```
import numpy as np
import matplotlib.pyplot as plt
```

Q1_a

```
In [8]:
         np.random.seed(24787)
         a = np.random.randint(low = 0, high = 8, size = (3,4,4))
         #printing the array and its shape
         print(a, "\n")
         print(a.shape)
        [[[2 6 4 1]
          [0 4 4 3]
          [6 6 1 2]
          [7 0 6 5]]
         [[1 \ 3 \ 3 \ 7]]
          [4 7 2 5]
          [0 4 6 7]
          [5 5 7 1]]
         [[7 2 4 5]
          [6 7 7 0]
          [6 2 0 4]
          [2 0 7 6]]]
        (3, 4, 4)
In [9]:
         four indices = np.where(a == 4)
         print("row indices:",four indices[1],"column indices",four indices[2])
         #four indices[2] denotes the depth index of value 4
        row indices: [0 1 1 1 2 0 2] column indices [2 1 2 0 1 2 3]
```

Q1_b, using tile to make a (3,8,8) array

Q1_c, calculating sum along depth of b

c, time taken manual = matmul(a,b)

In [31]:

```
In [16]:
          c = np.sum(b,axis=0)
          print(c)
          print(c.shape)
          [[10 11 11 13 10 11 11 13]
          [10 18 13 8 10 18 13 8]
          [12 12 7 13 12 12 7 13]
          [14 5 20 12 14 5 20 12]
          [10 11 11 13 10 11 11 13]
          [10 18 13 8 10 18 13 8]
          [12 12 7 13 12 12 7 13]
          [14 5 20 12 14 5 20 12]]
         (8, 8)
In [30]:
          import time
          np.random.seed(24787)
          a = np.random.randint(0,8,(1000,1000))
          b = np.random.randint(0,8,(1000,1000))
          #declaring array to hold the result
          c = np.zeros like(b)
          def matmul(a,b):
              start = time.time()
              for i in range(a.shape[0]):
                  for j in range(b.shape[1]):
                      c[i,j] = np.dot(a[i],b[:,j].T)
              time taken = time.time() - start
              return c, time taken
```

```
start = time.time()
 C = a@b
time taken inbuilt = time.time() - start
 print("Output by matmul function: \n",c)
 print("Output by @ operator: \n", C)
 print("Difference between matmul and @: \n", C - c)
 print("Time taken for matmul: ",time taken manual)
 print("Time taken for @: ",time taken inbuilt)
Output by matmul function:
 [[12146 12253 12302 ... 12123 12415 12239]
 [12251 12131 12180 ... 12691 12396 12497]
 [11434 11864 12043 ... 12348 11960 12207]
 [11774 11945 12276 ... 12339 12178 12059]
 [11627 12167 12254 ... 11929 11958 12078]
 [11560 12145 12077 ... 12210 12124 12031]]
Output by @ operator:
 [[12146 12253 12302 ... 12123 12415 12239]
 [12251 12131 12180 ... 12691 12396 12497]
 [11434 11864 12043 ... 12348 11960 12207]
 [11774 11945 12276 ... 12339 12178 12059]
 [11627 12167 12254 ... 11929 11958 12078]
 [11560 12145 12077 ... 12210 12124 12031]]
difference between matmul and @:
 [[0 \ 0 \ 0 \ \dots \ 0 \ 0 \ 0]
 [0 \ 0 \ 0 \ \dots \ 0 \ 0 \ 0]
 [0 \ 0 \ 0 \ \dots \ 0 \ 0 \ 0]
 . . .
 [0 \ 0 \ 0 \ \dots \ 0 \ 0 \ 0]
 [0 \ 0 \ 0 \ \dots \ 0 \ 0 \ 0]
 [0 0 0 ... 0 0 0]]
Time taken for matmul: 4.92743706703186
Time taken for @: 1.779259204864502
```

The difference between C and c = 0, which shows the correctness of our method. The matmul takes 4.92s almost 3 times for @ operator which takes 1.78s

