CS 541 - Artificial Intelligence

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1 Problem 1

The problem states that we should find the expected number of rolls (as a function of A and B) we will have to do until we stop.

For this, we need to find the expected value of rolling the fair six-sided die until we stop. Following information is given:

1. The die is rolled until we reach 1.
2. When we roll a 2, we lose A points.
3. When we roll a 6, we gain B points.
4. Points are neither lost nor gained when we roll a 3, 4, or a 5.

The expected value of a random variable X is denoted as $E(X)$ where,

$$E(X) = \sum_{i=1}^{n} p_i x_i \tag{1}$$

where p_i are the probabilities of outcomes x_i. There are four possible outcomes.

- x_1: rolling a 1, p_1: $\frac{1}{6}$
- x_2: rolling a 2, p_2: $\frac{1}{6}$
- x_3: rolling a 6, p_3: $\frac{1}{6}$
- x_4: rolling a 3, 4 or a 5, p_4: $\frac{1}{2}$ (individual probability $\frac{1}{6}$, total probability $\frac{1}{2}$)

With the information provided above, we can deduce that,

- $x_1 = 0$ since when we roll a 1, there are no points and the game stops.
- $x_2 = E(X) - A$ since we lose A points when we roll a 2, thus, affecting the expected value.
- $x_3 = E(X) + B$ since we gain B points when we roll a 6, thus, affecting the expected value.
- $x_4 = E(X)$ since no points are lost or gained when we roll a 3, 4, or a 5, thus, not affecting the expected value.

Putting values of p_i and x_i in Equation 1, we get,

$$E(X) = \frac{1}{6} \times 0 + \frac{1}{6} \times (E(X) - A) + \frac{1}{6} \times (E(X) + B) + \frac{1}{2} \times E(X)$$

$$E(X) = \frac{1}{6} \times E(X) - \frac{1}{6} \times A + \frac{1}{6} \times E(X) + \frac{1}{6} \times B + \frac{1}{2} \times E(X)$$

$$E(X) = \frac{5}{6} \times E(X) - \frac{1}{6} \times A + \frac{1}{6} \times B$$

$$E(X) - \frac{5}{6} \times E(X) = \frac{1}{6} \times B + \frac{1}{6} \times A$$

$$\frac{1}{6} \times E(X) = \frac{1}{6} \times B - \frac{1}{6} \times A \tag{2}$$

Simplifying Equation 2, we get,

$$E(X) = B - A$$

2 Problem 2

The problem states that we need to find the probability of getting a review from Chipotle, given that the review is from positive.

Following information has been given in the problem:

- Chipotle has 200 reviews - 120 positive (+), 80 negative (-).
- Five Guys has 100 reviews - 40 positive (+), 60 negative (-).

We need to find $P(Chipotle|+)$. Using Bayes rule,

$$P(Chipotle|+) = \frac{P(+|Chipotle) \times P(Chipotle)}{P(+)} \tag{3}$$

where $P(+|Chipotle)$ is the probability of getting a positive review given that it is from Chipotle, $P(Chipotle)$ is the probability that the review is from Chipotle and $P(+)$ is the probability of getting a postive review. From above information,

- $P(+|Chipotle) = 120/200 = 0.6$
- $P(Chipotle) = \frac{200}{200+100} = 0.6667$

We need to find $P(+)$. Using the law of total probability,

$$P(+) = P(+|Chipotle) \times P(Chipotle) + P(+|FiveGuys) \times P(FiveGuys)$$

We know that,

- $P(+|Chipotle) = 120/200 = 0.6$
- $P(Chipotle) = 200/300 = 0.6667$
- $P(+|FiveGuys) = 40/100 = 0.4$
- $P(FiveGuys) = 100/300 = 0.3333$

With above information, we get,

$P(+) = 0.6 \times 0.6667 + 0.4 \times 0.3333$
$P(+) = 0.40002 + 0.13332$
$P(+) = 0.53334$

Putting values of $P(+|Chipotle$, $P(Chipotle)$, and $P(+)$ into equation 3, we get,

$$P(Chipotle|+) = \frac{0.6 \times 0.6667}{0.53334} \tag{4}$$

Solving equation 4, we get, $P(Chipotle|+) \approx 0.75$. Thus, the probability that the review chosen is from Chipotle given that the review is positive is about 75%.

3 Problem 3

The problem states that we need to find the value of p that maximizes the equation $L(p)$.

Following information has been given in the problem:

- Probability of the coin turning heads is p where $0 < p < 1$.
- The coin is flipped 7 times with outcomes: H, H, T, H, T, T, H.
- Probability of obtaining the above sequence is $L(p) = p^4(1-p)^3$.

We need to find the value of p that maximizes the equation $L(p)$. Taking log on both sides, we get,

$$\log L(p) = \log(p^4(1-p)^3)$$

$$\log L(p) = \log p^4 + \log(1-p)^3$$

$$\log L(p) = 4\log p - 3\log(1-p)$$

Differentiating both sides of the equation with respect to p, we get,

$$\frac{d}{dp} logL(p) = 4\frac{d}{dp}\log p - 3\frac{d}{dp}\log(1-p)$$

To maximize the value of p, we need to set the derivative to zero. Thus,

$$\frac{d}{dp} logL(p) = 0$$

$$4\frac{d}{dp}\log p - 3\frac{d}{dp}\log(1-p) = 0$$

$$\frac{4}{p} - \frac{3}{1-p} = 0$$

Solving the equation, we get,

$$\frac{4(1-p) - 3p}{p(1-p)} = 0$$

$$4(1-p) - 3p = 0$$

$$4 - 4p - 3p = 0$$

$$4 - 7p = 0$$

$$4 = 7p$$

$$p = \frac{4}{7} \approx 0.571$$

Therefore, the value of p that maximizes the value of $logL(p)$ is 0.571.