

### CS 558, Fall 2019, Quiz 3

NAME:

**Problem 1.** Explain a) WHY after applying a mean filter, the maximum intensity of an image cannot increase? b) WHEN does the maximum stay the same after filtering? The filter is square ( $n \times n$ ), all weights are equal and they sum to 1.

**Problem 2.** a) WHY should image gradient estimation be performed over a pre-smoothed image? b) HOW can differentiation and smoothing be performed at the same time?

**Problem 3.** Let  $\mathbf{A} = [A]_{ij}$  be  $3 \times 3$  diagonal matrix with  $A_{ii} = i$  and  $\mathbf{p} = (x, y, z)^\top$  be a vector. Write the expression for the quadratic form  $\mathbf{p}^\top \mathbf{A} \mathbf{p}$

**Problem 1.**

A) average is never greater than the maximum element in a set.

B) When all points have the same intensity

**Problem 2.**

A) To reduce sensitivity to noise

B) By combining a pair of convolutional filters into a single one through composition (i.e. applying one filter to the other after adding zeros for padding)

**Problem 3.**

$$x*[A_{11}*x+A_{12}*y+A_{13}*z]+y*[A_{21}*x+A_{22}*y+A_{23}*z]+z*[A_{31}*x+A_{32}*y+A_{33}*z] = A_{11}*x^2 + A_{22}*y^2 + A_{33}*z^2 + (A_{12}+A_{21})*xy + (A_{13}+A_{31})*xz + (A_{23}+A_{32})*yz$$

Student Name: \_\_\_\_\_

1.- Choosing between the second moment matrix and the Hessian matrix, which one would be more efficient to estimate? Explain your reasoning.

**Second moment matrix only considers the derivatives  $I_x$  and  $I_y$**   
**Harris requires “mixed” derivatives  $I_{xy}$  as well as the second derivatives  $I_{xx}$  and  $I_{yy}$**

2. How is detecting a “corner” feature different than a “blob” feature?

**Corners are a function of the second moment matrix**  
**Blobs require finding an extrema of scale-space analysis (i.e. Laplacian of DoG filters)**

Student Name: \_\_\_\_\_

1.- Consider you have a laser scan of a room (consisting of a million unique 3D points). You want to fit a 3D plane model to the attained point cloud. Assume you only need 3 different points to define a single plane hypothesis, and that 50% of the data are contained within the desired plane. What is the probability of finding the correct plane after 2 RANSAC iterations? (Fractional answers are sufficient)

$e=0.5$	probability of outlier sample
$1-e$	probability of inlier sample
$(1-e)^3$	probability of a model sample comprised only of inliers
$1-(1-e)^3$	probability of a model sample containing at least one outlier
$(1-(1-e)^3)^2$	probability of both model samples containing at least one outlier
$1-(1-(1-e)^3)^2$	probability of both at least one model sample comprised only of inliers

2. Consider a 2D point in the plane with coordinates  $(x=3, y=2)$ . Considering the polar representation of a line, what are the values of rho for the following scenarios:

- |  |                   |
|--|-------------------|
| a) A vertical line                       | $\rho=3$          |
| b) A horizontal line                     | $\rho=2$          |
| c) A line with theta equal to 45 degrees | $\rho=\sqrt{1.5}$ |

Student Name: \_\_\_\_\_

1.- Consider a 2D rotation matrix operating on a 2D point (x,y), is there a point which after being pre-multiplied by the rotation matrix remains constant? Explain

**The point at the origin (0,0) will not be moved**

**A rotation will not change the norm a vector, the only vector with zero norm is the origin**

2.- Consider a 2D point in homogeneous coordinates (x,y,1), write a single linear operation matrix operation that yields the 2D vector (3x+5,4y+6) in non-homogeneous coordinates. How would your matrix change if you were required to generate the output in homogeneous coordinates?

**Matrix in non-homogeneous coordinates**

$$\begin{bmatrix} 3 & 0 \\ 0 & 4 \end{bmatrix} * \begin{bmatrix} x \\ y \end{bmatrix} + \begin{bmatrix} 5 \\ 6 \end{bmatrix}$$

**Matrix in homogeneous coordinates**

$$\begin{bmatrix} 3 & 0 & 5 \\ 0 & 4 & 6 \end{bmatrix} * \begin{bmatrix} x \\ y \\ 1 \end{bmatrix}$$