Exercises and solutions: *Matrix determinant*

The only way to learn mathematics is *to solve math problems*. Watching and re-watching video lectures is important and helpful, but it's not enough. If you really want to learn linear algebra, you need to solve problems *by hand*, and then check your work on a computer.

Below are some practice problems to solve. You can find many more by searching the Internet.

Exercises

1. Determine whether the following determinants are correct.

a) $\begin{vmatrix} 3 & 1 \\ 2 & 1 \end{vmatrix} = 1$ **b)** $\begin{vmatrix} 3 & 4 \\ 1 & 1 \end{vmatrix} = -1$ **c)** $\begin{vmatrix} 1 & 9 \\ 6 & 1 \end{vmatrix} = -3$ **d)** $\begin{vmatrix} 9 & 1 \\ 6 & 1 \end{vmatrix} = -3$

e) $\begin{vmatrix} 1 & 9 \\ 0 & 0 \end{vmatrix} = 0$ **f)** $\begin{vmatrix} 1 & 5 \\ 3 & 15 \end{vmatrix} = 15$ **g)** $\begin{vmatrix} 1 & 5 \\ 3 & 0 \end{vmatrix} = -8$ **h)** $\begin{vmatrix} -1 & 2 \\ 4 & -5 \end{vmatrix} = -3$

2. Which value(s) of λ would make the following matrices singular (that is, have zero determinant)?

a) $\begin{bmatrix} 1 & 2 \\ 4 & \lambda \end{bmatrix}$ **b)** $\begin{bmatrix} 0 & 14 \\ \lambda & 4 \end{bmatrix}$ **c)** $\begin{bmatrix} 4 & 1 \\ 1 & \lambda \end{bmatrix}$ **d)** $\begin{bmatrix} 10 & 1 \\ 3-\lambda & 1 \end{bmatrix}$

e) $\begin{bmatrix} 6 & 2 \\ 5 & 3\lambda \end{bmatrix}$ **f)** $\begin{bmatrix} \lambda & 4 \\ 1 & \lambda \end{bmatrix}$ **g)** $\begin{bmatrix} \lambda & 18 \\ 1/2 & \lambda \end{bmatrix}$ **h)** $\begin{bmatrix} 5-\lambda & -1/3 \\ -3 & 5-\lambda \end{bmatrix}$

3. Determine whether the following determinants are correct.

a) $\begin{vmatrix} 1 & 1 & 2 \\ 2 & 2 & 1 \\ 4 & 3 & 0 \end{vmatrix} = -2$ **b)** $\begin{vmatrix} 1 & 1 & 2 \\ 4 & 3 & 0 \\ 2 & 2 & 1 \end{vmatrix} = -3$ **c)** $\begin{vmatrix} 1 & 2 & 4 \\ 1 & 2 & 3 \\ 2 & 1 & 0 \end{vmatrix} = -3$ **d)** $\begin{vmatrix} 1 & -2 & 3 \\ -4 & 5 & -6 \\ 7 & -8 & 9 \end{vmatrix} = 8$

4. Which value(s) of λ would make the following matrices singular?

a) $\begin{bmatrix} 0 & 1 & 1 \\ 0 & 4 & 7 \\ 0 & 3 & \lambda \end{bmatrix}$ **b)** $\begin{bmatrix} 1 & 0 & -3 \\ -4 & -11 & 1 \\ 3 & \lambda & 0 \end{bmatrix}$ **c)** $\begin{bmatrix} 1 & 1 & 1 \\ 2 & 2 & 3 \\ 3 & \lambda & 4 \end{bmatrix}$ **d)** $\begin{bmatrix} 1-\lambda & 0 & 4 \\ -4 & 3-\lambda & 4 \\ 0 & 0 & 5-\lambda \end{bmatrix}$

Answers

1. Note: The Δ symbol indicates determinant.

a) Correct **b)** Correct **c)** No, $\Delta = -53$ **d)** No, $\Delta = 3$
e) Correct **f)** No, $\Delta = 0$ **g)** No, $\Delta = -15$ **h)** Correct

2. -

a) $\lambda = 8$ **b)** $\lambda = 0$ **c)** $\lambda = 1/4$ **d)** $\lambda = -7$
e) $\lambda = 5/9$ **f)** $\lambda = \pm 2$ **g)** $\lambda = \pm 3$ **h)** $\lambda = 6, 4$

3. -

a) No, $\Delta = -3$ **b)** No, $\Delta = 3$ **c)** Correct **d)** No, $\Delta = 0$

4. -

a) any λ **b)** $\lambda = 9$ **c)** $\lambda = 3$ **d)** $\lambda = 1, 3, 5$