END TERM EXAMINATION [MAY–2016]
SECOND SEMESTER [B.TECH]
ENGINEERING MECHANICS [ETME–110]

Time : 3 Hrs. **M.M. : 75**

Note: *Attempt any five questions including Q no.1 which is compulsory. Select one question from each unit.*

Q.1. Short questions: (2.5 × 10 = 25)

Q.1. (a) State the various assumptions for the analysis of a perfect truss.

Ans. Perfect Truss: When the truss is non collapsible on the removal of external supports, the truss is said to be perfect truss. For a perfect truss.

$$m = 2j - 3$$

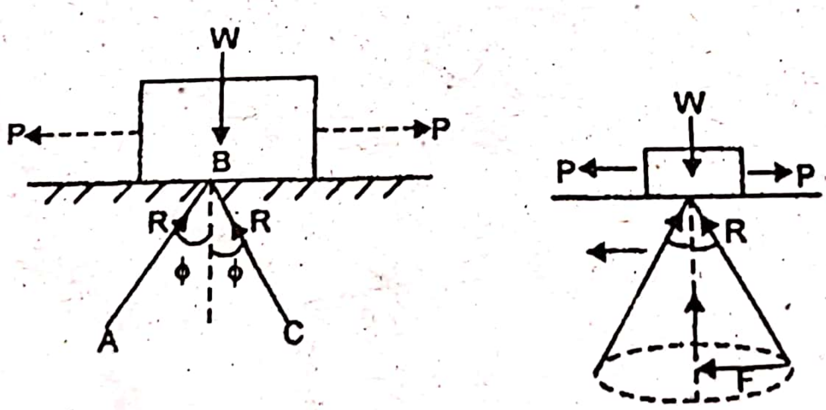
where m *is no.* of members and j is number of joints.

Assumptions:

1. Th joints of a simple truss are assumed to be pin connection and frictionless.
2. The loads on the truss are applied at joints only.
3. The members of a truss are straight two force members with the forces acting collinear with the centre line of the members.
4. The weights of the members are negligible unless otherwise mentioned.
5. The trusse is statically determinate.

Q.1. (b) Explain cone of friction and angle of friction.

Ans. Cone of friction: Consider a block of weight W resting on horizontal surface and acting upon fay force P. When we consider coplanar forces, in order fro motion not to occur in any direction, resultant R must lie within angle ABC, where ϕ *is* angle of friction



Consider force P is gradually changed through 360°. For motion not to occur, resultant reaction R must be contained with in the cone generated by revolving line AB about normal BN,

The inverted so formed with semi-central angle equal to angle of friction ϕ is called cone of friction. Now, for motion to occur resultant R will be on the surface of cone.

Q.1. (c) Explain the meaning of MA, VR and Efficiency of a screw jack.

Ans. Mechanical Advantage (MA) → is ratio of force the jack exerts on the load to input force on the lever ignoring friction is

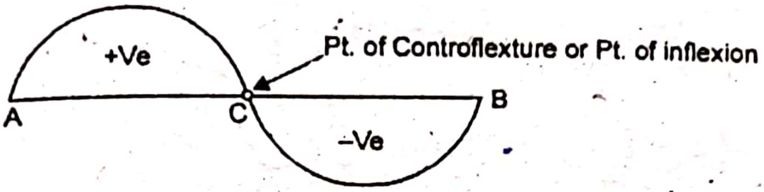
$$\frac{F_{Load}}{F_{in}} = \frac{2\pi r l}{l}$$

Velocity Ratio (VR) → is ratio of distance moved by effort to distance moved by load.

Efficiency → is ratio of Mechanical Advantage (MA) to Velocity Ratio

Q.1. (d) What is point of contraflexture in case of a beam?

Ans. Pt. of controflexture in S.F.D and B.M.D.: It is the point where Bending moment is zero after changing its sign from +ve to –ve or vice-versa also known as point of inflexion.



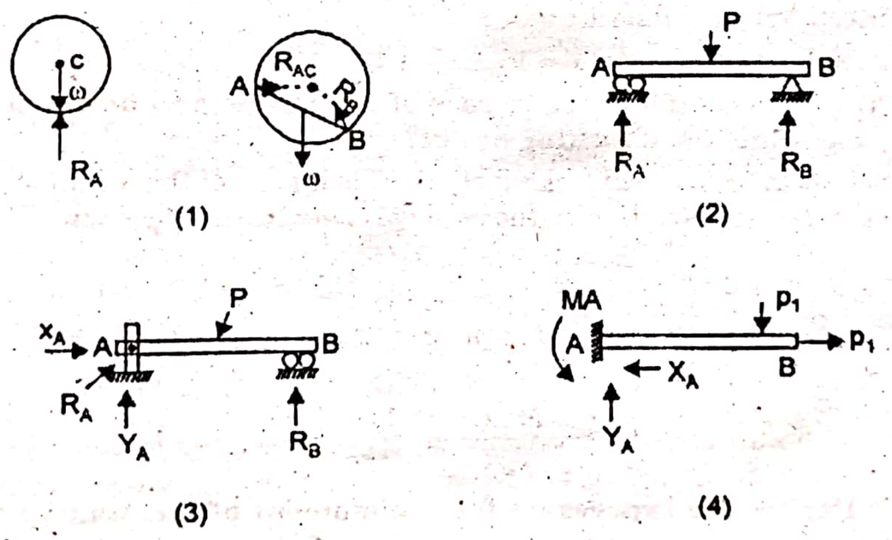
Q.1. (e) Discuss various type of supports used in engineering structures.

Ans. Types of support: Four types: 1. Frictionless support.

2. Roller and kinfe edge support

3. Hinged support

4. Built in support



Q.1. (f) What are the advantages of method of section over the method of joints in case of truss?

Ans. Method of section is useful when you want to know forces acting on a certain member in a truss. By making a "Cut" along the truss and member you want to calculate, you can solve forces in each member, but instead calculate forces in all members whereas **Method of joint** will find the forces in all members and you have to start at a joint and continuously work your way around the truss to calculate force in each member. This process is tedious.

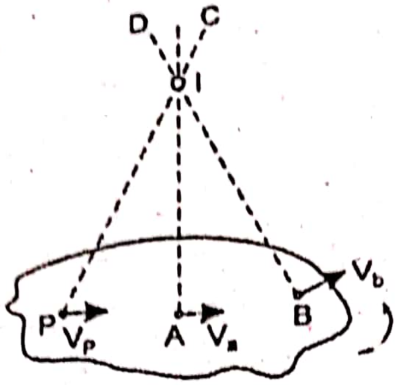
Q.1. (g) Define the coefficient of restitution and instantaneous centre of rotation.

Ans. Instantaneous center: The point at which is in intaneously understood here that the body of zero velocity called. I.C.S.

Location:

$$w = \frac{V_a}{I_A}.$$

$$V_a = wI_a$$



Coefficient of restitution: The coefficient of restitution is the ratio of magnitude of impulses during restitution period and the deformation period. Or it may be defined as the-ve value of teh ratio of velocity of sepration to velocity of approach.

$$e = (-)\frac{\text{Velocity of Separation}}{\text{Velocity of approach}}$$

For perfectly plastic impact e = 0

For perfectly elastic impact e = 1

Q.1. (*h*) What is self locking in case of a machine and how you will verify whether a machine is self locking or not?

Ans. Self locking: The self locking of a M/C is known of the machine (M/C) which cannot work in reverse direction on the removal of effort. Its efficiency will be less than 50%.

$$\eta < \frac{1}{2}$$

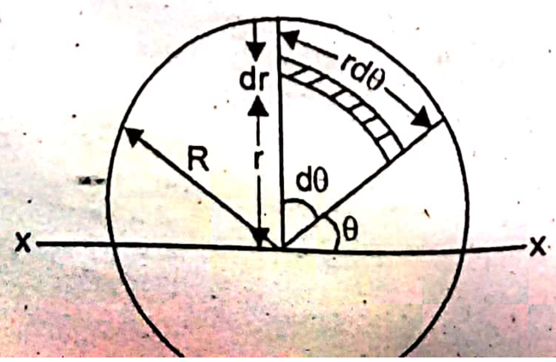
$$\frac{M.A.}{V.R} = \frac{1}{2}$$

Q.1. (*i*) Derive the expression for the moment of inertia of a semi circle about its base.

Ans. Consider an element of sides r $d\theta$ and dr within a circular lamina of radius R. Moment of this elemental area about diametical axis x–x is $= y^2\, da = r^3 \sin^2\theta\, d\theta\, dr$

Moment of inertia of entire Circular lamina

$$I_{xx} = \int_0^R \int_0^{2\pi} r^3 \sin^2\theta\, d\theta\, dr$$



If d is diame

Then

For semicirc

Q.1. (*j*) Disc body.

Ans. It is us measure of how c concentrated awa

Application

(*a*) Design of

(*b*) Design of

Q.2. Three i inclined surface the arrangemen

Ans. Limiting

Now from '3' w

$\Sigma F_x = 0$

$\Sigma F_y = 0$

R_{13}

$$= \int_0^R \int_0^{2\pi} r^3 \frac{(1-\cos 2\theta)}{2} d\theta dr$$

$$= \int_O^R \frac{r^3}{2} \left[\frac{\theta - \sin 2\theta}{2}\right]_0^{2\pi} dr$$

$$= \int_O^R \frac{r^3}{2} [2\pi] dr = 2\pi \left|\frac{r^4}{8}\right|_O^R = \frac{\pi R^4}{4}$$

If d is diameter of circular lamina then

$$I_{xx} = \frac{\pi}{4}\left(\frac{d}{2}\right)^4 = \frac{\pi d^4}{64}$$

Then

For semicircle with AB its base, Moments of inertia about AB is

$$I_{AB} = \frac{1}{2} \times \left(\frac{\pi d^4}{64}\right) = \frac{\pi d^4}{128}$$

Q.1. (j) Discuss the significance and applications of moment of inertia of a body.

Ans. It is used to calculate how much torque require to stop rotating body. It is a measure of how difficult is to rotate a particular body about given axis. Greater mass concentrated away from axis, greater the moment of inertia.

Applications:

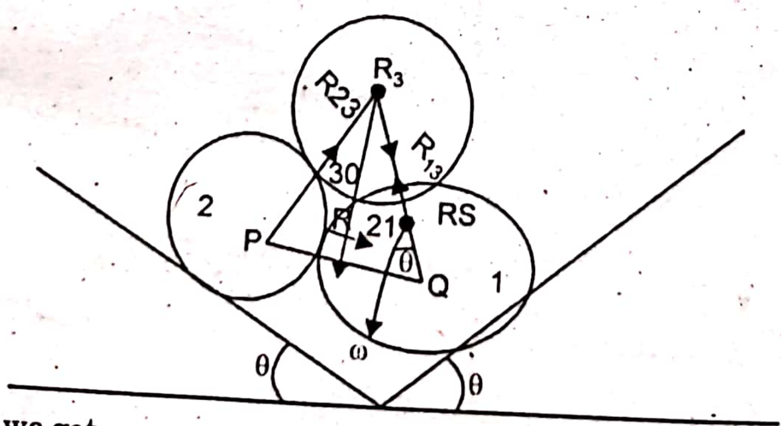
(a) Design of Car, Aeroplane,

(b) Design of machines.

UNIT-I

Q.2. Three identical spheres P,Q, R of weight W are arranged on smooth inclined surface as shown in the fig. Determine the angle θ which will prevent the arrangement from collapsing. (12.5)

Ans. Limiting case of collapse- Reaction between 2 and 1 becomes zero



Now from '3' we get

$\Sigma F_x = 0$ $\quad R_{23} \sin 30 - \sin 30\, R_{13} = 0$

$\Sigma F_y = 0$

$$R_{13} \cos 30 + R_{23} \cos 30 - \omega = 0$$

$$R_{13} = R_{23} = \frac{\omega}{2\cos 30}$$

From '1' we have

$$\Sigma F_x = 0\ R_{31}\sin 30 - R_s \sin\theta = 0$$
$$\Sigma F_y = 0\ R_s \cos\theta - R_{31}\cos 30 - \omega = 0$$
$$R_{31} = R_{13} = \frac{\omega}{2\cos 30}$$

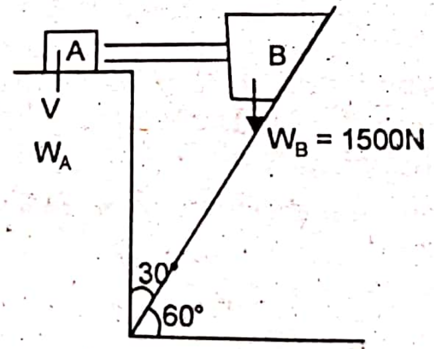
We get

$$\tan\theta = \frac{\tan 30}{3}$$
$$\tan\theta = \frac{1}{3\sqrt{3}}$$
$$\boxed{\theta = \tan^{-1}\frac{1}{3\sqrt{3}}}$$
$$\theta = 10.89°.$$

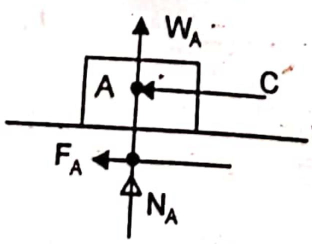
Q.3. Two blocks A and B are connected by a horizontal rod and are supported on two rough planes as shown in figure. If weight of block B is 1500 N and coefficient of friction of block A and B are 0.25 and 0.35 respectively, find the smallest weight of block A for which the equilibrium exist.

Ans.

$$\mu_A = .25$$
$$\mu_B = .35$$
$$W_B = 1500\ N$$
$$W_A = ?$$

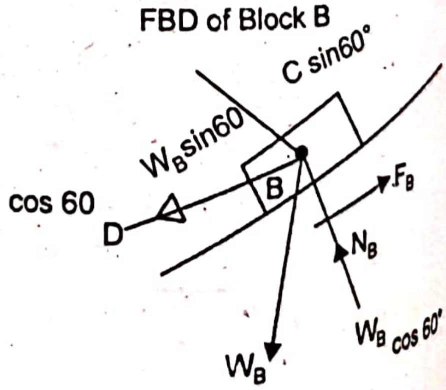


FBD of Block A



$$\Sigma F_x = 0$$
$$F_A = C$$

FBD of Block B



$$\Sigma F_x = 0$$

are supported
is 1500 N and
ively, find the
(12.5)

$$F_B - W_B \sin 60 + C \cos 60 = 0$$

$$\Sigma F_y = 0 \quad ...(1)$$

$$N_B - C \sin 60 - W_B \cos 60 = 0 \quad ...(2)$$

$$N_B = W_B \cos 60 + C \sin 60$$

$$N_B = 1500 \cos 60 + C \sin 60 \quad ...(3)$$

From eqn (1)

$$= \mu N_B - W_B \sin 60 + C \cos 60$$

$$= \mu (1500 \cos 60 + C \sin 60) \quad \text{(Putting eq. (3))}$$

$$-w_B \sin 60 + C \cos 60$$

$$= .36 (1500 \cos 60 + C \sin 60) - 1500 \sin 60 + C \cos 60$$

$$= .36 (1500 \times .5 + C \times .866) - 1500 \times .866 + C \times .5$$

$$= .36 \times 750 + .866C - 1299 + .5C$$

$$= 270 - 1299 + 1.366C$$

$$= 1029 = 1.366C$$

$$C = 753.294 \text{ N}$$

For Block A

$$F_A = C$$

$$F_A = 753.294$$

$$\Sigma F_y = 0$$

$$N_A = W_A$$

As

$$F_A = \mu_A N_A$$

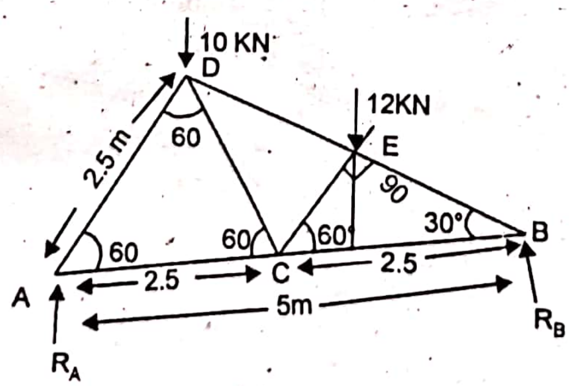
$$\frac{F_A}{\mu_A} = N_A$$

$$W_A = N_A = 3013 \text{ N}$$

$$W_A = 3013 \text{ N}$$

UNIT-II

Q.4. Determine the forces in the members BC, CE and DE of a truss loaded and supported as shown in Figure. (12.5)



Ans.

