Exercises and solutions: *Eigendecomposition*

The only way to learn mathematics is *to solve math problems*. Watching and re-watching video lectures is important and helpful, but it's not enough.

Below are some practice problems to solve. You can find many more by searching the Internet.

Exercises

1. Find the eigenvalues of the following matrices.

a) $\begin{bmatrix} 4 \end{bmatrix}$ **b)** $\begin{bmatrix} 0 & 3 \\ 5 & 0 \end{bmatrix}$ **c)** $\begin{bmatrix} 3 & 0 \\ 0 & 5 \end{bmatrix}$

d) $\begin{bmatrix} 2 & 5 \\ 6 & 3 \end{bmatrix}$ **e)** $\begin{bmatrix} -4 & 1 \\ 1 & 3 \end{bmatrix}$ **f)** $\begin{bmatrix} -2 & 2 \\ -3 & 2 \end{bmatrix}$

2. Diagonalize the following matrices by computing the eigenvalues and eigenvectors matrices.

a) $\begin{bmatrix} 1 & 1 \\ -3 & 5 \end{bmatrix}$ **b)** $\begin{bmatrix} -1 & 0 \\ -1 & 0 \end{bmatrix}$ **c)** $\begin{bmatrix} 4 & 0 & 1 \\ -2 & 1 & 0 \\ -2 & 0 & 1 \end{bmatrix}$

3. The following pairs of matrices show a matrix and its eigenvectors. Without computing eigenvalues, determine the missing eigenvector component.

a) $\begin{bmatrix} -2 & 2 \\ 2 & 1 \end{bmatrix}, \begin{bmatrix} -2 & 1 \\ * & 2 \end{bmatrix}$ **b)** $\begin{bmatrix} 52 & 16 \\ 16 & 28 \end{bmatrix}, \begin{bmatrix} 2 & * \\ -4 & -2 \end{bmatrix}$ **c)** $\begin{bmatrix} 3 & -3 \\ -3 & 3 \end{bmatrix}, \begin{bmatrix} * & 1 \\ 1 & -1 \end{bmatrix}$

4. Compute the eigenvalues of the following matrices. Do you notice any patterns?

a) $\begin{bmatrix} 2 & 2 \\ 0 & 1 \end{bmatrix}$ **b)** $\begin{bmatrix} 2 & 5 & -1 \\ 0 & 4 & 3 \\ 0 & 0 & 1 \end{bmatrix}$ **c)** $\begin{bmatrix} a & 34 & \sqrt{23} \\ 0 & b & e^{i\pi^3} \\ 0 & 0 & c \end{bmatrix}$ **d)** $\begin{bmatrix} a & 0 & 0 \\ 34 & b & 0 \\ \sqrt{23} & e^{i\pi^3} & c \end{bmatrix}$

Answers

1. -

a) 4 **b)** $\pm\sqrt{15}$ **c)** 3, 5

d) -3, 8 **e)** $(-1 \pm \sqrt{53})/2$ **f)** $\pm\sqrt{2}i$

2. Matrices below are eigenvalues, eigenvectors.

a) $\begin{bmatrix} 2 & 0 \\ 0 & 4 \end{bmatrix}, \begin{bmatrix} 1 & 1 \\ 1 & 3 \end{bmatrix}$ **b)** $\begin{bmatrix} -1 & 0 \\ 0 & 0 \end{bmatrix}, \begin{bmatrix} \pi & 0 \\ \pi & 1 \end{bmatrix}$ **c)** $\begin{bmatrix} 1 & 0 & 0 \\ 0 & 3 & 0 \\ 0 & 0 & 2 \end{bmatrix}, \begin{bmatrix} 0 & 1 & -1 \\ 1 & -1 & 2 \\ 0 & -1 & 2 \end{bmatrix}$

3. -

a) $* = 1$ **b)** $* = -4$ **c)** $* = 1$

4. The eigenvalues of a triangular matrix are the diagonal elements.

a) $\lambda = 2, 1$ **b)** $\lambda = 2, 4, 1$ **c)** $\lambda = a, b, c$ **d)** $\lambda = a, b, c$