Important Note for question1!

- Please **do not** change the default variable names in this problem, as we will use them in different parts.
- The default variables are initially set to "None".
- You only need to modify code in the "TODO" part. We added a lot of "assertions" to check your code. Do not modify them.

```
In [1]:
# load packages
import numpy as np
import pandas as pd
import time
from sklearn.naive_bayes import GaussianNB
```

P1. Load data and plot

TODO

• Load train and test data, and split them into inputs(trainX, testX) and labels(trainY, testY)

```
In [4]:
         # print(type(testX))
         # N train = trainX.shape[0]
         \# py0 = (trainY == 0).sum()/N train
         \# py1 = (trainY == 1).sum()/N train
         # print(py0+py1)
         trainX = trainX.to numpy()
         trainY = trainY.to numpy()
         testX = testX.to numpy()
         testY = testY.to numpy()
         print(trainX.shape)
         #trainX[idx].shape
         mean0 = np.mean(trainX[np.where(trainY == 0)],axis=0)
         var0 = np.var(trainX[np.where(trainY == 0)],axis=0)
         mean1 = np.mean(trainX[np.where(trainY == 1)],axis=0)
         var1 = np.var(trainX[np.where(trainY == 1)],axis=0)
         P \times y0 = (1/(np.sqrt(2*np.pi*var0)))*
                  np.exp(-0.5*np.power((trainX-mean0)/np.sqrt(var0),2))
         P \times y1 = (1/(np.sqrt(2*np.pi*var1)))*
                  np.exp(-0.5*np.power((trainX-mean1)/np.sqrt(var1),2))
         P \times y0 = np.product(P \times y0,axis=1)
         P x y0.shape
         (134000, 200)
         (134000,)
Out[4]:
```

P2. Write your Gaussian NB solver

- Finish the myNBSolver() function.
 - Compute P(y == 0) and P(y == 1), saved in "py0" and "py1"
 - Compute mean/variance of trainX for both y = 0 and y = 1, saved in "mean0", "var0", "mean1" and "var1"
 - Each of them should have shape (N_train, M), where N_train is number of train samples and M is number of features.
 - Compute P(xi | y == 0) and P(xi | y == 1), compare and save **binary** prediction in "train pred" and "test pred"
 - Compute train accuracy and test accuracy, saved in "train acc" and "test acc".

Return train accuracy and test accuracy.

```
In [9]:
         def myNBSolver(trainX, trainY, testX, testY):
             N train = trainX.shape[0]
             N test = testX.shape[0]
             M = trainX.shape[1]
             #### TODO ####
             # Compute P(y == 0) and P(y == 1)
             py0 = np.sum(trainY == 0)/N train
             py1 = np.sum(trainY == 1)/N train
             py0 test = np.sum(testY == 0)/N test
             py1 test = np.sum(testY == 1)/N test
             ##############
             print("Total probablity is %.2f. Should be equal to 1." %(py0 + py1))
             #### TODO ####
             # Compute mean/var for each label
             mean0 = np.mean(trainX[np.where(trainY == 0)],axis=0)
             mean1 = np.mean(trainX[np.where(trainY == 1)],axis=0)
             var0 = np.var(trainX[np.where(trainY == 0)],axis=0)
             var1 = np.var(trainX[np.where(trainY == 1)],axis=0)
             ##############
             assert(mean0.shape[0] == M)
             #### TODO ####
             # Compute P(xi|y == 0) and P(xi|y == 1), compare and make prediction
             # This part may spend 5 - 10 minutes or even more if you use for loop, so feel free to
             # print something (like step number) to check the progress
             \# P(xi|y == 0)
             P \times y0 = (1/(np.sqrt(2*np.pi*var0)))*
                       np.exp(-0.5*np.power((trainX-mean0)/np.sqrt(var0),2))
             P \times y0 = np.product(P \times y0,axis=1)
             \# P(xi|y == 0)
             P \times y1 = (1/(np.sqrt(2*np.pi*var1)))*\
                       np.exp(-0.5*np.power((trainX-mean1)/np.sqrt(var1),2))
             P \times y1 = np.product(P \times y1,axis=1)
```

```
\# P(xi|y == 0) for test
P \times y0 \text{ test} = (1/(np.sqrt(2*np.pi*var0)))*
          np.exp(-0.5*np.power((testX-mean0)/np.sqrt(var0),2))
P_x_y0_{\text{test}} = np.product(P_x_y0_{\text{test,axis}=1})
\# P(xi|y == 0) for test
P \times y1 \text{ test} = (1/(np.sqrt(2*np.pi*var1)))*
          np.exp(-0.5*np.power((testX-mean1)/np.sqrt(var1),2))
P \times y1 \text{ test} = np.product(P \times y1 \text{ test,axis=1})
#calculating P(y=0/x) and P(y=1/x), we are calculating only the numerator
# as to compare, the denominator is constant for both.
P \times y0 = P \times y0*py0
P \times y1 = py1
P \times y0 test *= py0 test
P x y1 test *= py1 test
train pred = np.empty((N train))
train pred = np.where(P \times y0>P \times y1,0,1)
test pred = np.empty((N test))
test pred = np.where(P_x y0_test>P_x_y1_test,0,1)
#############
assert(train pred[0] == 0 or train pred[0] == 1)
assert(test pred[0] == 0 or test pred[0] == 1)
#### TODO ####
# Compute train accuracy and test accuracy
train acc = np.sum(trainY == train pred)/N train
test acc = np.sum(testY == test pred)/N test
##############
return train acc, test acc
```

```
In [10]: # driver to test your NB solver
    train_acc, test_acc = myNBSolver(trainX, trainY, testX, testY)
    print("Train accuracy is %.2f" %(train_acc * 100))
    print("Test accuracy is %.2f" %(test_acc * 100))
```

