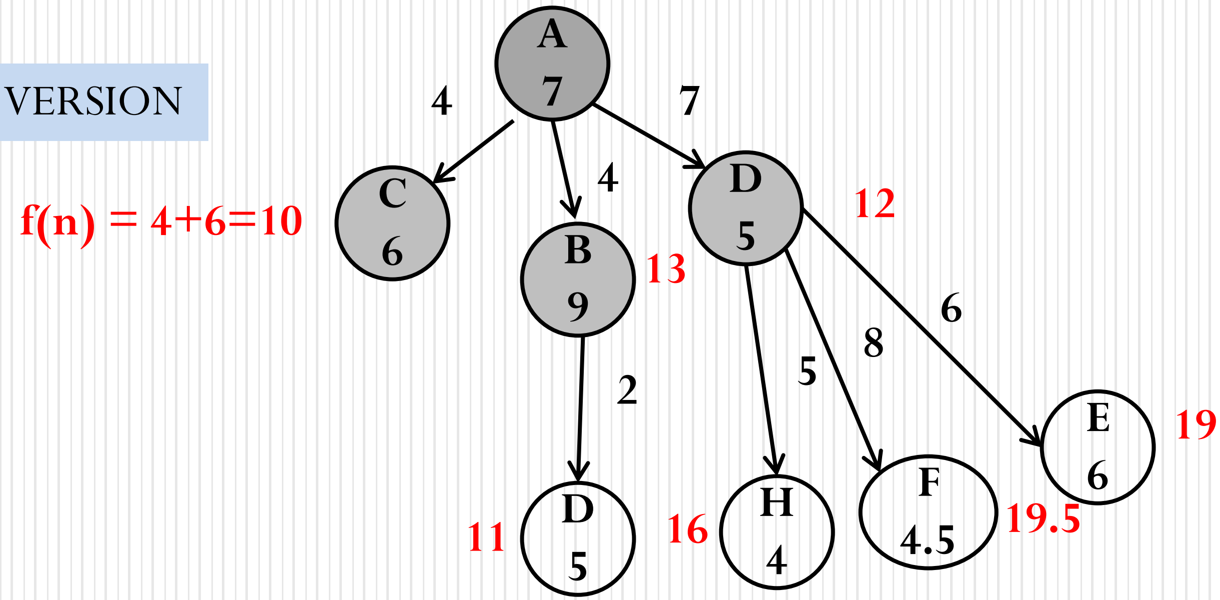

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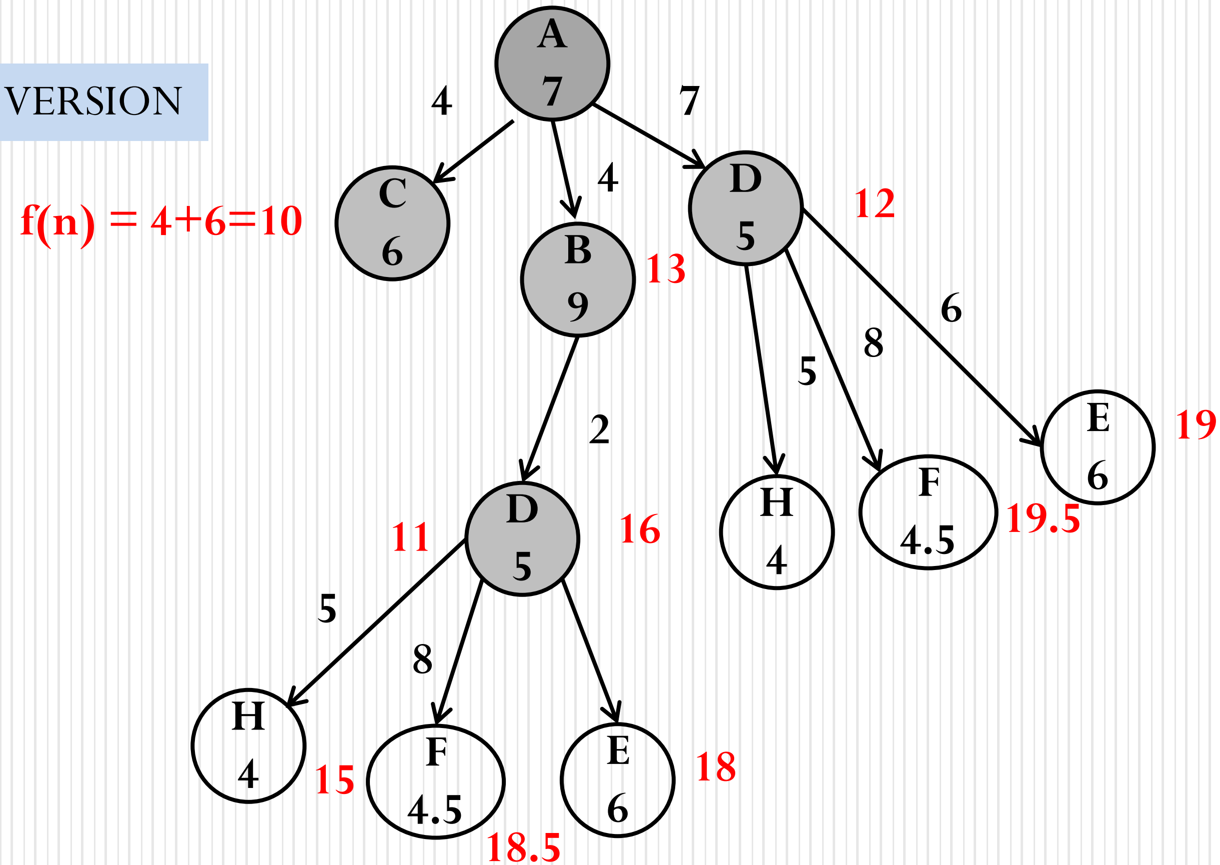

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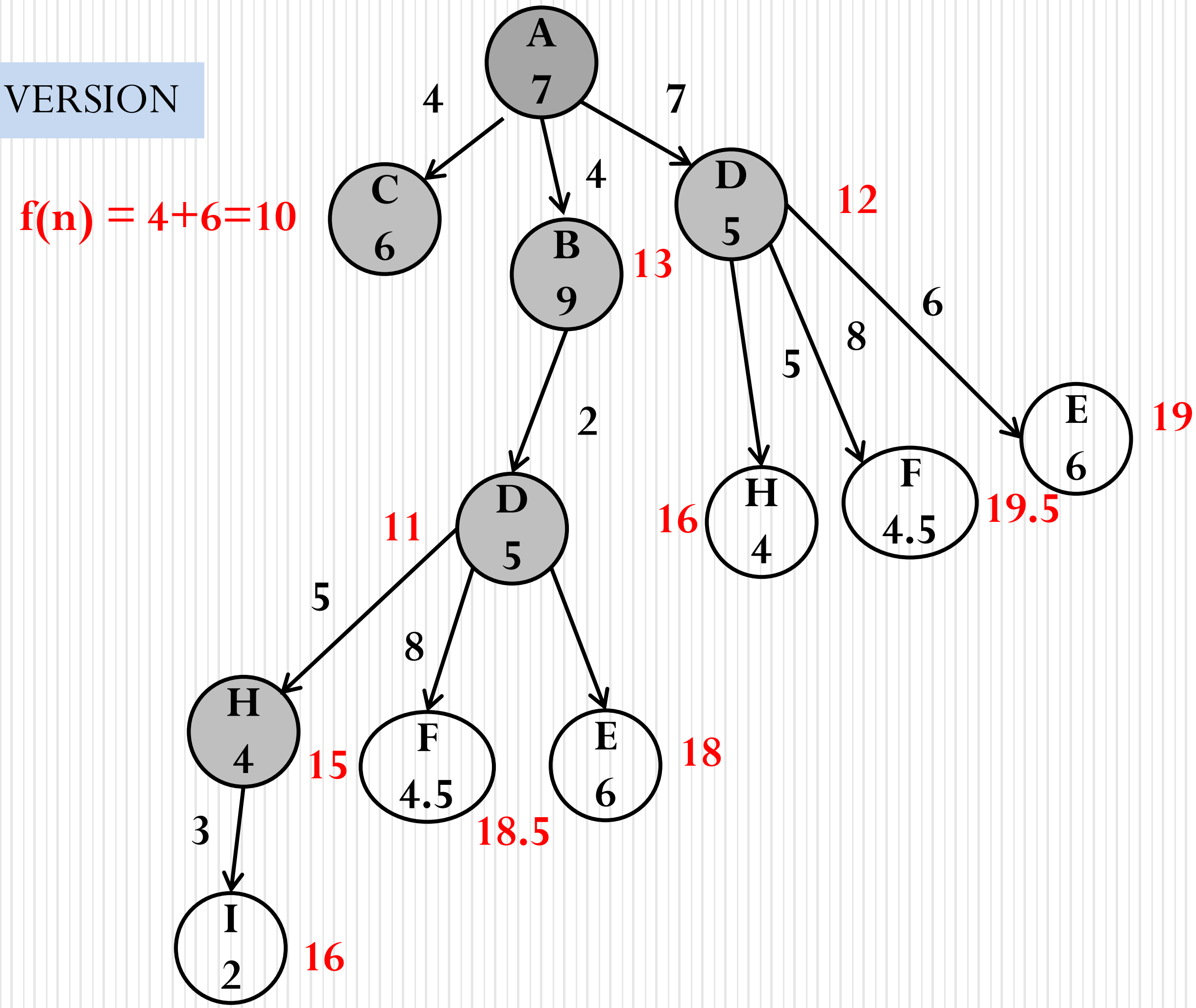

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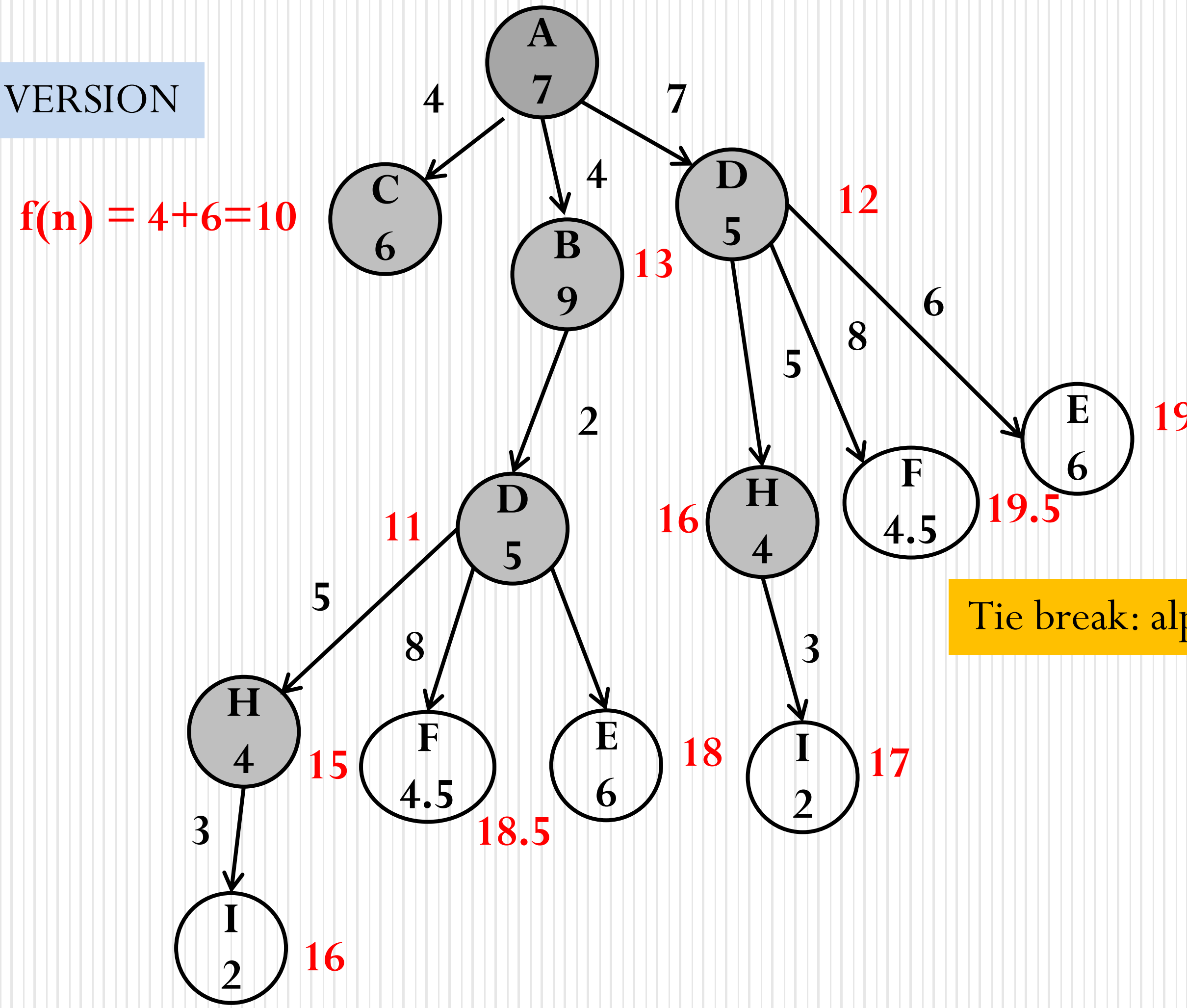
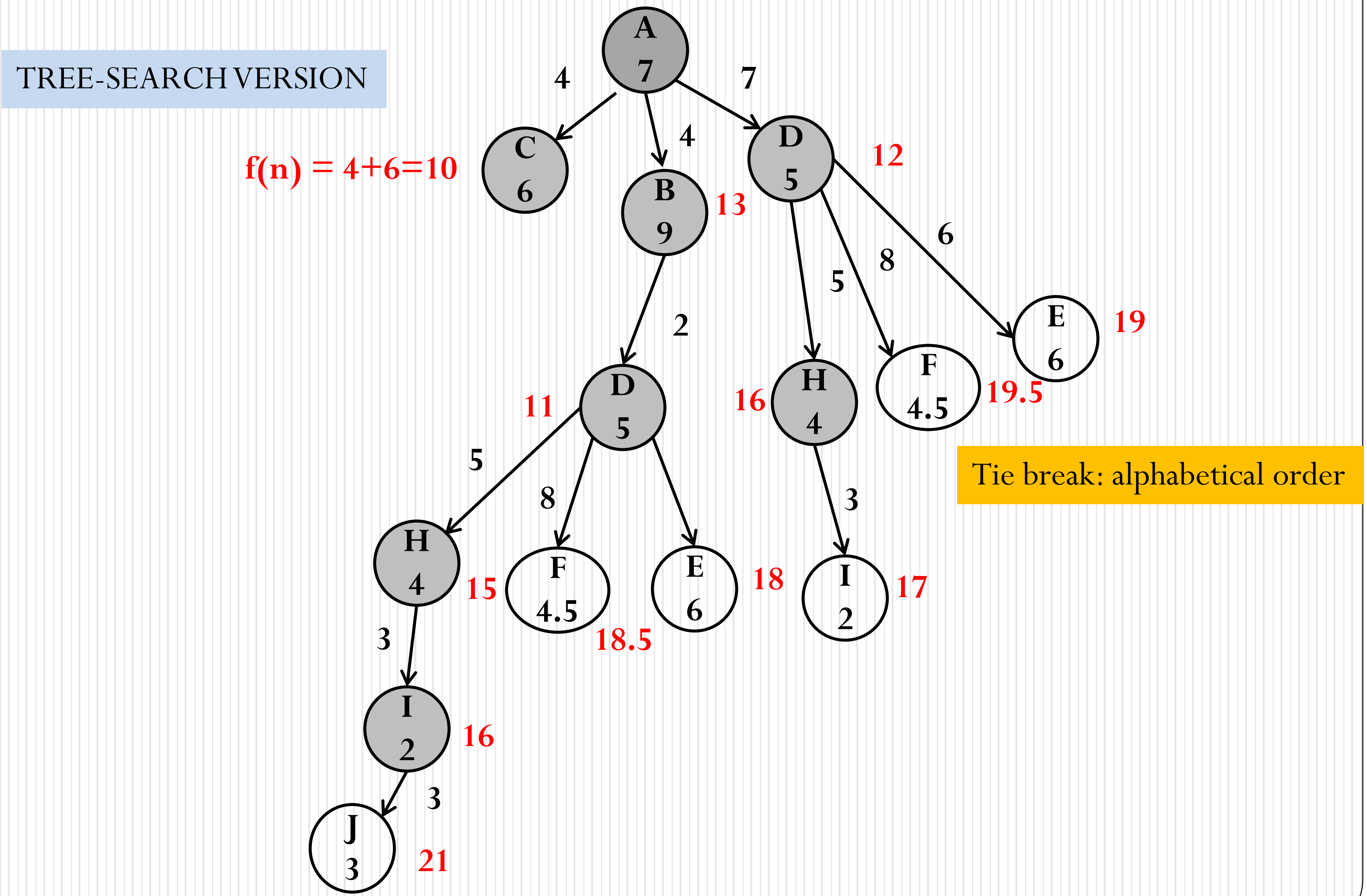