The reason for faster performance of @ operator, is beacuse in our method the loop runs for 1000000 times. While @ operator runs multiplication of various rows of a and columns of b parallelly

| In []: | | | |
|---------|--|--|--|
| | | | |
| | | | |

Q_2 Linear Regression

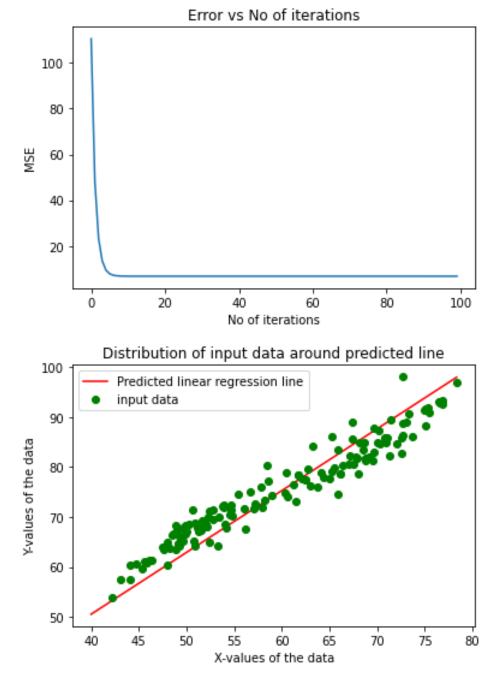
```
In [ ]:
         import numpy as np
         import matplotlib.pyplot as plt
In [2]:
         error list = []
In [3]:
         #function to calculate gradients
         def compute(train data,b0,b1):
             x values = train data[:,0]
             y values = train data[:,1]
             y calc = b1*x values + b0
             diff = y calc - y values
             sum error = np.sum(diff*diff)
             error list.append(sum error/(2*x values.shape[0]))
             gradient b0 = np.sum(diff)/x values.shape[0]
             gradient b1 = np.sum(diff*x values)/x values.shape[0]
             return gradient b0, gradient b1
In [4]:
         #fucntion to update the weights
         def weight update(train data,b0,b1,learning rate,no_of_iterations):
             for i in range(no of iterations):
                 #print(b0,b1,"\n")
                 gradient b0, gradient b1 = compute(train data,b0,b1)
                 b0 -= learning rate*gradient b0
                 b1 -= learning rate*gradient b1
             return b0,b1
```

Learning rate .0001

```
In [5]: #declaring hyper parameters
```

```
error list = []
b0, b1 = np.random.randint(20,size=2)
learning rate = 0.0001
data = np.load("data-2.npy")
no of iterations = 100
b0, b1 = weight update(data,1,1,learning rate,no of iterations)
x data = data[:,0]
y data = data[:,1]
x \text{ values} = \text{np.arange}(40, \text{np.max}(x \text{ data}), .001)
y values = b0 + b1 * x values
iteration list = np.arange(0,no of iterations,1)
#printing b0,b1
print("Final intercept:",np.round(b0,4),"and slope: ",np.round(b1,4))
print("Final error: ",error list[-1])
#plotting various fraphs
fig, ax1 = plt.subplots(1)
ax1.plot(iteration list,error list)
ax1.set xlabel("No of iterations")
ax1.set ylabel("MSE")
ax1.set title("Error vs No of iterations")
fig, ax = plt.subplots(1)
#plotting the obtained line
ax.plot(x values, y values, 'r', label="Predicted linear regression line")
ax.plot(x data,y data,'og',label="input data")
ax.set xlabel("X-values of the data")
ax.set ylabel("Y-values of the data")
ax.set title("Distribution of input data around predicted line")
ax.legend()
plt.show()
```