Total probablity is 1.00. Should be equal to 1.

```
Train accuracy is 92.22 Test accuracy is 92.08
```

P3. Test your result using sklearn

- Finish the skNBSolver() function.
 - fit model, make prediction and return accuracy for train and test sets.

```
In [57]:
          def skNBSolver(trainX, trainY, testX, testY):
              #### TODO ####
              # fit model
              # make prediction
              # compute accuracy
              model = GaussianNB()
              trained = model.fit(trainX,trainY)
              sk train acc = np.sum(trainY == trained.predict(trainX))/trainX.shape[0]
              sk test acc = np.sum(testY == trained.predict(testX))/testX.shape[0]
              ##############
              return sk train acc, sk test acc
In [58]:
          # driver to test skNBSolver
          sk train acc, sk test acc = skNBSolver(trainX, trainY, testX, testY)
          print("Train accuracy is %.2f" %(sk train acc * 100))
          print("Test accuracy is %.2f" %(sk test acc * 100))
         Train accuracy is 92.22
         Test accuracy is 92.05
In [ ]:
```

Note for question2

- Please follow the template to complete q2
- You may create new cells to report your results and observations

```
# Import modules
from sklearn.linear_model import LinearRegression, Lasso, Ridge
import numpy as np
import matplotlib.pyplot as plt
```

P1. Create data and plot

- implement the true function f(x) defined in the write-up
- use function name model()
- sample 30 random points with noise
- plot sampled points together with the model function

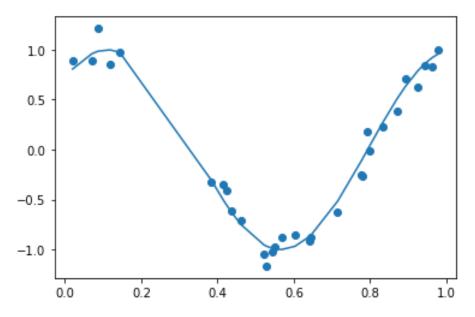
```
In [42]: # Define the function to generate data points
def generate_points(input_points):
    output_points = np.sin(2.2*np.pi*input_points + 0.8)
    return output_points

# Initialize random seed
np.random.seed(0)
input_points = np.random.uniform(0,1,size=(30,1))
input_points = np.sort(input_points,axis=0)
output_points = generate_points(input_points)

# Generate noisy data points: (x,y)
noise = np.random.normal(loc=0,scale=0.1,size=(30,1))
output_points_noise = output_points + noise
```

```
# Plot true model and sampled data points
plt.plot(input_points,output_points)
plt.scatter(input_points,output_points_noise)
```

Out[42]: <matplotlib.collections.PathCollection at 0x7f1cc621e6a0>



P2. Fit a linear model

- use sklearn to fit model: $h(x) = w_0 + w_1 x$
- report $w = [w_0, w_1]$
- plot the fitted model h(x) together with data points

