

# Introduction to Machine Learning part 2

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- Performance metrics
- Generalisation and overfitting
- Train, validate, test

# Performance Metrics: Classification

- True Positives (TP): The class is + and the prediction is +
- True Negatives (TN): The class is - and the prediction is -
- False Positive (FP): The class is - and the prediction is +
- False Negative (FN): The class is + and the prediction is -

|        |     | Predicted |           |
|--------|-----|-----------|-----------|
|        |     | Dog       | Cat       |
| Actual | Dog | <u>TP</u> | <u>FN</u> |
|        | Cat | <u>FP</u> | <u>TN</u> |

# Accuracy

■  $Accuracy = \frac{TP+TN}{TP+TN+FP+FN}$

$acc = \frac{n \text{ correctly classified}}{n}$

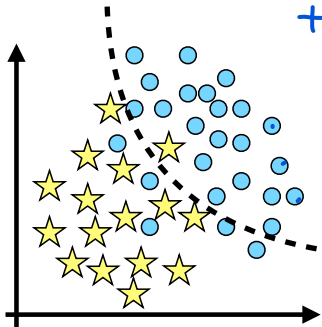
|        |     | Predicted |           |
|--------|-----|-----------|-----------|
|        |     | Dog       | Cat       |
| Actual | Dog | <u>TP</u> | FN        |
|        | Cat | FP        | <u>TN</u> |

# Precision

■  $Precision = \frac{TP}{TP+FP}$

$$\frac{21}{21 + 1} \approx \frac{21}{22}$$

|        |           |     |
|--------|-----------|-----|
| Actual | Predicted |     |
|        | Dog       | Cat |
| Dog    | TP        | FN  |
| Cat    | FP        | TN  |

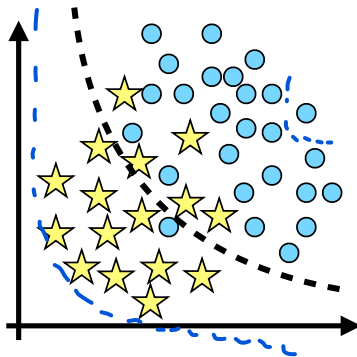


# Recall

■  $Recall = \frac{TP}{TP+FN}$

$$\frac{25}{26}$$

| Actual | Predicted |     |
|--------|-----------|-----|
|        | Dog       | Cat |
| Dog    | TP        | FN  |
| Cat    | FP        | TN  |



The  $F_1$  score is the harmonic mean of precision and recall

$$\begin{aligned} F_1 &= \frac{1}{precision^{-1} + recall^{-1}} \\ &= 2 \frac{precision \cdot recall}{precision + recall} \end{aligned}$$

# Metrics for more than two classes

$$\text{Accuracy} = \frac{\sum_{i=1}^n \mathbb{1}(y_i = f(\vec{x}_i))}{n} = \frac{\text{Correct class}^n}{n}$$

- This is the same as the binary case, even though the equation looks different

| Actual | Predicted |          |           |
|--------|-----------|----------|-----------|
|        | Dog       | Cat      | Bird      |
| Dog    | <u>10</u> | 2        | 0         |
| Cat    | 1         | <u>5</u> | 1         |
| Bird   | 2         | 0        | <u>12</u> |

$$10 + 5 + 12 = 27$$

$$\text{acc} = \frac{27}{33}$$



# Metrics for more than two classes

- Macro-averaging:

$$\text{Precision} = \frac{1}{k} \sum_{j=1}^k \text{Precision}_j \quad \text{Recall} = \frac{1}{k} \sum_{j=1}^k \text{Recall}_j$$

Predicted

|        | Dog | Cat | Bird |
|--------|-----|-----|------|
| Actual |     |     |      |
| Dog    | 10  | 2   | 0    |
| Cat    | 1   | 5   | 1    |
| Bird   | 2   | 0   | 12   |

$$\text{precision}_{\text{dog}} = \frac{10}{13}$$

Predicted

|        | Dog | Cat | Bird |
|--------|-----|-----|------|
| Actual |     |     |      |
| Dog    | 10  | 2   | 0    |
| Cat    | 1   | 5   | 1    |
| Bird   | 2   | 0   | 12   |

$$\text{Recall}_{\text{dog}} = \frac{10}{12}$$

# Metrics for more than two classes

## ■ Micro-averaging

|        |      | Predicted |     |      |
|--------|------|-----------|-----|------|
|        |      | Dog       | Cat | Bird |
| Actual | Dog  | 10        | 2   | 0    |
|        | Cat  | 1         | 5   | 1    |
|        | Bird | 2         | 0   | 12   |
|        |      |           |     |      |