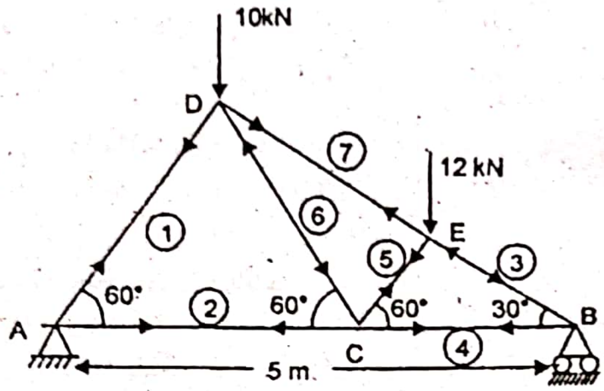
$$F_{PE} = ?$$

$$F_{EC} = ?$$

$$F_{BC} = ?$$

In ΔABD

$$AD = AB\cos 60^\circ = 5\times 0.5 - 2.5\text{ m}$$



From ΔACD

$$BC = 5-2.5 = 2.5\text{m}$$

In right triangle CEB

$$BE = BC\cos 30^\circ = 2.5\times\frac{\sqrt{3}}{2}$$

$$BE = 1.875\text{m}$$

Taking moment about A we get

$$R_B\times 5 = 10\times 1.25 + 12\times 3.125$$

$$R_B = \frac{50}{5} = 10\text{ kN}$$

$$R_A = \text{Total load} - R_B$$

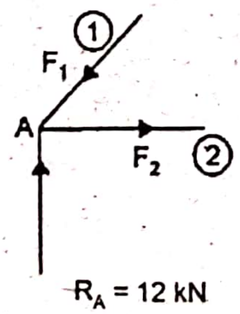
$$= 22 - 10 = 12\text{ kN}$$

Joint A: $F_1\sin 60 = 12$

$$F_1 = 13.58\text{ kN (C)}$$

Also $F_2 = F_1\cos 60$

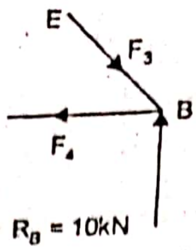
$$F_2 = 6.928\text{ (kN) (T)}$$



Joint B: $F_3\sin 30 = 10$

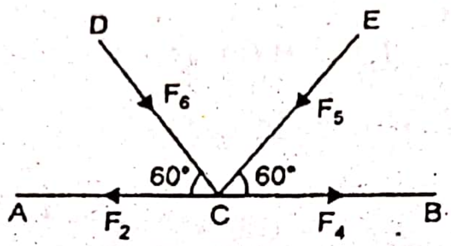
$$F_3 = 20\text{ kN (C)}$$

$$F_4 = F_3 \cos 30°$$
$$F_4 = 17.32 \text{ kN}$$



***Joint C:* Resolve force vertically**

$$F_6 \sin 60 + F_5 \sin 60 = 0$$
$$F_6 = -F_5$$

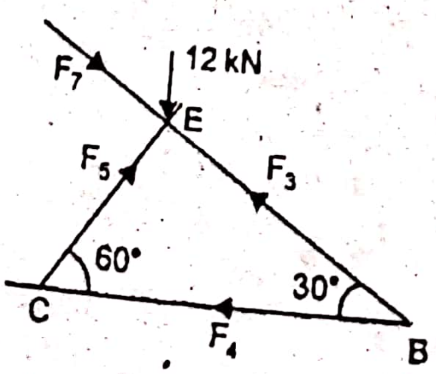


Resolve force horizontally

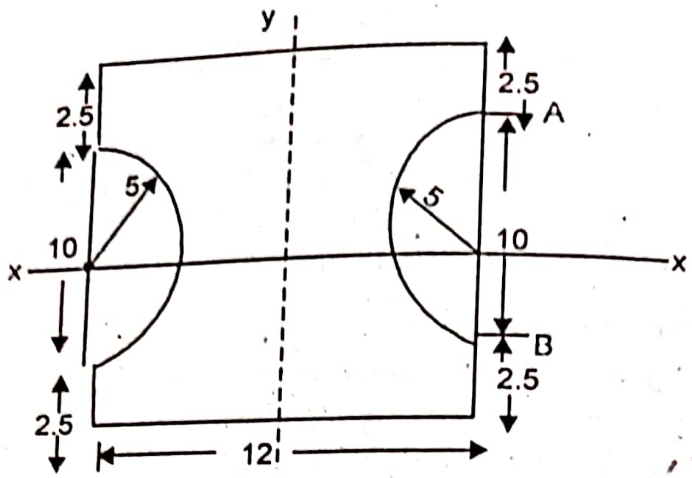
$$F_2 - F_6 \cos 60 = F_4 - F_5 \cos 60°$$
$$6.928 - \frac{F_6}{2} = 17.32 - \frac{F_5}{2}$$
$$\frac{-F_6 + F_5}{2} = 10.392$$
$$F_5 + F_5 = 20.784$$
$$F_5 = 10.392 \text{ (C)}$$
$$F_6 = -10.392 \text{ (T)}$$

Joint E:

$$F_7 = \text{Force in member 'FD'}$$
$$F_7 + 12 \cos 60° = F3$$
$$F_7 = F_3 - 12 \times 0.5$$
$$F7 = 14 \text{ kN (C)}$$



Q.5. Determine the moment of inertia of a cast iron section shown in Figure (12...) about both X and Y axis.



Ans. M.O.I of Lamina

$$I_{xx} = \text{M.O.I of Rectangle} - \text{M.O.I. of circle}$$

$$= \frac{6d^3}{12} - \frac{\pi r^4}{4}$$

$$= \frac{(15)^3 \times 12}{12} - \frac{\pi \times (5)^3}{4}$$

$$I_{xx} = 33.75 - 490.87$$

$$= 2884.13$$

$$I_{yy} = I_{yy}\text{, of rectangle} - I_{yy} \text{ of semi-circle.}$$

$$I_{yy} \text{ of rectangle} = \frac{15 \times 12^3}{12} = 2160$$

I_{yy} of semi circle

$$I = \frac{1}{2} \times \pi \times \frac{5^4}{4} = 245.43$$

Distance of its C_G from Diameter

$$h = \frac{4r}{3\pi} = \frac{4 \times 5}{3\pi} = 2.12 \text{ cm}$$

$$A = \frac{1}{2}\pi r^2 = \frac{1}{2} \times \pi \times 5^2 = 39.27 \text{ cm}$$

$I_{AB} = I_{GG} + AH^2$, MOI of semi circular part about centroidal axises

$$I_{GG} = 245.43 - 39.27 \times (2.12)^2 = 68.94$$

Again from Parallel axis Theorem

$$I_{yy} = I_{GG} + Ah_1^2$$

where h_1 = distance between axis and G axis = b – 2.12 = 3.88

$$I_{yy} = 68.94 + 39.27 \times 3.88^2$$

$$= 660.13$$

I_{yy} for 2 semi circular part = 2 × 660.13 = 1320.26

$$= 2160 - 1320.26 = 839.74$$

Q.6. In a system of connected bodies (Figure) the pulleys are frictionless and of negligible weight. Determine the value of weight A required to give 0.6g accelaration of weight *B* (*i*) in downward direction (*ii*) in upward direction. (12.5)

Ans.

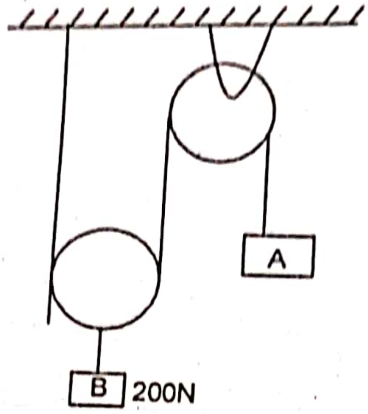
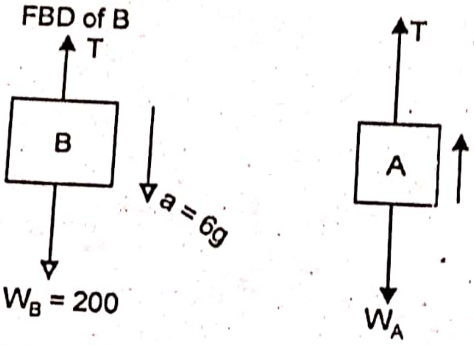


Fig.

As $a_3 = .6g$



Eq. of body B (Downward motion of block (B)

$$200 - T = \frac{200}{9.81} \times \frac{a}{2}$$

$$200 - T = \frac{200}{\cancel{9.81}} \times \frac{.6 \times \cancel{9.81}}{.2}$$

$$200 - T = 200 \times .3$$

$$200 - 200 \times .3 = T$$

$$200\,(.7) = T$$

$$T = 140 \text{ N}$$

(Upward Motion of A)

Upward Motion of A

$$T - W = \frac{W}{9.81} \times 6 \times 9.81$$

$$T = W + W \times .6$$

$$T = 1.6\,W$$

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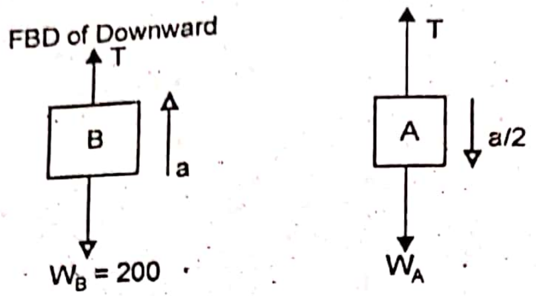
$$\text{As } (T = 140 \text{ N})$$
$$140 = 1.6\,\text{w}$$
$$W_A = 87.5 \text{ N (in upward motion)}$$

FBD of Downward motion



(Upward Motion of B)

$$T + \frac{200}{\cancel{9.81}} \times 6 \times \cancel{9.81} = 200$$
$$T = 200 - 200 \times .6$$
$$T = 200 \times .4$$
$$T = 80$$

Downward Motion of A

$$T = w + \frac{w \times a}{9.81 \times 2}$$
$$T = w + w\left(\frac{.6 \times \cancel{9.81}}{\cancel{9.81} \times 2}\right)$$
$$T = w + w \times .3$$
$$T = 1.3\,w$$
$$\frac{80}{1.3} = w$$
$$61.53 = w$$

Q.7. Two smooth spheres A and B having a mass of 2kg and 4 kg respectively collide with initial velocities as shown in Figure. If the coefficient of restitution for the spheres is e = 0.8, determine the velocities of each sphere after the impact. (12.5)

Ans.

$$e = .8$$
$$A = 2 \text{ kg}$$

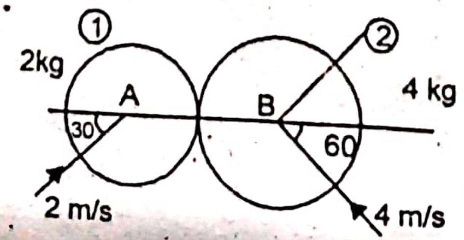


Fig.

$$B = 4\text{ kg}$$
$$V_A = 2\text{ m/s}$$
$$0_1 = 30$$
$$0_2 = 60$$

Tangential velocity ball A

$$(V_a)x = V_A \cos 30 = 2 \times \cos 30 = 1.732\text{ m/s}$$
$$(V_a)y = V_A \sin 30 = 2 \times \sin 30 = 1\text{ m/s}$$

Component of Tangential velocity of ball (B)

$$(V_b)x = 4 \times \cos 60^\circ = -4 \times \frac{1}{2} = -2$$

$$(V_b)y = 4 \times \sin 60^\circ = 4 \times \frac{\sqrt{3}}{2} = 2\sqrt{3}\text{ m/s}$$

Acc. to law of conservation of momentum along x axis

$$m_1 u_1 x + m_2 u_2 x = m_1 v_1 x` + m_2 v_2 x$$
$$2\times \sqrt{3} + 4 \times -2 = 2\times V_1 x + 4 \times V_2 x$$
$$\sqrt{3} + 2 \times -2 = V_1 x + 2V_2 x$$
$$\sqrt{3} - 4 = V_1 x + 2V_2 x$$
$$-2.26 = V_1 x + 2V_2 x \quad ...(1)$$

Given

$$e = .8$$

along x axis

$$e = \frac{V_2 x - V_1 x}{u_1 x - u_2 x}$$

$$.8 = \frac{V_2 x - V_1 x}{1.732 + 2}$$

$$.8 \times 3.732 = V_2 x - V_1 x$$
$$2.9856 = V_2 x - V_1 x \quad ...(2)$$

Solving (1) and (2)

ctively
tution
er the
(12.5)

$$\begin{array}{r} V_1 x + 2V_2 x = -2.26 \\ -V_1 x + V_2 x = 2.9856 \\ \hline 3V_2 x = .725 \\ V_2 x = .241 \end{array}$$

$$V_2 x = .241$$
$$V_1 x = -2.744$$

As line of impact is about x-axis, motion along dir^n $\perp$ to line of impact remain reserved.

Velocity of body along y axis remain same before and after collision.

$$\left.\begin{array}{l} U_{1y} = V_{1y} \\ U_{2y} = V_{2y} \end{array}\right\} \text{Velocity along y axis}$$

$$V_1 = \sqrt{V_1x^2 + V_1y^2}$$

$$V_1 = \sqrt{(2.744)^2 + 1}$$

$$V_1 = 2.917$$

$$V_2 = \sqrt{(V_2x)^2 + (V_1y)^2}$$

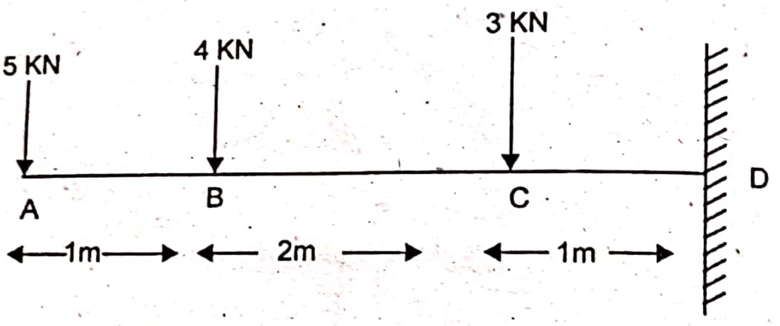
$$V_2 = \sqrt{(.241)^2 + (2\sqrt{3})^2}$$

$$V_2 = 3.47 \text{ m/s.}$$

UNIT-IV

Q.8. Draw the shear force and bending moment diagram for the cantilever beam loaded as shown in Figure. (12.5)

Ans.



From Free End

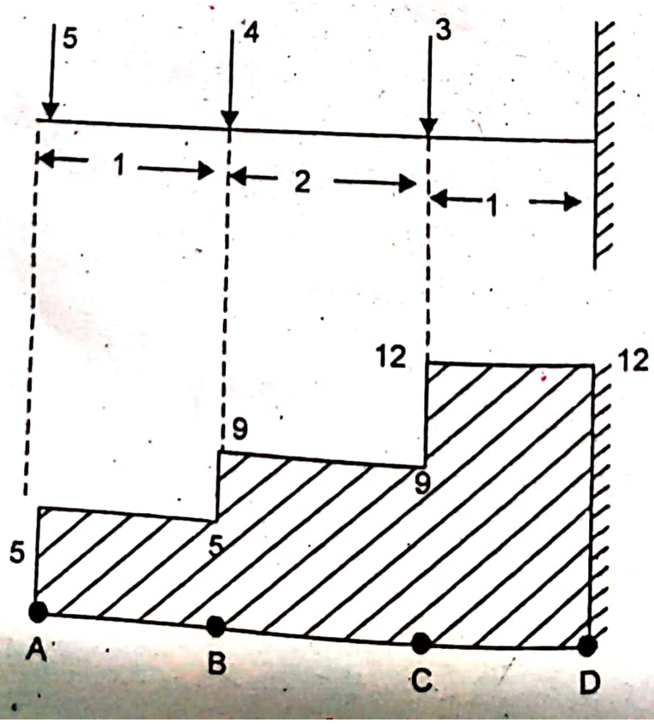
At point 'A' = +5 K N

At Point 'B' = 5 + 4 = 9 KN

At Point 'C' = 5 + 4 + 3 – 12 KN

At Point 'D' = 12 KN.

SFD



BMD

BMD

Q.9. A recipr
of length 20 cm
Find the angula

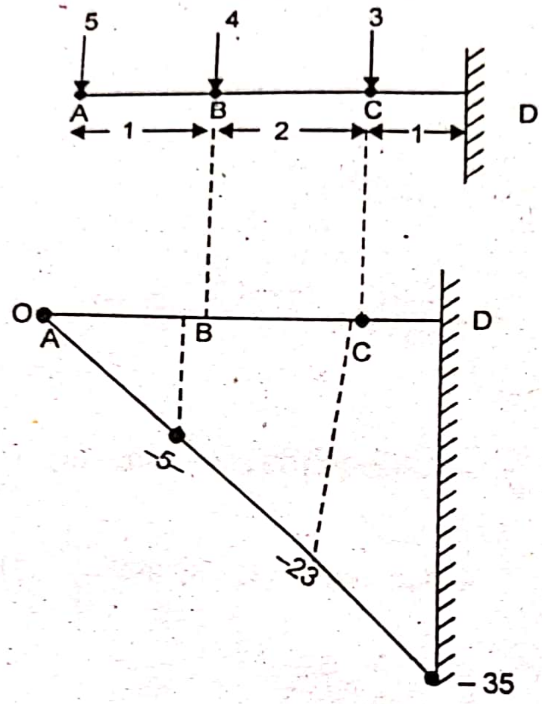
Ans.