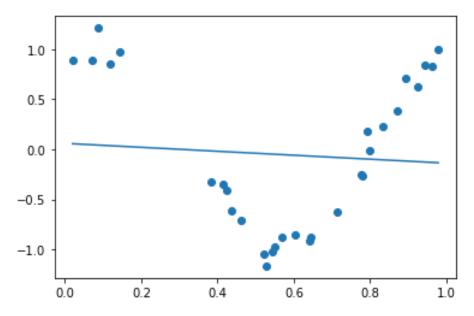
```
In [43]: # Fit a linear model in the original space
linear_model = LinearRegression()
linear_model.fit(input_points,output_points_noise)
w0, w1 = linear_model.intercept_, linear_model.coef_
print("w0: ",w0,"w1: ",w1)

# Plot fitted linear model
predicted = linear_model.predict(input_points)
```

```
plt.plot(input_points,predicted)
plt.scatter(input_points,output_points_noise)
```

```
w0: [0.06038094] w1: [[-0.19787027]]
```

Out[43]: <matplotlib.collections.PathCollection at 0x7f1cc61fc130>



P3. Fit a polynomial curve

- augment the original feature to $[x,x^2,\cdots,x^{15}]$
- fit the polynomial curve: $h(x) = \sum_{i=0}^{15} w_i x^i$
- report $w=[w_0,w_1,\cdots,w_{15}]$
- plot the fitted model h(x) together with data points

```
In [44]:
# Augment the original feature to a 15-vector
def total_feature_generate(input_points):
    total_feature = input_points
    for i in range(14):
        new_feature = np.power(input_points,i+2)
```

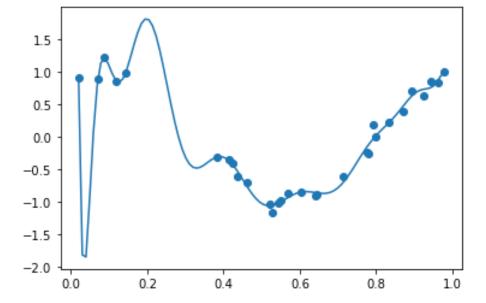
```
total_feature = np.concatenate((total_feature,new_feature),axis=1)
return total_feature
```

```
In [45]:
          # Fit linear model to the generated 15-vector features
          total feature = total feature generate(input points)
          poly linear model = LinearRegression()
          poly linear model.fit(total feature,output points noise)
          w = np.zeros(16)
          w[0] = poly_linear_model.intercept_
          w[1:] = poly linear model.coef
          print("Weights, the first term is the bias: \n",w)
          print("Score:",poly linear model.score(total feature,output points noise))
          predicted = poly linear model.predict(total feature)
          # Plot fitted curve and sampled data points
          x = np.linspace(np.min(input points), np.max(input points), 100)
          x = x.reshape(-1,1)
          #print(x)
          x total feature = total feature generate(x)
          y = poly linear model.predict(x total feature)
          #print(x)
          #plt.plot(input points, predicted)
          plt.plot(x,y)
          plt.scatter(input points,output points noise)
         Weights, the first term is the bias:
          [ 3.11668855e+01 -2.97811934e+03 1.03893518e+05 -1.87420290e+06
           2.03717119e+07 -1.44873537e+08 7.09317173e+08 -2.47066536e+09
```

Out[45]: <matplotlib.collections.PathCollection at 0x7f1cc61509d0>

Score: 0.9914608458991773

6.24563083e+09 -1.15676914e+10 1.56895523e+10 -1.54006637e+10 1.06457712e+10 -4.91379706e+09 1.35920276e+09 -1.70381611e+08]



