

Investigate the exponential distribution in R and compare it with the Central Limit Theorem

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November 04 2023

Overview

The purpose of this data analysis is to investigate the exponential distribution and compare it to the Central Limit Theorem. For this analysis, the λ will be set to 0.2 for all of the simulations. This investigation will compare the distribution of averages of 40 exponentials over 1000 simulations.

Simulations

Set the simulation variables λ , exponentials, and seed.

```
ECHO=TRUE
set.seed(1337)
lambda = 0.2
exponentials = 40
```

Run Simulations with variables

```
simMeans = NULL
for (i in 1 : 1000) simMeans = c(simMeans, mean(rexp(exponentials, lambda)))
```

Sample Mean versus Theoretical Mean

Sample Mean

Calculating the mean from the simulations will give the sample mean.

```
mean(simMeans)
```

```
## [1] 5.055995
```

Theoretical Mean

The theoretical mean of an exponential distribution is λ^{-1} .

λ^{-1}

```
## [1] 5
```

Comparison

There is only a slight difference between the simulations sample mean and the exponential distribution theoretical mean.

```
abs(mean(simMeans)-lambda^-1)
```

```
## [1] 0.05599526
```

Sample Variance versus Theoretical Variance

Sample Variance

Calculating the variance from the simulation means will give the sample variance.

```
var(simMeans)
```

```
## [1] 0.6543703
```

Theoretical Variance

The theoretical variance of an exponential distribution is $(\lambda * \sqrt{n})^{-2}$.

```
(lambda * sqrt(exponentials))^-2
```

```
## [1] 0.625
```

Comparison

There is only a slight difference between the simulations sample variance and the exponential distribution theoretical variance.

```
abs(var(simMeans)-(lambda * sqrt(exponentials))^-2)
```

```
## [1] 0.0293703
```

Distribution

This is a density histogram of the 1000 simulations. There is an overlay with a normal distribution that has a mean of λ^{-1} and standard deviation of $(\lambda * \sqrt{n})^{-1}$, the theoretical normal distribution for the simulations.

```
library(ggplot2)
ggplot(data.frame(y=simMeans), aes(x=y)) +
  geom_histogram(aes(y=..density..), binwidth=0.2, fill="#0072B2",
    color="black") +
  stat_function(fun=dnorm, arg=list(mean=lambda^-1,
    sd=(lambda*sqrt(exponentials))^-1),
    size=2) +
  labs(title="Plot of the Simulations", x="Simulation Mean")
```