Handwritten annotations on the confusion matrix:

- TP** (True Positive) is written above the Dog-Dog cell.
- FN** (False Negative) is written to the right of the Dog-Cat and Dog-Bird cells.
- FP** (False Positive) is written below the Cat-Dog and Bird-Dog cells.
- TN** (True Negative) is written to the right of the Bird-Bird cell.
- A large blue bracket spans the bottom of the matrix, encompassing the FP and TN regions.

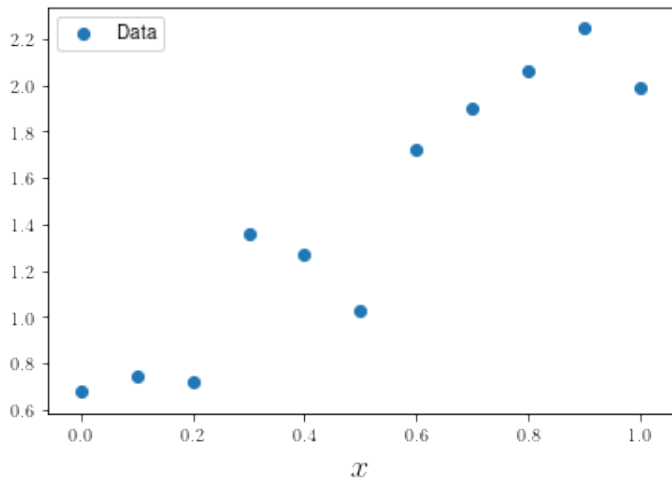
|        |       | Predicted |       |
|--------|-------|-----------|-------|
|        |       | D         | non D |
| Actual | D     | 10        | 2     |
|        | non D | 3         | 18    |

|        |       | B  | non B |
|--------|-------|----|-------|
| Actual | B     | 12 | 2     |
|        | non B | 1  | 18    |

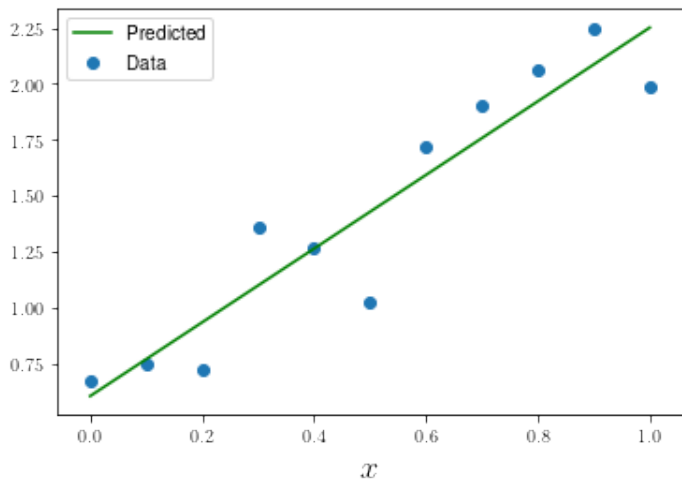
|        |       | C | non C |
|--------|-------|---|-------|
| Actual | C     | 5 | 2     |
|        | non C | 2 | 24    |

|        |   | +  | -  |
|--------|---|----|----|
| Actual | + | 27 | 6  |
|        | - | 6  | 60 |

