Summative Assessment EMATM0061 - Part B

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Section B

B.1 Faulty Security System Probability

Given

####Assumption - passage of time is per minute p^o - conditional probability that the sensor makes a sound if there is no person within one meter of the gate
p^1 - conditional probability that the sensor makes at least once, if there is at least one person present
q - probability that at least one person walks within one meter of the gate
ϕ - conditional probability that at one person has passed during the minute, and the alarm made the sound.

therefore, q^o - probability that nobody walks within one meter of the gate

Part (a):

Function used ->

$\phi = (p^1/q)/(p^o/q^o+p^1/q)$

```
c_prob_person_given_alarm = function(p0,p1,q){

  (p1/q) / ((p0/(1-q)) + (p1/q))

}
```

Part (b):

Value of ϕ
Given: $p^o = 0.05$,
$p^1 = 0.95$,
$q = 0.1$

```
phi <- c_prob_person_given_alarm(0.05,0.95,0.1)

print(phi)
```

```
## [1] 0.994186
```

Part (c):

Graph between ϕ and q for varying value of q
Given : $p^o = 0.05$,
$p^1 = 0.95$

```
library(ggplot2)

x1<-seq(from=0, to=1, by=.01)

print(x1)
```

```
##   [1] 0.00 0.01 0.02 0.03 0.04 0.05 0.06 0.07 0.08 0.09 0.10 0.11 0.12 0.13 0.14
##  [16] 0.15 0.16 0.17 0.18 0.19 0.20 0.21 0.22 0.23 0.24 0.25 0.26 0.27 0.28 0.29
##  [31] 0.30 0.31 0.32 0.33 0.34 0.35 0.36 0.37 0.38 0.39 0.40 0.41 0.42 0.43 0.44
##  [46] 0.45 0.46 0.47 0.48 0.49 0.50 0.51 0.52 0.53 0.54 0.55 0.56 0.57 0.58 0.59
##  [61] 0.60 0.61 0.62 0.63 0.64 0.65 0.66 0.67 0.68 0.69 0.70 0.71 0.72 0.73 0.74
##  [76] 0.75 0.76 0.77 0.78 0.79 0.80 0.81 0.82 0.83 0.84 0.85 0.86 0.87 0.88 0.89
##  [91] 0.90 0.91 0.92 0.93 0.94 0.95 0.96 0.97 0.98 0.99 1.00
```

```
plotphi<- as.data.frame(x1)
dim(plotphi)
```

```
## [1] 101   1
```