Final intercept: 1.0082 and slope: 1.2376 Final error: 6.7802409829839005

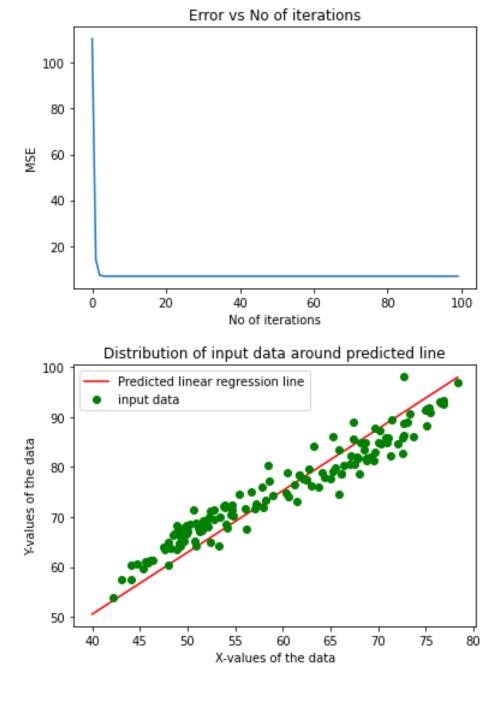


Learning rate 0.0002

```
In [6]:
    error_list = []
    b0, b1 = 1,1
```

```
learning rate = 0.0002
data = np.load("data-2.npy")
no of iterations = 100
b0, b1 = weight update(data,1,1,learning rate,no of iterations)
iteration list = np.arange(0,no of iterations,1)
print("Final intercept: ",np.round(b0,4),"and slope: ",np.round(b1,4))
print("Final error", error list[-1])
fig, ax1 = plt.subplots(1)
ax1.plot(iteration list,error list)
ax1.set xlabel("No of iterations")
ax1.set ylabel("MSE")
ax1.set title("Error vs No of iterations")
fig, ax = plt.subplots(1)
#plotting the obtained line
ax.plot(x values, y values, 'r', label="Predicted linear regression line")
ax.plot(x data,y data,'og',label="input data")
ax.set xlabel("X-values of the data")
ax.set ylabel("Y-values of the data")
ax.set title("Distribution of input data around predicted line")
ax.legend()
plt.show()
```

Final intercept: 1.0125 and slope: 1.2375 Final error 6.778377807023614



Learning rate 0.0006

```
In [7]:
    error_list = []
    b0, b1 = 1,1
```

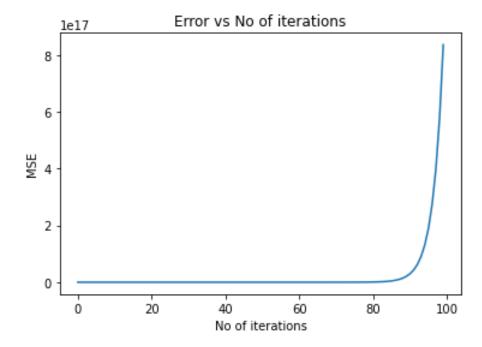
```
learning_rate = 0.0006
data = np.load("data-2.npy")
no_of_iterations = 100
b0, b1 = weight_update(data,1,1,learning_rate,no_of_iterations)
iteration_list = np.arange(0,no_of_iterations,1)

print("Final intercept: ",np.round(b0,4),"and slope: ",np.round(b1,4))
print("Final error",error_list[-1])

fig, ax1 = plt.subplots(1)
ax1.plot(iteration_list,error_list)
ax1.set_xlabel("No of iterations")
ax1.set_ylabel("MSE")
ax1.set_title("Error vs No of iterations")
```

Final intercept: -418375.976 and slope: -25679229.1794 Final error 8.365249706795523e+17

Out[7]: Text(0.5, 1.0, 'Error vs No of iterations')

