# Agenda

- Amortized complexity
  - Aggregate
  - Potential
  - Accounting

i-1

i

| 5 | 6 | 7 | |  $\rightarrow$  | 5 | 6 | 7 | 8 |  $\rightarrow$  | 5 | 6 | 7 | 8 | | | | |

i-2

$2^2$

B

A
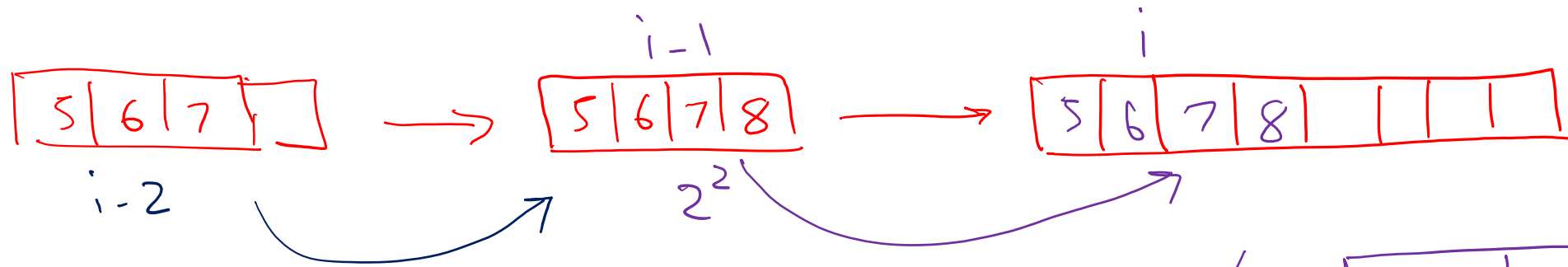
If we have a sequence of 'n' stack operations, what is the cost per operation?
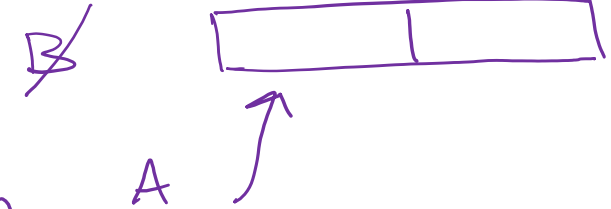
Crude: if i-1 is an exact power of 2, $(i-1)+1$
$= i$

o/w 1
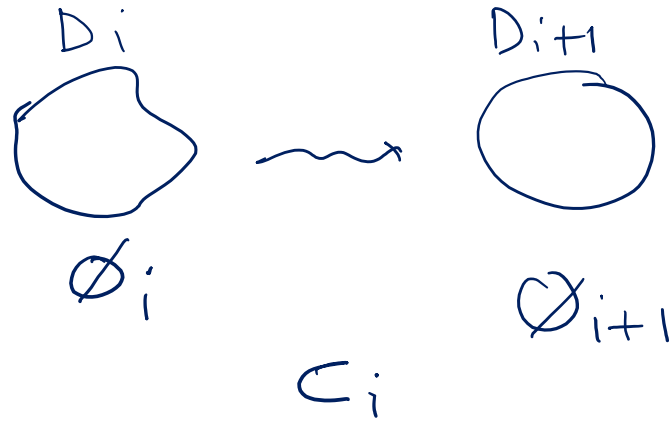
$$c_n = n \longrightarrow O(n^2)$$

worst case analysis

Aggregate method:

