Control Systems (EED302)
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Example: **Consider a state model for a system $\frac{d^3y}{dt^3} + 6\frac{d^2y}{dt^2} + 11\frac{dy}{dt} + 6y + u = 0$. Also give the block diagram representation of the state model.**

Sol: Let us choose y and their derivatives as state variables. The system is governed by third order differential equation, so the number of state variables required are three.

Let the state variables x_1, x_2 and x_3 are related to phase variables as follows.

$$x_1 = y$$

$$x_2 = \frac{dy}{dt} = \dot{x}_1$$

$$x_3 = \frac{d^2y}{dt^2} = \frac{dx_2}{dt} = \dot{x}_2$$

Put $y = x_1$, $\frac{dy}{dt} = x_2$, $\frac{d^2y}{dt^2} = x_3$ and $\frac{d^3y}{dt^3} = \dot{x}_3$ in the given equation

$$\therefore \dot{x}_3 + 6x_3 + 11x_2 + 6x_1 + u = 0$$

$$\Longrightarrow \dot{x}_3 = -6x_1 - 11x_2 - 6x_3 - u$$

The state equations are

$$\dot{x}_1 = x_2$$

$$\dot{x}_2 = x_3$$

$$\dot{x}_3 = -6x_1 - 11x_2 - 6x_3 - u$$

Arranging the state equations in the matrix form,

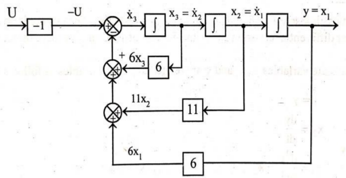
$$\begin{bmatrix} \dot{x}_1 \\ \dot{x}_2 \\ \dot{x}_3 \end{bmatrix} = \begin{bmatrix} 0 & 1 & 0 \\ 0 & 0 & 1 \\ -6 & -11 & -6 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} + \begin{bmatrix} 0 \\ 0 \\ -1 \end{bmatrix} [u]$$

Here y = output

But $y = x_1$

$$\therefore \text{The output equation is, } y = \begin{bmatrix} 1 & 0 & 0 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix}$$

The block diagram for the state model is



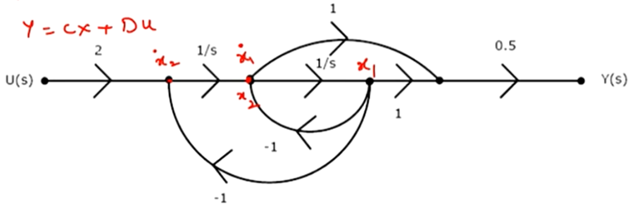
Example: SFG to state space representation

The signal flow graph of a system is shown below.

$\dot{X} = AX + BU$

$Y = CX + DU$

$\boxed{\dot{x}_1 = x_2}$



The state variable representation of the system can be

(A) $\dot{x} = \begin{bmatrix} 1 & 1 \\ -1 & 0 \end{bmatrix} x + \begin{bmatrix} 0 \\ 2 \end{bmatrix} u$

$y = [0 \quad 0.5]x$

(B) $\dot{x} = \begin{bmatrix} -1 & 1 \\ -1 & 0 \end{bmatrix} x + \begin{bmatrix} 0 \\ 2 \end{bmatrix} u$

$y = [0 \quad 0.5]x$

(C) $\dot{x} = \begin{bmatrix} 1 & 1 \\ -1 & 0 \end{bmatrix} x + \begin{bmatrix} 0 \\ 2 \end{bmatrix} u$

$y = [0.5 \quad 0.5]x$

(D) $\dot{x} = \begin{bmatrix} -1 & 1 \\ -1 & 0 \end{bmatrix} x + \begin{bmatrix} 0 \\ 2 \end{bmatrix} u$

$y = [0.5 \quad 0.5]x$

$\frac{dx}{dt} \overset{LT}{\Longleftrightarrow} sX(s)$

$\int x\,dt \underset{LT}{\Longleftrightarrow} \frac{1}{s} \cdot X(s)$

Integrate $\left(\frac{dx}{dt}\right) \underset{LT}{\Longleftrightarrow} \frac{1}{s} \cdot s \cdot X(s)$

$X(s).$

$\dot{x}_1 = \frac{1}{s}(\dot{x}_2) - x_1$

$\dot{x}_1 = x_2 - x_1 = -x_1 + x_2$ —

$\dot{x}_2 = -x_1 + 2U(s)$ —

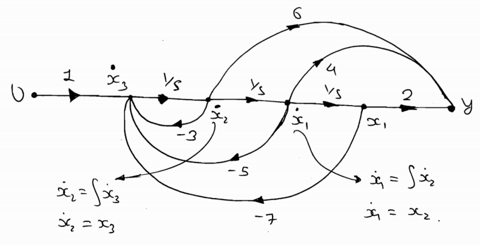
$\begin{bmatrix} \dot{x}_1 \\ \dot{x}_2 \end{bmatrix} = \begin{bmatrix} -1 & 1 \\ -1 & 0 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \end{bmatrix} + \begin{bmatrix} 0 \\ 2 \end{bmatrix} u$

$Y(s) = 0.5x_2 + 0.5x_1$

$= 0.5x_1 + 0.5x_2$

$y = [0.5 \quad 0.5] \begin{bmatrix} x_1 \\ x_2 \end{bmatrix}$

Example: SFG to state space form



$$\therefore \begin{bmatrix} \dot{x}_1 \\ \dot{x}_2 \\ \dot{x}_3 \end{bmatrix} = \begin{bmatrix} 0 & 1 & 0 \\ 0 & 0 & 1 \\ -7 & -5 & -3 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} + \begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix} [u].$$

$$[y] = [2 \quad 4 \quad 6] \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix}.$$