# Performance Metrics: Regression



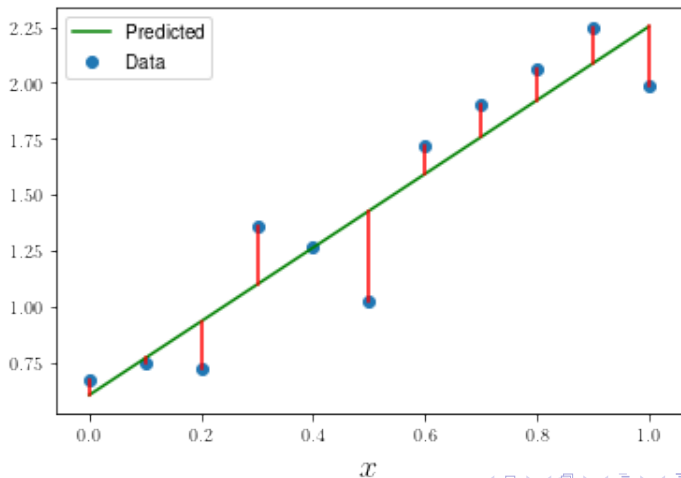
# Performance Metrics: Regression



# Performance Metrics: Regression

- Mean Squared Error (MSE):

$$\underline{E} = \frac{1}{N} \sum_{(\vec{x}, y)} (y - f(\vec{x}))^2$$

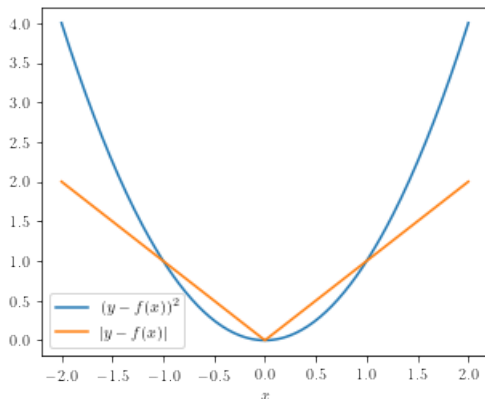


# Performance Metrics: Regression

- Root Mean Squared Error (RMSE):

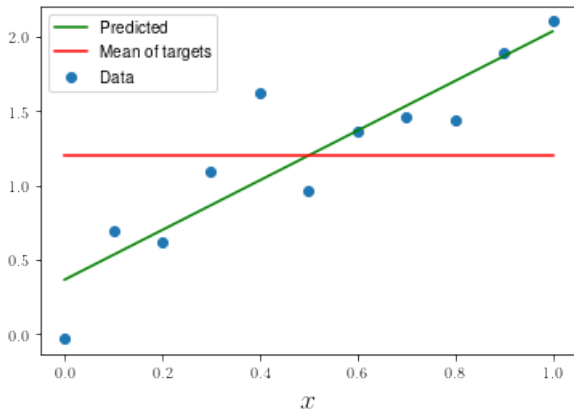
$$E = \sqrt{\frac{1}{N} \sum_{(\vec{x}, y)} (y - f(\vec{x}))^2}$$

- Mean Absolute Error (MAE):  $E = \frac{1}{N} \sum_{(\vec{x}, y)} \underline{|y - f(\vec{x})|}$



# Performance Metrics: Regression

- $R^2 = 1 - \frac{\sum_{(\vec{x}, y)} (y - f(\vec{x}))^2}{\sum_{(\vec{x}, y)} (y - \bar{y})^2}$   
where  $\bar{y}$  is the mean of the target values  $y$



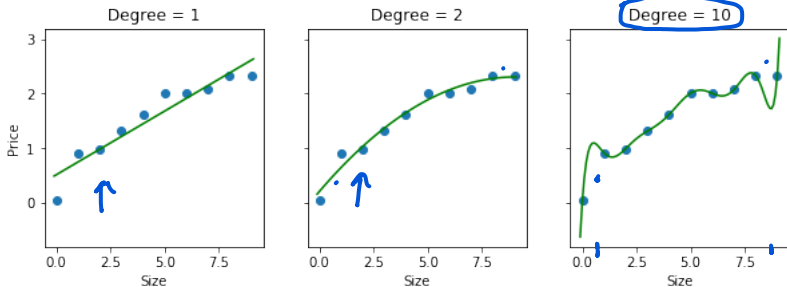
# Summary: Performance Metrics

- For classification problems, we can use metrics such as accuracy, precision, recall, or  $F_1$  score.
- These can be used for multi-class problems as well as binary problems
- Mean squared error or absolute error can be used for regression problems
- $R^2$  is often used for linear regression



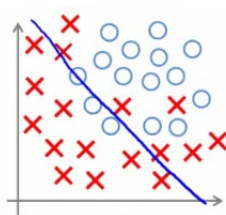
# Generalisation

- By fitting an approximation function with a high number of parameters it is possible to obtain very high accuracy for the data on which you train.
- However, this learnt approximation may not then perform well on different, previously unseen, data.
- This is called over-fitting



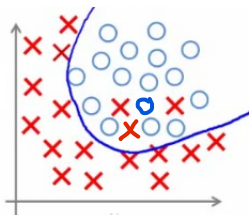
# Over-Fitting in Classification

- For classification problems we must be careful how we model the boundary between different classes in the feature space.