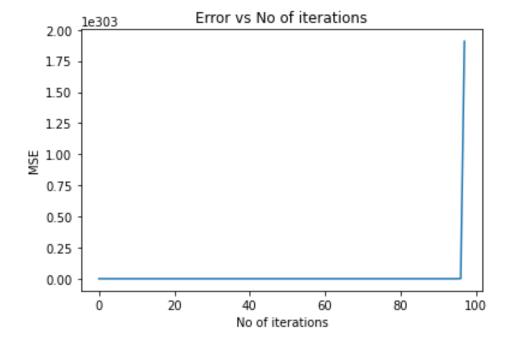


Learning rate 0.01

```
In [8]: error_list = []
```

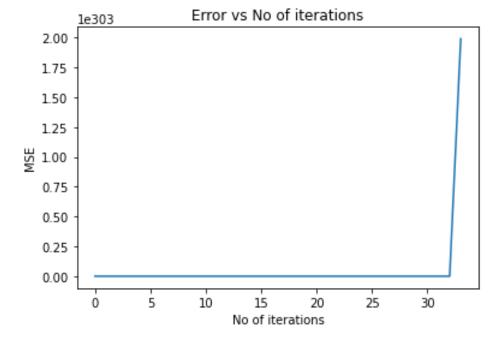
```
b0, b1 = 1,1
learning rate = 0.01
data = np.load("data-2.npy")
no of iterations = 100
b0, b1 = weight update(data,1,1,learning rate,no of iterations)
iteration list = np.arange(0,no of iterations,1)
print("Final intercept: ",np.round(b0,4),"and slope: ",np.round(b1,4))
print("Final error",error list[-1])
fig, ax1 = plt.subplots(1)
ax1.plot(iteration list,error list)
ax1.set xlabel("No of iterations")
ax1.set ylabel("MSE")
ax1.set title("Error vs No of iterations")
Final intercept:
                  -7.568487598507173e+152 and slope:
                                                      -4.6454019747292825e+154
Final error inf
/home/akshay/.local/lib/python3.8/site-packages/numpy/core/fromnumeric.py:86: RuntimeWarning: over
flow encountered in reduce
 return ufunc.reduce(obj, axis, dtype, out, **passkwargs)
<ipython-input-3-e36a763e0022>:7: RuntimeWarning: overflow encountered in multiply
 sum error = np.sum(diff*diff)
```

Out[8]: Text(0.5, 1.0, 'Error vs No of iterations')



Learning rate 10

```
In [9]:
         error list = []
         b0, b1 = 1,1
         learning rate = 10
         no of iterations = 100
         b0, b1 = weight update(data,1,1,learning rate,no of iterations)
         iteration list = np.arange(0,no of iterations,1)
         print("Final intercept:",np.round(b0,4),"and slope: ",np.round(b1,4))
         print("Final error",error list[-1])
         fig, ax1 = plt.subplots(1)
         ax1.plot(iteration list,error list)
         ax1.set xlabel("No of iterations")
         ax1.set ylabel("MSE")
         ax1.set title("Error vs No of iterations")
        Final intercept: nan and slope: nan
        Final error nan
        <ipython-input-3-e36a763e0022>:7: RuntimeWarning: overflow encountered in multiply
          sum error = np.sum(diff*diff)
        /home/akshay/.local/lib/python3.8/site-packages/numpy/core/fromnumeric.py:86: RuntimeWarning: over
        flow encountered in reduce
          return ufunc.reduce(obj, axis, dtype, out, **passkwargs)
        <ipython-input-3-e36a763e0022>:10: RuntimeWarning: overflow encountered in multiply
          gradient b1 = np.sum(diff*x values)/x values.shape[0]
        <ipython-input-4-e3c348689dc5>:6: RuntimeWarning: invalid value encountered in double scalars
          b0 -= learning rate*gradient b0
        <ipython-input-4-e3c348689dc5>:7: RuntimeWarning: invalid value encountered in double scalars
          b1 -= learning rate*gradient b1
Out[9]: Text(0.5, 1.0, 'Error vs No of iterations')
```



Best Learning rate according to me is either .0001 or 0.0002, they both converge 5-6 epochs, as you can see above .0006,.1,10 diverges. The final MSE of 0.0001 and 0.0002 are comparable. .0001 offers more smooth curve compared to .0002

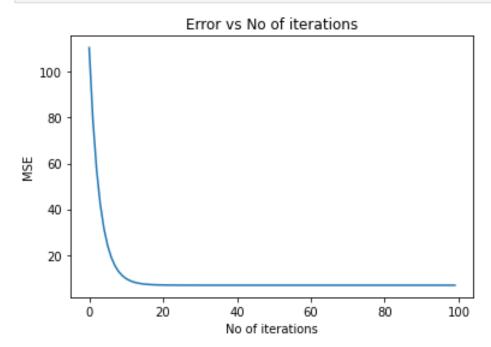
Showing Proof for the upper bound, obtained upper bound 0.0005