Tie break: alphabetical order

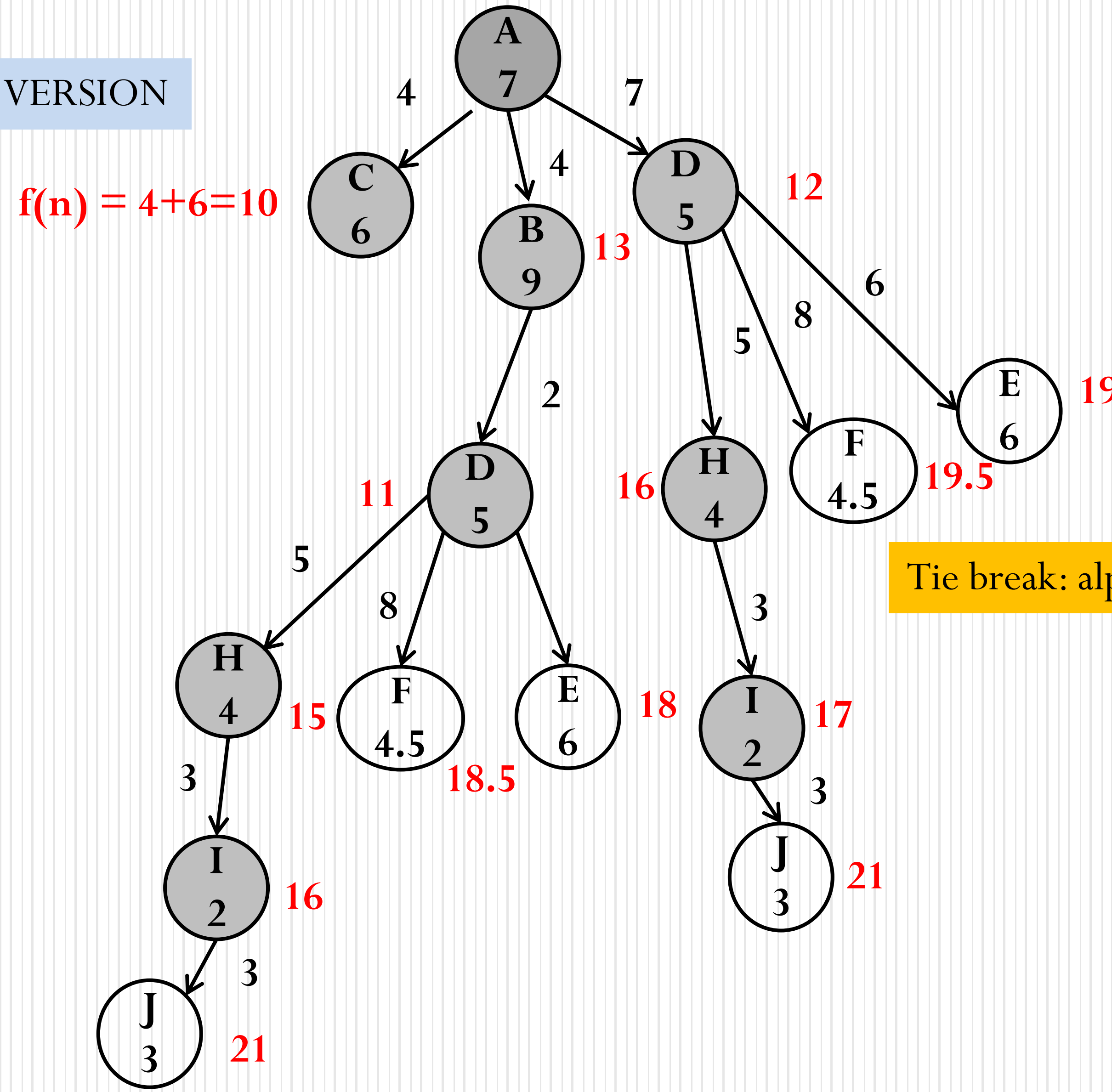
A * Example



TREE-SEARCH VERSION
f(n) = 4+6=10
A 7
C 6
B 9
D 5
4
4
7
13
12
2
6
8
5
E 6
19
F 4.5
19.5
H 4
16
D 5
11
5
8
H 4
15
F 4.5
18.5
E 6
18
3
I 2
17
3
I 2
16
3
J 3
21
Tie break: alphabetical order

A * Example

TREE-SEARCH VERSION



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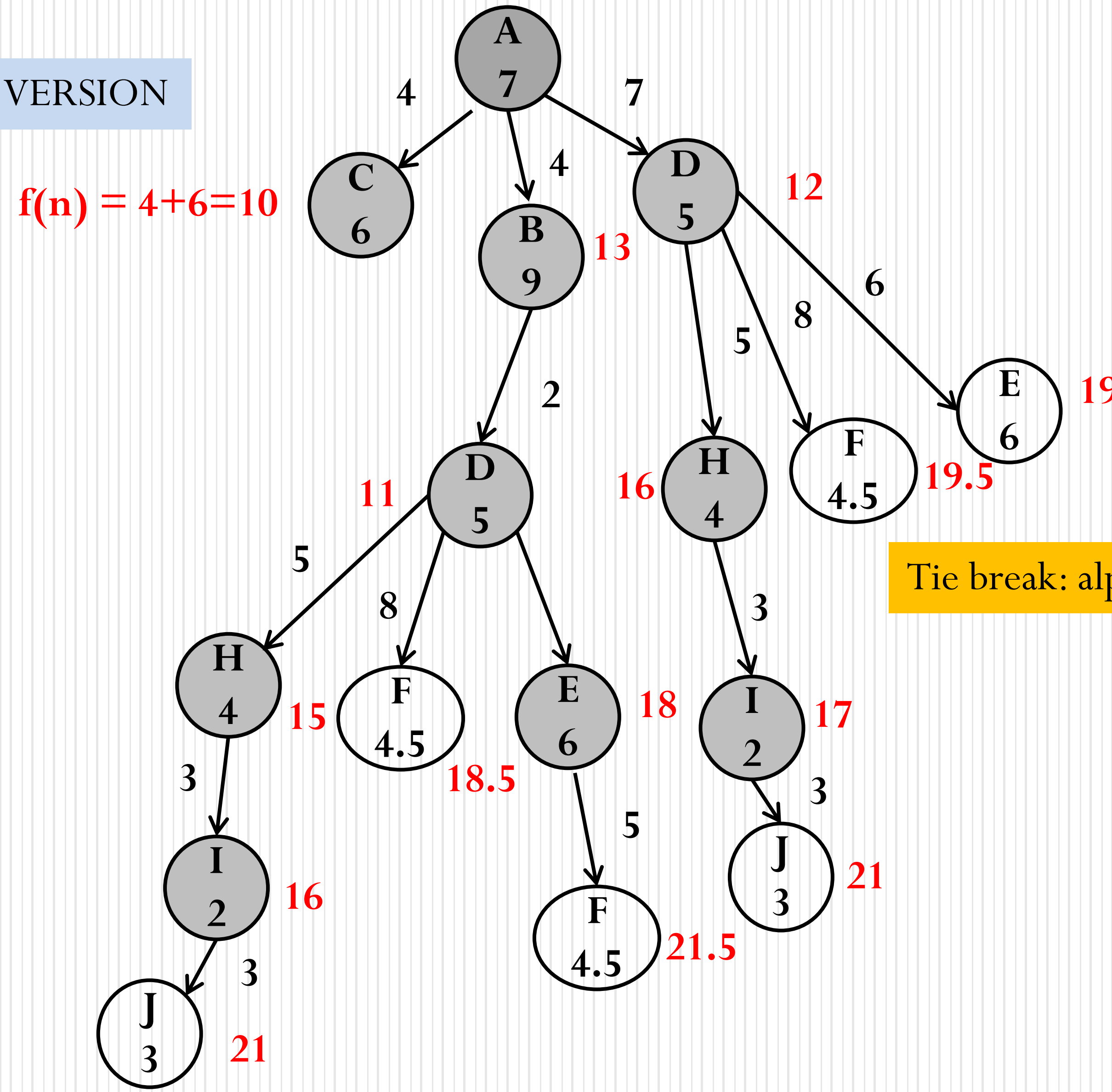

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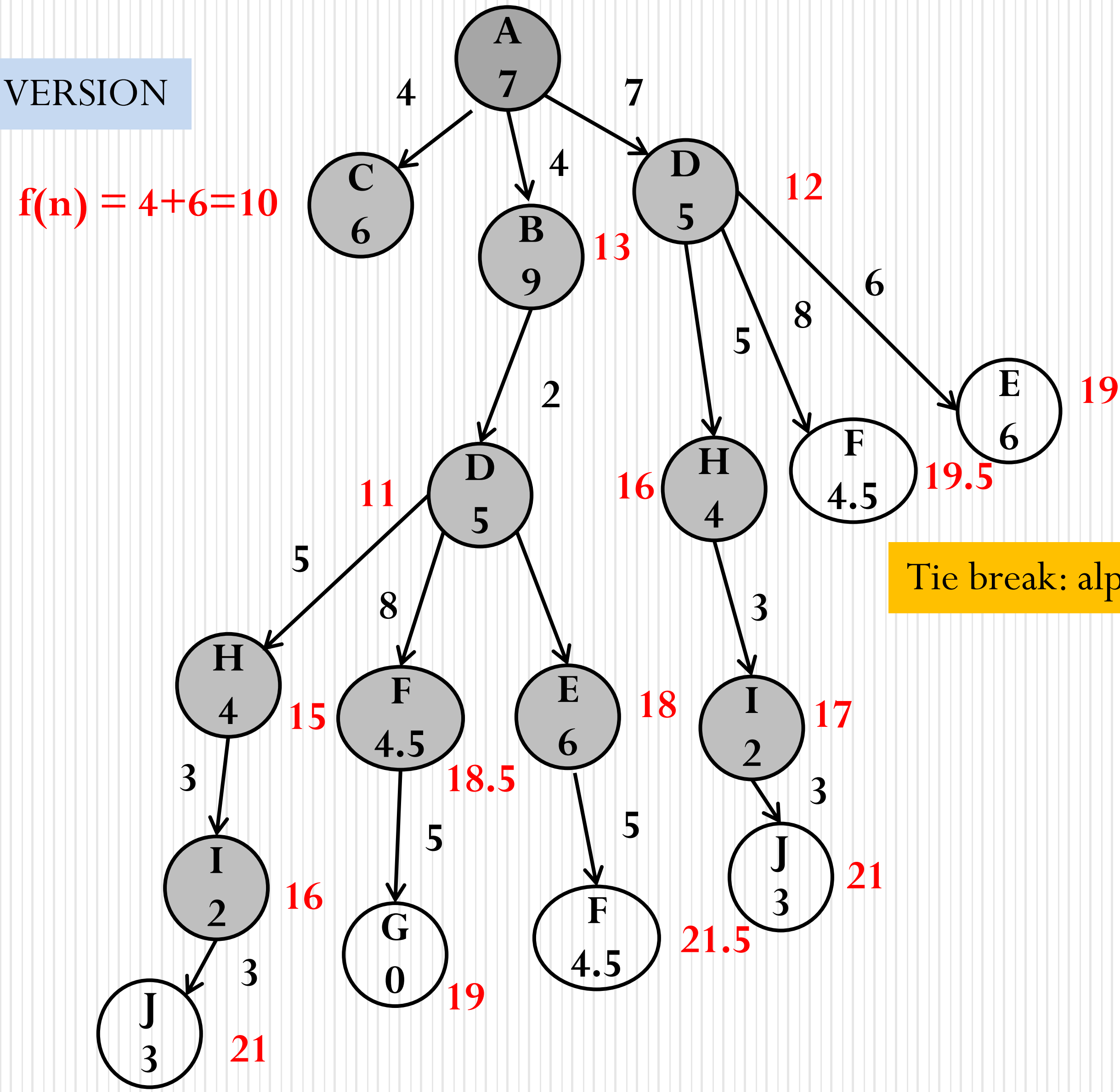