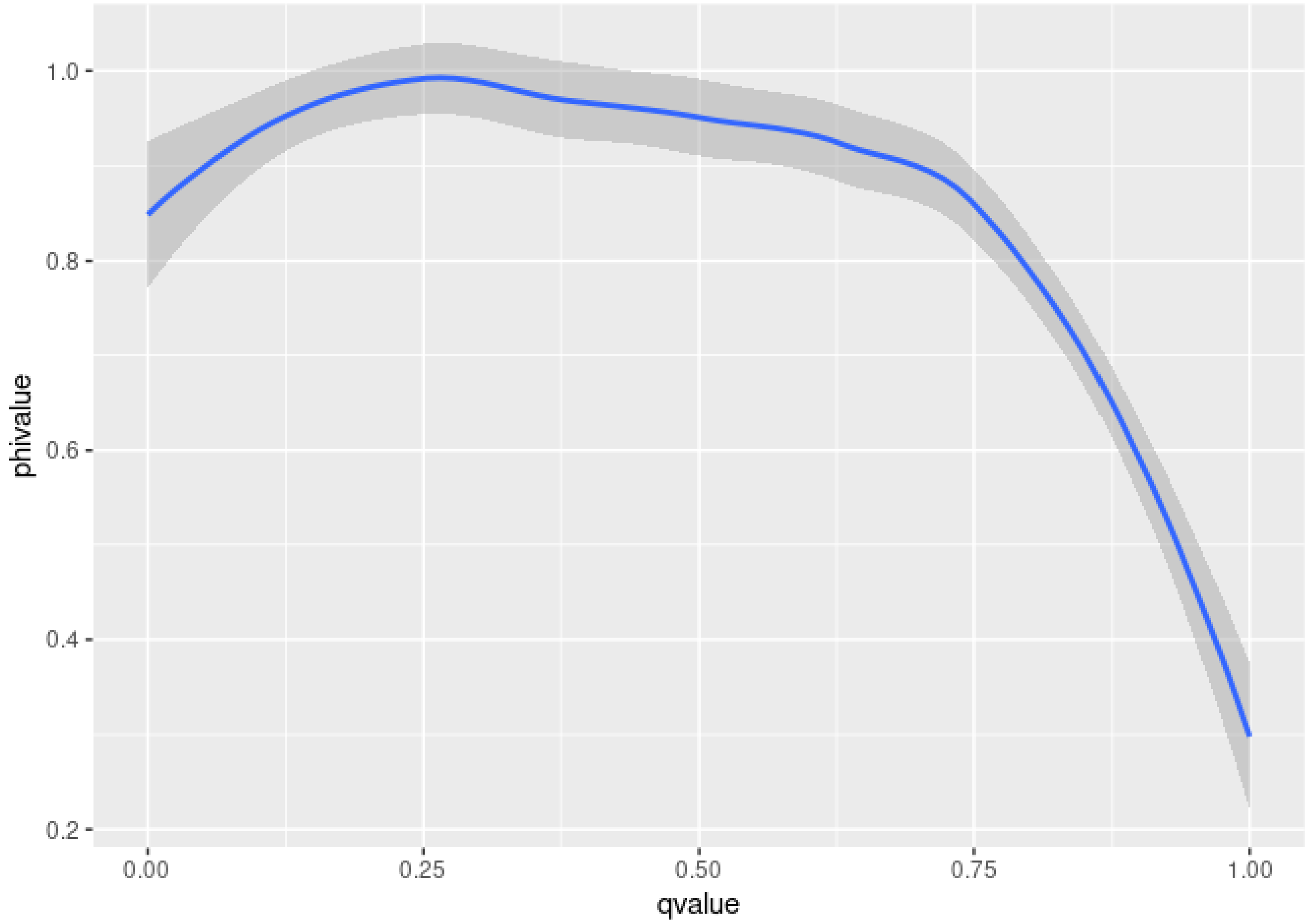
```
x2<-seq(from=0, to=0.1, by=.01)
c_prob = function(p0,p1,input_vector){
   (p1/input_vector) / ((p0/(1-input_vector)) + (p1/input_vector))
}

x2 <-c_prob(0.05,0.95,x1)
x2[is.na(x2)]<-0
phivalue<-as.data.frame(x2)
plotphi$phivalue<-x2
plotphi <-setNames(plotphi,c("qvalue", "phivalue"))
head(plotphi)
```

```
##   qvalue  phivalue
## 1   0.00 0.0000000
## 2   0.01 0.9994687
## 3   0.02 0.9989270
## 4   0.03 0.9983749
## 5   0.04 0.9978118
## 6   0.05 0.9972376
```

```
flow<- ggplot(plotphi, aes(x=qvalue,y=phivalue) ) +geom_smooth()
flow
```

```
## `geom_smooth()` using method = 'loess' and formula 'y ~ x'
```



B.2 Discrete function

Given -

$\alpha, \beta, \gamma \in$

$$0, 1$$

X(DNV)- {0,1,2,5}
P(X=1)= α
P(X=2)= β
P(X=5)= γ
P(X $\notin$ {0,1,2,5}) = 0

1. Probability mass function =

P(x)= {1 -$\alpha - \beta - \gamma$, if x=0}
{α, if x=1 }
{β, if x=2}
{γ, if x=5}
{0, otherwise}

2. E(X) = α + 2β + 5γ

3. E(X^2) - $E(X)^2$ = (α+4β+25γ) − (α+2β+5γ)^2

$X1, X2 \ldots,$ *X**n*
P(Xi=1)= α,
P(Xi=2)= β,
P(Xi=5)= γ and
P(X $\notin$ {0,1,2,5}) = 0