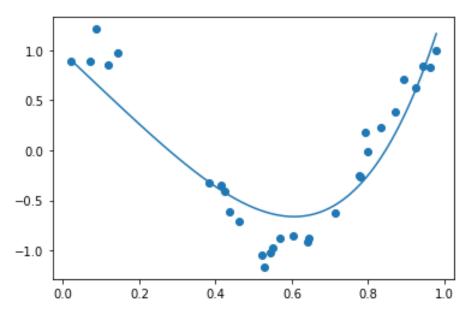
P4. Lasso regularization

- use sklearn to fit a 15-degree polynomial model with L1 regularization
- ullet report w
- plot the fitted model h(x) together with data points

```
In [46]: # Fit 15-degree polynomial with L1 regularization
    # Start with lambda(alpha) = 0.01 and max_iter = 1e4
    lasso_regression = Lasso(alpha=.01,max_iter=le4)
# Plot fitted curve and sampled data points
    lasso_regression.fit(total_feature,output_points_noise)
#w holds the weights
w = np.zeros(16)
w[0] = lasso_regression.intercept_
w[1:] = lasso_regression.coef_
print(w, "\nScore", lasso_regression.score(total_feature,output_points_noise))
predicted = lasso_regression.predict(x_total_feature)

plt.plot(x,predicted)
plt.scatter(input_points,output_points_noise)
```

Out[46]: <matplotlib.collections.PathCollection at 0x7f1cc6129e20>



1.06843622

-2.49636651

1.451898821

Score 0.9747580523845099

0.2167843

0.

-0.

-14.45629376

-0.

- 0 .

```
In [47]:
    lasso_regression = Lasso(alpha=.0001,max_iter=1e4)
    # Plot fitted curve and sampled data points
    lasso_regression.fit(total_feature,output_points_noise)
    #w holds the weights
    w = np.zeros(16)
    w[0] = lasso_regression.intercept_
    w[1:] = lasso_regression.coef_
    print(w,"\nScore",lasso_regression.score(total_feature,output_points_noise))
    predicted = lasso_regression.predict(x_total_feature)

plt.plot(x,predicted)
    plt.scatter(input_points,output_points_noise)
```

5.73351697

-0.

- 0 .

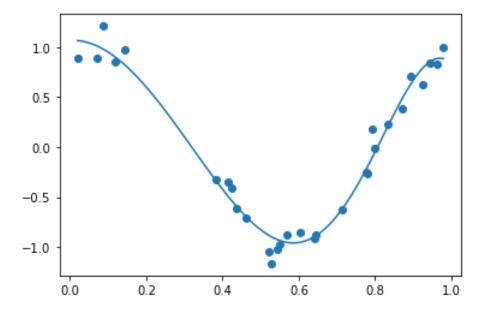
14.49985714

-5.1753093

0.

```
/home/akshay/anaconda3/lib/python3.8/site-packages/sklearn/linear_model/_coordinate_descent.py:53
0: ConvergenceWarning: Objective did not converge. You might want to increase the number of iterat ions. Duality gap: 0.02063625723763507, tolerance: 0.0017515538543778103
    model = cd_fast.enet_coordinate_descent(
```

Out[47]: <matplotlib.collections.PathCollection at 0x7f1cc6081b80>



Reporting best lambda and weights

```
In [48]:
          print("weights: The first value is the intercept:\n",w)
          print("lambda = .0001")
         weights: The first value is the intercept:
                          0.2167843 -14.45629376
                                                    5.73351697 14.49985714
          [ 1.06843622
                         0.
                                                                -5.1753093
                                     -0.
                                                   -0.
           -2.49636651 -0.
                                     -0.
                                                   -0.
            1.451898821
         lambda = .0001
```

Observations

As the value of lambda decreases the value the number of non-zero elements in the weights decreases. For example lambda = 0.0001 has 8 non-zero values as compared to 4 non zero values when lambda=.01. This means increasing

Understanding of lasso

As any regularization method lasso also tries to reduce overfitting. The a proportion (ie lambda times) magnitudes of the weights are subtracted from the weights in the training process. Lasso can lead to zero coefficients i.e. some of the features are completely neglected for the evaluation of output. This means that lasso can also help in completely removing some features which cause overfitting by making their weights 0.

Tweaking lambda

So when lambda is decreases, overfitting increases. I have displayed the scores for two different values of lambda. When lambda is very high the model is very underfitted.