From Law of

BMD

$$\text{At point } A = 0$$
$$\text{At Point } B = -5\times1 = -5 \text{ KN}$$
$$\text{At point } C = -5\times3 - 4\times2 = -15 - 8 = -23 \text{ KN}$$
$$\text{At point } D = -5\times4 - 4\times3 - 3\times1$$
$$-20 - 12 - 3 = -35 \text{ KN}$$

BMD



Q.9. A reciprocating Engine mechanism is shown in Figure. The crank OA is of length 20 cm and rotating at 500 rpm. The connection rod AB is 100 cm long. Find the angular velocity of the connection rod and velocity of piston B. (12.5)

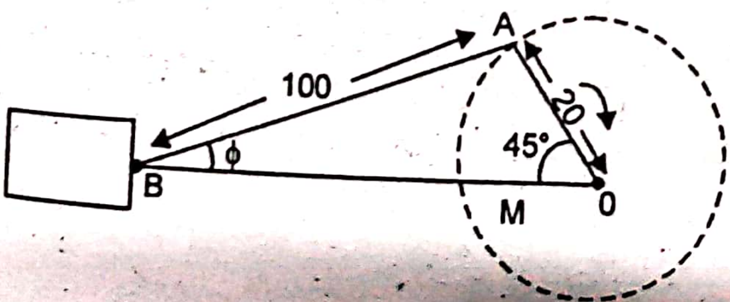
Ans.

$$N = 500 \text{ rpm}$$

From Law of Triangle

$$\frac{l}{\sin 0} = \frac{r}{\sin\phi}$$

$$\sin\phi = \frac{r\sin\theta}{l}$$



$$= \frac{20 \sin 45}{100}$$

$$\text{Sin } \phi = .141 \ \phi = 8.1°$$

x is distance between Crank Centre O and Piston B

$$x = \text{B M} + \text{OM}$$

$$x = \text{l} \cos \phi + \text{r} \cos \theta$$

$$\frac{dx}{dt} = r \sin \theta \frac{d\theta}{dt} + l \sin \phi \frac{d\phi}{dt}$$

$$\frac{dx}{dt} = r \sin \theta \times w + l \sin \phi \times \left(\frac{r \cos \theta}{l \cos \phi}\right) \times w$$

$$\text{As } \frac{d\phi}{dt} = \left(\frac{r \cos \theta}{l \cos \phi}\right) w$$

As $\omega = \frac{2\pi N}{60} = \frac{2\pi \times 500}{60} = 52.359$

Put value of ω in eq (1)

$$\frac{dx}{dt} = 20 \sin 45 \times 52.359 + 100 \times .141 \times \left(\frac{20 \cos 45}{100 \cos 8.1}\right) \times 52.39$$

$$= 740.369 + 141 \times 52.39 \times \frac{2}{10} \times .714$$

$$\frac{dx}{dt} = 1054.86 \text{ cm/s}$$

$$\frac{dx}{dt} = 10.54 \text{ m/s } \textbf{Ans.}$$

END TERM EXAMINATION [MAY-JUNE 2017]
SECOND SEMESTER [B.TECH.]
ENGINEERING MECHANICS [ETME–110]

Time : 3 hrs. **M.M. : 75**

Note: *Attempt any five questions including Q.no. 1 which is compulsory.*

Q.1. Short questions:

Q.1. (a) Classify the force systems. **(5)**

Ans. The force systems can be classified as

→ Concurrent forces in a plane

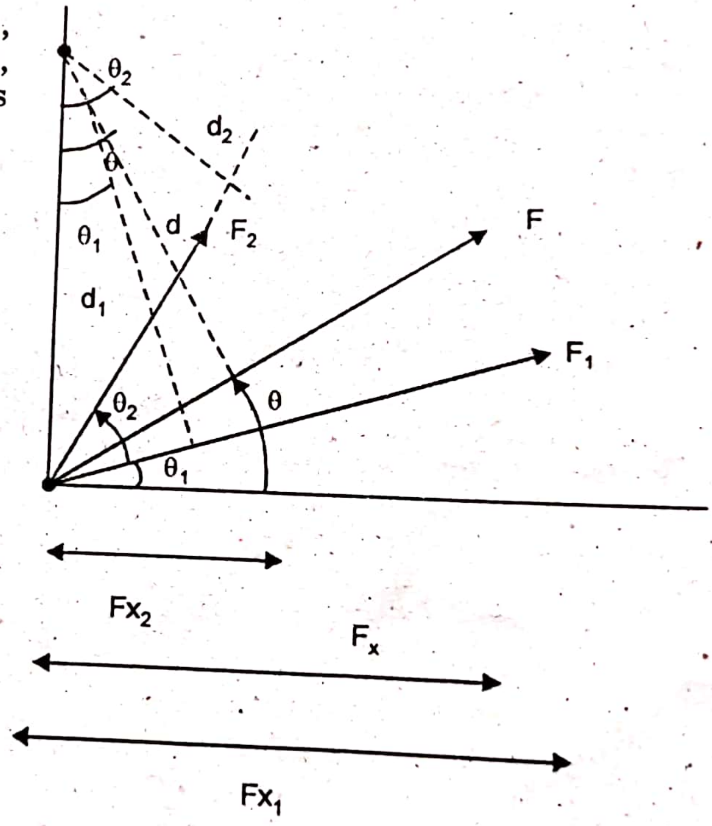
→ Parallel forces in a plane

All these forces whose line of action passes through a common point are known as concurrent forces. Forces for which line of action are parallel to each other are called parallel forces.

Q.1. (b) State and proof varignon's theorem with example. **(5)**

Ans. Varignon's Theorem

Moment of a resultant of two forces, about a point lying in the plane of forces, is equal to the algebraic sum of moments of these forces about the same point.



Proof →

Moment of force F about O

$$Fd = F(OA \cos\theta)$$

$$= OA\,(F \cos\theta)$$

$$Fd = OA\,f_x$$

Moment of the force F_1 about O

$$F_1d_1 = F_1\,(OA \cos\theta_1)$$

$$= OA\,(F_1 \cos\theta_1)$$

$$F_1d_1 = OA\,f_{x1} \qquad ...(1)$$

Moment of force F_2 about O

$$F_2d_2 = F_2(OA \cos\theta_2)$$

$$= OA\,(F_2 \cos\theta_2)$$

$$F_2d_2 = OA\,F_{x2} \qquad ...(2)$$

Adding (1) and (2)

$$F_1d_1 + F_2d_2 = OA\,(F_{x1} + F_{x2})$$

$$F_x = F_{x1} + F_{x2};\; OA\,(F_x) = OA\,(F_{x1} + F_{x2})$$

$$Fd = F_1d_1 + F_2d_2$$

Q.1. (c) Explain cone of friction. **(5)**

Ans. Cone of friction

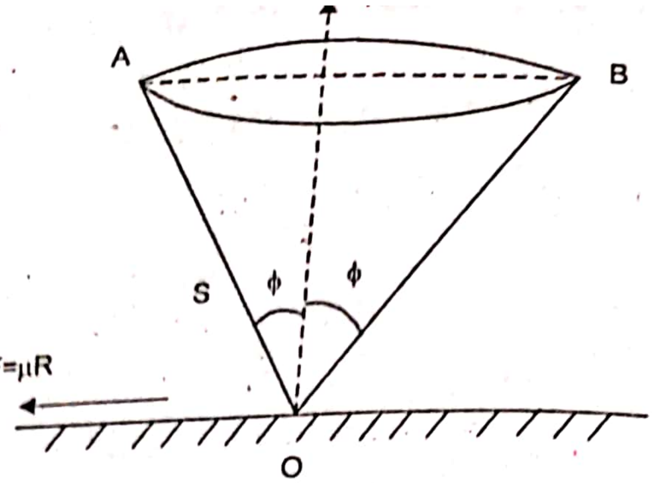
R → normal reaction

F → frictional force

ϕ semi vertex x angle

$$S = \sqrt{R^2 + F^2}$$

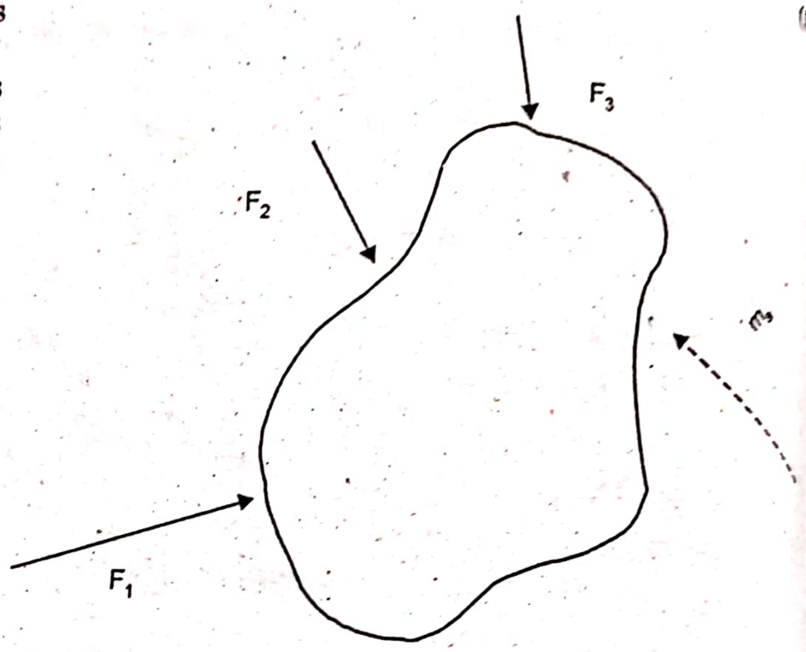
It is an imaginary cone AOB generated in case of non coplanar forces by revolving the resultant S about normal OR.



Q.1. (d) State and explain D'Alembert's principle.

Ans. D Alembert's principle

The principle states that the system of forces acting on a body in motion is in dynamic equilibrium with the inertia force of the body.



Consider a body subjected to a system of force F_1, F_2 and F_3. The resultant of these forces will cause the body to move along its sirection with an acceleration a. The body can be made to remain static if a force equal to ma is made to act in the reversed direction.

$$\Sigma F = 0 : R - ma = 0$$

or

$$R = ma$$

(–ma) is called as inertia force.

Q.1. (e) What are the assumptions involved in analysis of a perfect truss (5)

Ans. Assumptions of a Truss →

(1) Truss is statically determinate

(2) The external loads and reactions act only at the joints.

(3) All members are rigid and lie in the same plane.

(4) The self weight of the members is neglected as it is small compared to the loads they carry.

(5) All member are of uniform cross section.

Q.2. Derive the expression of the ratio of tensions for the belt pulley system. (12.5)

Ans. At Friction Ratio of Tensions

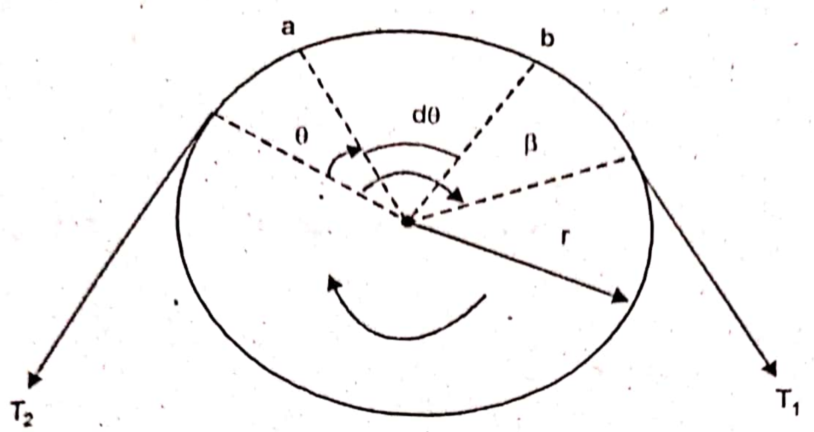
T1 ⟶ of belt (Belt leaving the pulley)

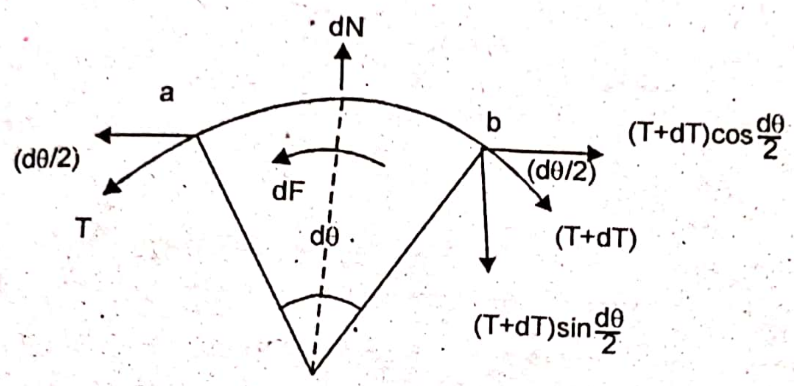
T2 ⟶ Tension on slack side of belt (Belt entering the pulley)

Frictional force

$dF = \mu\, dN$

Normal reaction = dN





$$\Sigma F_s = 0$$

$$(T+dt)\cos\frac{d\theta}{2} - T\cos\left(\frac{d\theta}{2}\right) - df = 0$$

$$\Sigma F_y = 0$$

$$dN - (T+dt)\sin\frac{d\theta}{2} - T\sin\frac{d\theta}{2} = 0$$

$$\cos\frac{d\theta}{2} \approx 1 \text{ and } \sin\frac{d\theta}{2} = \frac{d\theta}{2}$$

$$T + dt - T - dF = 0 \Rightarrow \alpha T - dF = 0$$

$$\Rightarrow dT - \mu dN = 0$$

$$dN - T\frac{d\theta}{2} - dT\frac{d\theta}{2} - \frac{Td\theta}{2}$$

[Neglected very small Term]

$$dN - Td\theta = 0$$

$$dT = \mu(Tdq)$$

$$\int_{T_2}^{T_1} \frac{dT}{T} = \mu\int_0^{\beta} d\theta$$

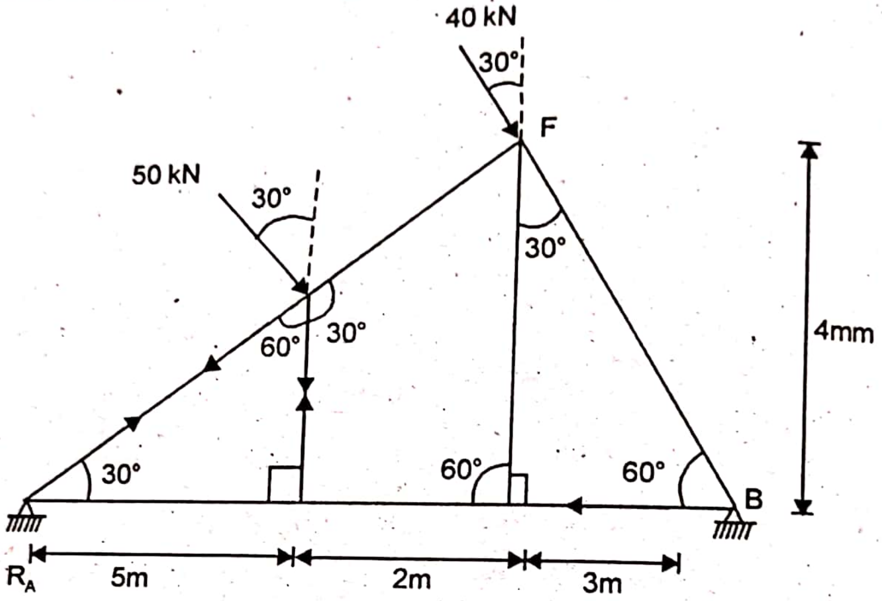
$$[GT]_{T_2}^{T_1} = \mu[\theta]_0^{\beta} \Rightarrow ln T_1 - ln T_2 = \mu\beta$$

$$In\left(\frac{T_1}{T_2}\right) = \mu\beta \Rightarrow \boxed{\frac{T_1}{T_2} = e^{\mu\beta}}$$

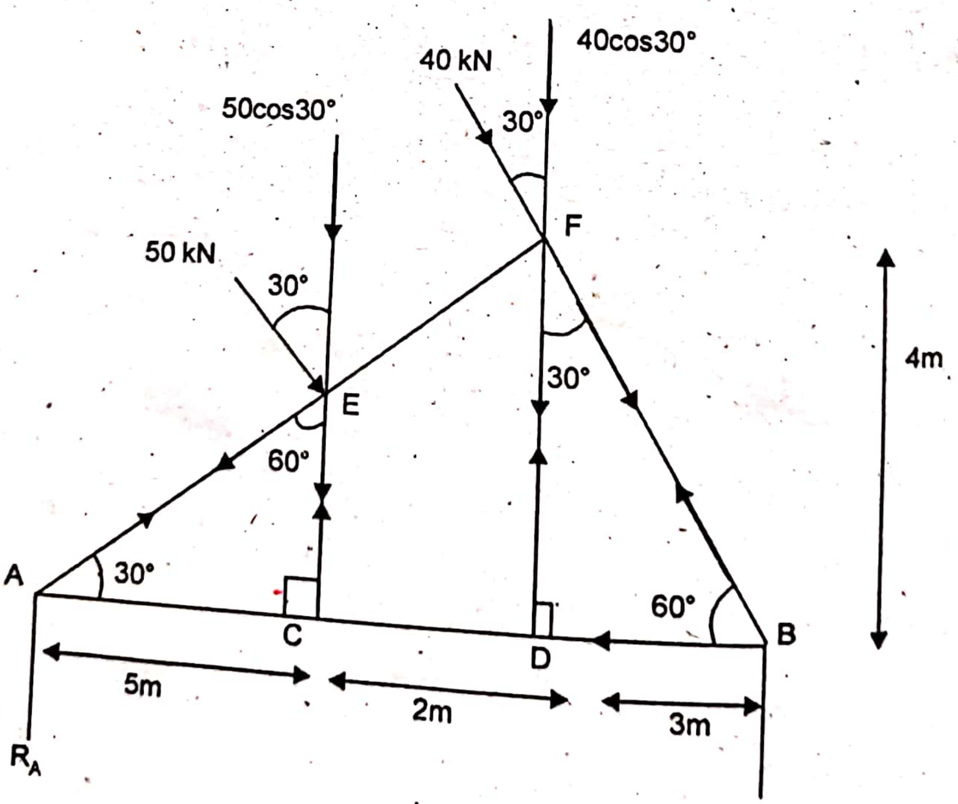
Valid only for →
(1) Flat belts over pulley
(2) Bend brake
(3) Rope wound over cylindrical drum
Ratio of Tensions – Rope and Belt drive

$$\frac{T_1}{T_2} = e^{(\mu \operatorname{cosec} \alpha)\beta} \quad 2\alpha \rightarrow \text{groove angle}$$

Q.3. Find the forces in all the members of the truss shown in figure. (12.5)



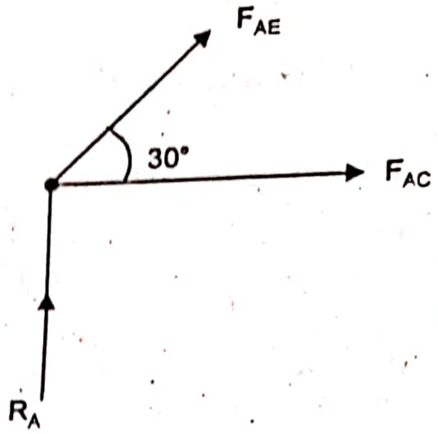
Ans.



$$50 \cos 30° = 43.30$$
$$40 \cos 30° = 34.64$$
$$R_A + R_B = 43.30 + 34.64$$
$$\Sigma M_A = 0$$

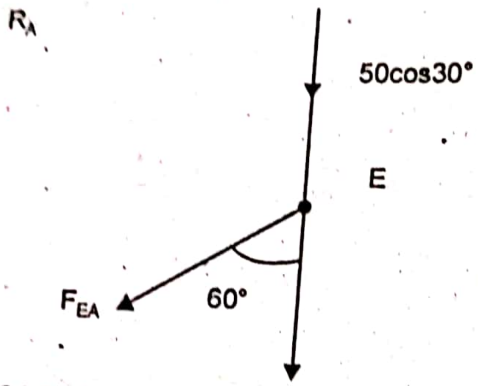
$$43.30 \times 5 + 34.64 \times 7 - R_B \times 10 = 0$$
$$R_B = 45.898 \text{ kN}$$
$$\therefore \quad R_A = 32.042 \text{ kN}$$

Joint A



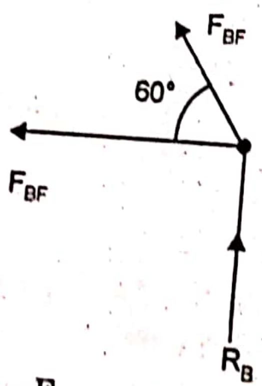
$$F_{AE} \sin 30° + R_A = 0$$
$$F_{AE} \times 0.5 = -32.042$$
$$F_{AE} = -64.084 \text{ kN}$$
$$F_{AE} \cos 30° + F_{AC} = 0$$
$$-64.084 \cos 30° + F_{AC} = 0$$
$$F_{AC} = 55.498 \text{ kN}$$

Joint E



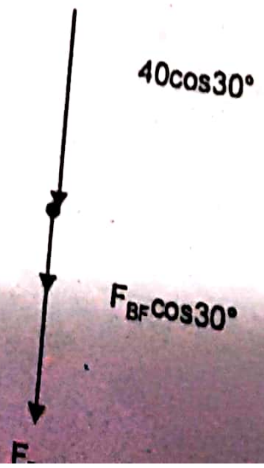
$$-50 \cos 30° + 64.084 - F_{EC} = 0$$
$$F_{EC} = 20.784 \text{ kN}$$

Joint B



$$-F_{BF} \cos 60° - F_{BA} = 0$$
$$-F_{BF} \times 0.5 - 55.498 = 0$$
$$F_{BF} = -110.996 \text{ kN}$$

Joint F



$$-40\cos 30° - F_{BF}\cos 30° - F_{BF} = 0$$

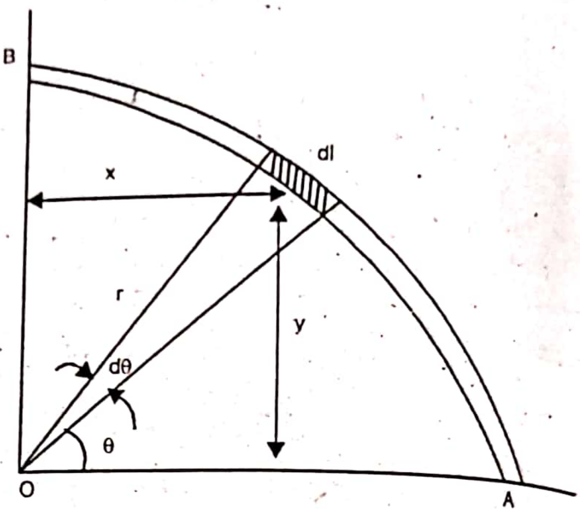
$$F_{FD} = 61.48 \text{ KN}$$

Q.4. Describe the method of finding centre of gravity of composite bent wires. **(12.5)**

Ans.