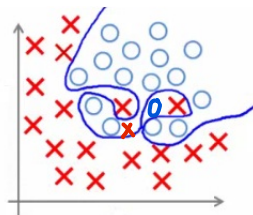


**Under-fitting**

(too simple to  
explain the  
variance)



**Appropriate-fitting**



**Over-fitting**

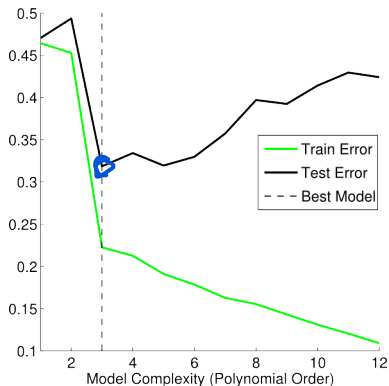
(forcefitting -- too  
good to be true)

# Summary: Generalisation and overfitting

- As well as attaining good scores on our training data, we want our algorithms to generalise well to unseen data
- Both regression and classification problems can overfit to data

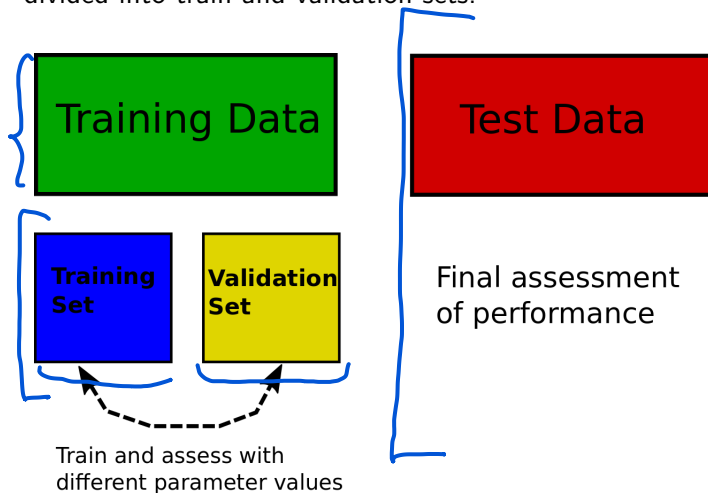
# Train and Test Data

- To evaluate the generalisation of a model available data should be divided into training and test sets.



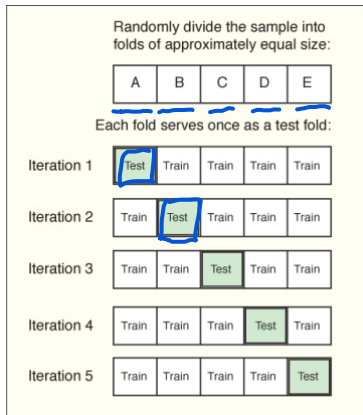
# Train, Validation and Test Data

- To fit model parameters the training data is sometimes further divided into train and validation sets.



# n-Folds

- How do you know that you have not just been lucky in your choice of training and test data?
- Answer: Repeat many times with different divisions into training and test.



Cross-validation  
can be used  
with the validation  
set.

- Splitting data into training and test sets helps ensure that our model can generalise.
- We can further split our training set into training and validation sets, to test the performance of our model on unseen data
- Cross-validation can give a more rounded view of performance

# Overall summary

We have looked at:

- Performance metrics
- Generalization and overfitting
- Training and test splits

You can practice working with these concepts in the...



- Available as pdf and also as jupyter notebook
- Covers evaluation metrics, generalisation, overfitting
- If you don't already know Python, please do work through the Introduction to Python available on BlackBoard
- You can ask questions in the lecture or in problem classes

## Simple supervised learning algorithms

- k-Nearest neighbours
- Linear regression
- Naive Bayes classifier