Lecture 15: Motion

Horn-Schunk method

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CS131 Computer Vision: Foundations and Applications

What will we learn today?

- Horn-Schunk method
  - Approach
  - Analysis

Reading: [Szeliski] Chapters: 8.4, 8.5

[Fleet & Weiss, 2005]
http://www.cs.toronto.edu/pub/jepson/teaching/vision/2503/opticalFlow.pdf

Horn-Schunk method for optical flow

- The flow is formulated as a global energy function which should be minimized:

$$E = \iint \left[(I_x u + I_y v + I_t)^2 + \alpha^2 (\|\nabla u\|^2 + \|\nabla v\|^2) \right] \mathrm{d}x\mathrm{d}y$$

Horn-Schunk method for optical flow

- The flow is formulated as a global energy function which should be minimized:
- The first part of the function is the brightness consistency.

$$E = \iint \left[\boxed{(I_x u + I_y v + I_t)}^2 + \alpha^2 (\|\nabla u\|^2 + \|\nabla v\|^2) \right] \mathrm{d}x\mathrm{d}y$$

Horn-Schunk method for optical flow

- The flow is formulated as a global energy function which should be minimized:
- The second part is the smoothness constraint. It's trying to make sure that the changes between pixels are small.

$$E = \iint \left[(I_x u + I_y v + I_t)^2 + \alpha^2 \left(\boxed{\|\nabla u\|^2 + \|\nabla v\|^2} \right) \right] \mathrm{d}x \mathrm{d}y$$

Horn-Schunk method for optical flow

- The flow is formulated as a global energy function which should be minimized:
- α is a regularization constant. Larger values of α lead to smoother flow.

$$E = \iint \left[(I_x u + I_y v + I_t)^2 + \boxed{\alpha^2} (\|\nabla u\|^2 + \|\nabla v\|^2) \right] \mathrm{d}x\mathrm{d}y$$

Horn-Schunk method for optical flow

- The flow is formulated as a global energy function which should be minimized:

$$E = \iint \left[(I_x u + I_y v + I_t)^2 + \alpha^2 (\|\nabla u\|^2 + \|\nabla v\|^2) \right] \mathrm{d}x \mathrm{d}y$$

- This minimization can be solved by taking the derivative with respect to u and v, we get the following 2 equations:

$$I_x (I_x u + I_y v + I_t) - \alpha^2 \Delta u = 0$$
$$I_y (I_x u + I_y v + I_t) - \alpha^2 \Delta v = 0$$

Horn-Schunk method for optical flow

- By taking the derivative with respect to u and v, we get the following 2 equations:

$$I_x(I_x u + I_y v + I_t) - \alpha^2 \Delta u = 0$$
$$I_y(I_x u + I_y v + I_t) - \alpha^2 \Delta v = 0$$

- Where $\Delta = \frac{\partial^2}{\partial x^2} + \frac{\partial^2}{\partial y^2}$ is called the Lagrange operator. In practice, it can be measured using:

$$\Delta u(x,y) = \overline{u}(x,y) - u(x,y)$$

- where $\bar{u}(x,y)$ is the weighted average of u measured at a neighborhood around (x,y).

Horn-Schunk method for optical flow

- Now we substitute $\Delta u(x,y) = \overline{u}(x,y) - u(x,y)$ in:

$$I_x(I_x u + I_y v + I_t) - \alpha^2 \Delta u = 0$$
$$I_y(I_x u + I_y v + I_t) - \alpha^2 \Delta v = 0$$

- To get:

$$(I_x^2 + \alpha^2)u + I_x I_y v = \alpha^2 \overline{u} - I_x I_t$$
$$I_x I_y u + (I_y^2 + \alpha^2)v = \alpha^2 \overline{v} - I_y I_t$$

- Which is linear in u and v and can be solved analytically for each pixel individually.

Iterative Horn-Schunk

- But since the solution depends on the neighboring values of the flow field, it must be repeated once the neighbors have been updated.

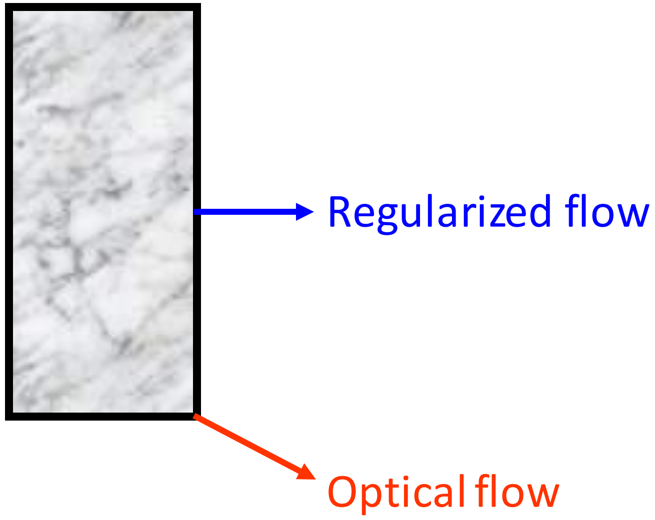
- So instead, we can iteratively solve for u and v using:

$$u^{k+1} = \overline{u}^k - \frac{I_x(I_x\overline{u}^k + I_y\overline{v}^k + I_t)}{\alpha^2 + I_x^2 + I_y^2}$$

$$v^{k+1} = \overline{v}^k - \frac{I_y(I_x\overline{u}^k + I_y\overline{v}^k + I_t)}{\alpha^2 + I_x^2 + I_y^2}$$

What does the smoothness regularization do anyway?

- It’s a sum of squared terms (a Euclidean distance measure).
- We’re putting it in the expression to be minimized.
- => In texture free regions, *there is no optical flow*
- => On edges, points will flow to nearest points, solving the aperture problem.

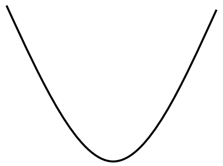


Slide credit: Sebastian Thrun

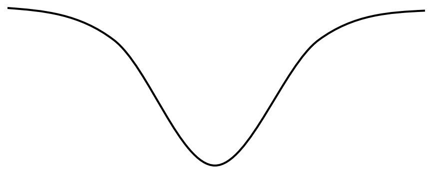
Dense Optical Flow with Michael Black's method

- Michael Black took Horn-Schunk's method one step further, starting from the regularization term:
- Which looks like a quadratic:

$$\|\nabla u\|^2 + \|\nabla v\|^2$$



- And replaced it with this:



- Why does this regularization work better?

Summary

- Horn-Schunk method
  - Approach
  - Analysis