

①

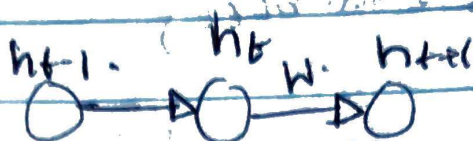
$$a) V_2 = \begin{bmatrix} -0.185 & 0.245 & -0.059 \\ -0.196 & -0.032 & 0.209 \end{bmatrix}$$

$$S_2 = \begin{bmatrix} 0.0106 & 0.0699 & 0.1099 \\ & & \end{bmatrix}$$

$$S_2 = \begin{bmatrix} 0.0106 & 0.070 & 0.110 \\ 0.141 & 0.170 & 0.021 \end{bmatrix}$$

$$W_2 = \begin{bmatrix} -0.182 & 0.308 & 0.674 \\ 0.502 & -0.858 & -0.934 \end{bmatrix}$$

a)



$$\frac{\partial L}{\partial h_t} = \frac{\partial L}{\partial h_{t+1}} \times \frac{\partial h_{t+1}}{\partial h_t}$$

$$h_{t+1} = \text{act}(\cancel{z_t})$$

$$h_{t+1} = \text{act}(\underbrace{z_t}_W)$$

$z_t$  as per question.

$$\frac{\partial h_{t+1}}{\partial h_t} = \sigma'(z_t) W^T$$

$$\boxed{\nabla h_t = \nabla h_{t+1} \times \sigma'(z_t) \times W^T}$$

Dimensional check

$$h_t = 64$$

$$W = 64 \times 64$$

$$(64 \times 32)$$

$$(64 \times 1) \times (1 \times 32)$$

$$(64 \times 32)$$

$$(32 \times 32) \times 64$$

b)  $\sigma'(0) = \frac{1}{4}$

$$\nabla h_t = \nabla h_{t+1} \times \frac{1}{4} \times w$$

$$\text{if } \boxed{w > 4}$$

$\nabla h_t > 1$ , so gradients will explode.

$$\text{if } \boxed{w < 4}$$

$\nabla h_t < 1$ , so the gradients will vanish.

$$\boxed{d = 4}$$



**False**

③ a)  $h_t = o_t \odot \tanh(c_t)$

$$o_t = \sigma(W_o x_t + U_o h_{t-1} + b_o)$$

$x_t = 0$

$$= \sigma[U_o h_{t-1} + b_o]$$

$$c_t = f_t \odot c_{t-1} + i_t \odot \tilde{c}_t \quad \text{--- (3)}$$

$$f_t = \sigma_x[U_f h_{t-1} + b_f] \quad x_t = 0 \quad \text{--- (1)}$$

$$i_t = \sigma[U_i h_{t-1} + b_i] \quad , \quad x_t = 0 \quad \text{--- (2)}$$

$$\tilde{c}_t = \tanh(U_c h_{t-1} + b_c) \quad \text{--- (4)}$$

Putting (1), (2), (4) in (3)

$$c_t = \sigma[U_o h_{t-1} + b_o] \odot c_{t-1} + \sigma[U_i h_{t-1} + b_i] \odot \tanh[U_c h_{t-1} + b_c]$$

$$h_t = \sigma[U_o h_{t-1} + b_o] \odot \left\{ \sigma[U_o h_{t-1} + b_o] \odot c_{t-1} + \sigma[U_i h_{t-1} + b_i] \odot \tanh[U_c h_{t-1} + b_c] \right\}$$

so  $h_t \neq h_{t-1}$ ,

Qualitatively due to input zero also, network may decide to forget something from long term memory as even input to the long term memory

**True**

3) b) By equation, the propagation of gradient-

$$\frac{\partial h_t}{\partial h_{t-1}} = \frac{\partial h_t}{\partial o_t} \times \frac{\partial o_t}{\partial h_{t-1}} + \frac{\partial h_t}{\partial c_t} \times \frac{\partial c_t}{\partial f_t} \times \frac{\partial f_t}{\partial h_{t-1}} + \dots$$

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$$\frac{\partial c_t}{\partial c_{t-1}} = f_t$$

~~Star~~ In the long term part of the LSTM, cell  $\frac{\partial c_t}{\partial c_{t-1}}$  becomes smaller when  $f_t$  is small.

By the concept of forget gate, if the previous input is to be forgotten  $f(t)$  should be small, so error does not affect previous states.

**True**

c)  $i_t, f_t = \text{output of sigmoid} [0, 1]$  so non-negative. for  $o_t$ , output gate also sigmoid ~~non~~ output. Sigmoid bounded between 0, 1 so non-negative output.

d) **False**

$f_t, i_t, o_t$  are independent of each other.  $f_t, i_t, o_t$  can be seen as individual probability of forgetting data, adding data and output data, but they do not sum to 1.