$c_i = i$ if $i-1$ is a power of 2

$$\sum_{i=1}^{n} c_i \leq n + \sum_{j=0}^{\lfloor \lg n \rfloor} 2^j = 1$$

$$\leq n + 2n \leq 3n$$

$\therefore$ Amortized cost $\triangleq \dfrac{\text{Total Cost}}{n} = O(1)$

# Potential Method

$$\hat{C_i} \overset{\Delta}{=} C_i + \emptyset_i - \emptyset_{i-1} \longrightarrow \Sigma \hat{C_i} = \Sigma C_i + \Sigma(\ )$$

$$= \text{Total Cost} + \emptyset_n - \emptyset_0$$

If $\emptyset_n - \emptyset_0 \geq 0$

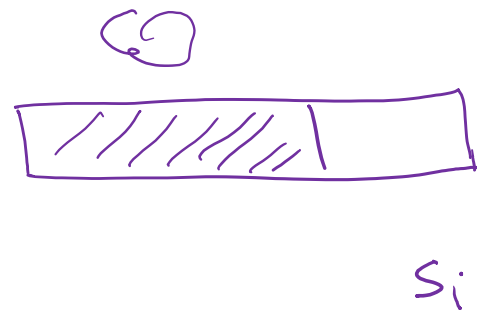Then    Total Cost $\leq \Sigma \hat{C_i}$

$\hookleftarrow \hat{C_1}$ is easy to evaluate.

$\Sigma \hat{C_i}$ is easy to evaluate

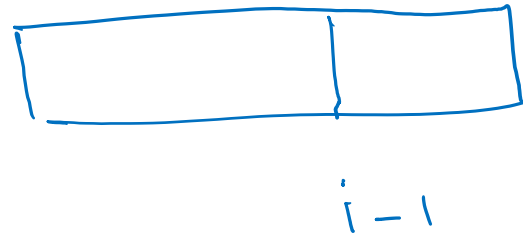$\therefore$ T.C. is easy to compute

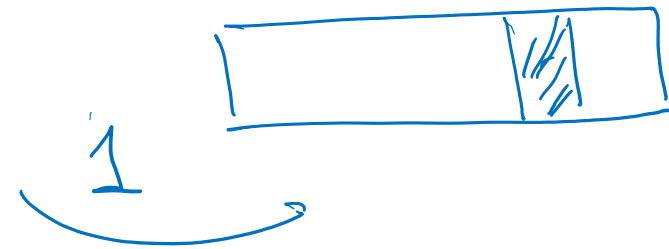$$\varnothing_i = 2(\#\text{items})_i - S_i$$