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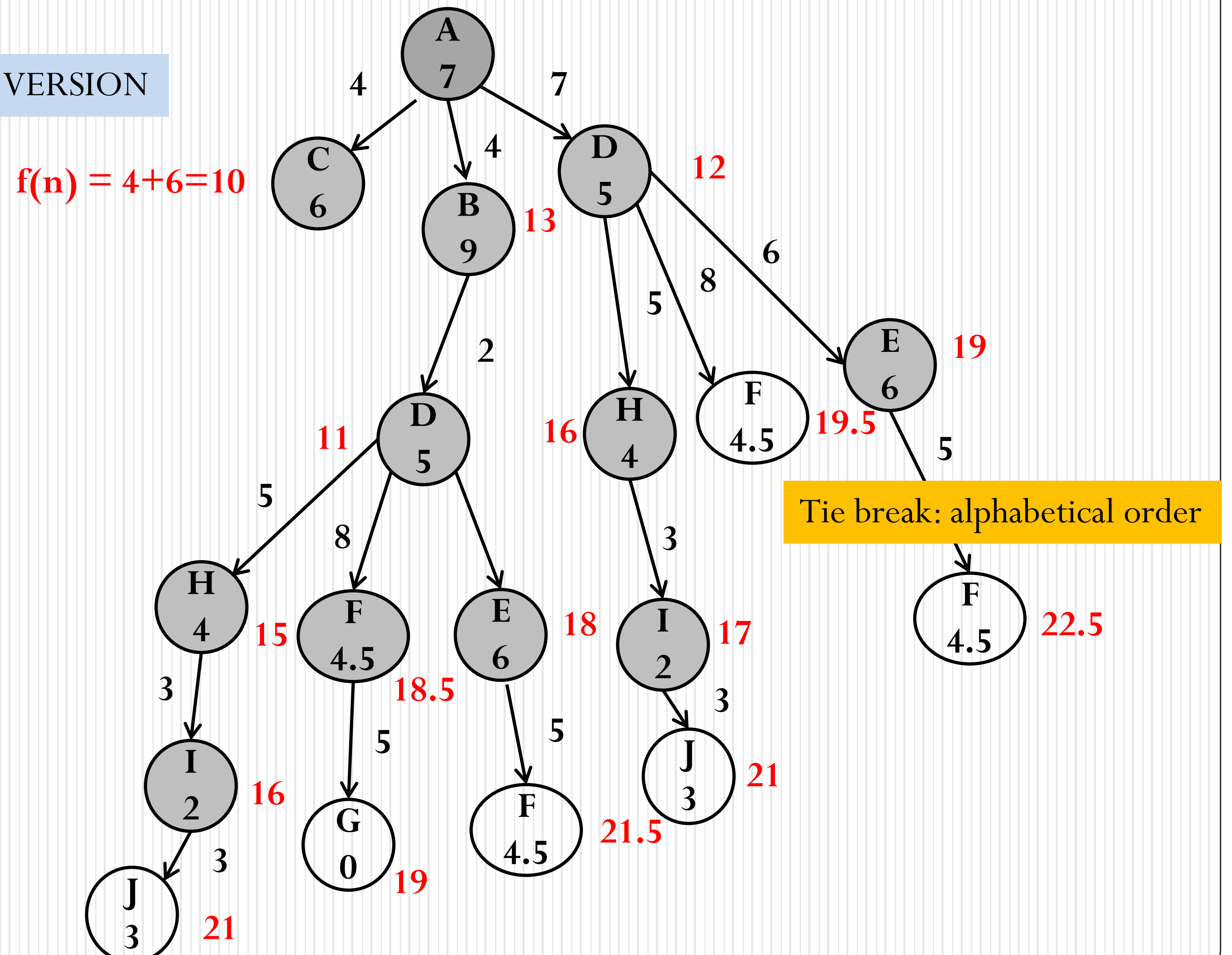
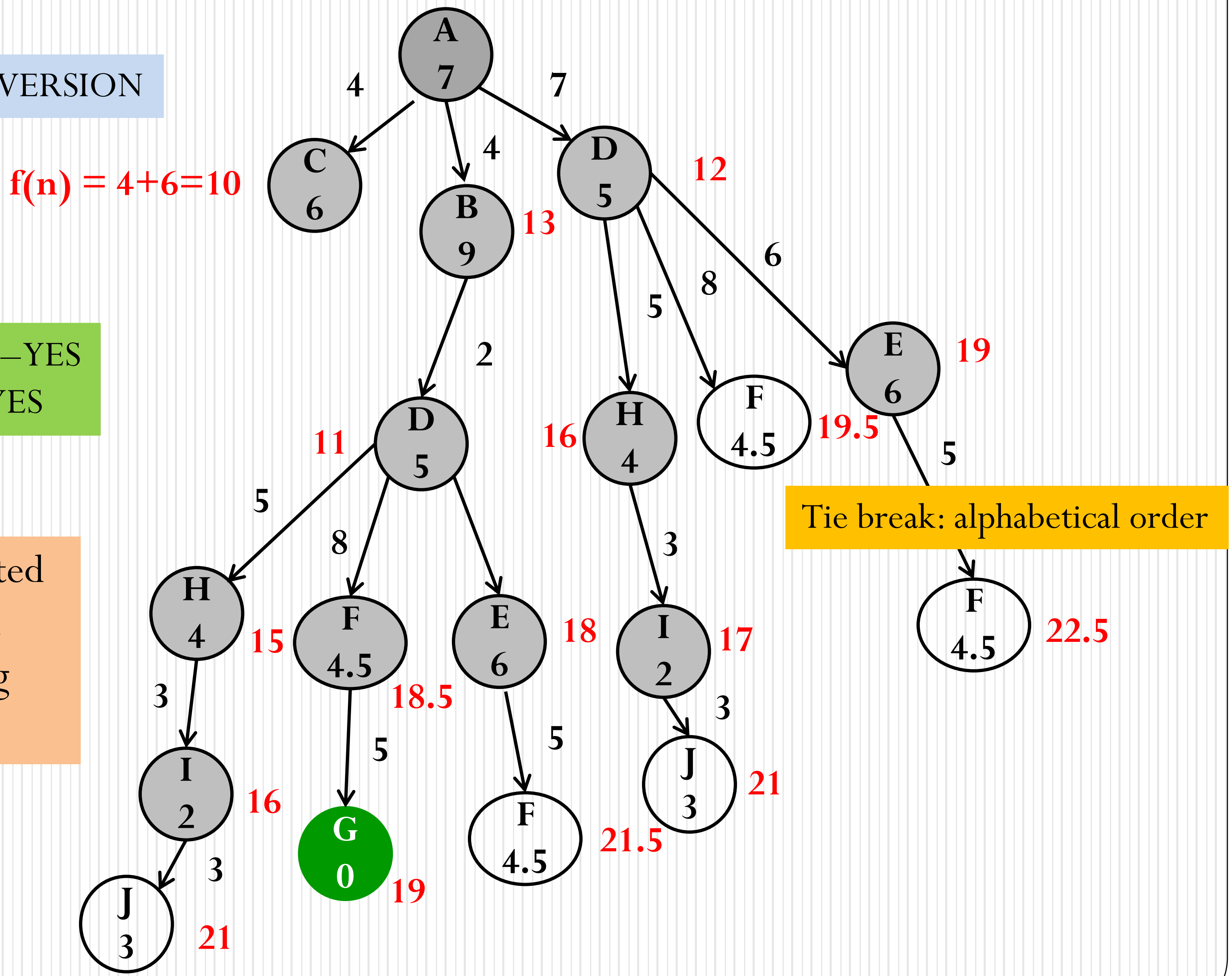


A * Example

TREE-SEARCH VERSION

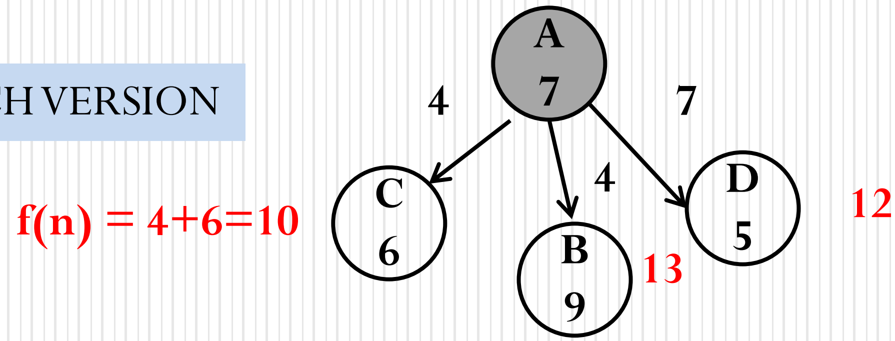
COMPLETE ? – YES
OPYIMAL ? - YES

Search is duplicated from D, identical sub-trees hanging below



A * Example

GRAPH-SEARCH VERSION



A * Example

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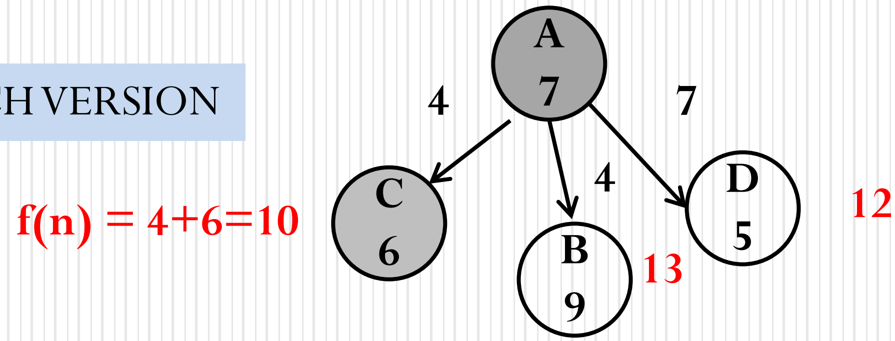

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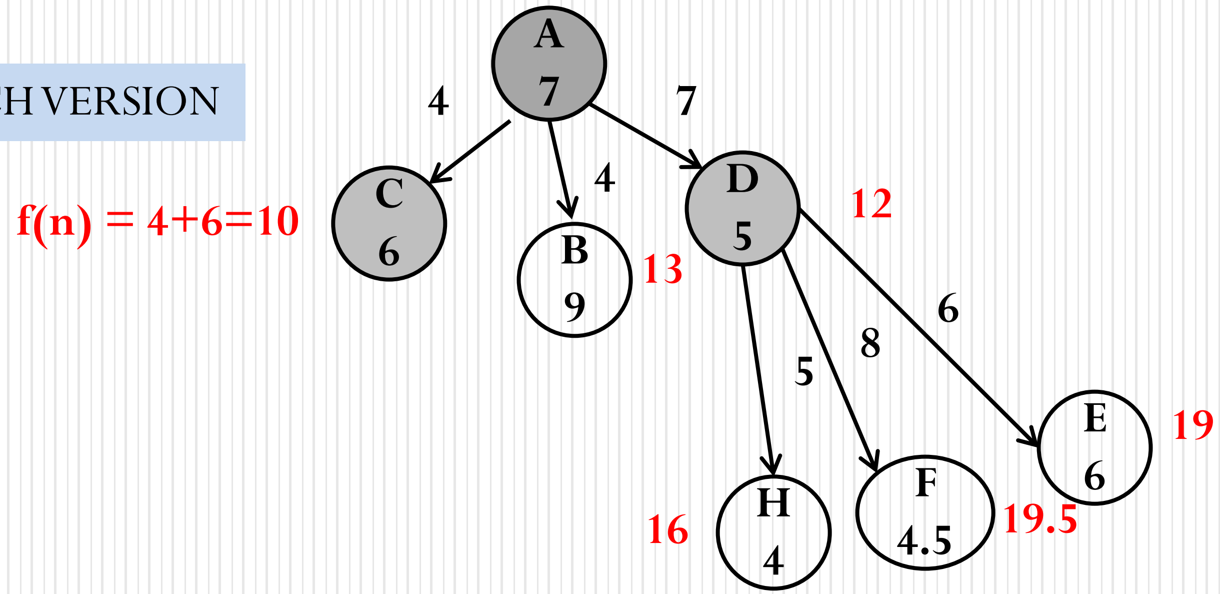

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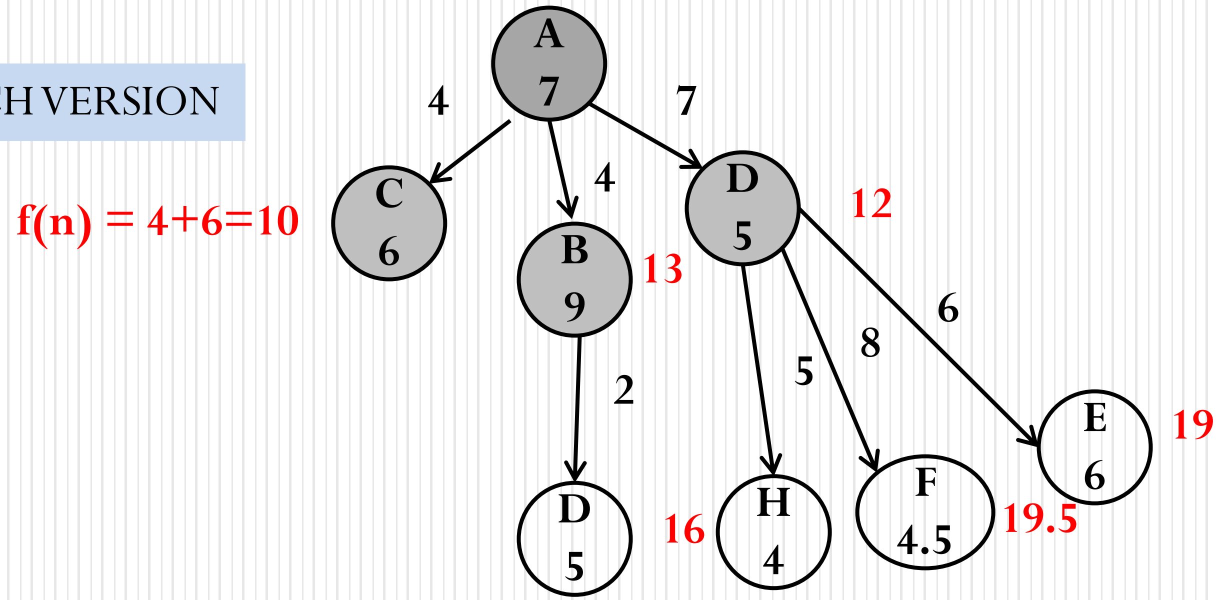

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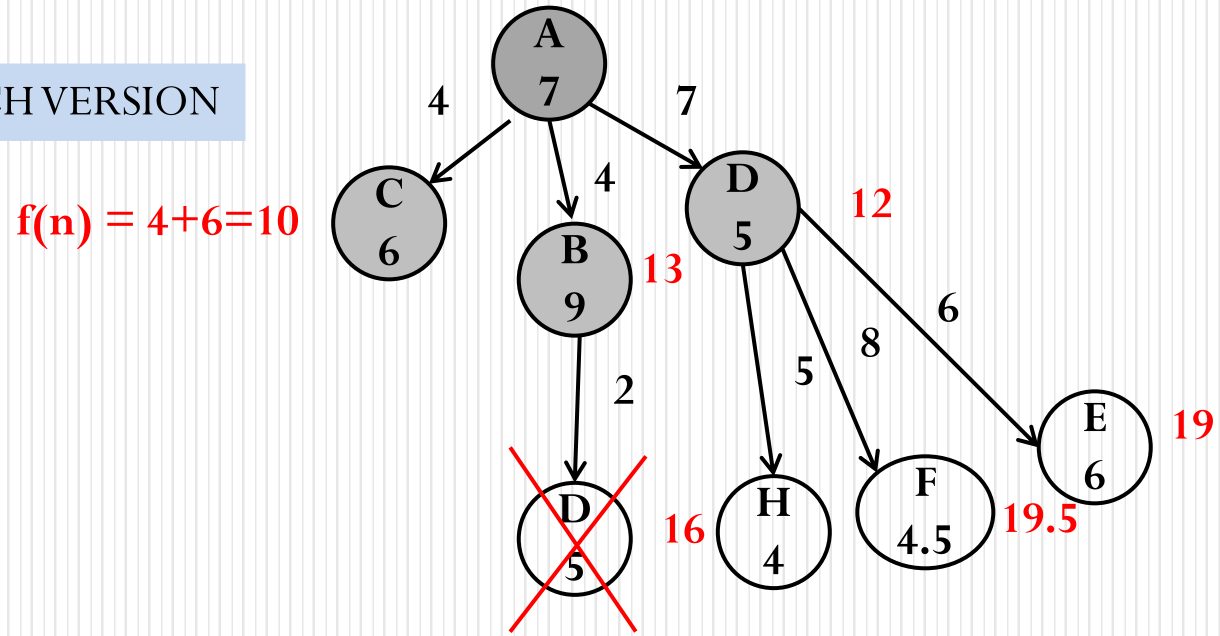