Length AB of wire is divided into a number of small dements. Element of length dl subtends an angle $d\theta$ at O

Centroidal distance of the dement from



$$x - G_x S \quad \bar{y} = \frac{\int y\,dl}{\int dl}$$

In term's of polar coordinates

$$y = r\sin\theta \text{ and } dl = r d\theta$$

$$\int y\,dl = \int_0^{\pi/2} (r\sin\theta) r d\theta = r^2 \int_0^{\pi/2} \sin\theta d\theta$$

$$= r^2(\cos\theta)_0^{\pi/2} = r^2$$

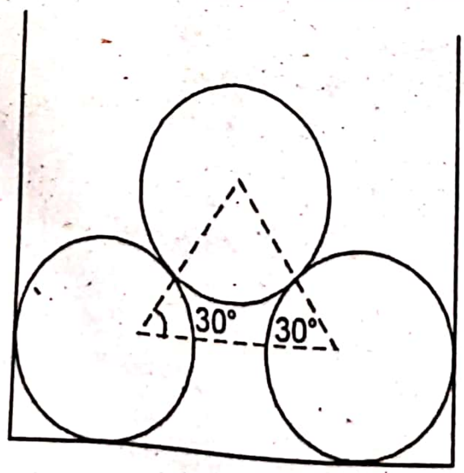
$$\int dl = \int_0^{\pi/2} r d\theta = r(\theta)_0^{\pi/2} = \frac{r\pi}{2}$$

$$\therefore \quad \bar{y} = \frac{r^2}{\frac{r\pi}{2}} = \frac{2r}{\pi}$$

Since the wire is bent in the form of a quadrant of are of circle which is symmtrical about x-axis and y-axis

$$\bar{x} = \bar{y} = \frac{2r}{\pi}$$

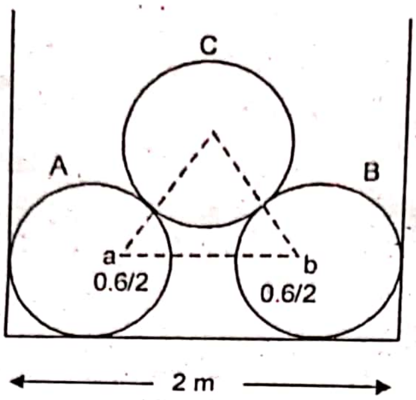
Q.5. (a) Three identical tubes of weights 8 kN each are placed as shown in figure. Determine the forces exerted by the tubes on the smooth walls and floor.



(6.5)

Ans.

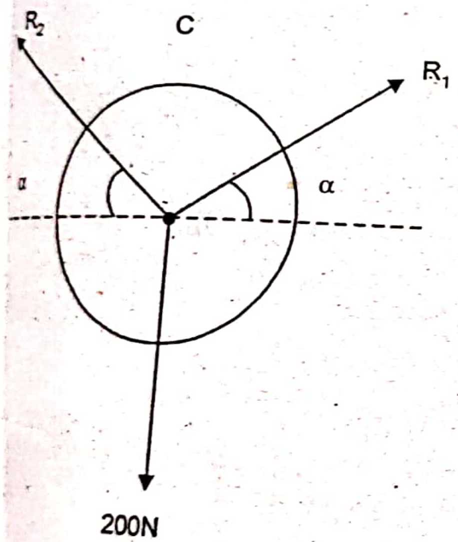
$$ab = 2 - \frac{0.6}{2} - \frac{0.6}{2} = 1.4$$

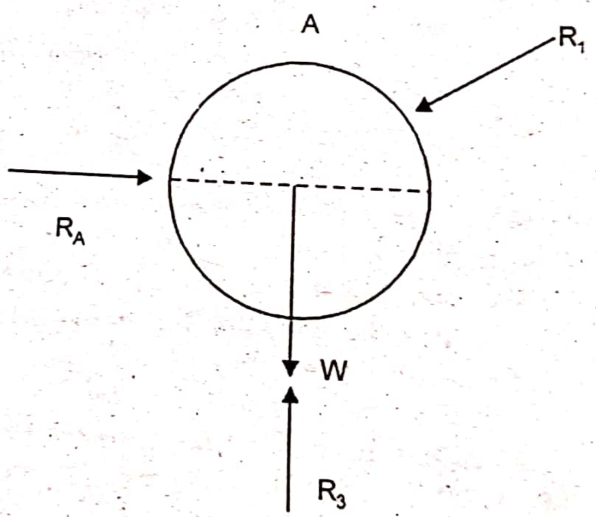


$$ac = 0.3 + 0.6 = 0.9 \text{ m}$$

$$\cos\alpha = \frac{1.412}{0.9} = .7777 \quad \alpha = 38.94$$

$$\frac{R_1}{\sin(90+\alpha)} = \frac{R_2}{\sin(90+\alpha)} = \frac{2000}{\sin(180-2\alpha)}$$





$$R_1 = R_2 = 2000 \times \frac{\sin(90+\alpha)}{\sin(180-2\alpha)} = 2000 \times \frac{\sin(90+38.94)}{\sin(180-2\times 38.9)}$$

$$= 2000 \times \frac{.7777}{-9777} = 1590.87 \text{ N}$$

$$\Sigma F_x = 0 \quad R_a - R_1\cos\alpha = 0$$

$$R_a = R_1 \cos\alpha = 1237.38 \text{ N}$$

$$\Sigma F_y = 0 \quad R_1 \sin\alpha + W - R_3 = 0$$

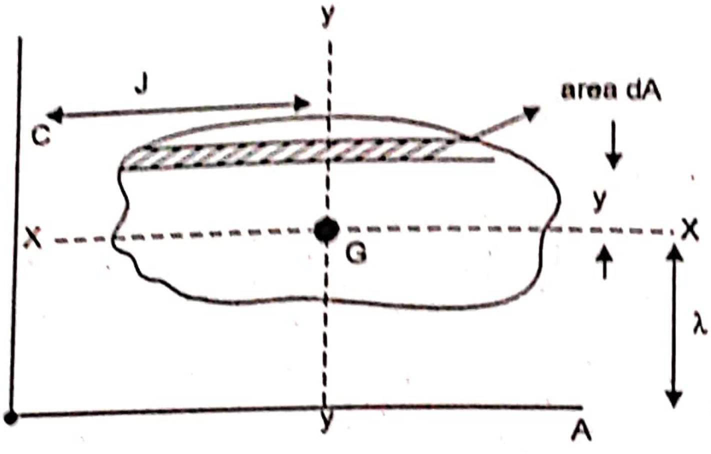
$$R_3 = 19999.87 \text{ N}$$

Q.5. (b) State and prove parallel axis theorem. **(6)**

Ans. Parallel axis Theorem

The moment of inertia of a plane lamina about any axis is equal to the sum of its M.I. about a parallel axis through its center to gravity and the product of tis area (A) and the square of the distance between the two area.

Proof →



Element of area dA is located at distance y from the x-axis.

Distance from axis AD = (h + y)

MOI of the element about DA = $dA(h + y)^2$

MOI of the entire lamna about AA

$$= \Sigma dA(h + y)^2$$
$$= \Sigma dAh^2 + \Sigma dAy^2 + \Sigma dA\,(2hy)$$
$$= h^2\Sigma dA + \Sigma dAy^2 + 2h\,\Sigma dAy$$

$$\Sigma dA = A$$

$$\therefore \quad h^2\Sigma dA = h^2A$$

ΣdAy^2 = MOI of lamina abput axis x – x

ΣdAy = 0 as $x - x$ is centroidal

$$I_{AA} = I_{xx} + Ah^2$$
$$I_{BB} = I_{yy} + Aj^2$$

Q.6. Derive the expression for the coefficient of restitution. (12.5)

Ans. Coefficient of Restitution

Two bodies A and B of mass m_1 and m_2 respectively.

V_1 and V_2 be the velocities of bodies before impact. $(V_1 > V_2)$

Velocity of approach = $U_1 - U_2$

Velocities after seperartion be $(V_2 - V_1)$

Newton's Law of collision states that when two moving bodies collide with each other their velocity bears constant ratio to their velocity of approach.

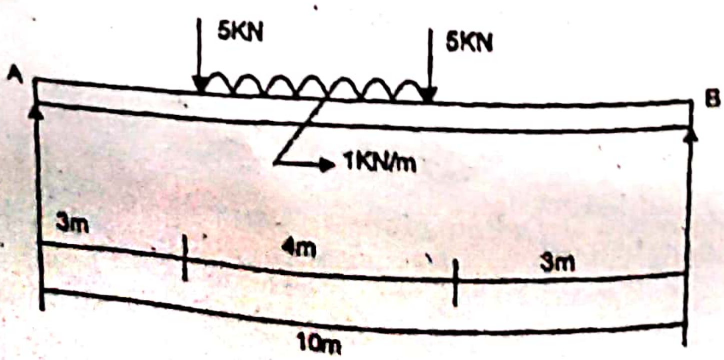
$$V_2 - V_1 = e(V_1 - V_2)$$

$$e = \frac{V_2 - V_1}{U_1 - U_2} = \frac{\text{Velocity of seperation}}{\text{Velocity of approach}}$$

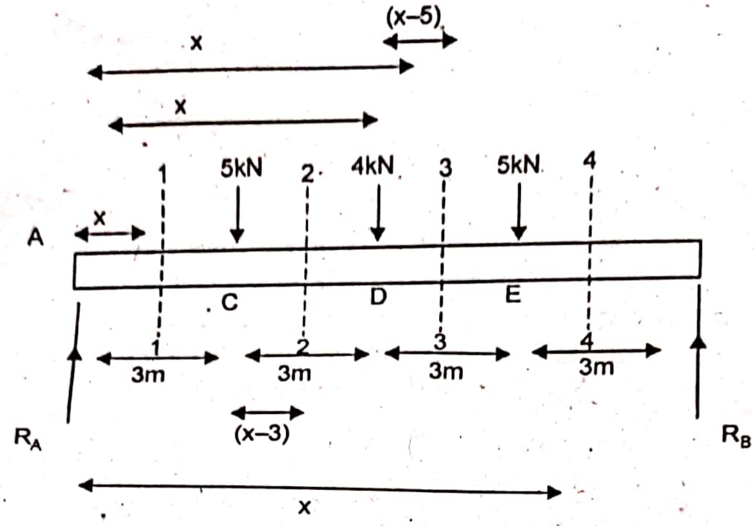
$e = 0 \rightarrow$ Bodies are in elastic

$e = 1 \rightarrow$ bodies perfectly elastic

Q.7. Draw the shear force and bending moment diagram for the beam shown in figure. (12.5)



Ans.



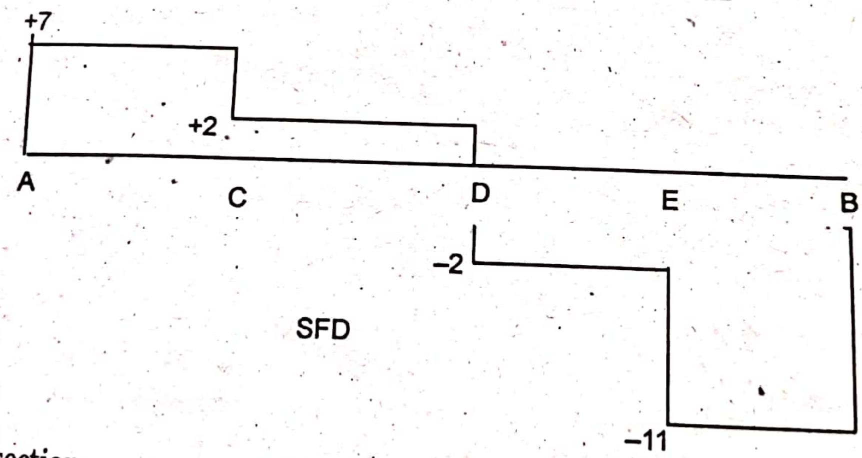
$$R_A + R_B = 14$$
$$\Sigma_{MA} = 0$$
$$-5 \times 3 - 4 \times 5 - 5 \times 7 + R_B \times 10 = 0$$
$$R_B = 7 \therefore R_B = 7$$

S.F for section $(1)-(1) = +R_B = 7$

S.F for section $(1)-(2) = +7-5 = +2$

S.F for section $(3)-(3) = +7-5-4 = -2$

S.F for section $(4)-(4) = +7-5-4-5 = -11$



SFD

B.m for section $(1)-(1) = +R_A \times x$

at $x = 0$ at $A = 7 \times 0 = 0$

$x = 3$ at $C = 7 \times 3 = 21$

B.m for section $(2)-(2) = 7 \times x - 5(x-3)$

at $x = 5$ at $D = 35 - 5 \times 2 = 25$

B.m for section $(3)-(3) = 7x3 - 5(x-3) - 4(x-5)$

at $x = 7$ at $E = 49 - 5 \times 4 - 4 \times 2 = 21$

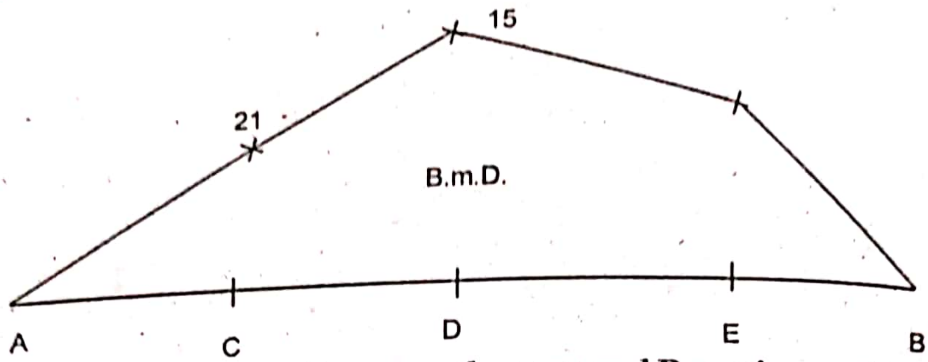
B.m for section (4) – (4)

at B

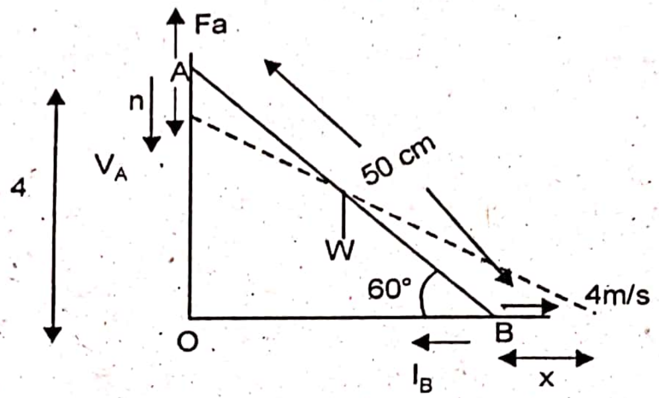
$= 7x - 5(x-3) - 4(x-5) - 5(x-7)$

$x = 10 -$ B.m $= 70 - 5 \times 7 - 4 \times 5 - 5 \times 3$

$= 70 - 35 - 20 - 15 = 0$



Q.8. (a) A straight rod AB, 50 cm long has one end B moving with a velocity of 4m/s, and the other end A moving along a vertical line YO as shown in figure. Find the velocity of the end A and of the midpoint of the rod when it is inclined at 60° with horizontal. **(6)**



Ans.

$$\text{Sin}60° = \frac{H}{50}$$

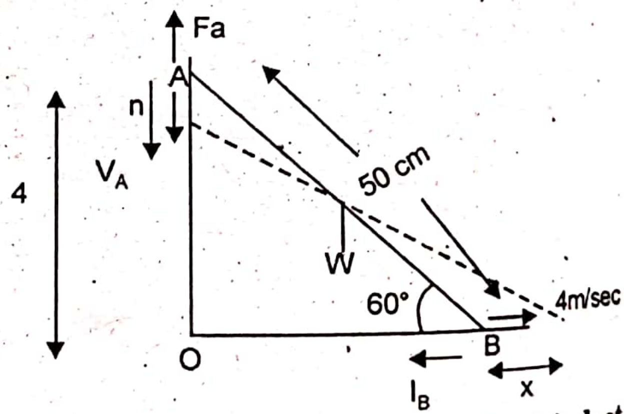
When the loaded moves

$$\text{Sin}60° = \frac{H-d}{50}$$

$$H = 43.30 \text{ cm}$$

$$F_a \times OB = F_B \times OA$$

$$\frac{F_a}{F_B} = \frac{OA}{OB}$$



Q.8. (b) A stone falls freely from rest and total distance covered by it in last second of its motion equals the distance covered by it in first three seconds of its motion. Determine the time in which the stone remains in air. **(6.5)**

Ans.

$$S_{nth} = S_{0-3}$$

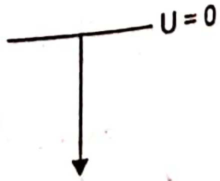
$$S_n - S_{n-1} = \frac{1}{2} \times g \times (3)^2$$

$$\frac{1}{2} \times 9 \times n^2 - \frac{1}{2} \times 9(n-1)^2 = \frac{1}{2} \times 9 \times 9$$

$$n_2 - (n-1)^2 = 9$$

$$n^2 - (n^2 + 1 - 2n) = 9$$

$$2n = 10 \quad \boxed{n = 5 \text{ seconds}}$$



END TERM EXAMINATION [MAY-JUNE 2018]
SECOND SEMESTER [B.TECH]
ENGINEERING MECHANICS [ETME–110]

Time : 3 hrs. **M.M. : 75**

Note: Attempt five questions in all including Q. No. 1 which is compulsory. Select one question from each unit. Assume suitable missing data if any.