4

$$F_t = \sigma(W_f x_1 + U_f h_t + b_f)$$

dimension =  $1 \times 1$

(12)  $2 \times 1$   
(1)

$$y_t = \sigma[W_p x_1 + U_p h_t + b_p]$$

dimension =  $1 \times 1$

$$O_t = \sigma[W_o x_1 + U_o h_t + b_o]$$

dimension =  $1 \times 1$

$\tilde{z}_t = (1 \times 1)$  dimension

$h_t = (1 \times 1)$

a) All of  $F_t$ ,  $P_t$ ,  $O_t$ ,  $h_t$  are  $(1 \times 1)$  dimension or a scalar

b) From python

$$h_1 = 0.2174$$

$$h_2 = [-0.18988]$$

c) MSE Loss:

$$= [0.5 - 0.2174]^2 + (0.8 + 0.18988)^2$$

$$= 1.0597$$



## Programming Exercises

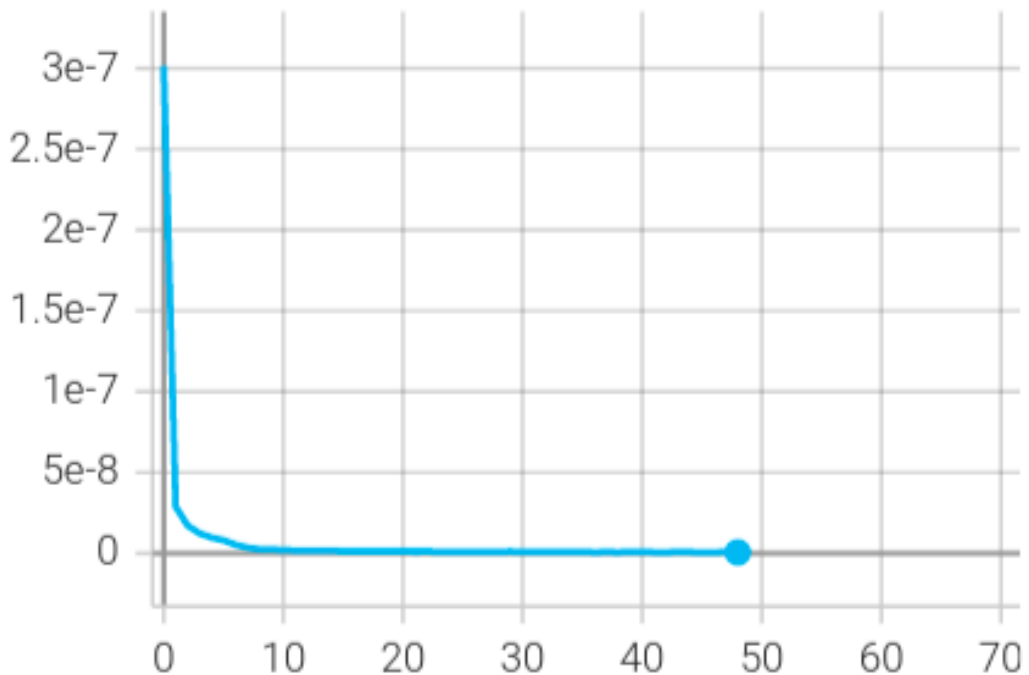
### Problem 1

Network structure:

- Three lstm layers, of the following features
  - 1st lstm, Input size 17, hidden size 128
  - 2nd lstm, Input size 128, hidden size 256
  - 3rd lstm, Input size 256, hidden size 512
- One MLP of the following feature
  - Input size 512, output size 17
- Number of trainable parameters: 83985
- Number of iterations: 50
- Learning rate = 0.001
- Optimizer: Adam
- Train loss criterion: MSELoss
- Train loss vs batch number
- Batch size: 256
- Scheduler: Step LR with 0.1 gamma, and 25 epochs step size
  - Train MSELoss vs epoch

