Q3. Consider two images I and J whose intensity values (in each location) are randomly drawn from the known probability mass functions (PMFs) p_I(i) and p_J(j) respectively. Derive an expression for the PMF of the image I + J. The expression resembles which operation?

Ans. For a given location (x,y), let us denote the intensity of image I by the random variable X and the intensity of the random variable by Y. The resultant intensity of the location in the image I + J will be X + Y = Z. The pmf of Z would be (assuming no clipping of intensities):

$$\begin{aligned} P(Z = z) &= P(X + Y = z) \\ &= \sum_{i=0}^{i=z} P(X = i)P(Y = z - i) \quad \text{by law of total probability} \end{aligned}$$

This represents the discrete convolution function.