Q. 1. Short questions:

(a) Define resultant and equilibrant. **(2.5)**

Ans. When a body is acted upon by a system of forces, then vectorial sum of all the forces is known as resultant. Hence resultant refers to the single force which produces the same effect as is done by the combined effect of several forces.

A number of forces may act on a body in such a manner that the body is not in equilibrium. The resultant of several forces may cause a change of state of rest or of uniform motion. A single force may have to be applied to the body to bring it in equilibrium state. That single force is known as equilibrant. Equilibrant is equal and opposite to the resultant of several forces acting on the body.

Q. 1. (b) What is a couple? What are its properties? **(2.5)**

Ans. A system of two equal parallel forces acting in opposite directions cannot be replaced by a single force. In such a case, the two forces form a *couple* which has a tendency to rotate the body. The perpendicular distance between the lines of action of the two forces is termed as the *arm* of the couple.

Moment of a Couple. The rotational tendency of the couple is measured by its moment. The moment of a couple is the product of the either one of the forces forming the couple and the arm of the couple.

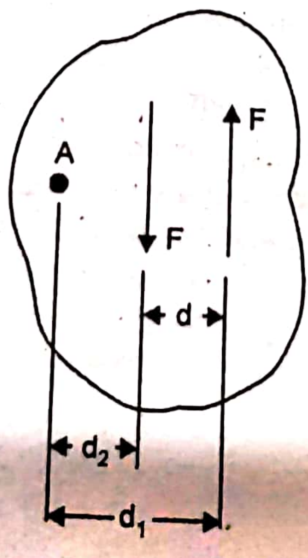
It has the same units as the moment of a force, N-m.

Moment of the couple $= F \times d$

Tendency to rotate in anticlockwise direction is assumed positive and in clockwise is negative.

To summarize:

1. The algebraic sum of the forces forming a couple is zero.
2. The algebraic sum of the moments of the two forces forming a couple is independent of the position of the moment centre chosen as shown below.



Q. 1. (c) Why co-efficient of dynamic friction is less than that of static friction? **(2.5)**

Ans. Dynamic friction is an opposing force that comes into place when the force applied on a body is causing the body to move.

The force here is greater than kinetic friction cause the body is already moving due to applied force.

Mean while static friction is the opposing force that comes into place when the force applied on a body is not causing the body to move.

So, kinetic friction or dynamic friction is less because it is not able to fully retain the body from moving but static friction and limiting friction is high because it is able to retain the body from moving.

Q. 1. (d) Explain the principle involved in graphical method in the analysis of trusses. **(2.5)**

Ans. The analytical methods, *i.e.*, the method of joints and the method of sections give absolutely correct results regarding forces set up in the members of a perfect truss when it is loaded. However, there arise certain situations where it becomes impossible to get the results from these methods. Recourse is then made to graphical methods which are quite convenient and provide reasonably accurate results.

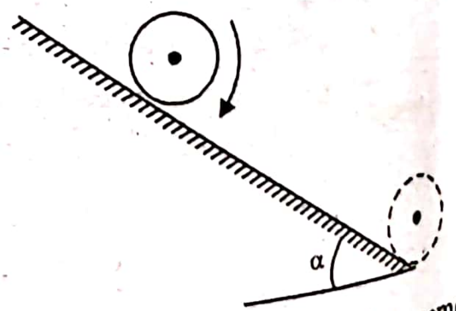
The given truss or frame is drawn accurately according to some linear scale. The loadsand support reactions are also indicated on the frame both in magnitude and direction. The naming of the various members of the frame is then done according to Bow's notation. In this technique, a force is designated by two letters which are written on either side of the line of action of the force.

Q. 1. (e) Define moment of inertia of an area. Why it is called the second moment of area? **(2.5)**

Ans. The mass and the surface area of a body are two of its important parameters. But in certain situations the distribution of these parameters within the body and their orientation with respect to some reference axis can be of as much importance as their absolute values.

Consider a solid cylinder and a hollow cylinder each of radius r, to slide down (without rolling) an inclined plane of angle α, from rest. Both the bodies shall be observed to reach the bottom of the plane at the same time, experiencing the same acceleration due to gravity (equal to $g \sin \alpha$) irrespective of the mass and the radius.

Now, let them roll down the same inclined plane without sliding. Which one would reach the bottom first? The answer, although not simple, is; the solid cylinder will reach first, followed by the hollow cylinder.



From the above observation we can say that, this phenomena has something to do with the distribution of the mass within the body.

The concept which gives a quantitative estimate of the relative distribution of area and mass of a body with respect to some reference axis is termed as the moment of inertia of the body.

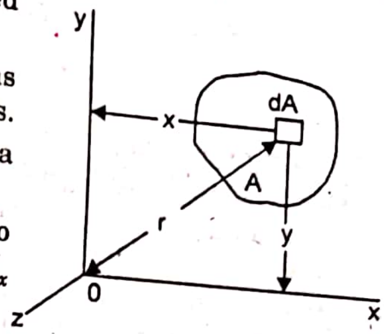
Analogy-wise the role played by the moment of inertia in the rotary motion is similar to the role played by the mass in the translatory motion.

The moment of inertia of an area is called as the area moment of inertia or the second moment of area.

The moment of inertia of tie mass of a body is called as the mass moment of inertia.

Consider a plane figure of area A in the x-y plane as shown in Fig. Divide this area A in to infinitesimal areas.

Let dA be any element of the area situated at a distance (x, y) from the axes.



The moment of inertia of the area A with respect to the x-axis (also called the second moment of the area) $= I_x$

$$= \int y^2 dA$$

The moment of inertia of the area A with respect to the y-axis (also called the second moment of the area) $= I_y = \int x^2 dA$

Integration should cover the entire area of the figure and its value shall depend upon the shape of the area and its orientation with respect to the axis.

Q. 1. (f) What is meant by dependent motion? Give example. (2.5)

Ans. Some times, the motion of one particle depends on the motion of another this is called dependent motion. One situation that creates dependent motion is when two particles are connected by cord around a pully.

Example: Four pulley system and three cord system.

Q. 1. (g) State D'Alembert's principle. Why it is called principle of dynamic equilibrium? (2.5)

Ans. The equation of motion of the particle P

$$\Sigma F = ma \quad \text{...(1)}$$

can be written in the form

$$\Sigma F - ma = 0 \quad \text{...(2)}$$

which means that the resultant of the external forces (ΣF) and the force ($-$ ma) is zero. The force ($- ma$) *is* called inertia force. The inertia of a body can be defined as the resistance to the change in the condition of rest or of uniform motion of a body.

The magnitude of the inertia force is equal to the product of the mass and acceleration of the particle and it acts in a direction *opposite* to the direction of acceleration of the particle.

The equations in the form

$$\Sigma F - ma = 0$$

Or

$$\Sigma F + \underbrace{(-ma)}_{\text{Inertia Force}} = 0 \quad \text{...(3)}$$

or in the component form

$$\Sigma F_x + \underbrace{(-ma_x)}_{\text{Inertia Force}} = 0 \quad \text{...(4)}$$

$$\Sigma F_y + \underbrace{(-ma_y)}_{\text{Inertia Force}} = 0 \quad \text{...(5)}$$

are called the equations of dynamic equilibrium of the particle.

So, to write the equation of dynamic equilibrium of a particle add a fictitious force equal to the inertia force to the external forces acting on the particle and equate the sum (resultant) to zero (Fig. 1). This concept is known as D'Alembert's Principle. It is very useful concept as moment equation of dynamic equilibrium is easy to conceive and write.

For the simplicity of representation, equations (4) and (5) can be written as

$$\Sigma F_x = 0 \qquad \text{...(6)}$$

$$\Sigma F_y = 0 \qquad \text{...(7)}$$

When expressed in the above manner, the terms ΣF_x. and ΣF_y are to be *redefined*. That is, ΣF_x and ΣF_y now include the inertia forces also.

Further the equations 6 and 7 have an appearance similar to the equations of static equilibrium.

It should be clearly understood that the equation of motion of a particle and the equation of dynamic equilibrium of a particle are the two methods of expression which differ only in the concept used and in the manner of writing the equations. The final result, however, shall be the same. Whichever method is followed should be followed consistently and clearly.

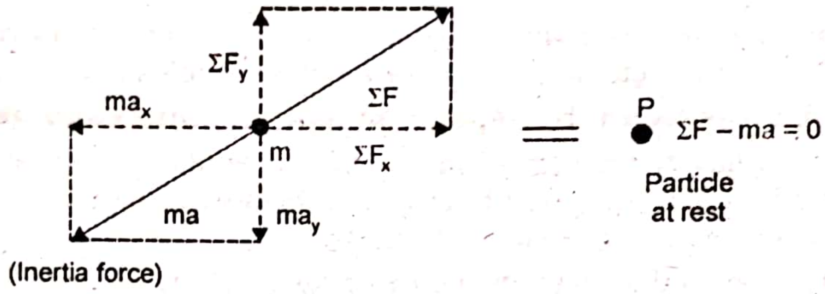


Fig. 1.

Q. 1. (h) How will you locate the instantaneous centre of rotation? (2.5)

Ans. (*i*) When the directions of the velocities of two points *A* and *B* in the body are known and are unequal.

Consider two points *A* and *B* on the rigid body in plane motion. Let at any instant their velocities be v_a and v_b respectively Fig. 1.

Draw *AC* perpendicular to the velocity v_a at the point *A*. If this velocity v_a is the result of rotation about some instantaneous centre then, the centre must lie along AC.

Next draw *BD* perpendicular to the velocity v_b at *B*. By the same argument, the instantaneous centre must lie along *BD*. Their point of intersection I therefore, determines the instantaneous centre of rotation of the body at that instant.

The angular velocity of the body ω can be determined as

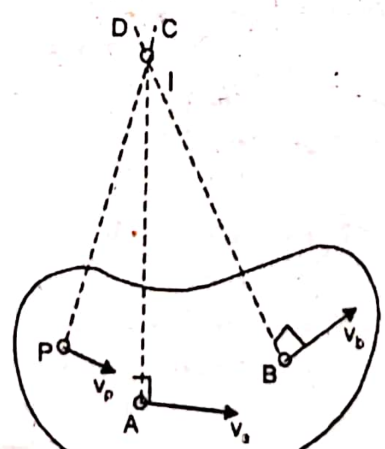


Fig. 1.

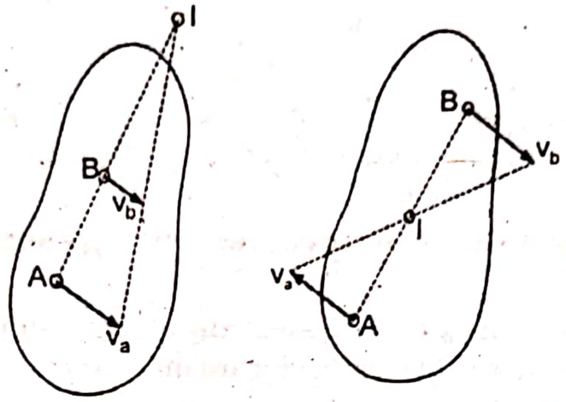
$$\omega = \frac{v_a}{IA}$$

The velocity of any point P is given at that instant by

$$v_p = \omega IP$$

(ii) When the velocity v_a and v_b of the two points in the body are parallel but unequal in magnitude.

The instantaneous centre I can be found by determining the point of intersection of the line AB with the line joining the extremities of the vectors v_a and v_b as shown in Fig. 2. (a) and (b).



(a) **(b)**

Fig. 2.

(*iii*) When the velocities v_a and v_b of the two points are equal and parallel then the instantaneous centre is at the infinity and all the points of the body have the same velocity as shown in Fig. 3.

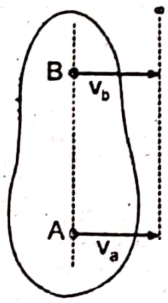


Fig. 3.

Q. 1. (i) Explain centre of percussion. **(2.5)**

Ans. Centre of Percussion. It refers to the fixed pivot point that oscillates in an object with zero net force. At the pivotal point translation and rotational movement cancel out each other when an object is hit by some impact. Consider the case of a bat which is struck by the cricket ball. When ball strikes the middle of the bat at it centre of gravity, the bat move forward in the direction of force. But when the ball strikes below or the upper part of the bat other than the C.G. position, it results only rotational and translation motion because of the vibration of the bat.

Fig. (1) shows when the ball hits the bat at its centre of gravity.

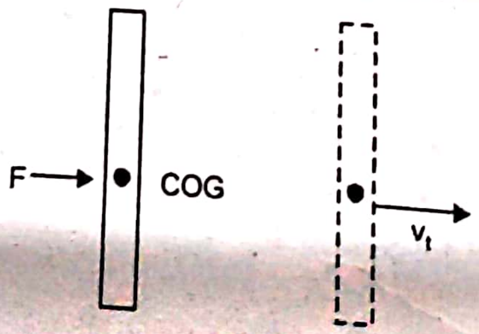


Fig. 1.

Fig. (2) below shows when the ball hits the bat below or upper part.

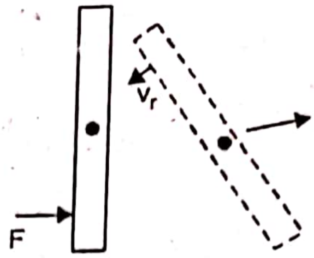


Fig. 2.

In fig. (2) it is clearly shown that when the ball hits other than the C.G. position, translation velocity (v_t) and rotational velocity (v_r) are equal and opposite. This is point known as the centre of percussion.

Q. 1. (j) Define the term shear force and bending moment at the cross-section of a beam.

Ans. Shear force: It may be defined as the algebric sum of all the vertical forces which is coming either upward or the backward direction.