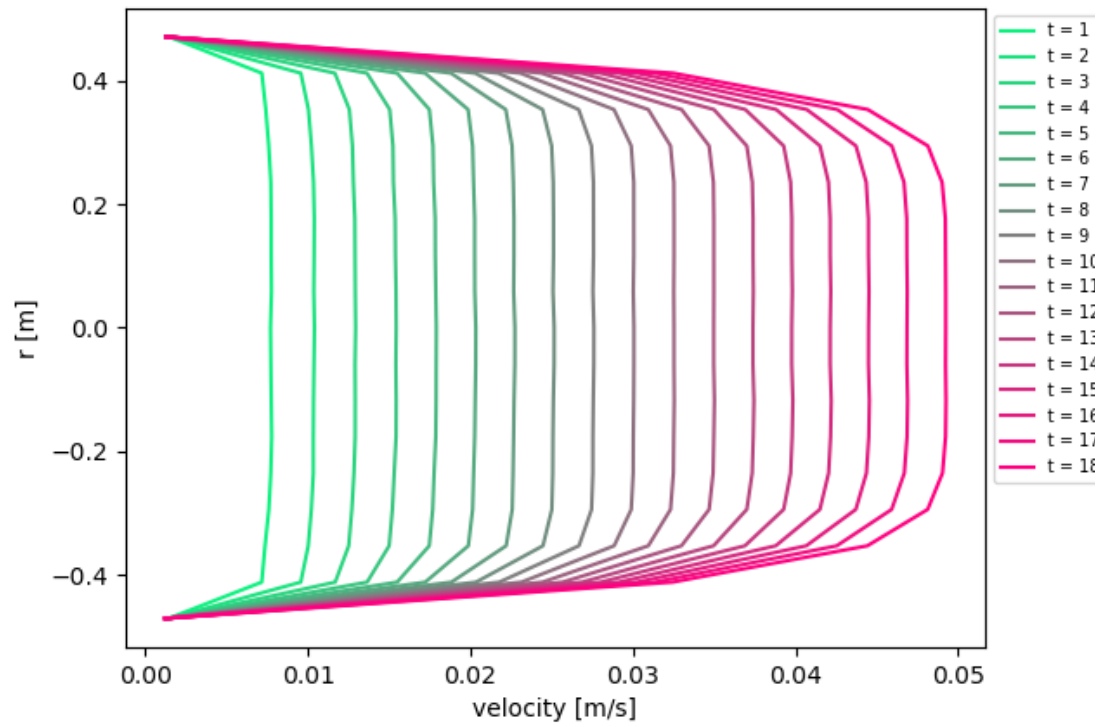
### MSE train loss

tag: MSE train loss

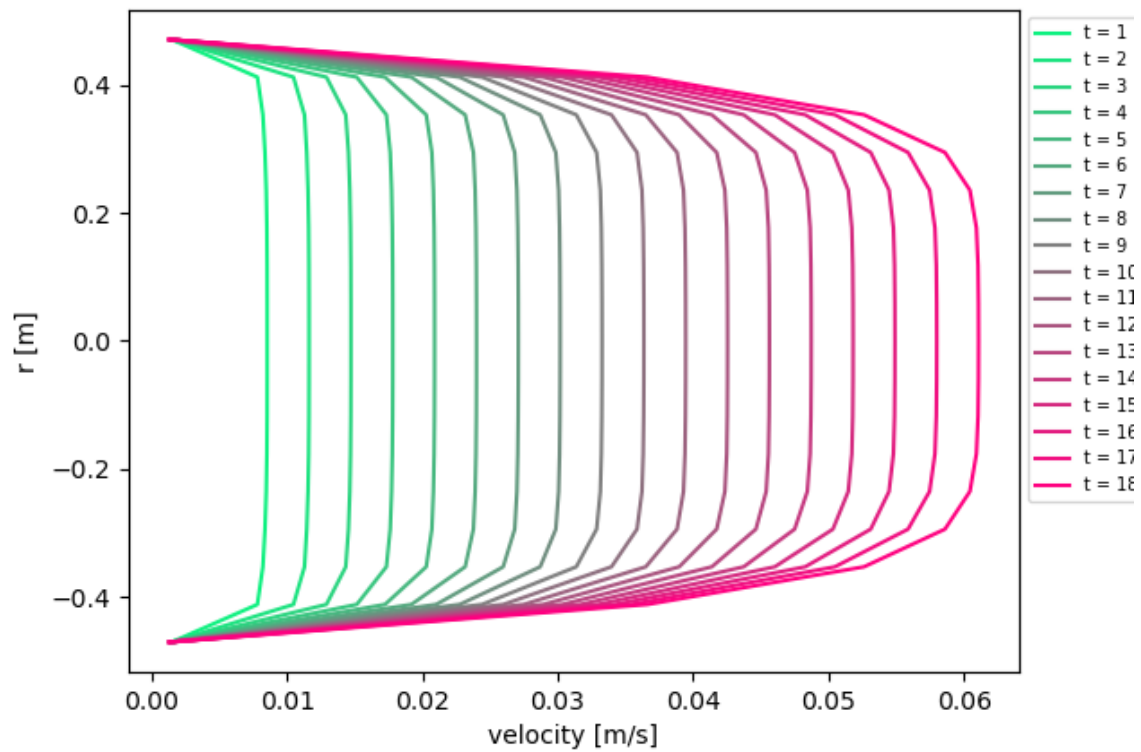


- Final test MSELoss: **0.00028252945048734546**
- Final test L1Loss: **0.06915978819597512**

## Results diagram



*Prediction*



*Ground truth*