P5. Ridge regularization

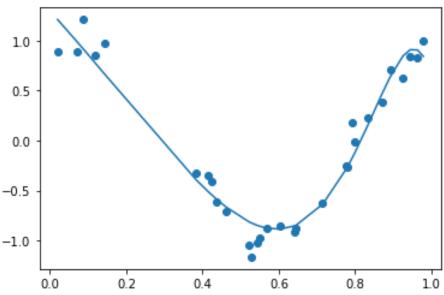
- use sklearn to fit a 15-degree polynomial model with L2 regularization
- ullet report w
- plot the fitted model h(x) together with data points

```
In [49]: # Fit 15-degree polynomial with L2 regularization
    # Start with lambda(alpha) = 0.01 and max_iter = 1e4
    ridge_regression_model = Ridge(alpha=0.01,max_iter=1e4)
    ridge_regression_model.fit(total_feature,output_points_noise)

predicted = ridge_regression_model.predict(total_feature)
    w = np.zeros(16)
    w[0] = ridge_regression_model.intercept_
    w[1:] = ridge_regression_model.coef_
    plt.plot(input_points,predicted)
    plt.scatter(input_points,output_points_noise)
```

```
# Plot fitted curve and sampled data points and compare to L1 regularization from P4
print("weights: (The first value is the intercept):\n",w)
print(ridge_regression_model.score(total_feature,output_points_noise))

weights: (The first value is the intercept):
[ 1.297752    -4.21231562   -1.58994435    1.38570425    2.48459438    2.39535518
    1.78896615    1.05511527    0.38500882   -0.14394475   -0.51380601   -0.73493061
    -0.82911584   -0.82098452   -0.73390704   -0.58833042]
0.9533749611502453
```

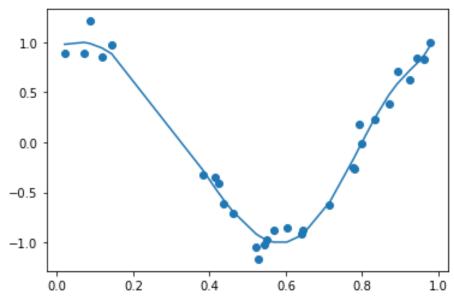


The best value of lambda found is .0001

```
ridge_regression_model = Ridge(alpha=0.0001,max_iter=1e4)
ridge_regression_model.fit(total_feature,output_points_noise)

predicted = ridge_regression_model.predict(total_feature)
w = np.zeros(16)
w[0] = ridge_regression_model.intercept_
w[1:] = ridge_regression_model.coef_
plt.plot(input_points,predicted)
plt.scatter(input_points,output_points_noise)
# Plot fitted curve and sampled data points and compare to L1 regularization from P4
print("weights: (The first value is the intercept):\n",w)
print(ridge_regression_model.score(total_feature,output_points_noise))
```

weights: (The first value is the intercept):
 [0.94933395 1.90502172 -17.24276774 4.25002429 11.61238507
 9.20596081 3.83848301 -1.02446738 -4.18410939 -5.53450391
 -5.38543602 -4.13771324 -2.15164759 0.29118807 2.98672066
 5.79387602]
0.9804373188863559



Note for question3

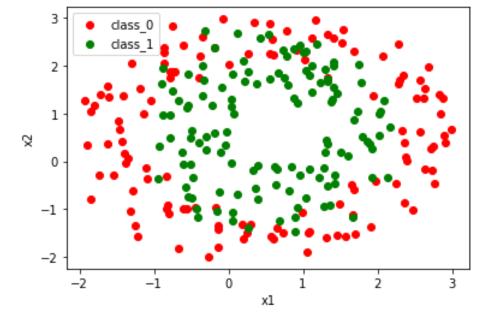
- Please follow the template to complete q3
- You may create new cells to report your results and observations

```
import libraries
import numpy as np
import matplotlib.pyplot as plt
import pandas as pd
```

P1. Load data and plot

- load q3_data.csv
- plot the points of different labels with different color

```
In [36]:
          # Load dataset
          data = pd.read_csv("q3_data.csv")
          data = data.to numpy()
          # Plot points
          class_1_idx = np.where(data[:,2] == 1)
          class_1 = data[class_1_idx,:2].squeeze()
          class 0 = data[np.where(data[:,2] == 0),:2].squeeze()
          fig, ax = plt.subplots(1)
          ax.scatter(class_0[:,0], class_0[:,1], color='r', label="class_0")
          ax.scatter(class_1[:,0], class_1[:,1], color='g', label="class_1")
          ax.set xlabel("x1")
          ax.set ylabel("x2")
          ax.legend()
          print(class 1.shape, class 0.shape)
         (126, 2) (125, 2)
```