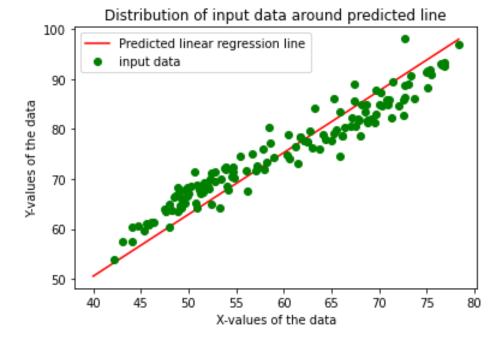
For 0.0005 below cell

```
In [10]:
    error_list = []
    b0, b1 = 1,1
    learning_rate = 0.0005
    no_of_iterations = 100
    b0, b1 = weight_update(data,1,1,learning_rate,no_of_iterations)
    iteration_list = np.arange(0,no_of_iterations,1)

fig, ax1 = plt.subplots(1)
    ax1.plot(iteration_list,error_list)
```

```
ax1.set_xlabel("No of iterations")
ax1.set_ylabel("MSE")
ax1.set_title("Error vs No of iterations")
fig, ax = plt.subplots(1)
#plotting the obtained line
ax.plot(x_values,y_values,'r',label="Predicted linear regression line")
ax.plot(x_data,y_data,'og',label="input data")
ax.set_xlabel("X-values of the data")
ax.set_ylabel("Y-values of the data")
ax.set_title("Distribution of input data around predicted line")
ax.legend()
plt.show()
```



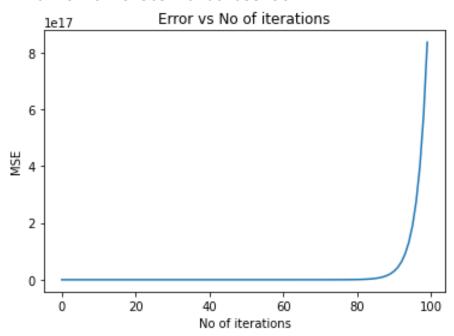


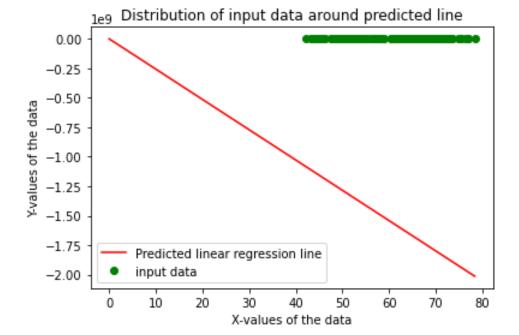
For 0.0006 the below diagram diverges

```
In [11]:
          error list = []
          b0, b1 = 1,1
          learning rate = 0.0006
          no of iterations = 100
          b0, b1 = weight update(data,1,1,learning rate,no of iterations)
          x values = np.arange(0,np.max(x data),.001)
          y values = b0 + b1 * x_values
          iteration list = np.arange(0,no of iterations,1)
          print("Final intercept: ",np.round(b0,4),"and slope: ",np.round(b1,4))
          print("Final error",error list[-1])
          fig, ax1 = plt.subplots(1)
          ax1.plot(iteration list,error list)
          ax1.set xlabel("No of iterations")
          ax1.set ylabel("MSE")
          ax1.set title("Error vs No of iterations")
          fig, ax = plt.subplots(1)
          ax.plot(x values, y values, 'r', label="Predicted linear regression line")
          ax.plot(x data,y data,'og',label="input data")
```

```
ax.set_xlabel("X-values of the data")
ax.set_ylabel("Y-values of the data")
ax.set_title("Distribution of input data around predicted line")
ax.legend()
plt.show()
```

Final intercept: -418375.976 and slope: -25679229.1794 Final error 8.365249706795523e+17