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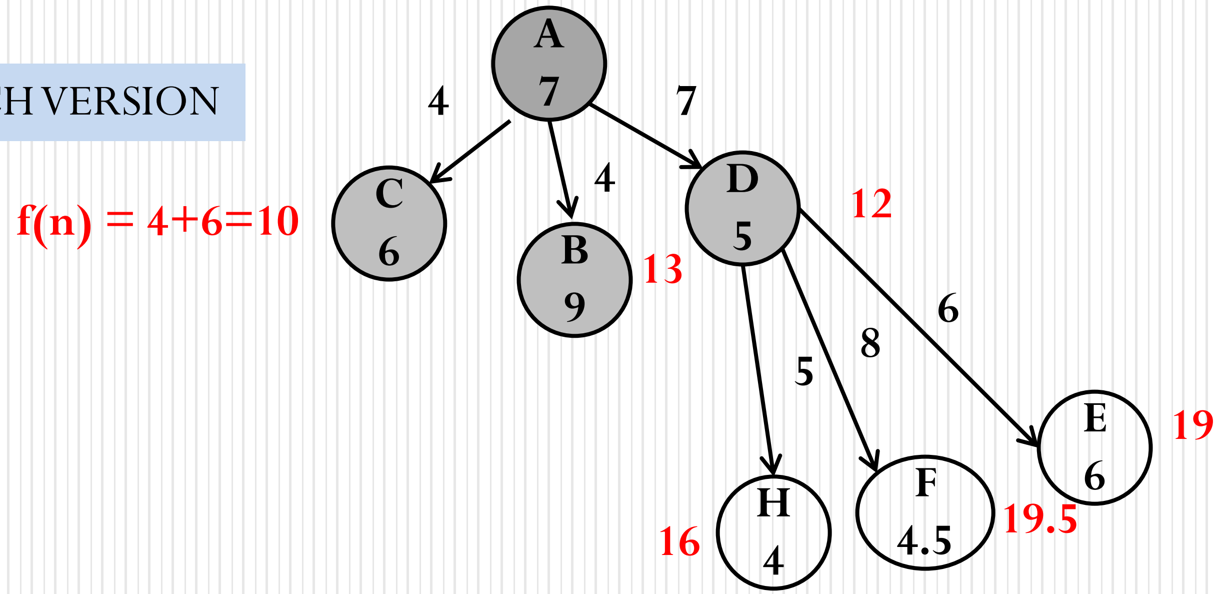

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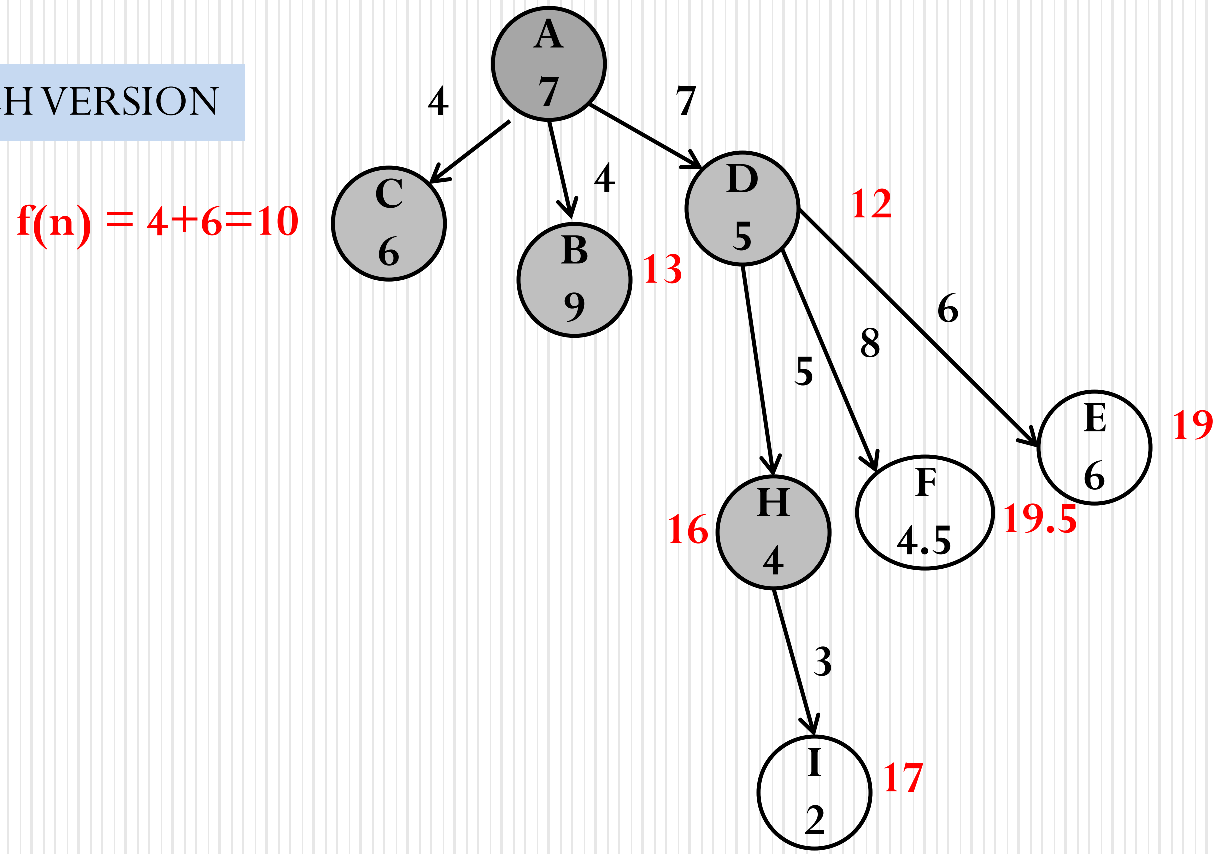

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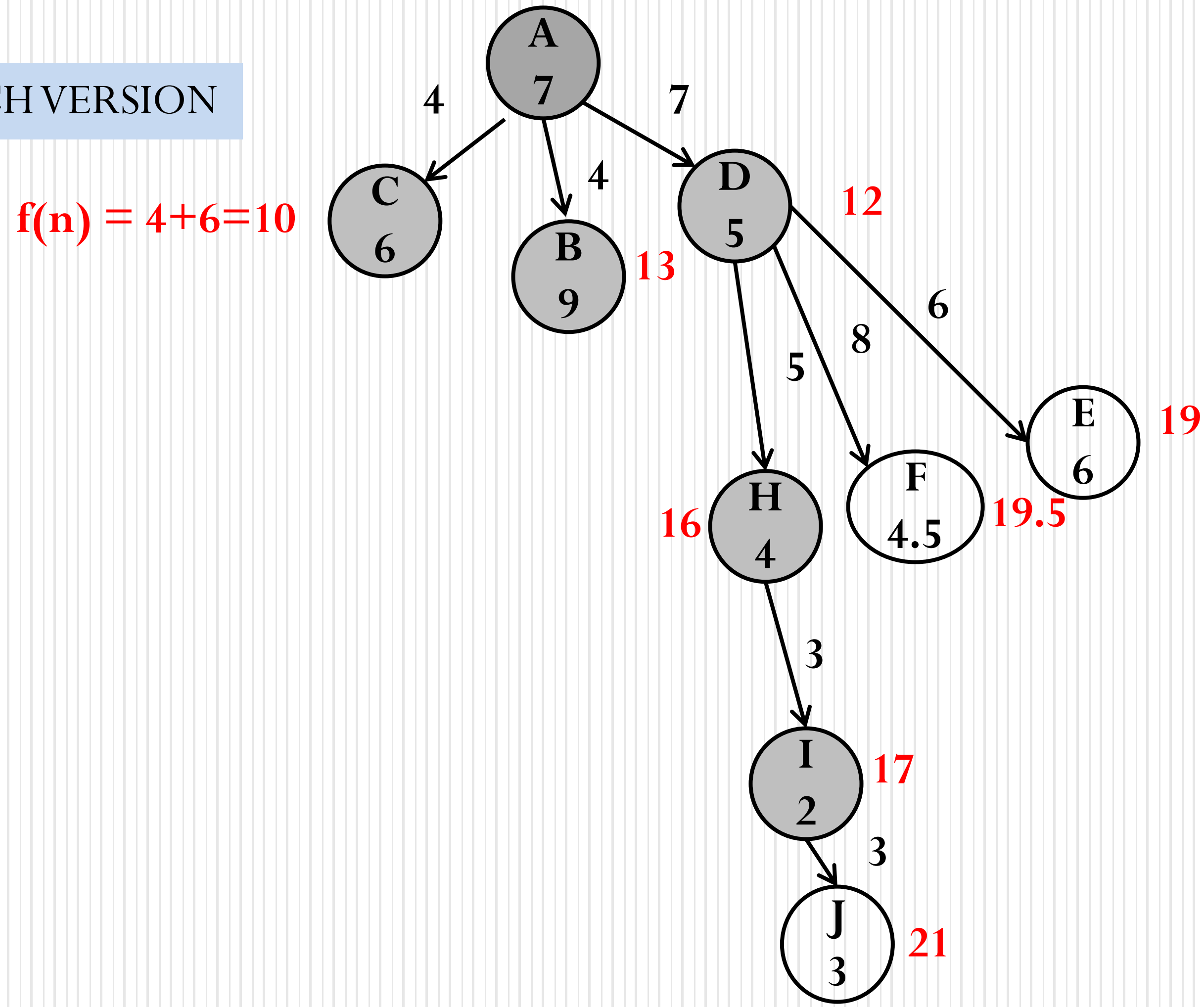
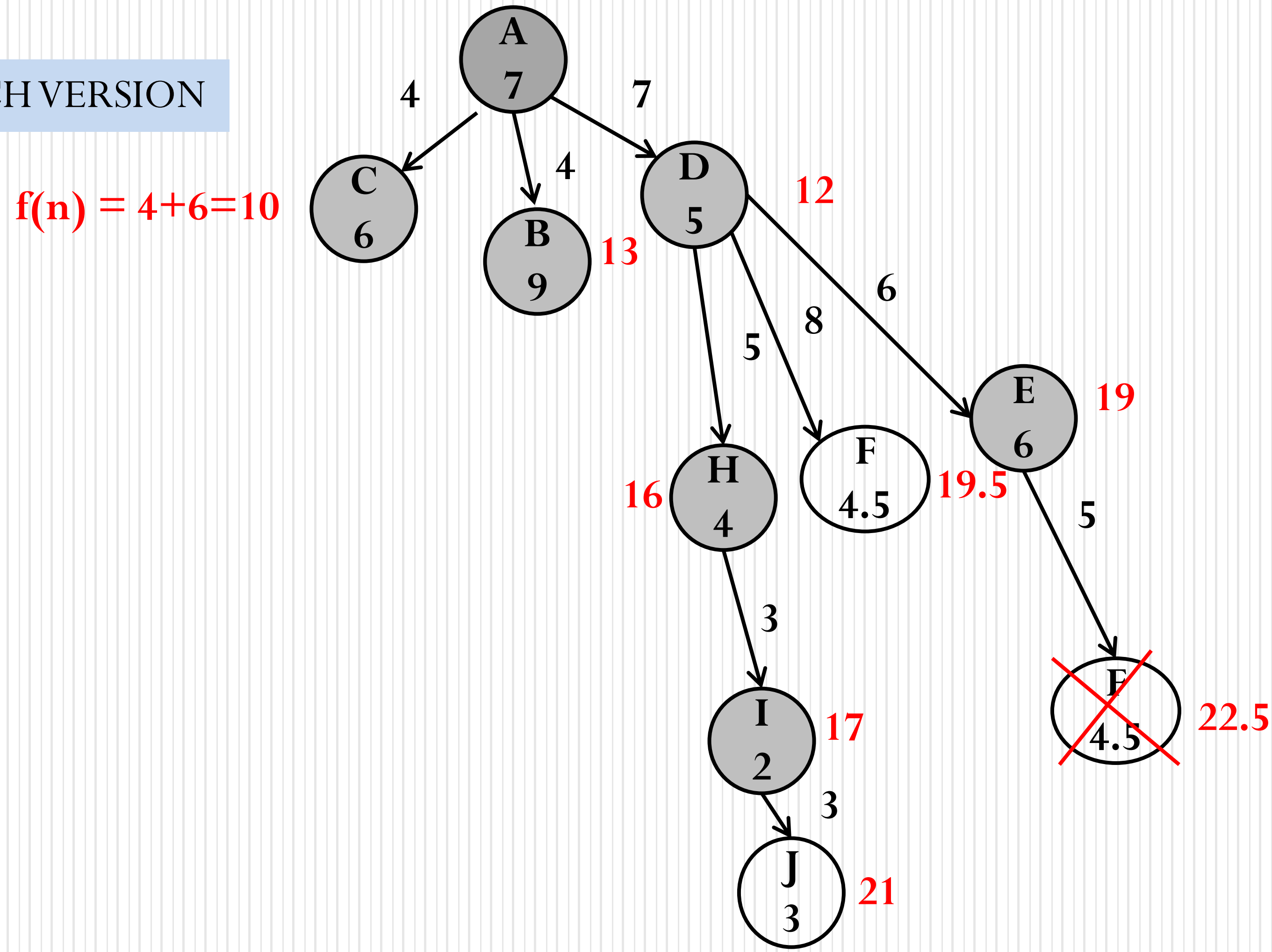