P2. Feature mapping

TODO

• implement function map_feature() to transform data from original space to the 28D space specified in the write-up

```
In [4]: # Transform points to 28D space
def map_feature(data):
    #taking the first column of input ie x1 and x2
    x1 = data[:,0]
    x1 = np.expand_dims(x1,axis=1)
    x2 = data[:,1]
    x2 = np.expand_dims(x2,axis=1)

    curr = np.concatenate((x1,x2),axis=1)
    #created an array named final, whose first column is 1
    final = np.full((x1.shape[0],1),1)
    #concatenate 1,x1,x2 array
    final = np.concatenate((final,curr),axis=1)

for i in range(2,7,1):
    m = int(np.floor(curr.shape[1]/2)+1)
```

```
n = int(np.floor(curr.shape[1]/2))
x1_part = curr[:,0:m]*x1
x2_part = curr[:,n:curr.shape[1]]*x2
#make the new feature matrix columns by multiplying previous generated features
#with x1 and x2
curr = np.concatenate((x1_part,x2_part),axis=1)
#append the currently generated array to the final
final = np.concatenate((final,curr),axis=1)
return final

final = map_feature(data)
```

```
In [5]:
         # Define your functions here
         def sigmoid(x):
             #returns the sigmoid of input data
              sig x = 1/(1 + np.exp(-x))
              return sig x
         def calculate gradients(Y,X,sig x,lambd,updated weights):
             #calculating the gradients
             grad = X*(sig x-Y)
             #defining regularization functions
             regularization = updated weights.copy()
             #implementing regularization only for weights except bias
             regularization[0] = 0
             regularization[1:] *= lambd
             loss = -Y * np.log10(sig x) - (1-Y)*np.log10(1-sig x)
             loss = np.sum(loss,axis=0)
             loss regularization = np.sum(np.power(updated weights,2),axis=0)/(2*Y.shape[0])
             loss += loss regularization
             global loss list
             loss list.append(loss)
             current grads = np.sum(grad,axis=0)
             current grads = np.expand dims(current grads,axis=1)
             #calculating gradients using L2 regularization
             current grads = (current grads+regularization)/Y.shape[0]
```

```
return current grads
         def update weights(prev weights, current grads, learning rate):
             #updating the weights
             prev weights -= learning rate*current grads
             return prev weights
         def main(X, Y, weights,alpha,learning rate, num steps):
             updated weights = weights
             for j in range(num steps):
                 sig x = sigmoid(X@updated weights)
                 predicted = np.where(sig x<0.5,0,1)
                 accuracy = np.sum(predicted == Y)/Y.shape[0]
                 #print(accuracy)
                 #print(sig x.shape, X.shape, Y.shape)
                 current grads = calculate gradients(Y,X,sig x,alpha,updated weights)
                 #current grads = np.expand dims(current grads,axis=1)
                 updated weights = update weights(updated weights, current grads, learning rate)
             return updated weights
         def predict(final weights,X):
             sig x = sigmoid(X@final weights)
             return sig x
In [6]:
         def logistic regression regularized(alpha,initial weights,lr,number of iterations):
             X = final
             Y = data[:,2]
             Y = np.expand dims(Y,axis=1)
             final weights = main(X,Y,weights=initial weights,alpha=alpha,learning rate=lr,num steps=number
             #print(final weights.shape, X.shape)
             predicted = predict(final weights,X)
             predicted = np.where(predicted<0.5,0,1)</pre>
             accuracy = np.sum(predicted == Y)/Y.shape[0]
             print("Accuracy for lambda=",alpha," : ",accuracy)#,"Coefficients: ",final weights)
             return final weights
```

```
nx, ny = (3, 2)
x = np.linspace(0, 1, nx)
y = np.linspace(0, 1, ny)
xv, yv = np.meshgrid(x, y)
X, Y, XV, YV
h = .01
x \min, x \max = data[:, 0].min(), data[:, 0].max()
y_min, y_max = data[:, 1].min(), data[:, 1].max()
x1, y1 = np.meshgrid(np.arange(x min, x max, h),
                         np.arange(y min, y max, h))
xx = x1.reshape(-1,1)
yy = y1.reshape(-1,1)
mesh data = np.concatenate([xx,yy],axis = 1)
mesh data = map feature(mesh data)
def plot_countour(final_weights):
    predicted mesh = predict(final weights, mesh data)
    predicted mesh = np.where(predicted mesh<0.5,0,1)</pre>
    predicted mesh = predicted mesh.reshape(x1.shape)
    fig, ax = plt.subplots(1)
    ax.scatter(class 0[:,0], class 0[:,1], color='r', label="class 0")
    ax.scatter(class 1[:,0], class 1[:,1], color='g', label="class 1")
    ax.set xlabel("x1")
    ax.set ylabel("x2")
    ax.legend()
    ax.contour(x1,y1,predicted mesh,levels=[1])
```