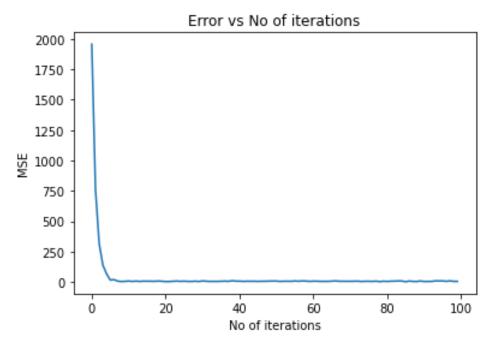
Conclusion

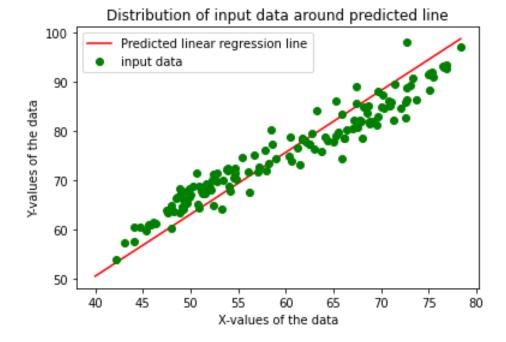
The error increases when Ir is .0006, when the Ir is 0.0006 the regression line obtained diverges from the data points so much

```
In [12]: #funtion used for mini batch gradient
def batch_weight_update(train_data,b0,b1,learning_rate,no_of_iterations,batch_size):
    for i in range(no_of_iterations):
        np.random.shuffle(train_data)
        gradient_b0, gradient_b1 = compute(train_data[:batch_size],b0,b1)
        b0 -= learning_rate*gradient_b0
        b1 -= learning_rate*gradient_b1
    return b0,b1
```

```
In [13]:
    error_list = []
    b0, b1 = np.random.rand(2,1)
    learning_rate = 0.0001
    no_of_iterations = 100
```

```
batch size = 20
b0, b1 = batch weight update(data,b0,b1,learning rate,no of iterations,batch size)
#plotting the obtained line
x \text{ values} = \text{np.arange}(40, \text{np.max}(x \text{ data}), .001)
y values = b0 + b1 * x values
iteration list = np.arange(0,no of iterations)
fig, ax1 = plt.subplots(1)
ax1.plot(iteration list,error list)
ax1.set xlabel("No of iterations")
ax1.set ylabel("MSE")
ax1.set title("Error vs No of iterations")
fig, ax = plt.subplots(1)
ax.plot(x values, y values, 'r', label="Predicted linear regression line")
ax.plot(x data,y data,'og',label="input data")
ax.set xlabel("X-values of the data")
ax.set ylabel("Y-values of the data")
ax.set title("Distribution of input data around predicted line")
ax.legend()
plt.show()
```





Minibatch converges more with less epochs and less data. And the final MSE we got is around 5.81 is lower than batch gradient descent, and also the error oscillates a little bit for minibatch gradient

Verifying with sklearn

In []:

```
import sklearn
from sklearn.linear_model import LinearRegression
x_values = data[:,0].reshape(-1,1)
y_values = data[:,1].reshape(-1,1)
reg = LinearRegression().fit(x_values,y_values)
print(reg.coef_,reg.intercept_)
y_pred = reg.predict(x_values)
sklearn.metrics.mean_squared_error(y_pred,y_values)

[[0.961082]] [17.98049938]

Out[14]: 6.195654144451672
```

HW2-Q2-Normal-Equation-Derivation

Akshay Antony

October 2, 2021

Normal equations:

J: cost(mean square derror)

 θ^T weights

Y: labels

X: input - data

$$h_{\theta}(x) = \theta^T X$$

$$J = (1/2m) * (\theta^T X - Y)^T (\theta^T X - Y)$$

$$J = (1/2m) * (((\theta^T X)^T . \theta^T X) - Y^T \theta^T X - (\theta^T X)^T Y + Y^T . Y)$$

Neglecting the constant 1/2m

$$J = X^T \theta \theta^T X - Y^T \theta^T X - X^T \theta Y + Y^T Y$$
 Taking derivative wrt θ

$$\partial J/\partial \theta = 2X^TX\theta - X^TY - X^TY$$

equating to 0

$$2X^T X \theta = 2X^T Y$$

$$\theta = (X^T X)^{-1} X^T Y$$

Gradient Descent and Update:

$$y' = b_1 * x + b_0$$

where y' is the predicted value

Sum of squared error is: $\sum_{i=1}^{m} (y - (b_1 x + b_0))^2$

Taking the Mean squared error as:

$$(1/2m) * \sum_{i=1}^{m} (y - (b_1 x + b_0))^2$$

To minimize the MSE, we take the differential w.r.t b_1, b_0

Taking derivative w.r.t b_1

$$= (1/m)(\sum_{i=1}^{m} 2(y - (b_1x + b_0)) * b_1 - (1)$$

$$\frac{\partial J}{\partial b_1} = (1/m)(\sum_{i=1}^m (y - (b_1 x + b_0)) * b_1$$