Bending moment: It is the algebric sum all the moment.

$$\Sigma Mx = 0$$

UNIT–I

Q. 2. (a) A bracket is subjected to a force as shown in Fig. 2 (a). Find an equivalent force couple at A. **(2.5)**

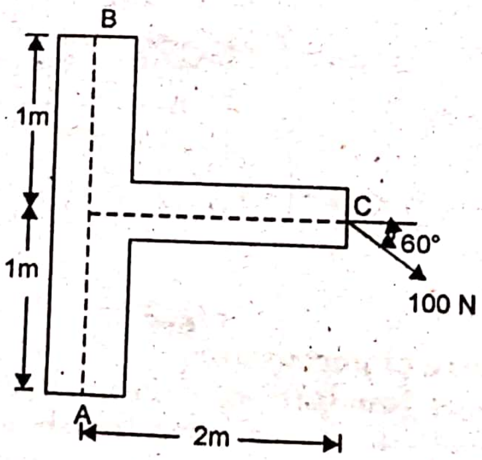
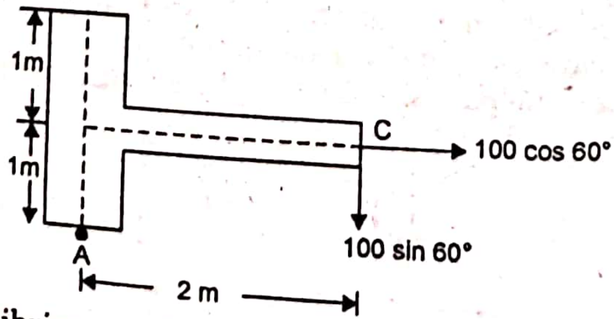


Fig. 2. (a)

Ans. F.B.D. of bracket



Applying the equilibrium condition.

$$\Sigma F_x = 0.$$

$$F_x = 100 \cos 60° = 50$$

$$\Sigma f_y = 0,$$
$$F_y = -100 \sin 60 = 86.6$$

Taking moment about point A.

$$M_A = 100 \sin 60° \times 2 - 100 \cos 60° \times 1$$
$$= 173.2 - 50$$

$$\boxed{M_A = 123.2} \text{ N. clock wise}$$

$$R = \sqrt{F_x^2 + F_y^2}$$
$$= \sqrt{50^2 + (-86.6)^2}$$
$$\boxed{R = 100\,N}$$

$$\theta = \tan^{-1}\left(\frac{F_y}{F_x}\right)$$
$$= \tan^{-1}\left(\frac{-86.6}{50}\right)$$
$$\boxed{\theta = 60°}$$

Q. 2. (b) Find the reactions at the supports A and C of the bent shown in fig. 2 (b). (6)

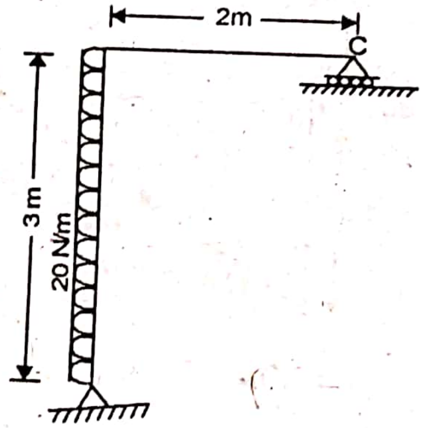
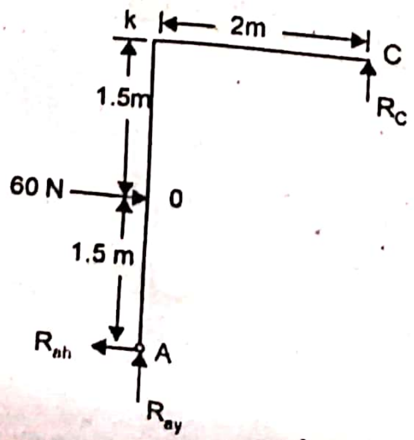


Fig. 2. (b)

Ans. F.B.D of the system.



$$\Sigma F_x = 0$$

$$\boxed{R_{ah} = 60N}$$

$$\Sigma F_y = 0$$

$$R_{ay} + R_C = 0 \quad \text{...(i)}$$

Moment about A.

$$R_C \times 2 = 60 \times 1.5$$

$$\boxed{R_C = 45N}$$

from e.q. (i)

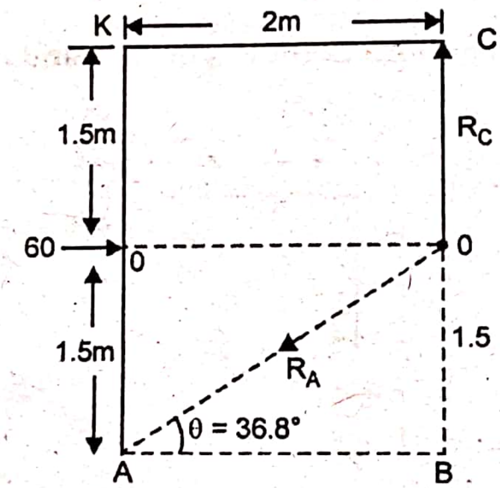
$$\boxed{R_{ay} = -45N}$$

$$R_A = \sqrt{{R_{ah}}^2 + {R_{ay}}^2}$$

$$= \sqrt{60^2 + (-45)^2}$$

$$\boxed{R_A = 75N}$$

Alternate Method

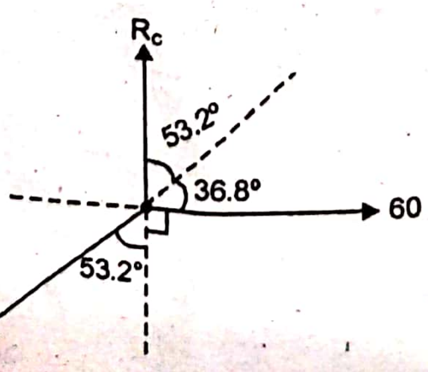


In ΔAOB,

$$\theta = \tan^{-1}\left(\frac{1.5}{2}\right)$$

$$\boxed{\theta = 36.8°}$$

From Lami theorem



$$\frac{R_A}{\sin 90} = \frac{R_C}{\sin 143.1} = \frac{60}{\sin 126.8}$$

$$R_A = \frac{\sin 90° \times 60}{\sin 126.8} = 75\,N$$

...(i)

$$\boxed{R_C = 45\,N}$$

Q. 3. (a) What is a screw jack? Explain the principle on which it works. **(3.5)**

Ans. The screw jack is a device for lifting heavy loads, by applying a comparatively smaller effort at its handle. The principle, on which a screw jack works, is similar to that of an inclined plane.

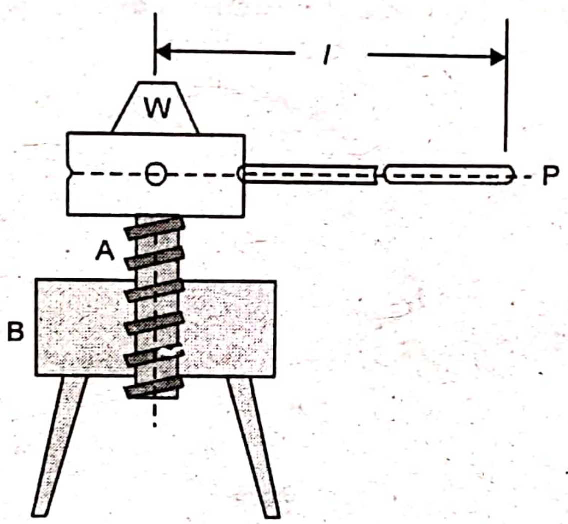


Fig. 1. Screw jack

Fig. 1 shows common form of a screw jack, which consists of a threaded rod A, called screw rod or simply screw. The screw has square threads, on its outer surface, which fit into the inner threads of the jack B. The load, to be raised or lowered, is placed on the head of the screw, which is rotated by the application of an effort at the end of the lever for lifting or lowering the load.

If one complete turn of a screw thread, be imagined to be unwound, from the body of the screw and developed, it will form an inclined plane as shown in Fig. 2.

Let p = Pitch of the screw,

d = Mean diameter of the screw

r = Mean radius of the screw, and

α = Helix angle.

From the geometry of the figure, we find that

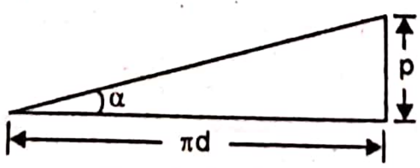


Fig. 2. Helix angle

$$\tan\alpha = \frac{p}{\pi d} = \frac{p}{2\pi r} \qquad \text{(where } d = 2r\text{)}$$

Now let

P = Effort applied at the mean radius of the screw jack to lift the load,

W = Weight of the body to be lifted, and

μ = Coefficient of friction, between the screw and nut.

Let

ϕ = Angle of friction, such that $\mu = \tan \phi$.

As a matter of fact, the principle, on which a screw jack works, is similar to that of an inclined plane. Thus the force applied on the lever of a screw jack is considered to be horizontal.

$$P = W \tan(\alpha + \phi)$$

Q. 3. (b) A rectangular prism weighing 150 N is lying on an inclined plane whose inclination with the horizontal is shown in fig. 3. (b). The block is tied by a horizontal string which has a tension of 50 N. using first principles, find (i) the frictional force on the block (ii) the normal reaction at the inclined plane (iii) the co-efficient of friction between the surfaces of contact. (9)

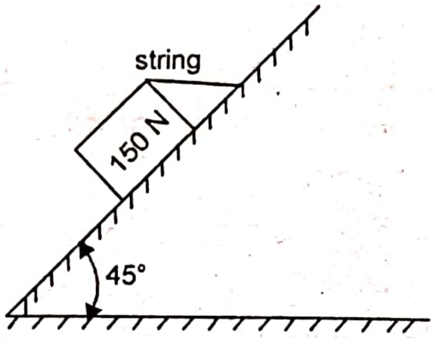
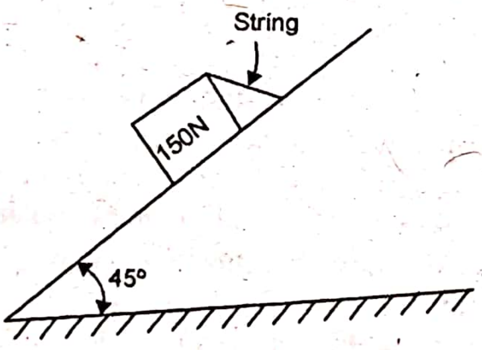


Fig. 3.(b)

Ans.



The various forces acting on the block are

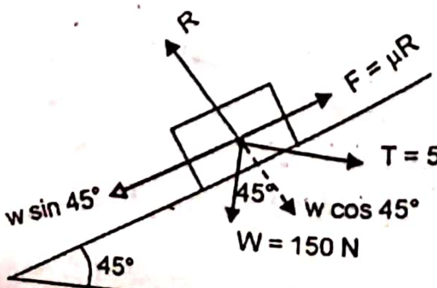
(*i*) W = 150 N

(*ii*) T = 50 N in the string.

(*iii*) Normal reaction R at the contact surface.

(*iv*) F = μR (friction force)

Now F.B.D of the system.



Applying the equilibrium condition

$$\Sigma F_x = 0,$$

$$\mu R - W \sin 45 + T \cos 45 = 0$$

$$\mu R = 150 \sin 45° - 50 \cos 45° = 106.0 - 35.3$$

$$\mu R = 70.7 \quad \text{...(i)}$$

$$\Sigma F_y = 0,$$

$$R - W \cos 45° - T \sin 45° = 0$$

$$R = 150 \cos 45° + 50 \sin 45°$$

$$R = 106 + 35.3 = 141.3$$

$$\boxed{R = 141.3 N}, \text{ from eq. (i) } \mu = \frac{70.7}{141.3}$$

$$\boxed{\mu = 0.5}$$

$$\boxed{F = 70.7 N}$$

$$\boxed{R = 141.3 N} \textbf{ Ans.}$$

UNIT–II

Q. 4. (a) Explain the principles involved in (i) method of joints (ii) method of sections in the analysis of trusses. (4)

Ans. Method of joints

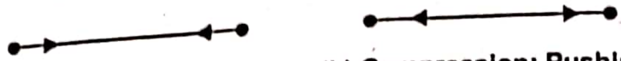
Every joint is treated separately as a free body in equilibrium, i.e., the sum of all the vertical forces as well as the horizontal forces acting on the joint is equated to zero.

$$\Sigma F_v = 0 \text{ and } \Sigma F_h = 0$$

The steps involved are:

- All the pin joints are labelled.
- A free body diagram of the entire frame is drawn and the reaction at the supports is determined using the conditions of equilibrium.
- Each joint is treated separately as a free body. A certain direction of forces acting on the joint is assumed and the magnitude of forces is worked out by applying the conditions of equilibrium. If the magnitude of a particular force comes out positive, the assumption in respect of its direction is correct. However if the magnitude of the force comes out negative, the actual direction of force is reversed, i.e., opposite to what has been assumed.
- Start is made from a joint where there are not more than two members in which the forces are unknown, and the process is repeated from one joint to another until all the unknowns have been determined.
- Assumption is made about the nature of forces in a member and the force is determined, if the answer is positive, the assumption is correct. A negative answer would imply that the nature of force is opposite to that assumed.

Finally, the force in the member will be tensile if the member pulls the joint to which it is connected (force is directed away from the pin) whereas the force in the member will be compressive if the member pushes the joint to which it is connected (force is towards the pin). These aspects have been illustrated in Fig. A member under tension is called a *tie*, and a member under compression is called a strut.



(a) Tension: Pulling at each end

(b) Compression: Pushing at each end

The following principles help to identify the members not subjected to any force when the truss is loaded:

(*i*) A single force cannot form a system in equilibrium. This implies that if there is only one force acting at a joint, then for the equilibium of this joint, this force equals zero.

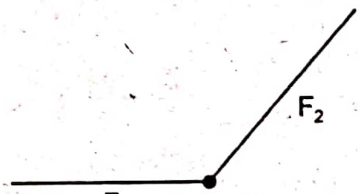
With reference Fig. (a) $$F_1 = 0$$

$F_1 = 0$

Fig. (a)

(*ii*) When two members meeting at a joint are not collinear and there is no external force acting at the joint, then the forces in both the members are zero. With reference to Fig. (*b*):

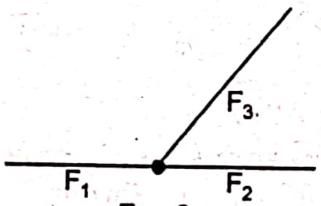
$$F_1 = F_2 = 0$$



$F_1 = F_2 = 0$

Fig. (b)

(*iii*) When three members are meeting at an unloaded joint and out of them two are collinear, then the force in the third member will be zero.



$F_3 = 0$

Fig. (c)

With reference to Fig. (c): $$F_3 = 0.$$

Method of sections

The various steps involved are:

(*i*) The truss is split into two parts by passing an imaginary section.

(*ii*) The imaginary section has to be such that it does not cut more than three members in which the forces are to be determined.

(*iii*) The conditions of equilibrium

$$\Sigma F_x = 0; \ \Sigma F_y = 0 \text{ and } \Sigma M = 0$$

are applied for the one part of the truss and the unknown force in the member is determined.

(*iv*) While considering equilibrium, the nature of force in any member is chosen arbitrarily to be tensile or compressive.

If the magnitude of a particular force comes out positive, the assumption in respect of its direction is correct. However, if the magnitude of the forces comes out negative, the actual direction of the force is opposite to that what has been assumed.

The method of section is particularly convenient when the forces in a few members of the frame are required to be worked out.

Q. 4. (b) For the simply supported truss shown in fig. 4(b), find the forces in the members BD, DE and EG. **(8.5)**

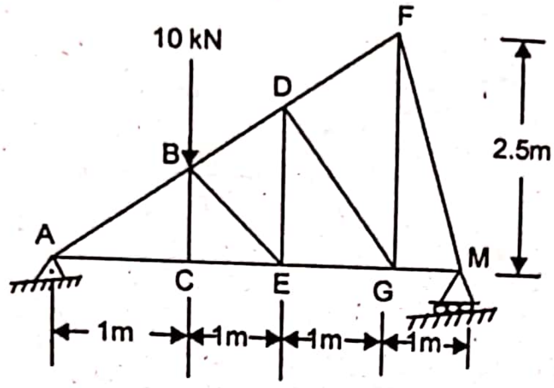
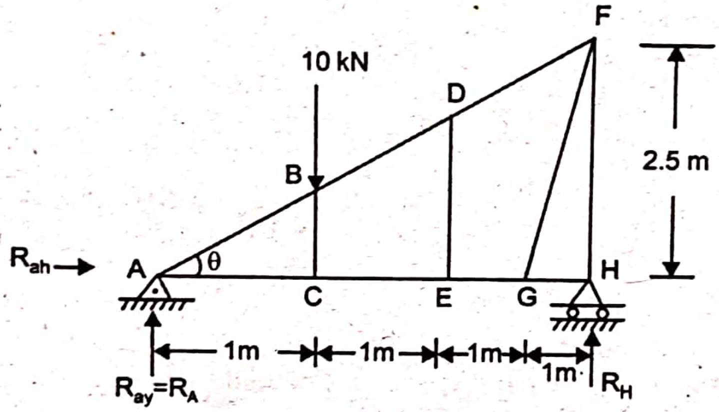


Fig. 4. (b)

Ans.



No horizontal force acting on the system so, $R_{ah} = 0$

In ΔAFG,

$$\tan\theta = \frac{2.5}{3} = 39.8°$$

$$\boxed{\theta = 39.8°}$$

Applying the equilibrium condition.

$$\Sigma Fy = 0.$$

$$R_A + R_H = 10 \quad ...(1)$$

$$\Sigma M = 0,$$

Taking moment about point A,

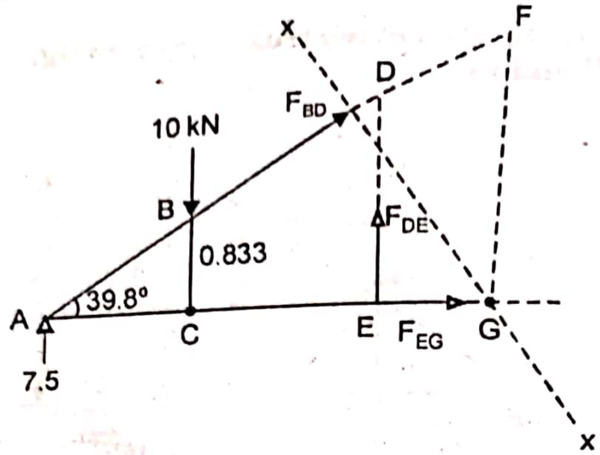
$$10 \times 1 = R_H \times 4$$

$$R_H = \frac{10}{4} = 2.5\,kN$$

$$R_A = 7.5 \text{ kN} \quad \text{(from eq. (i))}$$

Now Truss is solved by section method.

Choose the left side of cut-section of truss. For finding the forces.



Taking moment point A.

$$10 \times 1 = F_{DE} \times 2$$

$$\boxed{F_{DE} = 5\,kN}$$

Moment about B,

$$M_B = 0,$$

$$F_{ED} \times 1 + F_{EG} \times 0.833 = 0 + 7.5 \times 1$$

$$5 + 0.833\, F_{EG} = 7.5$$

$$F_{EG} = \frac{2.5}{0.833} = 3\,\text{kN}$$

$$\boxed{F_{EG} = 3\,\text{kN}}$$

Moment about G point.

$$7.5 \times 3 - 10 \times 2 + F_{DE} \times 1 + F_{BD} \cos 39.8° \times 0.833 + F_{BD} \sin 39.8° \times 2 = 0$$

$$22.5 - 20 + 5 + 0.63\, F_{BD} + 1.2\, F_{BD} = 0$$

$$7.5 + 1.83\, F_{BD} = 0$$

$$F_{BD} = -\frac{7.5}{1.83} = -4\,\text{kN}$$

$\boxed{F_{BD} = 4\,\text{kN}}$ comp.

$\boxed{F_{DE} = 5\,\text{kN}}$ Tension.

$\boxed{F_{EG} = 3\,\text{kN}}$ Tension.

$\boxed{F_{BD} = 4\,\text{kN}}$ Comp.

Q. 5. (a) Drive from first principles, the centroid of a cone. *(6)*

Ans. Let ABC be the right circular cone of height h and base circle radius R. Consider an elemental ring of radius r between two planes parallel to the base BC and at distance y and $(y + dy)$ from it. Presuming that the material of the cone is homogeneous (*i.e.*, its density is same through out), the centre of gravity will be its centre of volume.