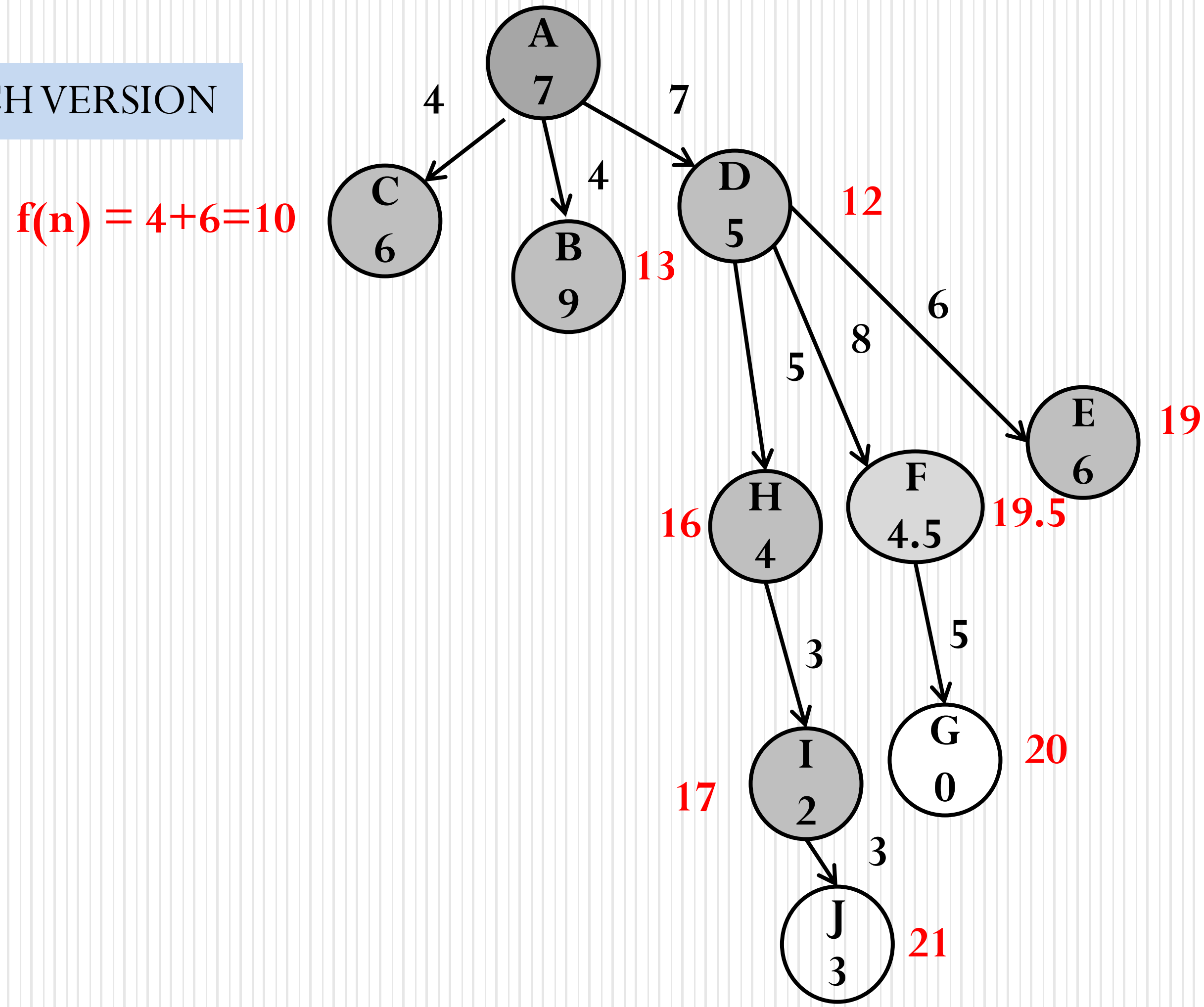
A * Example

GRAPH-SEARCH VERSION



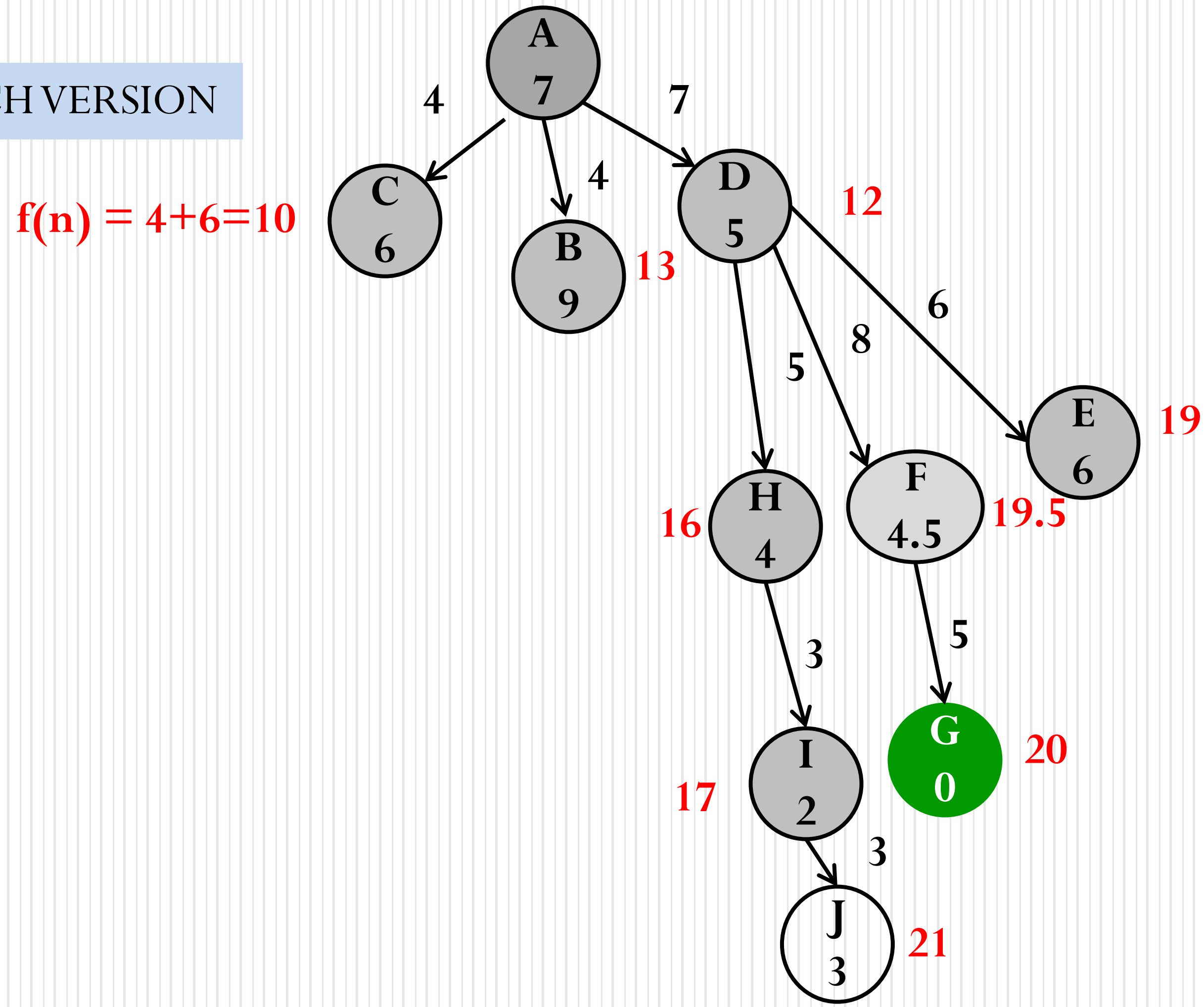
A * Example

GRAPH-SEARCH VERSION



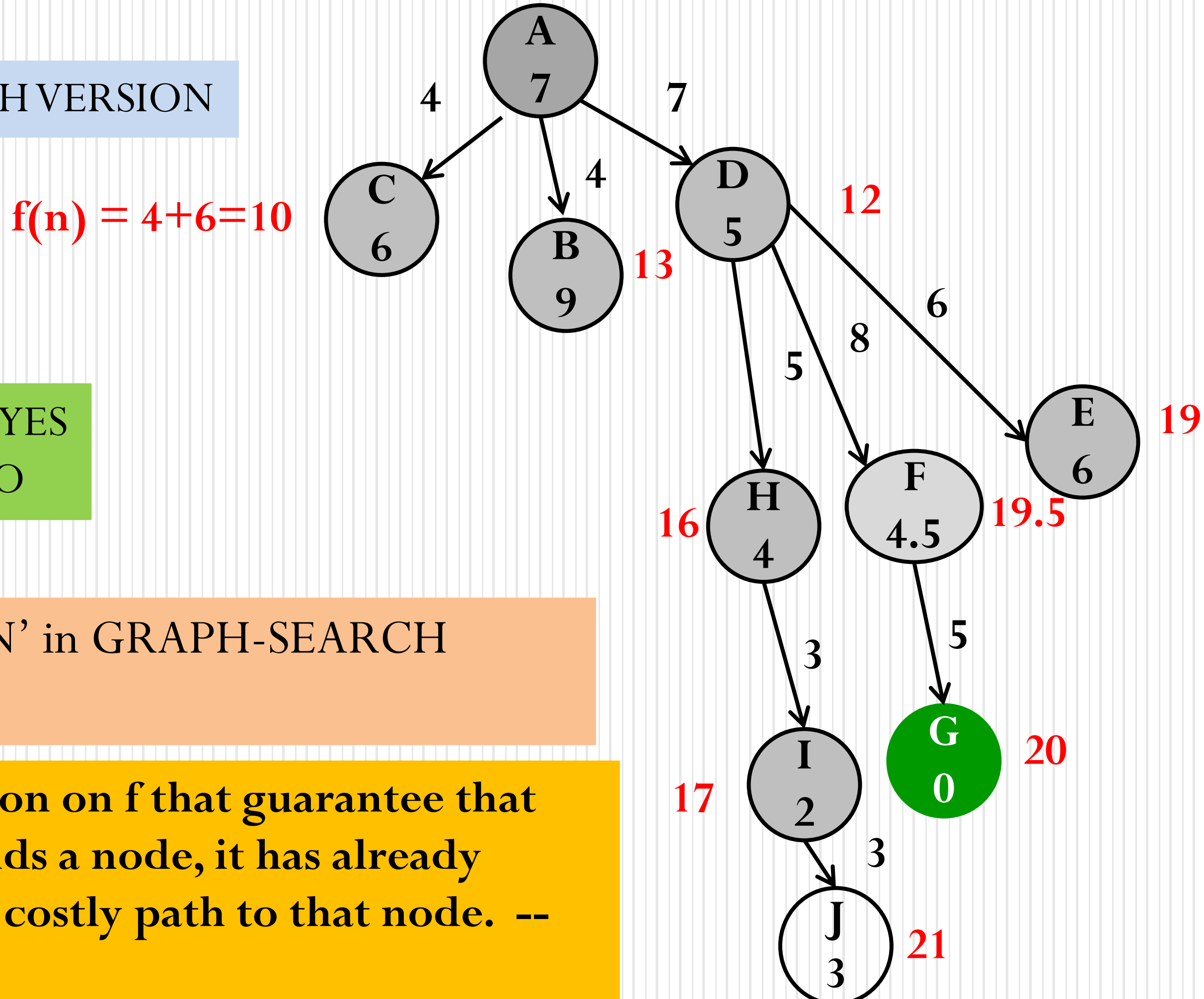
A * Example

GRAPH-SEARCH VERSION



A * Example

GRAPH-SEARCH VERSION



COMPLETE ? – YES
OPTIMAL ? - NO

Allow 'REOPEN' in GRAPH-SEARCH Version

Impose condition on f that guarantee that when A* expands a node, it has already found the least costly path to that node. -- CONSITENCY

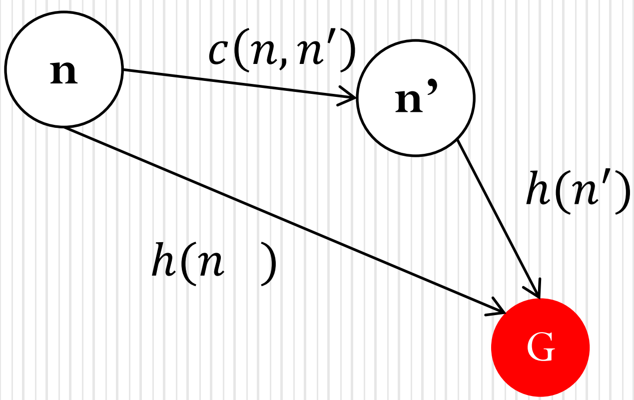
A* Search

- If n' is successor of n , h is **consistent** if $\forall\, n$, n'

$$h(n) \leq c(n, n') + h(n')$$

where $c(n, n')$ is the step cost from n to n'

i.e. the estimated cost of reaching the goal from n is no greater than the step cost of getting to n' plus the estimated cost of reaching the goal from n'

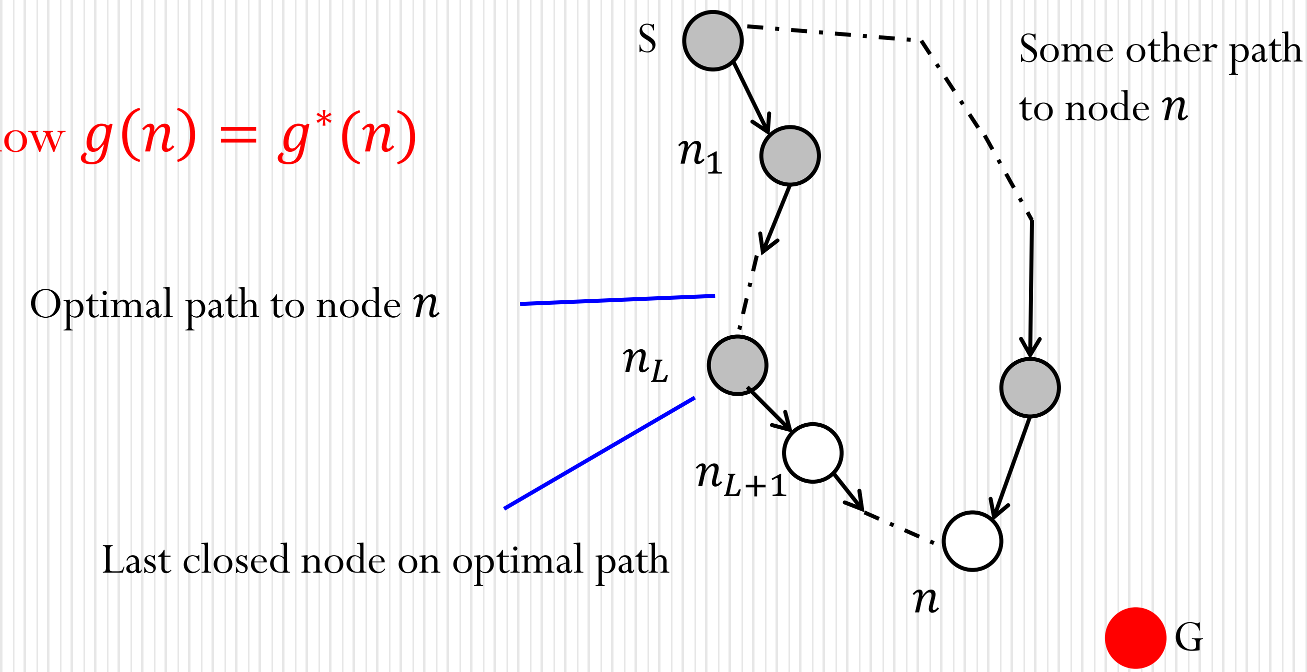


If $h(n)$ is consistent, then the values of f(n) along any path are non-decreasing

A* Search

- Theorem: if the consistency condition on h is satisfied, then when A* expands a node n, it has already found an optimal path to n.
- Proof:

  we will try to show $g(n) = g^*(n)$



A* Search

- Proof (contd.) : let A* is about to pick up node n with a value g(n). Let there be a (known) optimal path from S to n via n_L and n_{L+1}.
- Let A* expand node n with cost g(n).
- Let n_L be the last node on the optimal path from S to n that has been expanded.
- Let n_{L+1} be the successor of n_L that must be on OPEN.