Similarly

$$\frac{\partial J}{\partial b_0} = (1/m) * (\Sigma + i = 1^m (y - (b_1 x + b_0))) - (2)$$

Applying gradient descent to update b_1, b_0

$$b_1 = b_1 - \frac{\partial J}{\partial b_1} * \alpha$$
 where α is the learning rate $b_0 = b_0 - \frac{\partial J}{\partial b_0} * \alpha$

$$b_0 = b_0 - \frac{\partial \dot{J}}{\partial b_0} * \alpha$$

There are mainly 3 types of gradient descent:

- 1. Batch gradient descent: Here gradient descent is done after all the data in a batch is passed once. That means the parameters are updated only after the whole training set is passed
- 2. Mini batch gradient descent: Here the whole training set is divided into mini-batches and passed to the model. The parameters are updated by gradient descent after each mini-batch is processed.
- 3. Stochastic Gradient Descent: Here the parameters are updated after each training example in the training data. That means for the whole data set, this update happens m times according to equations 1 and 2.

Question 3: Logistic Regression

```
In [1]:
#Import all the required libraries
import numpy as np
import matplotlib.pyplot as plt
import pandas as pd
```

Load the data

```
In [2]:
# load the data
data_input_0 = pd.read_csv("/home/akshay/Downloads/MAIL/Assignment 2/class0-input.csv")
data_input_1 = pd.read_csv("/home/akshay/Downloads/MAIL/Assignment 2/class1-input.csv")
data_labels = pd.read_csv("/home/akshay/Downloads/MAIL/Assignment 2/labels.csv")

# Perform important operations on the data
X = pd.concat([data_input_0,data_input_1],axis=0)
X = X.to_numpy()
X = np.float64(X)
Y = data_labels.to_numpy()
Y = np.float64(Y)
```

Check the shape

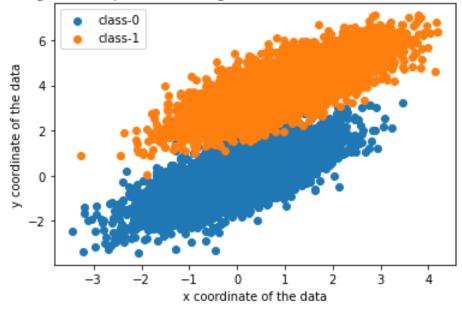
```
In [3]: # Shape of X
print(X.shape)
# Shape of Y
print(Y.shape)
(10000, 2)
(10000, 1)
```

Visualize the data

```
In [4]: # Use different colors for each class
# Use plt.scatter
fig, ax = plt.subplots(1)
ax.scatter(X[:5000,0],X[:5000,1],label="class-0")
ax.scatter(X[5000:10000,0],X[5000:10000,1],label="class-1")
# Dont forget to add axes titles, graph title, legend
ax.set_xlabel("x coordinate of the data")
ax.set_ylabel("y coordinate of the data")
ax.set_title("plotting all the input data, orange is class 1 data and blue is class 2 data")
ax.legend()
```

Out[4]: <matplotlib.legend.Legend at 0x7f442b303760>

plotting all the input data, orange is class 1 data and blue is class 2 data



Define the required functions

```
In [5]:
# Pass in the required arguments
# Implement the sigmoid function
def sigmoid(x):
    sig_x = 1/(1 + np.exp(-x))
    return sig_x
```

```
In [6]: # Pass in the required arguments
         # The function should return the gradients
         def calculate gradients(Y,X,sig x):
             grad x1 = (Y - sig x).squeeze()*X[:,0]
             grad x2 = (Y - sig x).squeeze()*X[:,1]
             grad x0 = (Y - sig x)
             current_grads = np.asarray([[np.sum(grad_x0)/Y.shape[0]],[np.sum(grad_x1)/Y.shape[0]]
                                          , [np.sum(grad x2)/Y.shape[0]])
             #print(current grads)
             return current grads
In [7]:
         # Update the weights using gradients calculated using above function and learning rate
         # The function should return the updated weights to be used in the next step
         def update weights(prev weights, current grads, learning rate):
             prev weights += learning rate*current grads
             return prev weights
In [8]:
         # Use the implemented functions in the main function
         # 'main' fucntion should return weights after all the iterations
         # Dont forget to divide by the number of datapoints wherever necessary!
         # Initialize the intial weigths randomly
         def main(X, Y, weights, learning rate = 0.0005, num steps = 50000):
             updated weights = weights
             for j in range(num steps):
                 sig x = sigmoid(X@updated weights[1:3] + updated weights[0])
                 \#predicted = np.where(sig x <= 0.5, 0, 1)
                 current grads = calculate gradients(Y, X, sig x)
                 updated weights = update weights(updated weights, current grads, learning rate)
             return updated weights
In [9]:
         # Pass in the required arguments (final weights and input)
         # The function should return the predictions obtained using sigmoid function.
         def predict(final weights,X):
             sig x = sigmoid(X@final weights[1:3] + final weights[0])
             return sig x
```