$$J_i \uparrow$$

Let's compute $\hat{C}_i$

(J) No resize

$i-1$

$J_{i-1}$

$S_{i-1}$
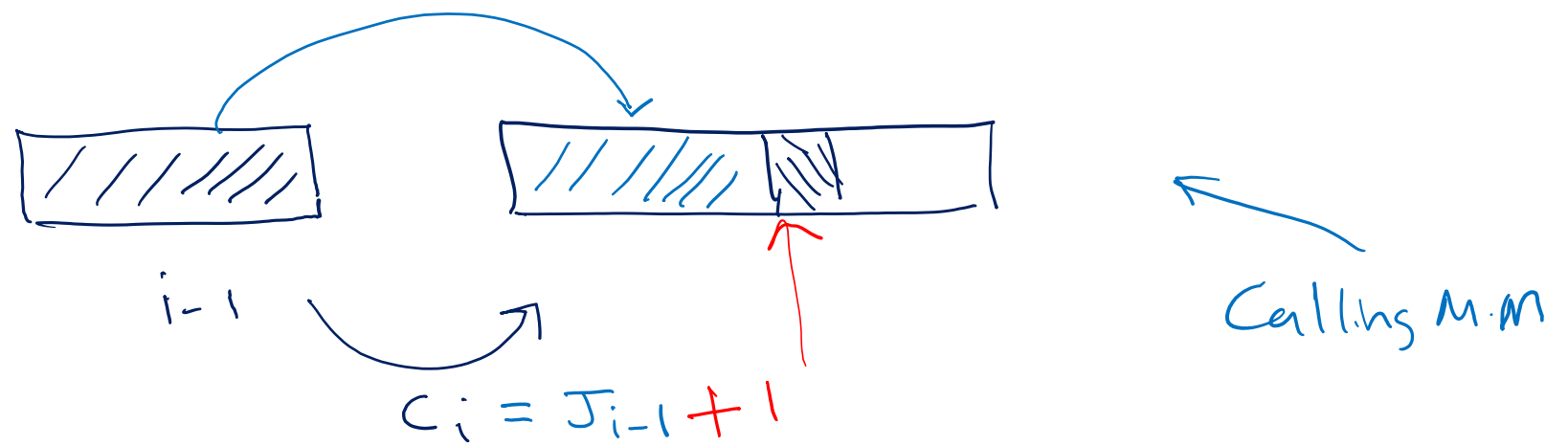
$$J_i = J_{i-1} + 1$$

$$S_i = S_{i-1}$$

$$\hat{C}_i = 1 + (2J_i - S_i) - (2J_{i-1} - S_{i-1})$$
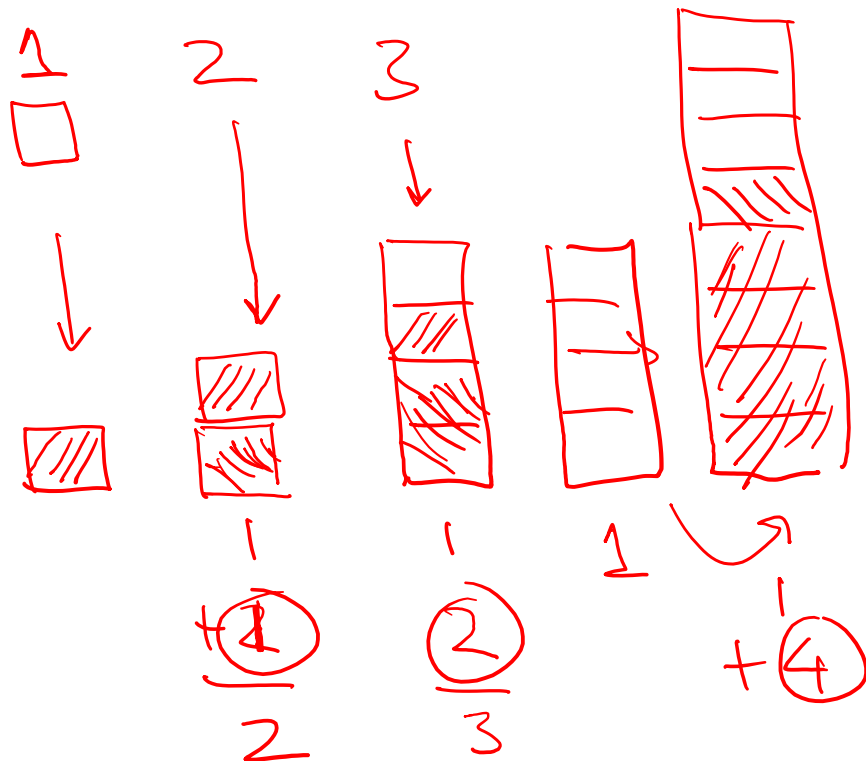$$= 1 + 2J_{i-1} + 2 \cancel{S_{i-1}} - 2J_{i-1} - \cancel{S_{i-1}} = 3$$

# If Resize

$C_i = J_{i-1} + 1$

$S_{i-1}$
$= J_{i-1}$

Calling M.M

$S_i = 2S_{i-1}$
$J_i = J_{i-1} + 1$
$S_i = 2S_{i-1}$
$= 2J_{i-1}$

$$\hat{C_i} = 1 + C_i + \phi_i - \phi_{i-1} = (J_{i-1}) + (2J_i - S_i) - (2J_{i-1} - S_{i-1})$$

$$= 1 + J_{i-1} + [2(J_{i-1} + 1) - 2J_{i-1}] - 2J_{i-1} + J_{i-1}$$

$$= 1 + J_{i-1} + 2J_{i-1} + 2 - 2J_{i-1} - 2J_{i-1} + J_{i-1}$$

$$= 1 + 2 = 3$$

You

1   2   3

Banker

$+1$ ①
$\frac{}{2}$

② ③
$\frac{}{3}$

1   1

$+$④

Amortization

worst case cost
of a seq of $n$ ops.

② ② ② ②

⑧

②