Exercises and solutions: *Matrices*

The only way to learn mathematics is *to solve math problems*. Watching and re-watching video lectures is important and helpful, but it's not enough. If you really want to learn linear algebra, you need to solve problems *by hand*, and then check your work on a computer.

Below are some practice problems to solve. You can find many more by searching the Internet.

Exercises

1. Perform the following matrix operations, when the operation is valid.

$$\mathbf{A} = \begin{bmatrix} 2 & 4 & 3 \\ 0 & 1 & 3 \end{bmatrix}, \quad \mathbf{B} = \begin{bmatrix} -2 & -1 & 3 \\ 6 & -7 & 7 \end{bmatrix}, \quad \mathbf{C} = \begin{bmatrix} 0 & -6 \\ -3 & -2 \\ -2 & 7 \end{bmatrix} \quad \mathbf{D} = \begin{bmatrix} 1 & 2 \\ 3 & 4 \\ 2 & 4 \end{bmatrix}$$

a) $\mathbf{A} + 3\mathbf{B}$ **b)** $\mathbf{A} + \mathbf{C}$ **c)** $\mathbf{C} - \mathbf{D}$ **d)** $\mathbf{D} + \mathbf{C}$
e) $\mathbf{A^T} + \mathbf{D}$ **f)** $(\mathbf{A} + \mathbf{B})^\mathbf{T} + 2\mathbf{C}$ **g)** $3\mathbf{A} + (\mathbf{B^T} + \mathbf{C})^\mathbf{T}$ **h)** $-4(\mathbf{A^T} + \mathbf{C})^\mathbf{T} + \mathbf{D}$

2. Identify the following types of matrices from the list provided in the video "A zoo of matrices." Note that some matrices can be given multiple labels.

a) $\begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 2 \end{bmatrix}$ **b)** $\begin{bmatrix} 1 & 2 & 3 \\ 0 & 4 & 5 \\ 0 & 0 & 6 \end{bmatrix}$ **c)** $\begin{bmatrix} 1 & 2 & 3 \\ 2 & 4 & 5 \\ 3 & 5 & 6 \end{bmatrix}$

d) $\begin{bmatrix} a & b & c \\ -b & d & e \\ -c & -e & f \end{bmatrix}$ **e)** $\begin{bmatrix} 0 & b & c \\ -b & 0 & e \\ -c & -e & 0 \end{bmatrix}$ **f)** $\begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 32 & 0 & 0 \\ 0 & 0 & 42 & 0 \end{bmatrix}$

3. Perform the indicated matrix operations (*tr(M)* indicates the trace of matrix M).

$$\mathbf{A} = \begin{bmatrix} 2 & 4 \\ 0 & 3 \end{bmatrix}, \quad \mathbf{B} = \begin{bmatrix} -2 & -1 \\ 6 & 7 \end{bmatrix}$$

a) $tr(\mathbf{A})$ **b)** $tr(-3\mathbf{A})$ **c)** $tr(\mathbf{A} + \mathbf{B})$
d) $tr((\mathbf{A} + \mathbf{B})^\mathbf{T})$ **e)** $tr(\mathbf{B^T} + \mathbf{A})$ **f)** $tr(\mathbf{B} + \mathbf{A^T})$

4. Perform the indicated matrix operations using the following matrices and scalars. Determine the underlying principle regarding trace, matrix addition, and scalar multiplication.

$$\mathbf{A} = \begin{bmatrix} 5 & -3 \\ 2 & -3 \end{bmatrix}, \quad \mathbf{B} = \begin{bmatrix} -4 & -1 \\ 1 & 3 \end{bmatrix}, \quad \mathbf{C} = \begin{bmatrix} a & c \\ b & d \end{bmatrix}, \quad \lambda = 5, \quad \alpha = -3$$

a) $tr(\mathbf{A})$ **b)** $tr(\mathbf{B})$ **c)** $tr(\mathbf{A} + \mathbf{B})$ **d)** $tr(\lambda\mathbf{C})$
e) $\lambda\, tr(\mathbf{C})$ **f)** $\lambda\, tr(\alpha\mathbf{C})$ **g)** $\alpha\, tr(\lambda\mathbf{C})$ **h)** $tr(\alpha\mathbf{A} + \lambda\mathbf{B})$
i) $(\lambda\alpha)\, tr(\mathbf{A} + \mathbf{B})$ **j)** $tr(\lambda\mathbf{A} + \lambda\mathbf{B})$ **k)** $\lambda\, tr(\mathbf{A} + \mathbf{B})$ **l)** $tr(\mathbf{A} + \mathbf{B}^T)$

Answers

1. -

a) $\begin{bmatrix} -4 & 1 & 12 \\ 18 & -20 & 24 \end{bmatrix}$ **b)** Not valid. **c)** $\begin{bmatrix} -1 & -8 \\ -6 & -6 \\ -4 & 3 \end{bmatrix}$ **d)** $\begin{bmatrix} 1 & -4 \\ 0 & 2 \\ 0 & 11 \end{bmatrix}$

e) $\begin{bmatrix} 3 & 2 \\ 7 & 5 \\ 5 & 7 \end{bmatrix}$ **f)** $\begin{bmatrix} 0 & -6 \\ -3 & -10 \\ 2 & 24 \end{bmatrix}$ **g)** $\begin{bmatrix} 4 & 8 & 10 \\ 0 & -6 & 23 \end{bmatrix}$ **h)** Not valid.

2. -

a) square, diagonal **b)** square, upper-triangular **c)** symmetric
d) square **e)** skew-symmetric **f)** rectangular, diagonal

3. -

a) 5 **b)** -15 **c)** 10
d) 10 **e)** 10 **f)** 10

4. -

a) 2 **b)** -1 **c)** 1 **d)** $5a + 5d$
e) $5(a + d)$ **f)** $5(-3a - 3d)$ **g)** $-3(5a + 5d)$ **h)** -11
i) -15 **j)** 5 **k)** 5 **l)** 1