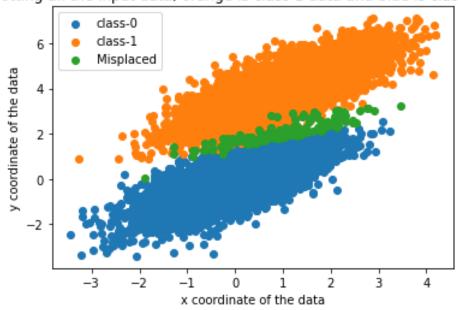
Visualize the misclassification

```
In [10]:
          # Use the final weights to perform prediction using predict funtion
          # Convert the predictions to '0' or '1'
          # Calculate the accuracy using predictions and labels
          \#initial\ weights = np.random.rand(3,1)
          initial weights = np.asarray([[0.],[0.],[0.]])
          final weights = main(X,Y,weights=initial weights)
          predicted = predict(final weights,X)
          predicted = np.where(predicted<=0.5,0,1)</pre>
          accuracy = np.sum(predicted == Y)/Y.shape[0]
          print("Accuracy: ",accuracy,"Intercept: ",final weights[0],"Coefficients: ",final weights[1],final
         Accuracy: 0.984 Intercept: [-2.20981838] Coefficients: [-0.59123076] [1.55533894]
In [11]:
          # Use different colors for class 0, class 1 and misclassified datapoints
          # Use plt.scatter
          # Dont forget to add axes titles, graph title, legend
          class0 x,class0 y,class1 x, class1 y, mis class x, mis class y = [], [], [], [], []
          for i in range(predicted.shape[0]):
              if(predicted[i] == Y[i] == 0):
                  class0 x.append(X[i,0])
                  class0 y.append(X[i,1])
              elif(predicted[i] == Y[i] == 1):
                  class1 x.append(X[i,0])
                  class1 y.append(X[i,1])
              else:
                  mis class x.append(X[i,0])
                  mis class y.append(X[i,1])
          #print(len(mis class x))
          fig, ax = plt.subplots(1)
          ax.scatter(class0 x,class0 y,label="class-0")
          ax.scatter(class1 x,class1 y,label="class-1")
          ax.scatter(mis class x,mis class y,label="Misplaced")
          # Dont forget to add axes titles, graph title, legend
          ax.set xlabel("x coordinate of the data")
          ax.set ylabel("y coordinate of the data")
```

```
ax.set_title("plotting all the input data, orange is class 1 data and blue is class 2 data")
ax.legend()
```

Out[11]: <matplotlib.legend.Legend at 0x7f442914d2b0>

plotting all the input data, orange is class 1 data and blue is class 2 data



Compare the results with sklearn's Logistic Regression

```
In [12]: # import sklearn and necessary libraries
# Print the accuracy obtained by sklearn and your model

In [13]: import sklearn
from sklearn.linear_model import LogisticRegression

Y = Y.reshape(10000)

log_reg_obj = LogisticRegression()
log_reg_obj.fit(X,Y)
print(log_reg_obj.coef_,log_reg_obj.intercept_,log_reg_obj.score(X, Y))

[[-3.92166117 6.5756403 ]] [-11.2220325] 0.9948
```

Accuracy given by my model: .984

SKLearn Accuracy: .9948

Both accuracies are equal. To increase the accuracy

- 1. We can pass in the data in batches
- 2. Optimizing the hyperparameters including Ir

| In [|]: | |
|------|----|--|
| In [|]: | |