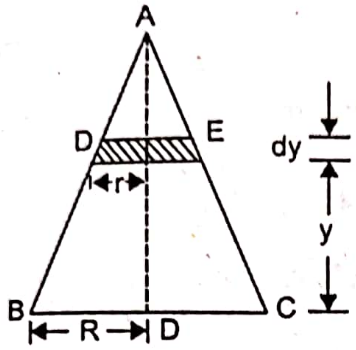
For the elemental ring,

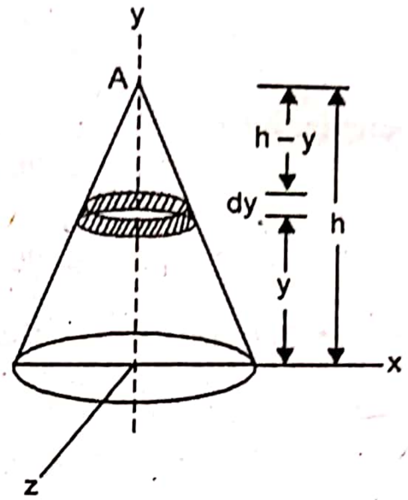
$$\text{volume} = \pi r^2\, dy$$

$$\text{moment about } x\text{-axis} = \pi r^2 dy \times y$$

Since the integration is to be done with reference to y within the limits 0 to h, it is necessary to express r in terms of y. For that we obtain the following correlation from the similarity of triangles ADE and ABC

$$\frac{r}{R} = \frac{h-y}{h};\ r = R\left(1-\frac{y}{h}\right)$$





$\therefore$ moment of elemental volume about x-axis $= \pi R^2\left(1-\frac{y}{h}\right)^2 dy \times y$

Volume of cone ABC $= \frac{1}{3}\pi R^2 h$

If $\bar{y}$ is the distance of the centre of volume from the base, then from the moment principle

$$\frac{1}{3}\pi R^2 h \times \bar{y} = \int_0^h \pi R^2\left(1-\frac{y}{h}\right)^2 y\,dy = \pi R^2 \int_0^h \left(y + \frac{y^3}{h^2} - 2\frac{y^2}{h}\right)dy$$

$$= \pi R^2 \left|\frac{y^2}{2} + \frac{y^4}{4h^2} - \frac{2}{3}\frac{y^3}{h}\right|_0^h = \pi R^2 \times \frac{h^2}{12}$$

$$\therefore \qquad \bar{y} = \left(\pi R^2 \times \frac{h^2}{12}\right) \times \frac{3}{\pi R^2 h} = \frac{h}{4}$$

Thus the centre of gravity of right circular cone of height h is at a distance h/4 from the base or 3h/4 from the apex.

Q. 5. (b) A rectangular hole is made in a triangular area as shown in fig. 5. (b). Find the M.O.I of the shaded area about the centroidal horizontal and vertical axes. Width of the hole is 20 cm.

(6.5)

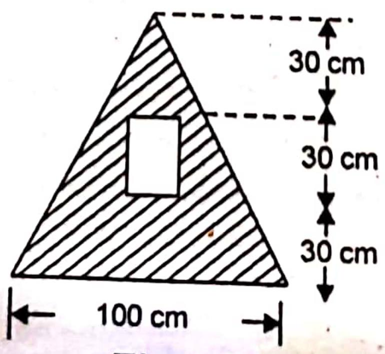


Fig. 5. (b)

(6)

us R.
he base BC
the cone is
e its centre

Ans. As the section is symmetrical about Y-Y axis, therefore centre of gravity of the section will lie on this axis. Let $\bar{y}$ be the distance between the centre of gravity of the section and the base BC.

(i) Triangular section

$$a_1 = \frac{100 \times 90}{2} = 4500\,\text{mm}^2$$

and $$y_1 = \frac{90}{3} = 30\,\text{mm}$$

(ii) Rectangular hole

$$a_2 = 30 \times 20 = 600\,\text{mm}^2$$

and $$y_2 = 30 + \frac{30}{2} = 45\,\text{mm}$$

We know that distance between the centre of gravity of the section and base BC of the triangle,

$$\bar{y} = \frac{a_1 y_1 - a_2 y_2}{a_1 - a_2} = \frac{(4500 \times 30) - (600 \times 45)}{4500 - 600} = 27.7\,\text{mm}$$

Moment of inertia of the section about X-X axis.

We also know that moment of inertia of the triangular section through its centre of gravity and parallel to X-X axis,

$$I_{G1} = \frac{bd^3}{36} = \frac{100 \times (90)^3}{36} = 2025 \times 10^3\,\text{mm}^4$$

and distance between the centre of gravity of the section and X-X axis,

$$h_1 = 30 - 27.7 = 2.3\,\text{mm}$$

$\therefore$ Moment of inertia of the triangular section about X-X axis

$$= I_{G1} + a_2 h_1^2 = 2025 \times 10^3 + [4500 \times (2.3)^2] = 2048.8 \times 10^3\,\text{mm}^4$$

Similarly moment of inertia of the rectangular hole through its centre of gravity and parallel to the X-X axis

$$I_{G2} = \frac{bd^3}{12} = \frac{20 \times (30)^3}{12} = 45 \times 10^3\,\text{mm}^4$$

and distance between the centre of gravity of the section and X-X axis

$$h_2 = 45 - 27.7 = 17.3\,\text{mm}$$

$\therefore$ Moment of inertia of rectangular section about X-X axis

$$= I_{G2} + a_2 h_2^2 = (45 \times 10^3) + [600 \times (17.3)^2] = 224.6 \times 10^3\,\text{mm}^4$$

Now moment of inertia of the whole section about X-X axis.

$$I_{xx} = (2048.8 \times 10^3) - (224.6 \times 10^3) = 1824.2 \times 10^3\,\text{mm}^4$$

Moment of inertia of the section about the base BC

We know that moment of inertia of the triangular section about the base BC

$$I_{G1} = \frac{bd^3}{12} = \frac{100 \times (90)^3}{12} = 6075 \times 10^3\,\text{mm}^4$$

Similarly moment of inertia of the rectangular hole through its centre of gravity and parallel to X-X axis,

$$I_{G2} = \frac{bd^3}{12} = \frac{20 \times (30)^3}{12} = 45 \times 10^3 \text{ mm}^4$$

and distance between the centre of gravity of the section about the base BC,

$$h_2 = 30 + \frac{30}{2} = 45 \text{ mm}$$

$\therefore$ Moment of inertia of rectangular section about the base BC,

$$= I_{G2} + a_2 h_2^2 = (45 \times 10^3) + [600 \times (45)^2] = 1260 \times 10^3 \text{ mm}^4$$

Now moment of inertia of the whole section about the base BC,

$$I_{BC} = (6075 \times 10^3) - (1260 \times 10^3) = 4815 \times 10^3 \text{ mm}^4$$

UNIT–III

Q. 6. (a) A stone is dropped from the top of a building. It was found the during the last one second of its journey, it has covered one-seventh of the height of the building. Find the height of the building. **(6)**

Ans. Distance travelled in last one second $= \frac{h}{7}$, $g = 9.81 \text{ m/s}^2$

$$h = ut + \frac{1}{2}gt^2$$

$$= 0 + \frac{1}{2} \times 9.81 \times t^2 \qquad (\because u = 0)$$

$$h = 4.9t^2 \qquad \text{...(i)}$$

If the distance travelled $(t-1)$ second is h', then we have

$$h' = (u(t-1) + \frac{1}{2}g(t-1)^2$$

$$h' = 0 + \frac{1}{2}g(t-1)^2$$

$$h' = 4.9(t-1)^2 \qquad \text{...(ii)}$$

Now

$$h - h' = \frac{h}{7} \qquad \text{(from eq. (i) and (ii))}$$

$$4.9t^2 - 4.9(t-1)^2 = \frac{1}{7} \times 4.9t^2$$

$$t^2 - (t-1)^2 = \frac{t^2}{7}$$

$$7t^2 - (t-1)^2 = t^2$$

$$t^2 - 14t + 7 = 0$$

$$t = \frac{-(-14) \pm \sqrt{14^2 - 4\times1\times7}}{2\times1}$$

$$t = 13.4 \text{ and } t = 0.55$$

But t is greater than unity

So,

$$\boxed{t = 13.4\,\text{sec.}}$$

Now, from eq. (i)

$$h = 4.9\times(13.4)^2$$

$$h = 879.8 \text{ m}$$

$$\boxed{h = 880 \text{ m}}$$

Q. 6. (b) A car enters a curve of 200 m radius at a speed of 45 kmph. If the car increases its speed at a rate of 2m/s², What will be its total acceleration when the car has travelled 450 m along the curve. (6.5)

Ans. r = 200 m, v = 45 km/hr, $a_t = 2\text{m/s}^2$, a = ?, s = 450 m.

$$a_n = \frac{v^2}{r} = \frac{12.5^2}{200} = \quad \left(\because v = \frac{45\times5}{18} = 12.5\,\text{m/s}\right)$$

$$\boxed{a_n = 0.78 \text{ m/s}^2}$$

$$a = \sqrt{a_t^2 + a_n^2}$$

$$= \sqrt{2^2 + 0.78^2}$$

$$\boxed{a = 2.14\,\text{m/s}^2}$$

Now,

$$v^2 - u^2 = 2as$$

$$v^2 - 0 = 2\times a_t \times 450$$

$$v^2 = 900\, a_t$$

$$12.5^2 \Rightarrow \boxed{a_t = 0.17\,\text{m/s}^2}$$

then

$$a = \sqrt{0.17^2 + 0.78^2}$$

$$\boxed{a = 0.80\,\text{m/s}^2} \textbf{ Ans.}$$

Q. 7. (a) Explain work of a force. (4)

Ans. Work of a force: If a particle is subjected to a force F and the particle is displaced by an infinitesimal displacement ds the work done dU by the force is given by

$$\boxed{dU = Fds\cos\alpha}$$

where α is angle between force and the displacement vector. (Fig. a)

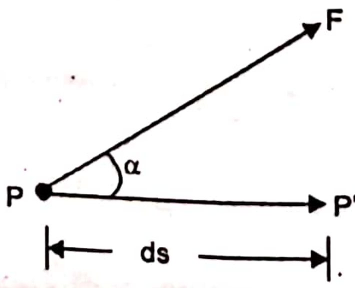


Fig. (a)

Thus, the work done by a force during a infinitesimal displacement is equal to the product of the displacement ds and the component of the force F $\cos\alpha$ in the direction of displacement. The work done by a force during finite displacement from position P_1 to P_2 can be obtained by integrating the above equation.

$$U_{1-2} = \int_{s_1}^{s_2} (F\cos\alpha)ds$$

Where the displacements s_1 and s_2 are measured a long the path is shown in fig. (b).

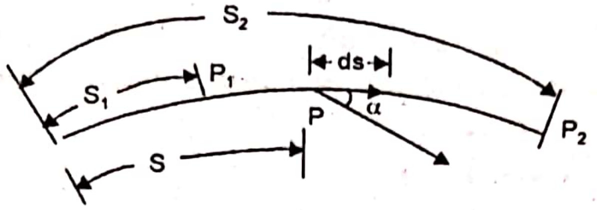


Fig. (b)

Q. 7. (b) State and explain the law of conservation of linear momentum. (4)

Ans. Consider the motion of a particle of mass (m) acted upon by a force (F), shown in fig.

The equation of motion of the particle in x and y directions are:

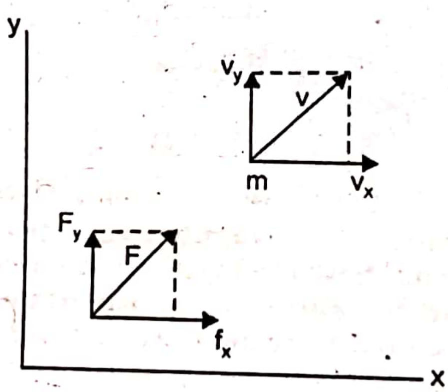
$$F_x = ma_x, \quad \text{and } F_y = ma_y$$

or

$$F_x = m\frac{dv_x}{dt} \quad \text{and } F_y = m\frac{dv_y}{dt}$$

or,

$$F_x = \frac{d}{dt}(mv_x) \quad \text{and } F_y = \frac{d}{dt}(mv_y)$$



A single equation in the vector form can be written as

$$\boxed{F = \frac{d}{dt}(mv)}$$

Which state that the force F acting on the particle in equal to the rate of change of momentum of the particle. The vector (mv) is called the momentum or linear momentum. It has the same direction as the velocity of particle. The unit of momentum if

$$mv = kg\left(\frac{m}{s}\right) = kg \times \frac{m}{s^2} \times s = \text{NS}.$$

Conservation of momentum

Final momentum – Initial momentum = Impuls of force

$$\Sigma m v_2 - \Sigma m v_1 = \int_0^t F dt$$

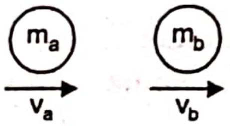
It can be observed from above eq. that when sum of the impulses due to external forces is zero the momentum of the system remains constant or is conserved.

$$\boxed{\Sigma m v_2 = \Sigma m v_1}$$

Final momentum of the system = Initial momentum of system.

Q. 7. (c) Prove that the two elastic bodies of equal masses exchange velocities in the case of direct central impact. (4.5)

Ans. Impact of two equal masses



$$m_a = m_b = m$$

Conservation of momentum

$$m_a v_a + m_b v_b = m_a v_b' + m_b v_b'$$

$$v_a + v_b = v_a' + v_b' \quad \text{...(i)}$$

Coefficient of restitution relation gives

$$e = 1 = \frac{v_a' - v_b'}{v_b - v_a}$$

$$v_b - v_a = v_a' - v_b' \quad \text{...(ii)}$$

or

$$v_b + v_b' = v_a' + v_a$$

Resolve the eq. (i) and (ii)

$$\boxed{v_b' = v_a}$$

$$\boxed{v_a' = v_b}$$

After an elastic impact the two masses exchange velocities.

Q. 8. (a) A roller of radius 10 cm rides between-two horizontal bars moving in the opposite directions as shown in Fig. 8 (a). Assuming no slip at the point of contacts A and B, locate the instantaneous centre of the roller. Where will be the instantaneous centre when both the bars are moving in the same direction? (4)

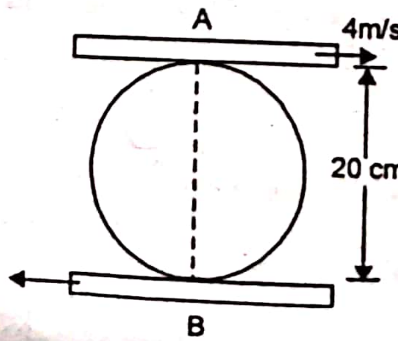
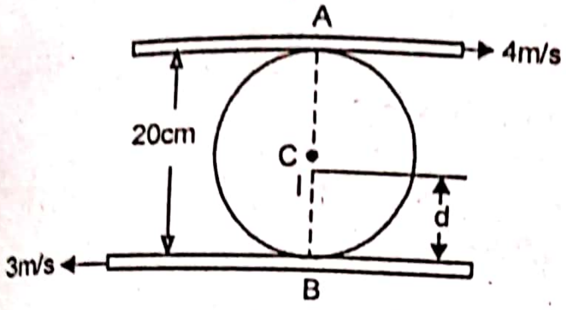
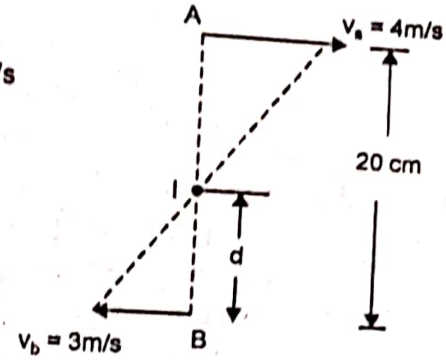


Fig. 8. (a)

Ans. The instantaneous centre I is the point of intersection of the line joining A and B with the line joining the extremities the velocity vectors $\overline{v}_a$ and $\overline{v}_b$ as shown in fig.





Every point on the roller shall appear to rotate about the I centre with angular velocity (ω), therefore,

$$\omega = \frac{v_a}{IA} = \frac{v_b}{IB}$$

or,

$$\frac{4}{IA} = \frac{3}{IB}$$

or,

$$4IB = 3IA$$

$$4IB - 3IA = 0 \qquad \text{...(i)}$$

Also,

$$IA + IB = 0.2 \Rightarrow IA = 0.2 - IB$$

from eq. (1)

$$4IB - 3\,(0.2 - IB) = 0$$

$$4IB - 0.6 + 3IB = 0$$

$$7IB = 0.6 \Rightarrow IB = \frac{0.6}{7} = 0.085\,\text{m}$$

or

$$\boxed{IB = 8.5\ \text{cm}}$$

And

$$\boxed{IA = 11.5\ \text{cm}}$$

Now

$$IB = d = 8.5\ \text{cm}.$$

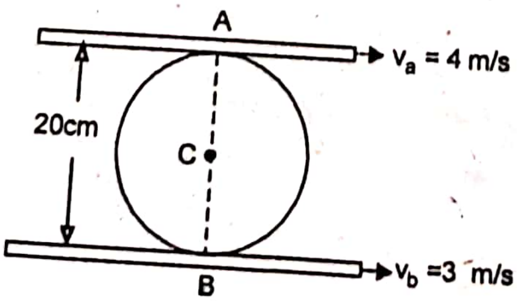


Fig. (a)

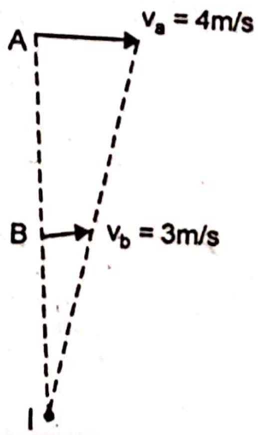


Fig. (b)

when moving in same direction,

then, $$\omega = \frac{v_a}{IA} = \frac{v_b}{IB}, \frac{4}{IA} = \frac{3}{IB}$$

$$4IB - 3IA = 0 \quad \text{...(1)}$$

Now, $$IA - IB = 0.2 \quad \text{...(2)} \quad \text{from fig. (b).}$$

∴ $$IA = 0.2 + IB$$

Put the value IA in eq. (1)

$$4IB - 3\,(0.2 + IB) = 0$$

$$4IB - 0.6 - 3IB = 0$$

$$\boxed{IB = 0.6} \ \boxed{IB = 60\,\text{cm}}$$

And $$IA = 0.8, \text{ or } \boxed{IA = 80\,\text{cm}}$$

Now, $$IB = d = 60 \text{ cm.}$$

Q. 8. (b) In the mechanism shown in 8(b), AB rotates clockwise with an angular velocity of 10 rad/sec. Find the angular velocities of bars BC and CD, when the bar AB makes an angle of 30° with the horizontal, bar CD makes an angle of 60° and the bar BC is horizontal. (8.5)

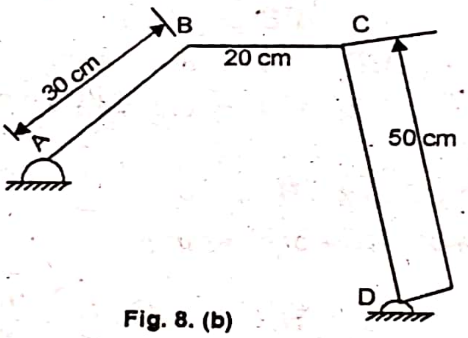
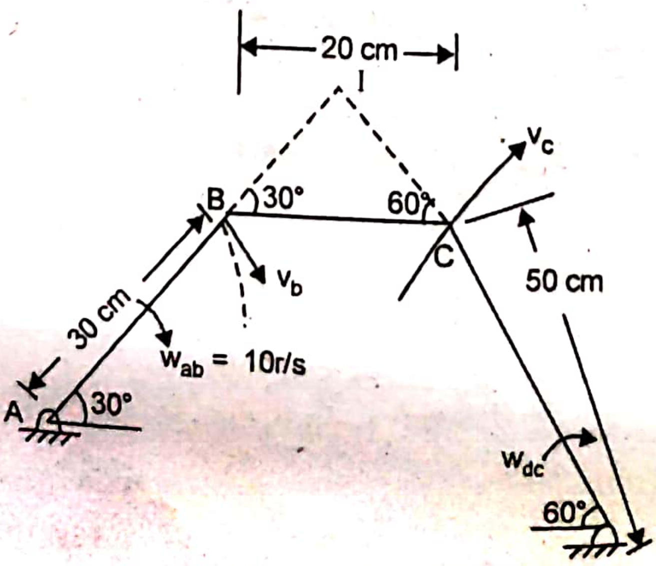


Fig. 8. (b)

Ans. In fig. four bar mechenism, the bar AB rotates about the fixed end A and the bar DC about the fixed end D.

