Q2. Suppose an image is subjected to histogram equalization. Prove that another round of histogram equalization will produce the exact same result as the first round.

Ans. Histogram equalization refers to the process of enhancing the contrast of an image but setting intensity at the pixel values as their cumulative distributive function times the (max intensity value - 1). Let I be a image such that the number of pixels of intensity value i is $n(i)$ and the max intensity value is L. After histogram equalization, we will have new value of the intensity of the pixels with initial intensity i as

$$i_{\text{new}} = \frac{\sum_{k=0}^{k=t} n(k)}{\sum_{k=0}^{k=L-1} n(k)} * (L-1) \quad \text{where } t = \text{no. of pixels with intensity value} < i$$

The number of pixels with this intensity value is the same as $n(i)$ as this new intensity value just gets mapped to pixels with original intensity i (Assuming no round off errors). Also , the number of pixels with intensity value less than i_{new} would be the same as the number of pixels with intensity value i. A second histogram equalization will give

$$i_{\text{second}} = \frac{\sum_{k=0}^{k=t^{'}} n_{\text{new}}(k)}{\sum_{k=0}^{k=L-1} n_{\text{new}}(k)} * (L-1) \quad \text{where } t^{'} = \text{no. of pixels with intensity value} < i_{\text{new}}$$

But $t^{'} = t$ and $n_{\text{new}}(k) = n(k)$. Hence,

$$i_{\text{second}} = \frac{\sum_{k=0}^{k=t} n(k)}{\sum_{k=0}^{k=L-1} n(k)} * (L-1) = i_{\text{new}}$$