- The following property holds
  - $h(n_L) \leq h(n_{L+1}) + C(n_L, n_{L+1})$
  - $h(n_L) + g(n_L) \leq h(n_{L+1}) + C(n_L, n_{L+1}) + g(n_L)$ (add $g(n_L)$)
  - $h(n_L) + g(n_L) \leq h(n_{L+1}) + g(n_{L+1})$
  - $h(n_L) + g^*(n_L) \leq h(n_{L+1}) + g^*(n_{L+1})$ (as both are in optimal path)

A* Search

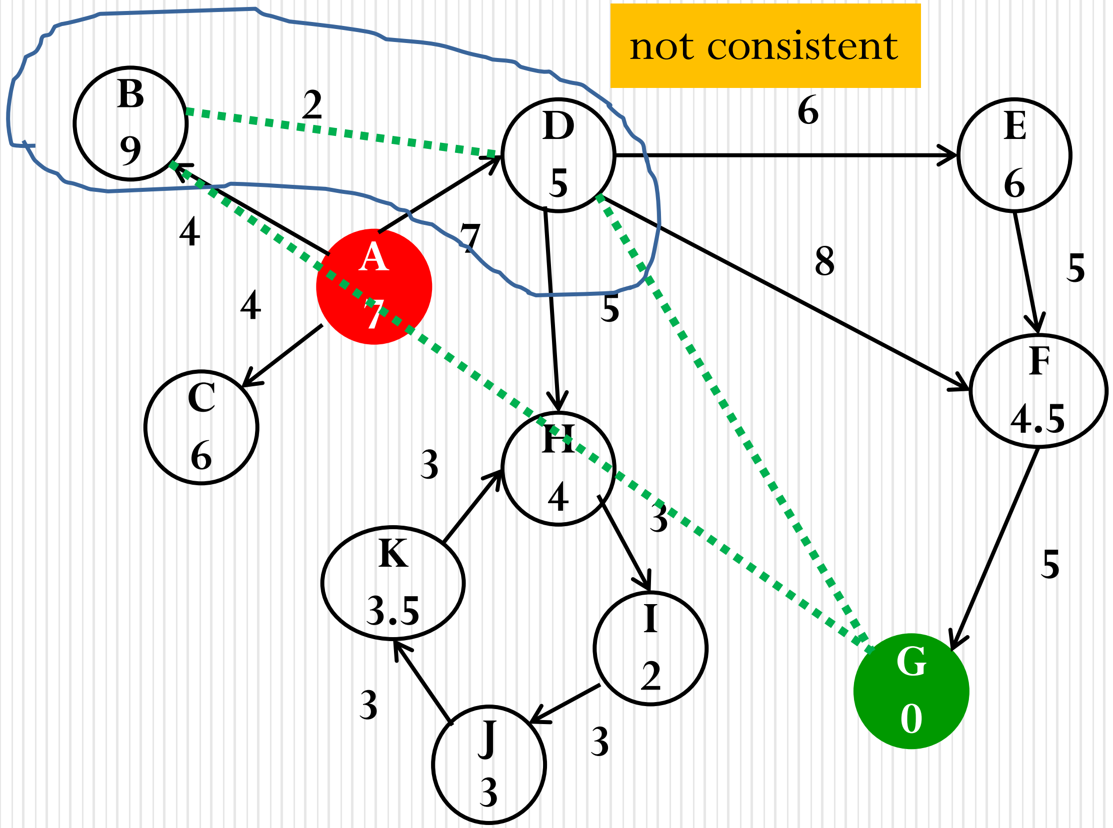
- By transitivity of $\leq$, the above property holds true for any two nodes on the optimal path, in particular it holds for n_{L+1} and node n.
  - $h(n_{L+1}) + g^*(n_{L+1}) \leq h(n)+g^*(n)$ **---------(1)**
  - $f(n_{L+1}) \leq h(n)+g^*(n)$ (because n_{L+1} is on the optimal path to n)
- But since A* is about to pick node n instead,
  - $f(n) \leq f(n_{L+1})$ or
  - $h(n) + g(n) \leq h(n_{L+1}) + g^*(n_{L+1})$ **---------(2)**

  Combining (1) and (2)
  - $h(n) + g(n) \leq h(n_{L+1}) + g^*(n_{L+1}) \leq h(n)+g^*(n)$
  - $h(n) + g(n) \leq h(n)+g^*(n)$
  - $g(n) \leq g^*(n)$

  g(n) cannot be less than g*(n) as g*(n) is the optimal cost.

  $\mathbf{g(n) = g^*(n)}$

A * Example



not consistent

h(D) can be changed to 8 for consistency property

Iterative Deepening A* Search

- Heuristic search can perform as bad as Breadth first search (specially in terms of space) if a *good* heuristic is not chosen.
- Basic idea of IDA* is to achieve similar benefits as iterative deepening DFS for heuristic search.
  - While finding minimal cost path the memory grows only linearly with depth of goal.
- Perform a depth-first search at each iteration with cut-off value as f cost.

Iteration 1

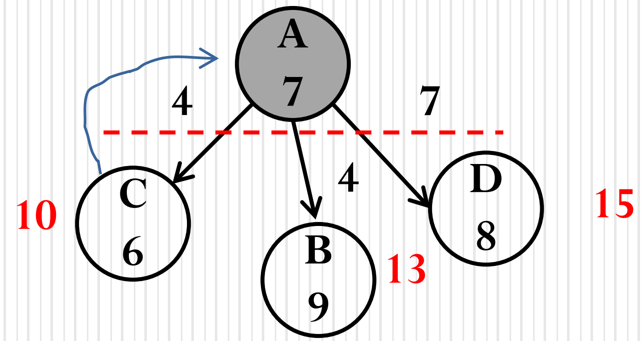
cost cut-off is $\mathrm{h}(n_0), n_0$ is the start node.

Successive Iterations

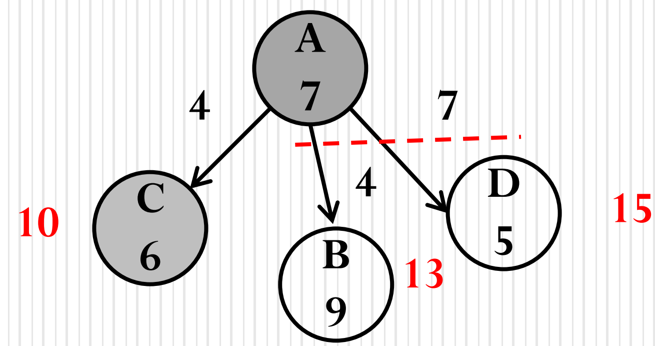
cut-off value is the smallest $\boldsymbol{f}$ cost of any node that exceeded the cut-off on previous iteration.

Iterative Deepening A*

- Expand nodes in depth-first fashion- backtracking whenever the f value of a successor of an expanded node exceeds the cut-off value



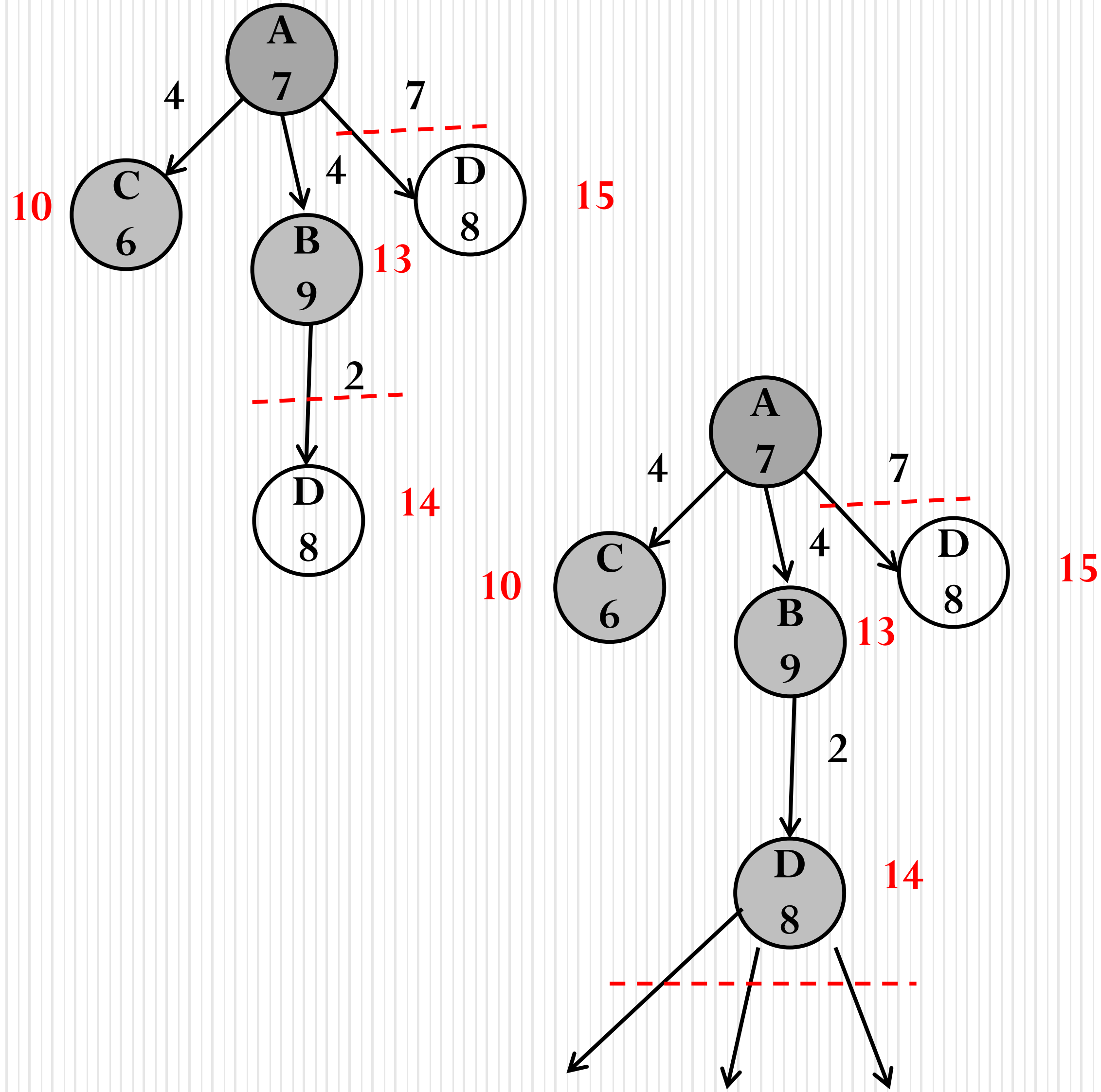
- **Iteration 1:**
  - cut-off = f(A) = h(A) = 7



- **Iteration 2:**
  - cut-off = f(C) = 10
  - Lowest of the f value of the nodes visited (but not expanded) in the previous DFS

Iterative Deepening A*

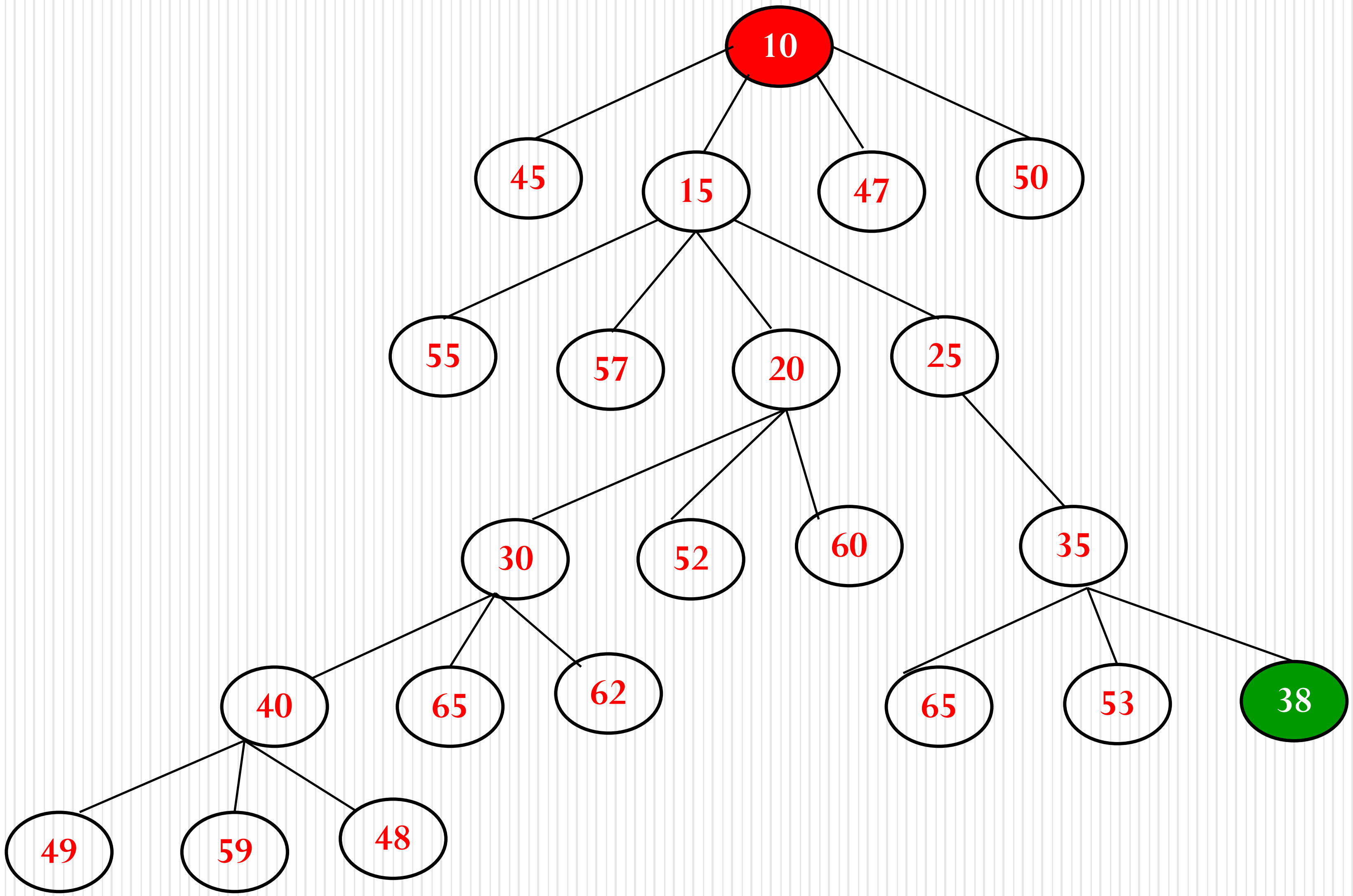
- **Iteration 3:**
  - cut-off = f(B) = 13
- **Iteration 4:**
  - cut-off = f(D) = 14



Recursive Best First Search

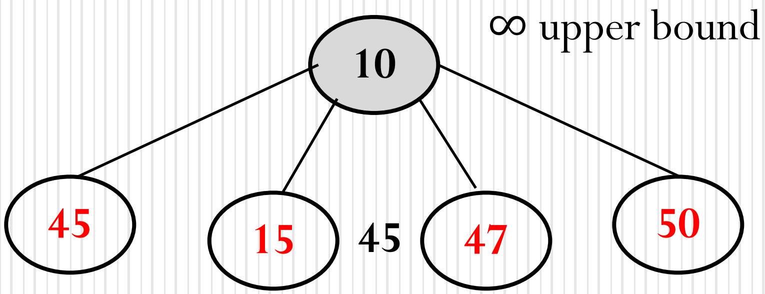
- Mimics best first search with linear space
- Similar to recursive depth-first but do not continue indefinitely down the current path
  - keeps track of the f value of the best alternative path available
  - if current f value exceeds this alternative f value then backtrack to alternative path
  - upon backtracking replace the f value of each node along the path with a ***backed-up*** value- the best f value of the children
- Like A* (Tree search) , RBFS is optimal if the heuristic is admissible.
- Space complexity is O(bd) and time complexity depends on the heuristic function and how often the best path changes as nodes are expanded.

