

② C (covariance matrix) is symmetric

→ Its eigenvectors are orthogonal

Let $(\lambda_1, v_1), (\lambda_2, v_2)$ be eigenval, eigenvector pairs for C

$$C v_1 = \lambda_1 v_1, \quad C v_2 = \lambda_2 v_2$$

$$\therefore v_2^T C v_1 = \lambda_1 v_2^T v_1$$

$$\rightarrow (C^T v_2)^T v_1 = \lambda_1 v_2^T v_1$$

$$(C v_2)^T v_1 = \lambda_1 v_2^T v_1 \quad (C \text{ is symmetric})$$

$$(\lambda_2 v_2)^T v_1 = \lambda_1 v_2^T v_1$$

$$v_2^T v_1 \lambda_2 = v_2^T v_1 \lambda_1$$

Since we have $\lambda_1 \neq \lambda_2$

so $v_2^T v_1 = 0$ or v_1 & v_2 are orthogonal

Maximising $f^T C f$

→ f is eigenvector & by choosing the largest eigenvalue & thus corresponding eigenvector. We want to maximise $f^T C f$ in perpendicular dirⁿ

assuming $\bar{f}^T f = 1$

Subject _____

Date: __/__/__

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$$\tilde{J}(\lambda) = f^T C f - \lambda (f^T f - 1)$$

~~on derivation~~ on differentiating

$$Cf = \lambda f \Rightarrow f^T C f = \lambda$$

Now, as $f^T C f$ is maximised when f satisfies
 $Cf = \lambda f$

so

$f^T C f$ is the eigenvalue, as the largest is already chosen, we will choose 2nd largest eigenvalue, but we need the corresponding eigenvector to be normal to the initial one.

Since C is symmetric & its eigenvectors are orthogonal

$\therefore f$ will be the eigenvector corresponding to the second largest eigenvalue.