P4. Tune the strength of regularization

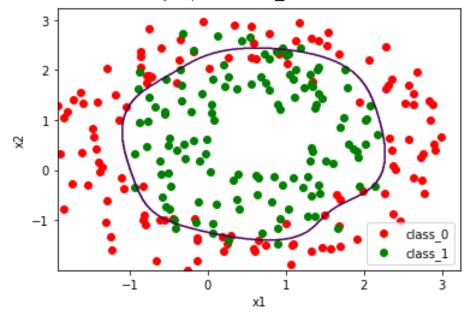
- tweak the hyper-parameter λ to be [0, 1, 100]
- · draw the decision boundaries

```
In [38]: # lambda = 0 loss_list = []
```

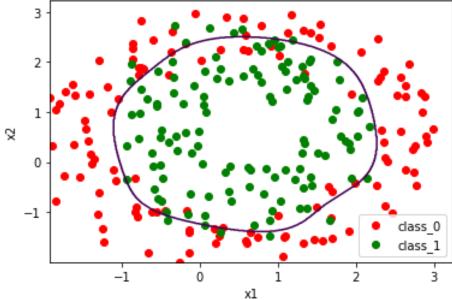
```
Accuracy for lambda= 0 : 0.8685258964143426 final weights: [ 3.41421127  0.09472157  0.63773323 -0.30190954 -0.9114259  -0.12601866  0.28713364  1.04951738 -0.32212943 -1.01455583 -1.30320873  0.20956718 -0.53474073 -0.01875392 -0.3928393  1.26532246  0.57789427  0.35449801  0.41480507  1.21375678  0.87640593 -0.34797747 -0.3189492 -0.15930069 -0.19518332 -0.2517759 -0.38654135 -0.25222523]
```

<ipython-input-37-f12fe0a94fe7>:29: UserWarning: No contour levels were found within the data rang
e.

ax.contour(x1,y1,predicted mesh,levels=[1])



```
In [39]: # lambda = 1
loss_list = []
lr = .005
number_of_iterations = 65000
initial_weight = np.zeros((28,1))
final_weights = logistic_regression_regularized(1,initial_weight,lr,number_of_iterations)
```



Accuracy for lambda= 100 : 0.8207171314741036

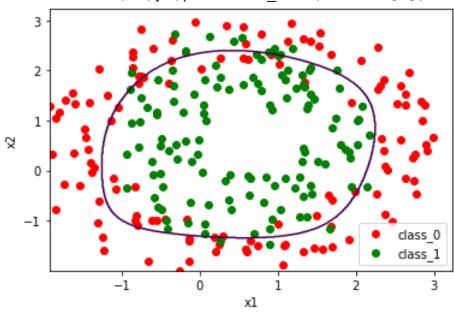
```
In [40]: # lambda = 100
loss_list = []
lr = .005
number_of_iterations = 65000
initial_weight = np.zeros((28,1))
final_weights = logistic_regression_regularized(100,initial_weight,lr,number_of_iterations)
plot_countour(final_weights)
print("final_weights: ",final_weights[:,0])
```

final weights: [1.61749735 0.10032936 0.07711003 -0.03337203 0.02790076 -0.00267251

0.13152594 0.05460935 0.10774059 0.11905399 -0.05316493 0.05228614

<ipython-input-37-f12fe0a94fe7>:29: UserWarning: No contour levels were found within the data rang
e.

ax.contour(x1,y1,predicted mesh,levels=[1])



Answer for part (d) here:

But increasing lambda decreases the over fitting, and also it brings down the value of coefficients of the weights of the model and makes it close to 0. So for lambda = 100, the accuracy very lower compared to lambda=0,1, as well as the weights have a lower magnitutude. It shows the data is underfitted. So as the lambda increases overfitting reduces