RBFS Example



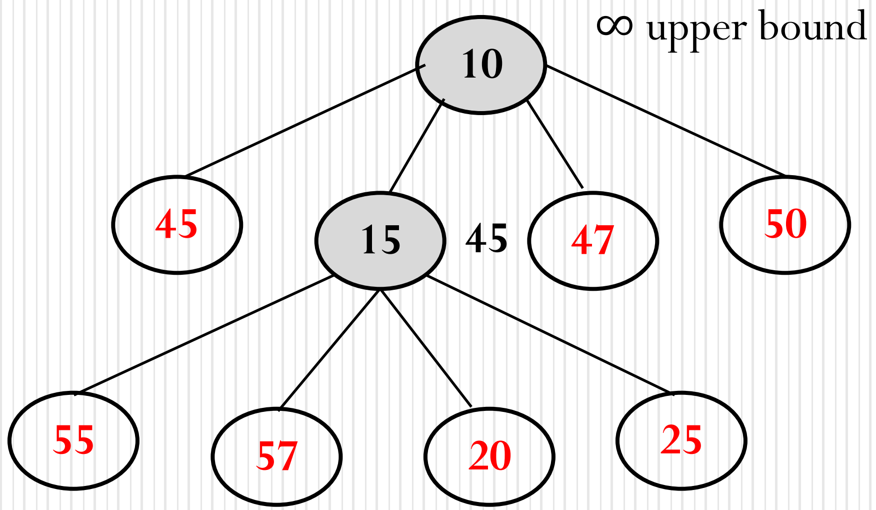
RBFS Example

Upperbound (n) =
min [upperbound(n's parent), current value of lowest cost brother]

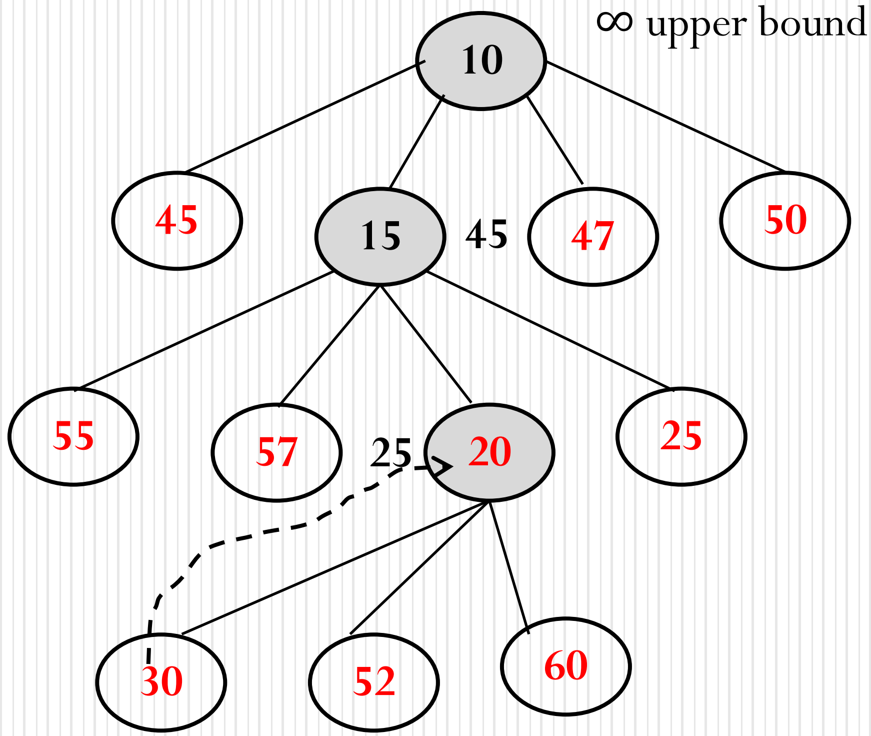


$f_{backed\text{-}up}(n) = f(n)$ /*if n is leaf*/
$f_{backed\text{-}up}(n) = f_{backed\text{-}up}(child)$,
child is successor of n /*if n is not a leaf node*/