Therefore, the velocity v_b of the end B is normal to AB and the velocity v_c of the end C is normal to DC as shown in fig.



IN ΔBIC

$$\frac{IB}{\sin 60} = \frac{IC}{\sin 30} = \frac{BC}{\sin 90}, \quad BC = 20 \text{ cm}$$

$$IB = \sin 60° \times BC \times 1$$
$$= 0.866 \times 20 \times 1$$
$$= 17.32 \text{ CM.}$$
$$IC = \sin 30° \times BC \times 1$$
$$= 0.5 \times 20 = 10 \text{ cm.}$$

Consider the rotation of the bar AB about A

$$v_b = w_{ab}\,(AB) = 10 \times (0.3)$$
$$= 3 \text{ m/s}$$

similar, $v_b = w_{bc}\,(IB)$, w_{bc} is the angular velocity of bar BC.

$$w_{bc} = \frac{v_b}{IB} = \frac{3}{17.3} = \frac{3}{0.173} = 17.34 \text{ r/s.}$$

$$v_c = w_{bc}\,(IC) = (17.34) \times 0.1$$
$$= 1.73 \text{ r/s.}$$

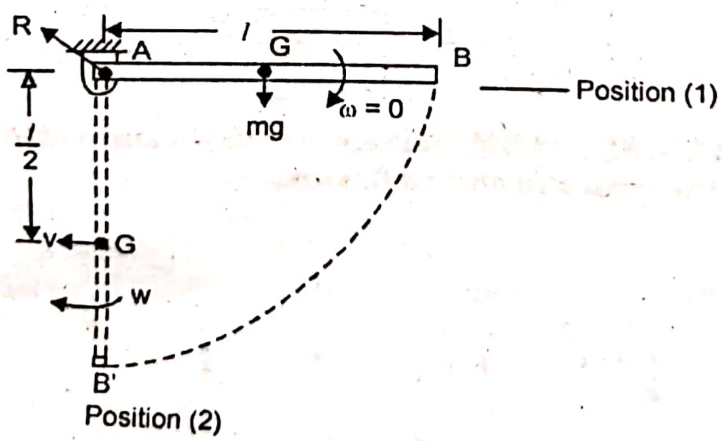
Consider the rotation of the bar DC about D.

$$w_{dc} = \frac{v_c}{CD} = \frac{1.73}{0.5} = 3.46$$

$$\boxed{w_{dc} = 3.46 \text{ r/s}}$$

Q. 9. (a) A uniform bar of mass m and length l hangs from a frictionless hinge. It is released from rest from the horizontal position. Find the angular and linear velocity of its mass centre when it is in vertical position. (4)

Ans.



As only conservative forces are involved, the principle of conservation of energy can be applied.

Position (1)

Then total energy $v = 0, \; w = 0, P.E = 0, K.E = 0$

$$E_1 = 0$$

Position (2)

Let the linear velocity of mass centre be v and the angular velocity be ω.

$$PE = -mgh = \frac{-mgl}{2}$$

$$K.E. = \frac{1}{2}mv^2 + \frac{1}{2}I_G w^2,\ w = \frac{v}{l/2},\ I_G = \frac{ml^2}{12}$$

$$KE = \frac{1}{2}mv^2 + \frac{1}{2}\frac{ml^2}{12}\left(\frac{2v}{l}\right)^2$$

Total energy,

$$E_2 = -\frac{1}{2}mgl + \frac{1}{2}mv^2 + \frac{1}{6}mv^2$$

Then principle of conservation of energy gives,

$$E_1 = E_2$$

$$0 = -\frac{mgl}{2} + \frac{1}{2}mv^2 + \frac{1}{6}mv^2$$

$$\frac{2}{3}mv^2 = \frac{mgl}{2}$$

$$v^2 = \frac{3}{4}gl$$

$$v = \sqrt{\frac{3}{4}gl}$$

$$\omega = \frac{v}{l/2} = \frac{\sqrt{\frac{3}{4}\cdot gl}}{l/2}$$

$$\boxed{w = \sqrt{\frac{3g}{l}}}$$

Q. 9. (b) Draw the *SF* and BM diagrams for the beam loaded as shown in Fig. 9 (b). also locate the points of contra flexure. **(8.5)**

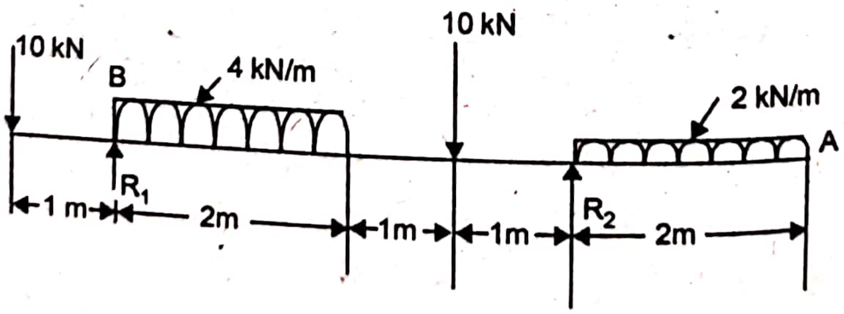


Fig.

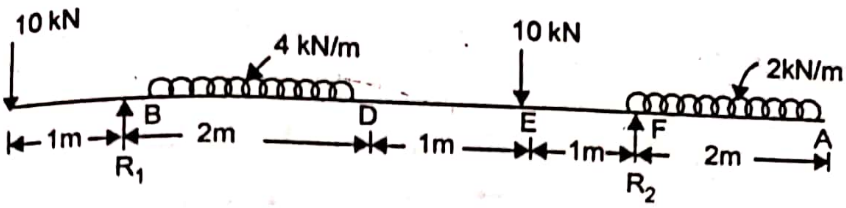
Ans.

Total load acting on the beam $= 10 + 8 + 10 + 4$

$= 32$ KN.

Now, $[\because \Sigma Fy = 0]$

$$R_1 + R_2 = 32 \text{ KN} \quad ...(1)$$



Taking moment about point C,

$$0 - R_1 \times 1 + 8 \times 2 + 10 \times 4 - 5R_2 + 4 \times 6 = 0$$

$$-R_1 + 16 + 40 - 5R_2 + 24 = 0$$

$$R_1 + 5R_2 = 80, \quad \text{from eq. (1)}$$

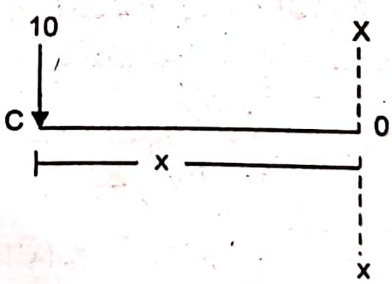
$$\boxed{R_1 = 20\text{kN}}, \quad \boxed{R_2 = 12\text{kN}}$$

Shear force at,

$$\begin{aligned} \text{At C} &= -10 \text{ kN} \\ \text{Before B} &= -10 \text{ kN} \\ \text{After B} &= -10 + 20 = +10 \text{ kN} \\ \text{Before D} &= 10 - 8 = +2 \text{ kN} \\ \text{Before E} &= +2 \text{ kN} \\ \text{After E} &= +2 - 10 = -8 \text{ kN} \\ \text{Before F} &= -8 \text{ kN} \\ \text{After F} &= -8 + 12 = +4 \text{ kN} \\ \text{Before A} &= +4 - 4 = 0 \text{ kN.} \end{aligned}$$

Bending moment:

At portion C to B



$$M_0 = -10x \text{ (linear eq.)}$$

If

$$x = 0,$$

$$\boxed{M_C = 0}$$

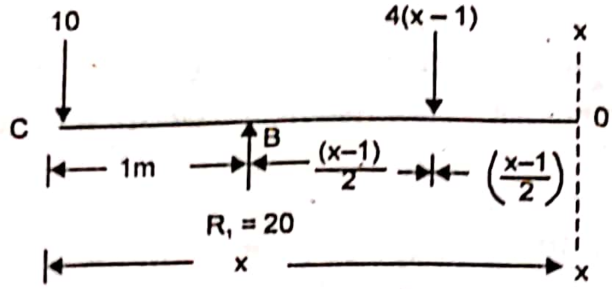
iF

$$x = 1\text{m}$$

$$M_B = -10 \times 1 = -10 \text{ kNm}$$

$$\boxed{M_B = -10 \; kNm}$$

At portion B to D



$$m_O = -10x + 20(x-1) - 4(x-1) \times \frac{(x-1)}{2}$$

$$= -10x + 20(x-1) - 4\,\frac{(x-1)^2}{2} \qquad \text{(parabolic eq.)}$$

if $\quad x = 3\text{m}$

then

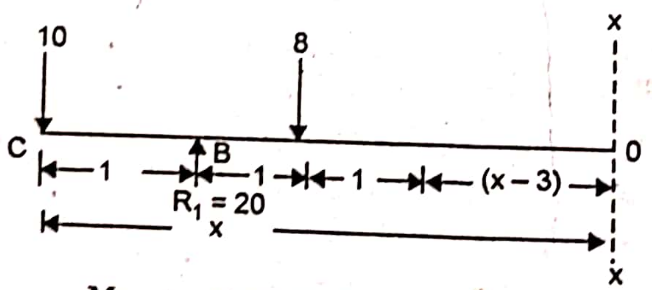
$$M_D = -10 \times 3 + 20(2) - \frac{4(2)^2}{2}$$

$$= -30 + 40 - 2 \times 4$$

$$= 10 - 8 = +2 \text{ kNm}$$

$\boxed{M_D = +\ 2\text{kNm}}$ sign in changing.

At portion D to E.



$$M_O = -10x + R_1(x - 3+2) - 8(1 + x - 3)$$

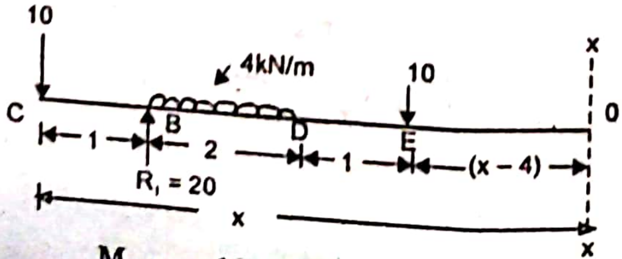
$$= -10x + 20x - 20 - 8x + 16 \qquad \text{(linear equation).}$$

if, $\quad x = 4\text{m}$

$$M_D = -40 + 80 - 20 - 32 + 16$$

$$= 40 - 20 - 16 \Rightarrow \boxed{M_D = 4\,\text{kNm}}$$

At portion E to F.



$$M_0 = -10x + R_1(3 + x - 4) - 8(2 + x - 4) - 10(x-4)$$

$$= -10x + 20(x-1) - 8(x-2) - 10(x-4) \qquad \text{(linear equation)}$$

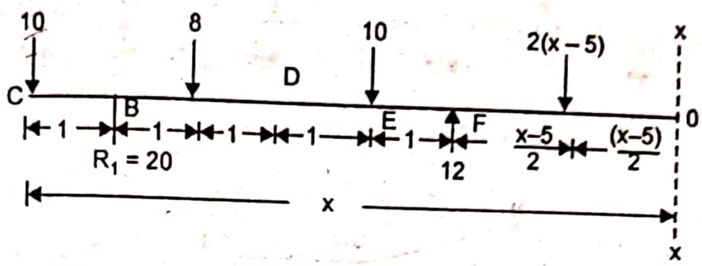
$$x = 5$$

if

$$M_F = -10 \times 5 + 20 \times 4 - 24 - 10 \times 1$$
$$= -50 + 80 - 24 - 10$$
$$= 30 - 34 = -4 \text{ kNm.}$$

$$\boxed{M_F = -4\text{kNM}}$$

BM. at portion F to A.



$$M_0 = -10x + 20(4 + x - 5) - 8(3 + x - 5) - 10(1 + x - 5) + 12(x - 5) - \cancel{2}(x-5)\frac{(x-5)}{\cancel{2}}.$$

(parabolic eq.)

if

$$x = 7.$$
$$M_A = -70 + 20(6) - 8(5) - 10(3) + 12(2) - (7-5)^2$$
$$= -70 + 120 - 40 - 30 + 24 - 4$$
$$= -140 + 120 + 20$$
$$= -140 + 140 = 0$$

$$\boxed{M_A = 0}$$

Note: The sign in changing between portion B to D and portion E to F then location of point of contra flexure between B to D and E to F.

Now, from portion B to D,

$$-10x + 20(x-1) - \frac{4(x-1)^2}{2} = 0$$
$$-10x + 20x - 20 - 2(x^2 + 1 - 2x) = 0$$
$$-10x + 20x - 20 - 2x^2 - 2 + 4x = 0$$
$$10x - 20 - 2x^2 - 2 + 4x = 0$$
$$14x - 20 - 2x^2 - 2 = 0$$
$$-2x^2 + 14x - 22 = 0$$
$$x^2 - 7x + 11 = 0$$

$\boxed{x = 2.38\text{m}}$ (By solving eq.)

From portion E to F

$$-10x + 20(x-1) - 8(x-2) - 10(x-4) = 0$$
$$-10x + 20x - 20 - 8x + 16 - 10x + 40 = 0$$
$$\cancel{10x} - 20 - 8x + 16 - \cancel{10x} + 40 = 0$$

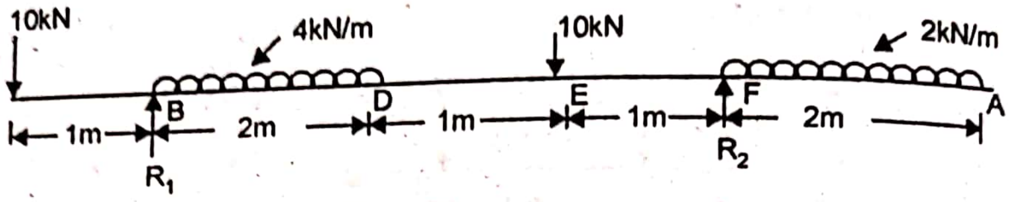
$$-8x + 20 + 16 = 0$$

$$8x = 36$$

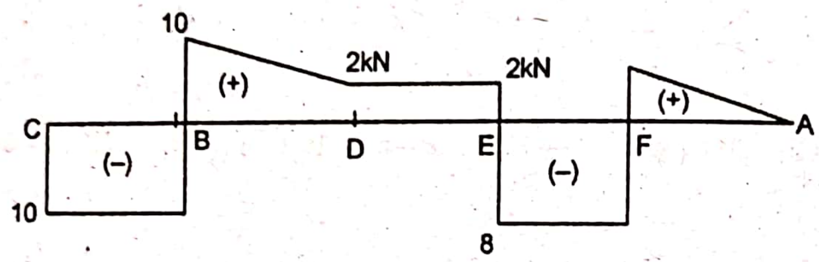
$$x = 36/8$$

$$\boxed{x = 4.5\,\text{m}}$$

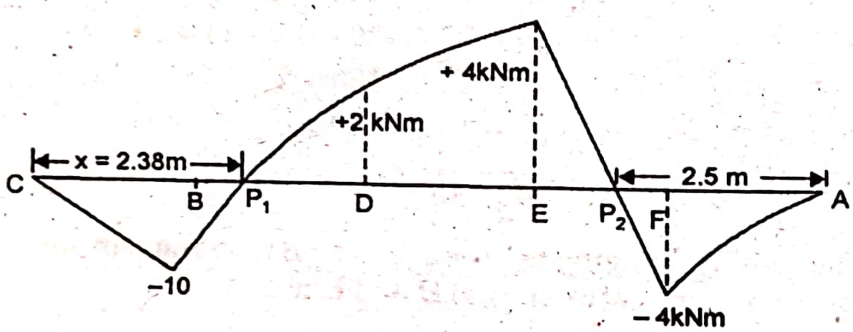
Beam



S.F.D.



B.M.D.



Contraflexure $\boxed{P_1 = 2.38\,\text{m}}$

$\boxed{P_2 = 4.5\,\text{m}}$