CS 541 - Artificial Intelligence

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1 Problem 1

We are given the following Knowledge Base:

$$KB = \{A \to (B \vee C),\ B \to D,\ C \to E\}$$

We need to derive E. Using modus ponens, we can make the following inferences from our knowledge base:

1. $A \to (B \vee C)$ (Given)

2. A (Assumption)

3. $B \vee C$ (Modus Ponens using 1 and 2)

4. $B \to D$ (Given)

5. $C \to E$ (Given)

6. B (Assumption)

7. D (Modus Ponens using 4 and 6)

8. C (Assumption)

9. E (Modus Ponens using 5 and 8)

We made two assumptions in this derivation: first, we assumed that "A" is true in step 2, and then we assumed both "B" and "C" are true in steps 6 and 8, respectively. By using modus ponens and these assumptions, we were able to derive that "E" is true.

We can also use resolution to derive E using contradiction. Following are the steps:

Step 1: Apply modus ponens to the first premise:

From $A \to (B \vee C)$ and A, we can derive $(B \vee C)$.

Step 2: Apply resolution between $(B \vee C)$ and $C \to E$:

$(B \vee C), \neg(C \to E) = (B \vee C), \neg(\neg C \vee E)$ (Implication rule) $= (B \vee C), (C \wedge \neg E)$ (De Morgan's law)

Step 3: Apply resolution between $(B \vee C)$ and $B \to D$:

$(B \vee C), \neg(B \to D) = (B \vee C), \neg(\neg B \vee D)$ (Implication rule) $= (B \vee C), (B \wedge \neg D)$ (De Morgan's law)

Step 4: Apply resolution between $(B \wedge \neg D)$ and $(C \wedge \neg E)$:

$(B \wedge \neg D), (C \wedge \neg E) = \neg(B \to D), \neg(C \to E)$ (Implication rule)

Step 5: Apply modus ponens to the resulting clauses:

From $\neg(B \rightarrow D)$ and B, we can derive $\neg D$.

From $\neg(C \rightarrow E)$ and C, we can derive $\neg E$.

Since we have $\neg E$, we have reached a contradiction with our assumption $\neg E$ in step 5. Therefore, the assumption $\neg E$ is false, which means E must be true.

Hence, we have derived E from the given knowledge base using modus ponens and resolution.

2 Problem 2

We are given the following Knowledge Base:

$$KB = \{B \rightarrow A,\ (B \rightarrow C) \rightarrow D\}$$

We need to derive D. We can apply resolution to our knowledge base in the following way:

1. $B \rightarrow A$ (Given)
2. $(B \rightarrow C) \rightarrow D$ (Given)
3. $\neg B \vee A$ (Implication using 1)
4. $\neg(B \rightarrow C) \vee D$ (Implication using 2)
5. $\neg(\neg B \vee \neg C) \vee D$ (Implication using De Morgan's law on 4)
6. $(B \wedge C) \vee D$ (Double negation on 5)

We used implication and De Morgan's law to transform our knowledge base into a set of clauses that we can use for resolution. In step 6, we applied resolution to the clauses "(B and C)" and "D" to derive the new clause "(B and C) or D". This new clause is logically equivalent to our original knowledge base, and it tells us that either "B" and "C" are true or "D" is true.

To derive "D" from this new clause, we can use modus ponens with the following steps:

1. $(B \wedge C) \vee D$ (Derived using resolution)
2. $B \rightarrow A$ (Given)
3. $B \wedge C$ (Assumption)
4. B (Simplification from 3)
5. A (Modus ponens using 2 and 4)
6. $\neg D \rightarrow \neg(B \wedge C)$ (Contrapositive of 1)
7. D (Modus tollens using 1 and 6)

We made two assumptions in this derivation: first, we assumed that "B and C" is true in step 3, and then we assumed that "notD" is true in step 6. By using modus ponens and these assumptions, we were able to derive that "D" is true.

3 References

[1] Marntirosian, K., Schrijvers, T., Oliveira, B. C. D. S., Karachalias, G. (2020). Resolution as intersection subtyping via modus ponens. arXiv preprint arXiv:2010.06216.