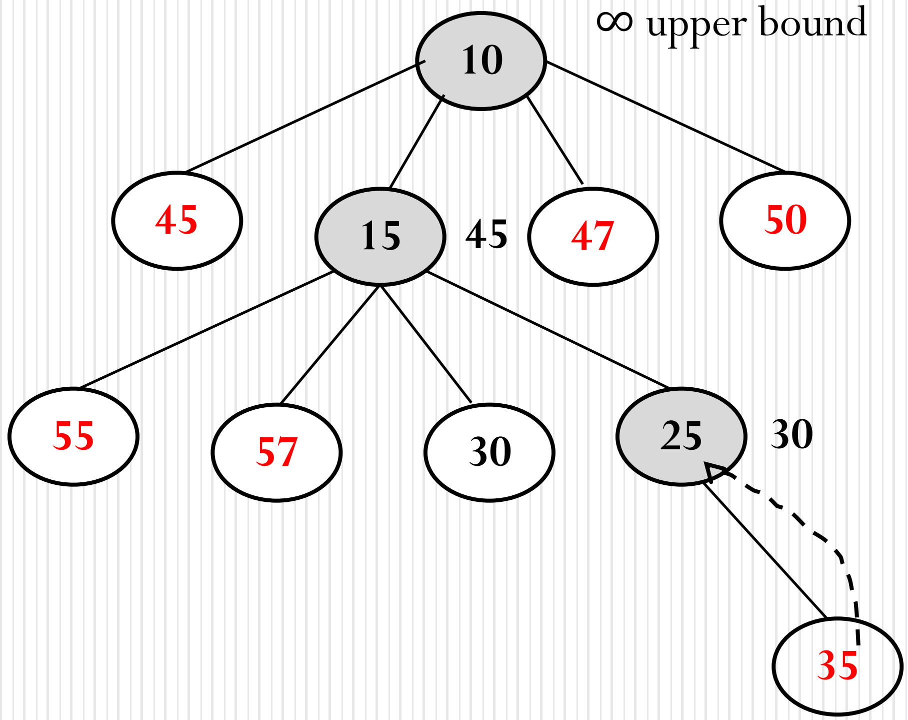
RBFS Example



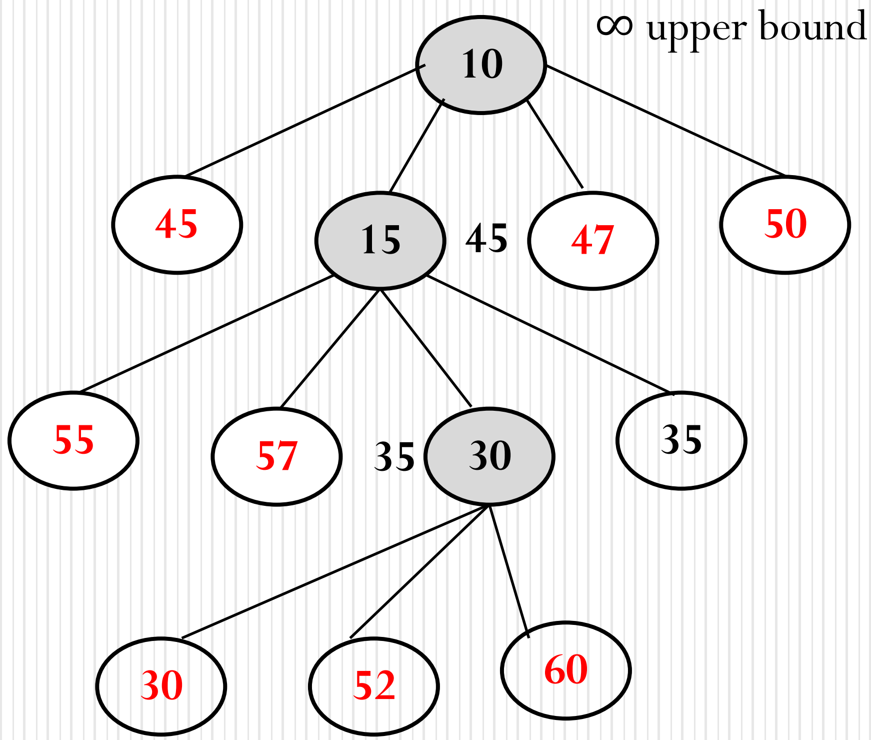
RBFS Example



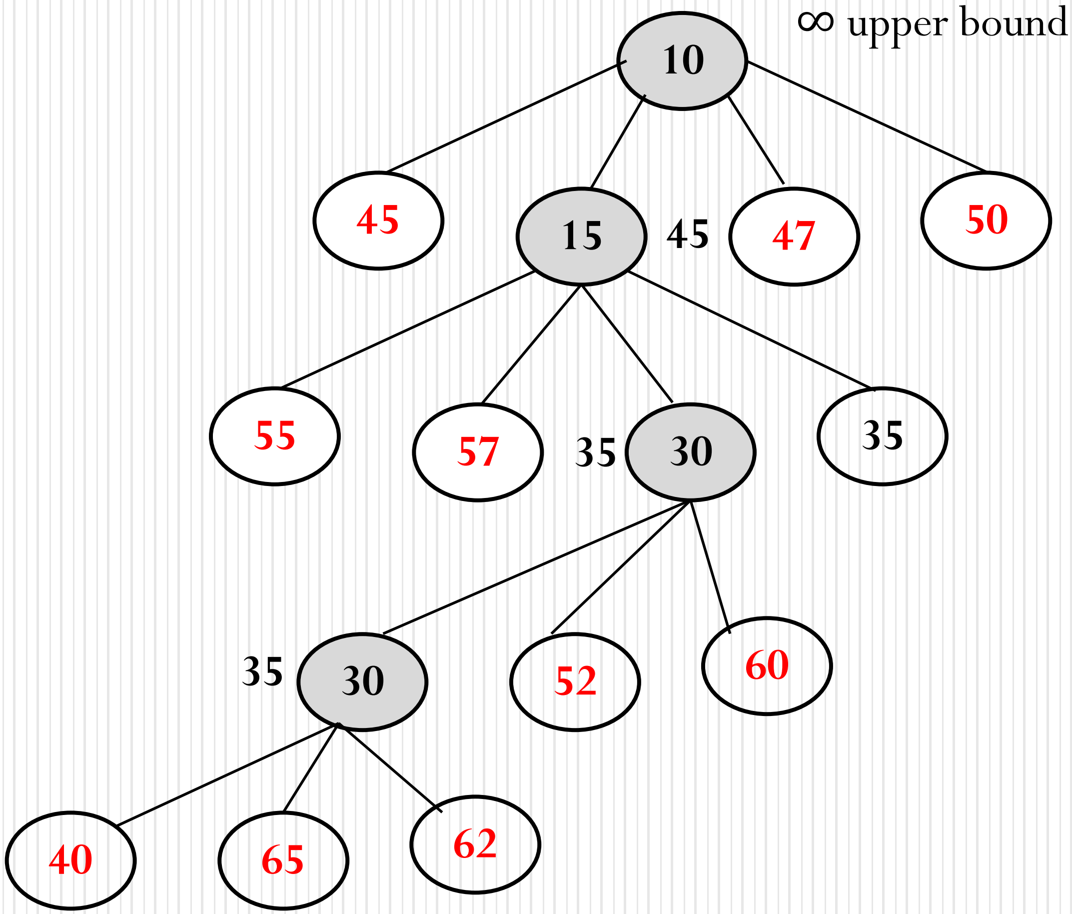
RBFS Example



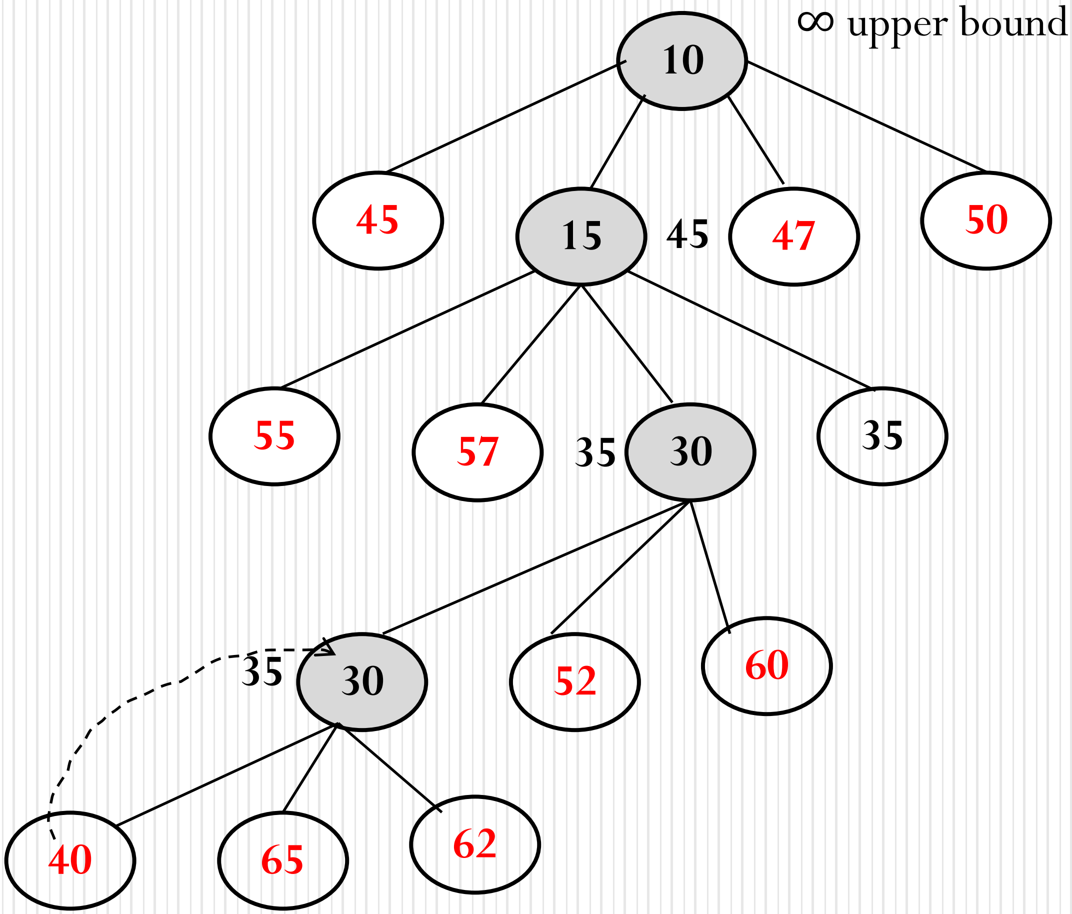
RBFS Example



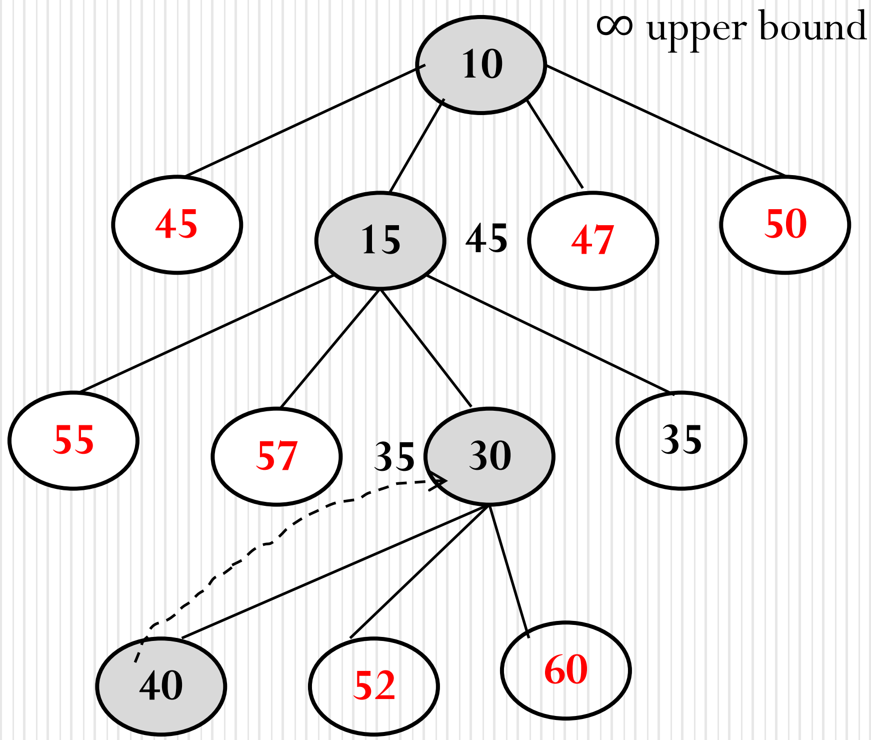
RBFS Example



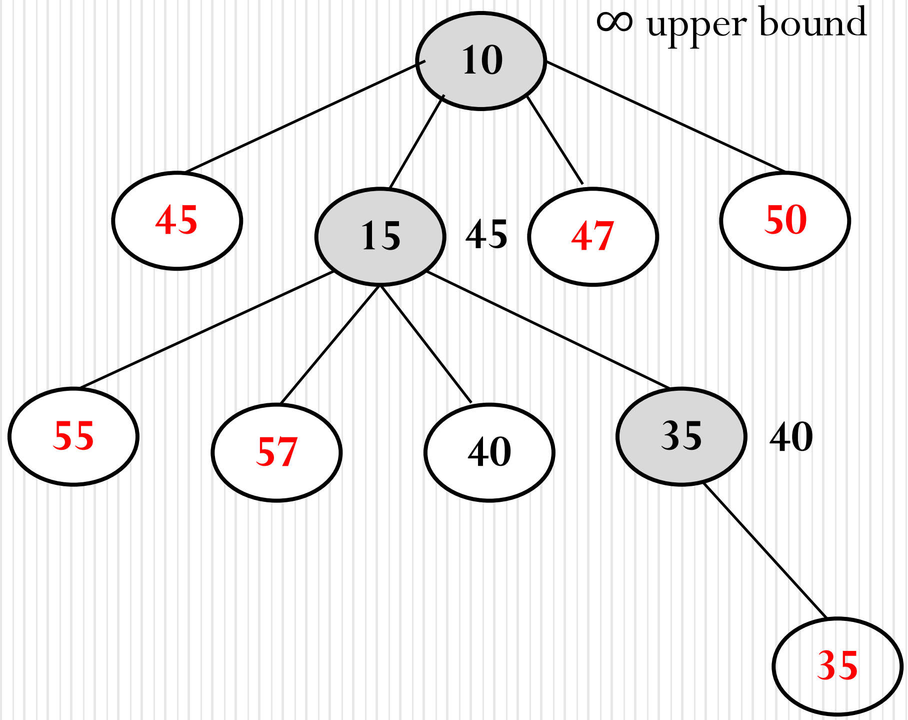
RBFS Example



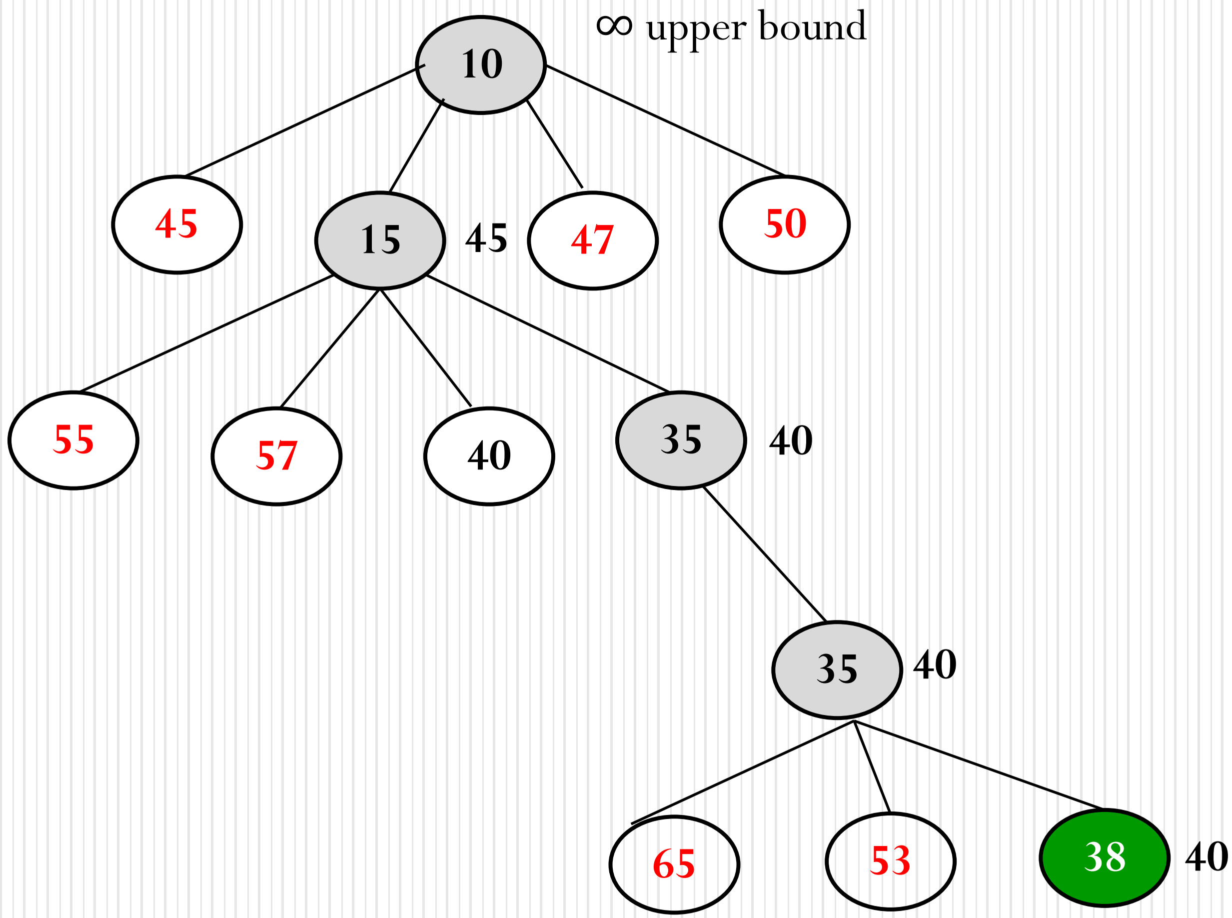
RBFS Example



RBFS Example



RBFS Example



More about Heuristics

- How to generate heuristics?
  - Relaxing problem definitions
  - Solution cost of a subproblem of a given problem
  - Learning from experience

- When more then one heuristic is available for the same problem, which one to choose?

Heuristics

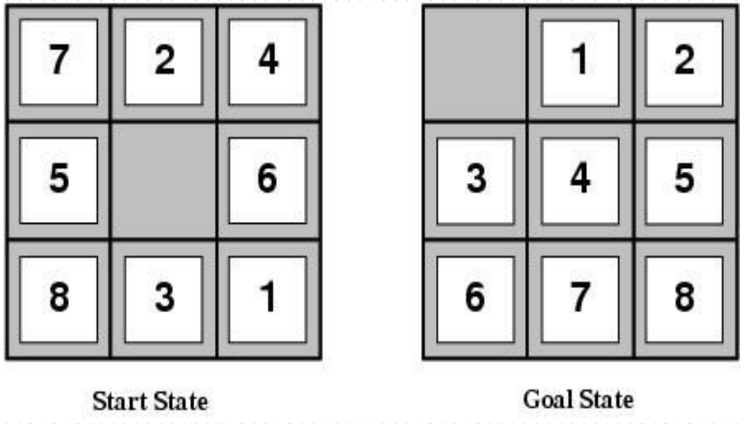
- Generating heuristics from Relaxed Problems
  - A problem with fewer restrictions on the actions is called a relaxed problem
  - The cost of an optimal solution to a relaxed problem is an admissible heuristic for the original problem.
- Example: 8-puzzle problem
  - A tile can move from square A to square B if A is horizontally or vertically adjacent to B and B is blank.
  - If the rules of the 8-puzzle are relaxed so that a tile can move anywhere, then $h_1(n)$ = number of misplaced tiles.

Heuristics

- If the rules are relaxed so that a tile can move to any adjacent square, then $h_2(n)$ = sum of horizontal and vertical distances of the tiles from their goal positions (Manhattan distance)

$h_1(n) = 8$, admissible because any tile that is out of place must be moved at least once.

$h_2(n) = 3+1+2+2+2+2+3+3+3+2 = 18$, admissible because any move can slide tile one step closer to the goal.



Heuristics

- Generating Heuristics from subproblems
  - Admissible heuristics can also be derived from the solution cost of a subproblem of a given problem.
  - This cost is a lower bound on the cost of the real problem.

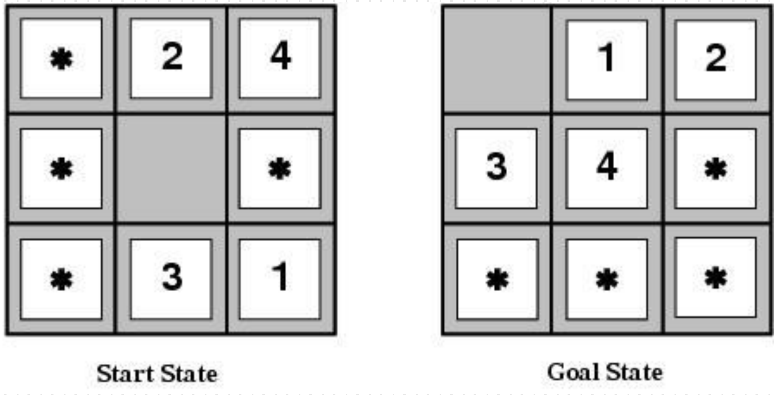


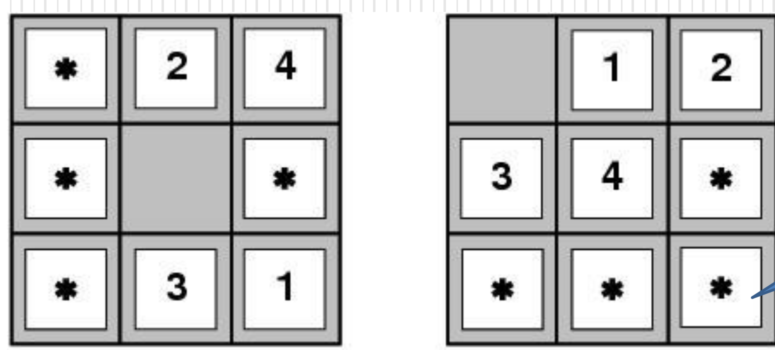
Fig. A subproblem of the 8-puzzle instance. The task is to get the tiles 1,2,3, and 4 into their correct positions without worrying about other tiles

Heuristics

- Pattern databases (PDB) store the exact solution costs for every possible subproblem instance (every possible configuration of four tiles and the blank).
  - The complete heuristic is constructed using the patterns in the DB

PDB consists of the set of all patterns which can be obtained by permutations of a target pattern

Pattern: partial specification of state i.e. tiles occupying certain states are unspecified



Start State   Goal State

Target Pattern: partial specification of goal state

Heuristics

- Learning Heuristics from Experience
  - Allow the agent to learn heuristic function h(n) from experience.
  - Experience – solving lots of 8-puzzles
  - Each optimal solution to an 8-puzzle problem = examples from which h(n) can be learned.
  - Example : (state from solution path, actual cost of solution path from that point)
  - Apply a learning algorithm to such examples to construct a function h(n) that predicts solution costs for other states that arise during search.

state1	**g(state1)**
state2	**g(state2)**

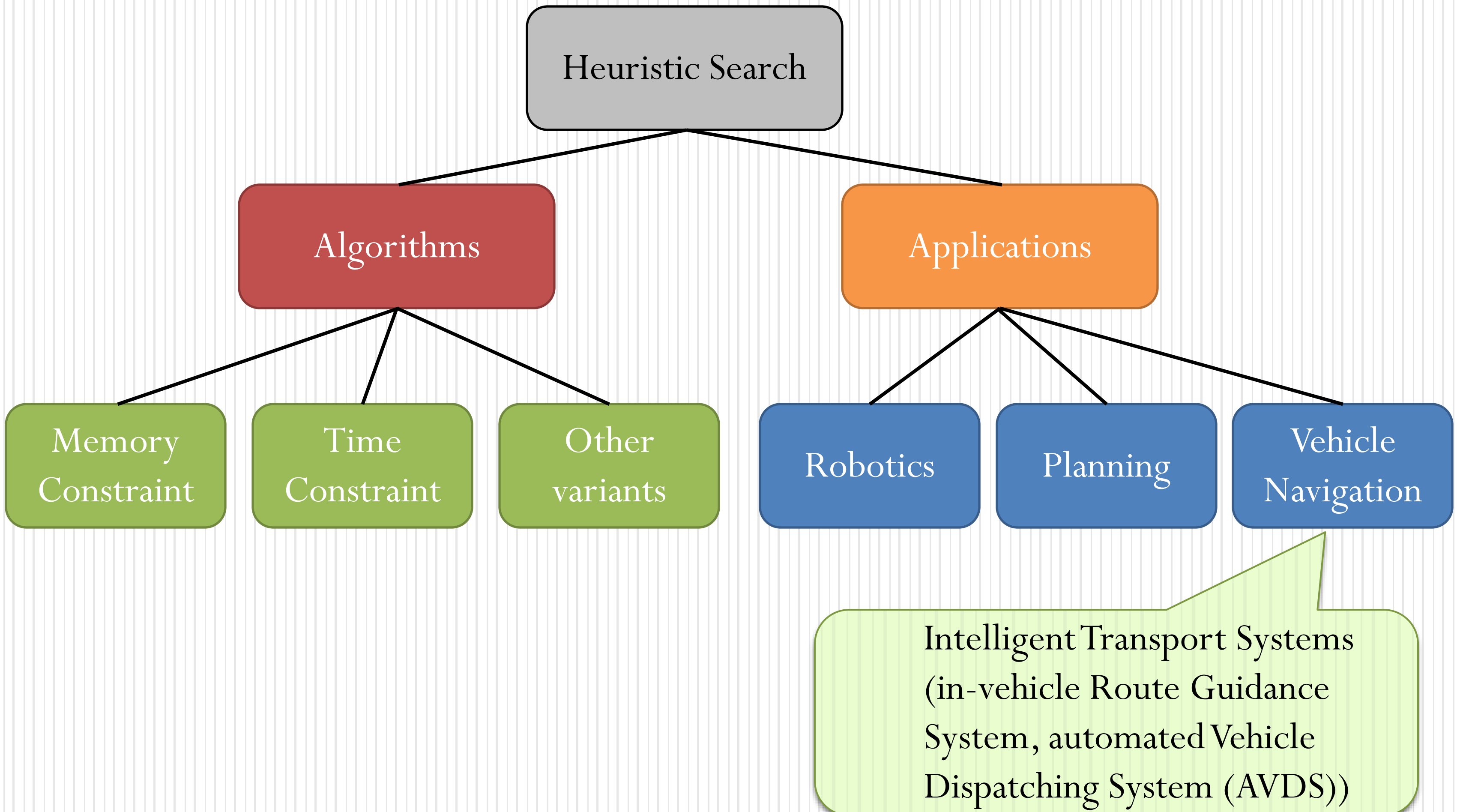
feature 11	**feature 12**	**g(state1)**
feature 21	**feature 22**	**g(State2)**

Heuristics

- Could try to learn a heuristic function based on "features" rather than state itself
  - For example, $x1(n)$ = number of misplaced tiles, $x2(n)$ = number of goal-adjacent-pairs that are currently adjacent
  - $h(n) = w_1\ x1(n) + w_2\ x2(n)$
    - Weights could be learned to identify which features are predictive of path cost
- **More than one heuristic for the same problem?**
  - If for any node n, $h_2(n) > h_1(n)$ then we say that $h_2(n)$ dominates $h_1(n)$
  - use h_2 provided it is consistent and that the computation time for the heuristic is not too long.
  - Given h_1, h_2, ..h_m, if none of them dominates the other then use

    $$h(n) = \max\{h_1(n), \ldots, h_m(n)\}$$

Heuristic search: Trends



Summary

- We discussed various informed search strategies that an agent can use to select actions before execution.
- Heuristics and heuristic (informed) search: Greedy Best-first search, A* search, IDA* , RBFS
- Performance of search strategies in terms of completeness, optimality, time complexity, and space complexity.
- How to generate admissible heuristics?