let $\tilde{X}$ =
$frac1n\ sum\_i = 1^n Xi$ be the sample mean

4. E(X) = $frac1n\ sum\_i = 1^n Xi$ * (α+2β+5γ)
5. E(X^2) - $E(X)^2$ = $frac1n\ sum\_i = 1^n Xi$ * ((α+4β+25γ) − (α+2β+5γ)^2)
6.

```
library('dplyr')
```

```
##
## Attaching package: 'dplyr'

## The following objects are masked from 'package:stats':
##
##     filter, lag

## The following objects are masked from 'package:base':
##
##     intersect, setdiff, setequal, union
```

```
library('purrr')
sample_X_0125<-function(n,alpha,beta,gamma){
  sample_X<-data.frame(U=runif(n))%>%
  mutate(X=case_when(
  (0<=U)&(U<alpha)~1,
  (alpha<=U)&(U<alpha+beta)~2,
  (alpha+beta<=U)&(U<alpha+beta+gamma)~5,
  (alpha+beta+gamma<=U)&(U<=1)~0))%>%pull(X)
  return(sample_X)
}
```

7.

```
n<-100000
alpha<-1/10
beta<-2/10
gamma<-3/10

sample_X<-sample_X_0125(n,alpha,beta,gamma)
mean(sample_X)
```

```
## [1] 1.99806
```

8.

```
alpha<-1/10
beta<-2/10
gamma<-3/10
simulation_by_n<-data.frame(n=seq(0,10000,100))%>%
  mutate(sample_X=map(.x=n,~sample_X_0125(.x,alpha,beta,gamma)))%>%
  mutate(sample_avg=map_dbl(.x=sample_X,~mean(.x)))%>%
  select(-sample_X)%>%
  mutate(expectation=alpha+2*beta+5*gamma)
  simulation_by_n%>%head(5)
```

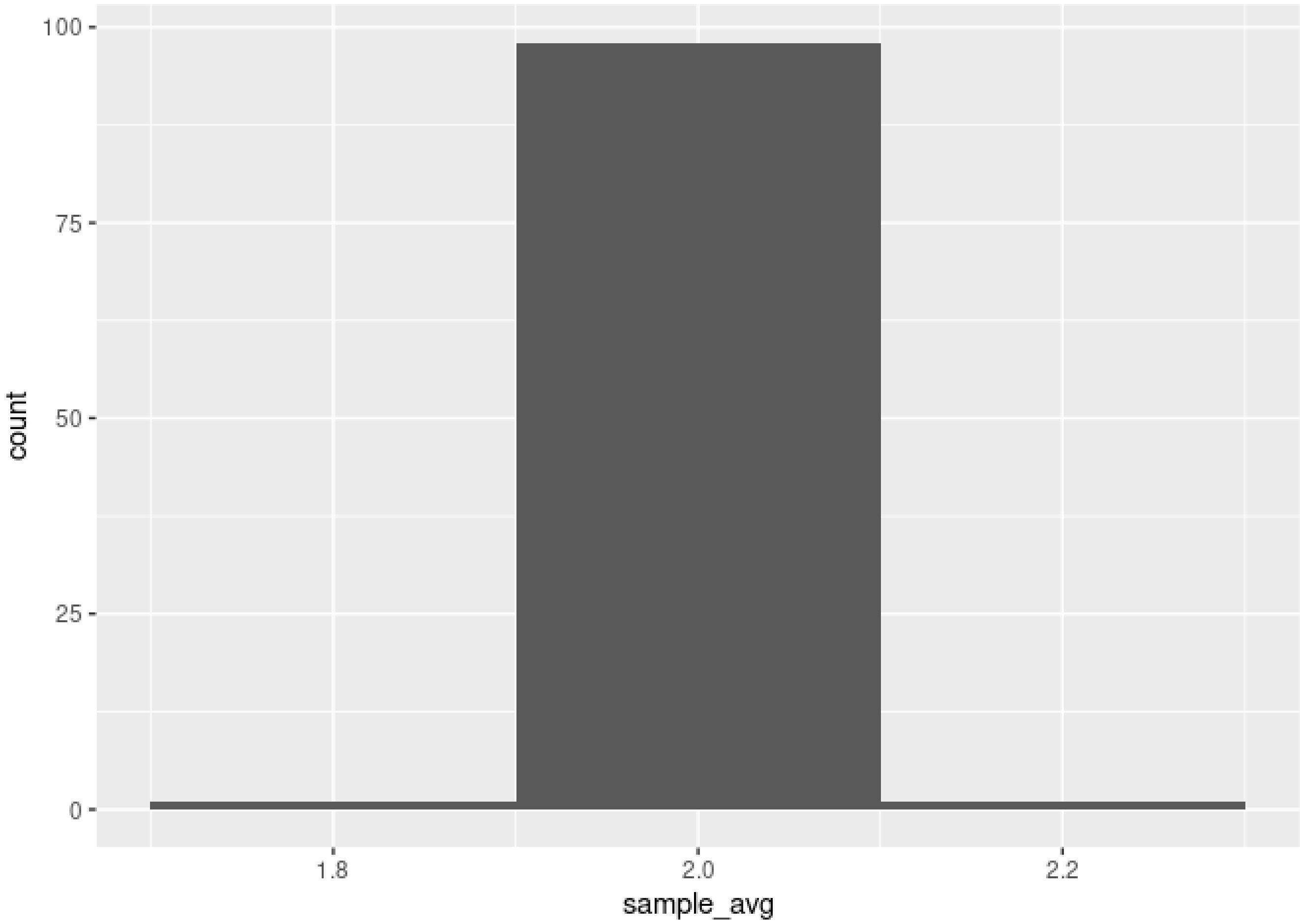
```
##     n sample_avg expectation
## 1   0        NaN           2
## 2 100   2.140000           2
## 3 200   1.885000           2
## 4 300   1.943333           2
## 5 400   2.007500           2
```

1.

```
plotit<- ggplot(simulation_by_n, aes(x=sample_avg)) + geom_histogram(binwidth=0.2)

plotit
```

```
## Warning: Removed 1 rows containing non-finite values (stat_bin).
```



8.

```
expectation<- alpha + 2*beta + 5*gamma
print(expectation)
```

```
## [1] 2
```

```
var(sample_X)
```

```
## [1] 4.40424
```

B.3 Exponential Distribution.

1. Formula for population mean and variance

Population mean for exponential random variable X with paramater λ = $frac1lambda$

Variance for exponential variable random variable X with parameter λ = $frac2lambda^2$

2. Formula for cumulative distribution function and quantile function for exponential random variables with paramter λ

Cummulative function is given by =
*F**λ(x) = {0, if x<=0} {1 - e⁻λ**x* , if x>0}

Quantile function is given by =
$F^{-1}\lambda(x)$ = inf{ x ∈ R : F λ x <=p} {-∞, if p=0} { $frac1lambda$ ln(1-p), if p ∈ (0,1]}

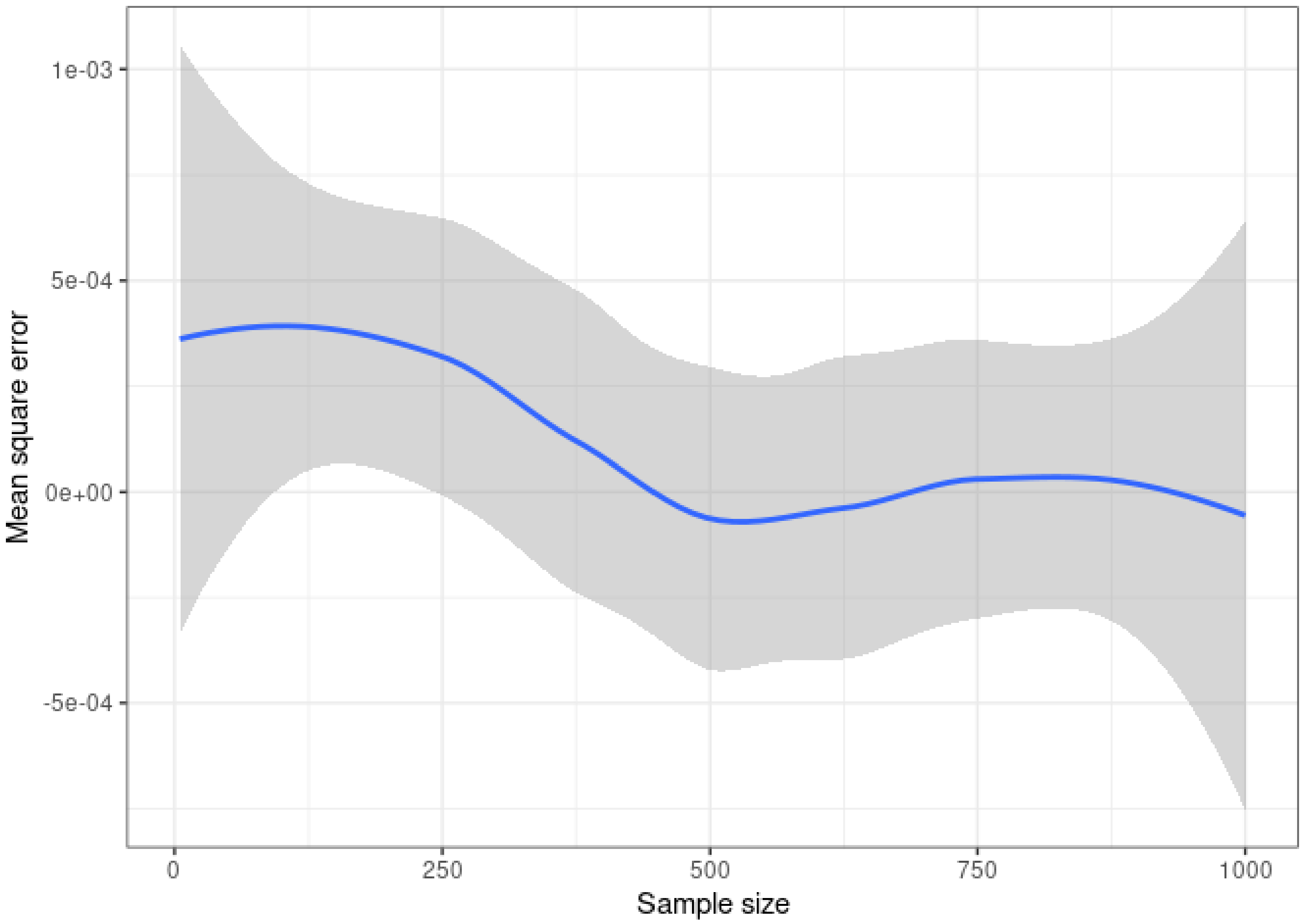
3.

Maximum Likelihood Estimation = $\bar{X}$

4.

```
library(dplyr)
set.seed(0)
num_trials_per_sample_size<-100
min_sample_size<-5
max_sample_size<-1000
sample_size_inc<-5
lambda_0<-0.01
poisson_simulation_df<-expand.grid(trial=seq(num_trials_per_sample_size),
sample_size=seq(min_sample_size,max_sample_size,
sample_size_inc))%>%
# create data frame of all pairs of sample_size and trial
mutate(simulation=pmap(.l=list(trial,sample_size),
.f=~rpois(.y,lambda=lambda_0)))%>%
# simulate sequences of Gaussian random variables
mutate(lambda_mle=map_dbl(.x=simulation,.f=mean))%>%
# compute the sample sd
group_by(sample_size)%>%
summarise(msq_error=mean((lambda_mle-lambda_0)*2))
poisson_simulation_df%>%
ggplot(aes(x=sample_size,y=msq_error))+
geom_smooth()+
theme_bw()+
xlab("Sample size")+ylab("Mean square error")
```

```
## `geom_smooth()` using method = 'loess' and formula 'y ~ x'
```



5.

```r
library(readr)
bird_data<-read.csv("bird_data_EMATM0061.csv")

alpha<-0.00000004192201
sample_size<-length(bird_data$Time)
sample_mean<-mean(bird_data$Time)
sample_sd<-sd(bird_data$Time)
t<-qt(1-alpha/2,df=sample_size-1)
confidence_interval_l<-sample_mean-t*sample_sd/sqrt(sample_size)
confidence_interval_u<-sample_mean+t*sample_sd/sqrt(sample_size)
confidence_interval<-c(confidence_interval_l,confidence_interval_u)
print(confidence_interval)
```

```
## [1] 1476693 1554413
```

```r
percentage<-(confidence_interval[1] / confidence_interval[2]) * 100
print(percentage)
```

```
## [1] 95
```

```r
print("Confidence level of 95% achieved for alpha=0.00000004192201")
```

```
## [1] "Confidence level of 95% achieved for alpha=0.00000004192201"
```