Lecture 15: Motion

Optical flow

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CS131 Computer Vision: Foundations and Applications

CS 131 Roadmap

Pixels	Images	Recognition	Videos	Cameras
Convolutions Edges Features	Priors Color Segmentation Resizing	Machine learning Classification Detection	Motion Tracking	Pinhole Camera Camera Parameters Stereo Vision

What will we learn today?

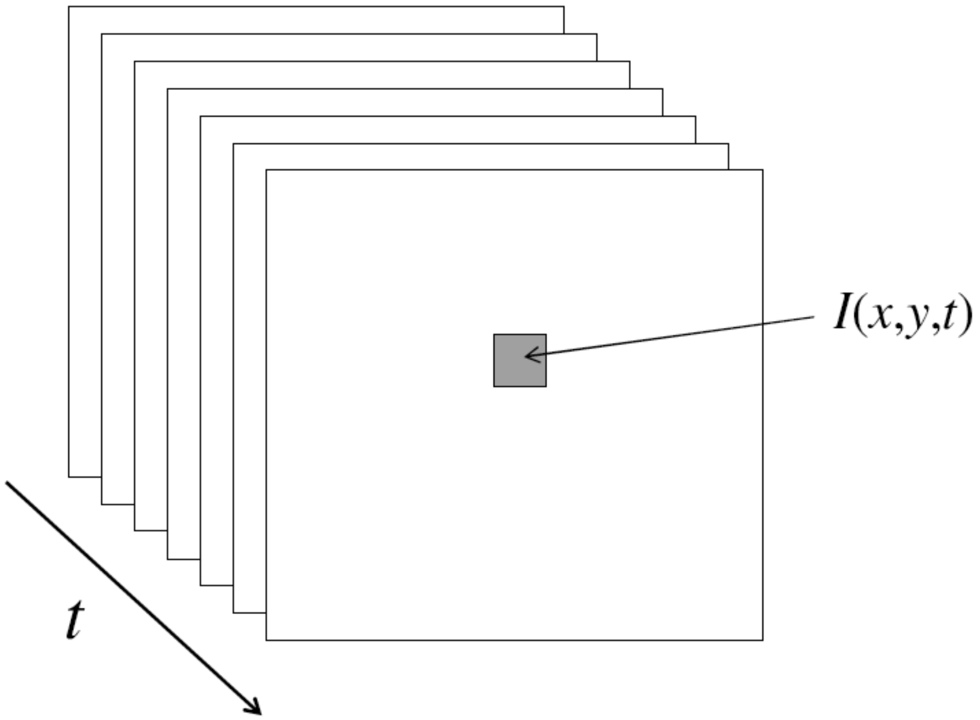
- Optical flow
  - Definition
  - Key assumptions in estimating optical flow
  - The aperture problem

Reading: [Szeliski] Chapters: 8.4, 8.5

[Fleet & Weiss, 2005]
http://www.cs.toronto.edu/pub/jepson/teaching/vision/2503/opticalFlow.pdf

From images to videos

- A video is a sequence of frames captured over time
- Now our image data is a function of space (x, y) and time (t)



Why is motion useful?

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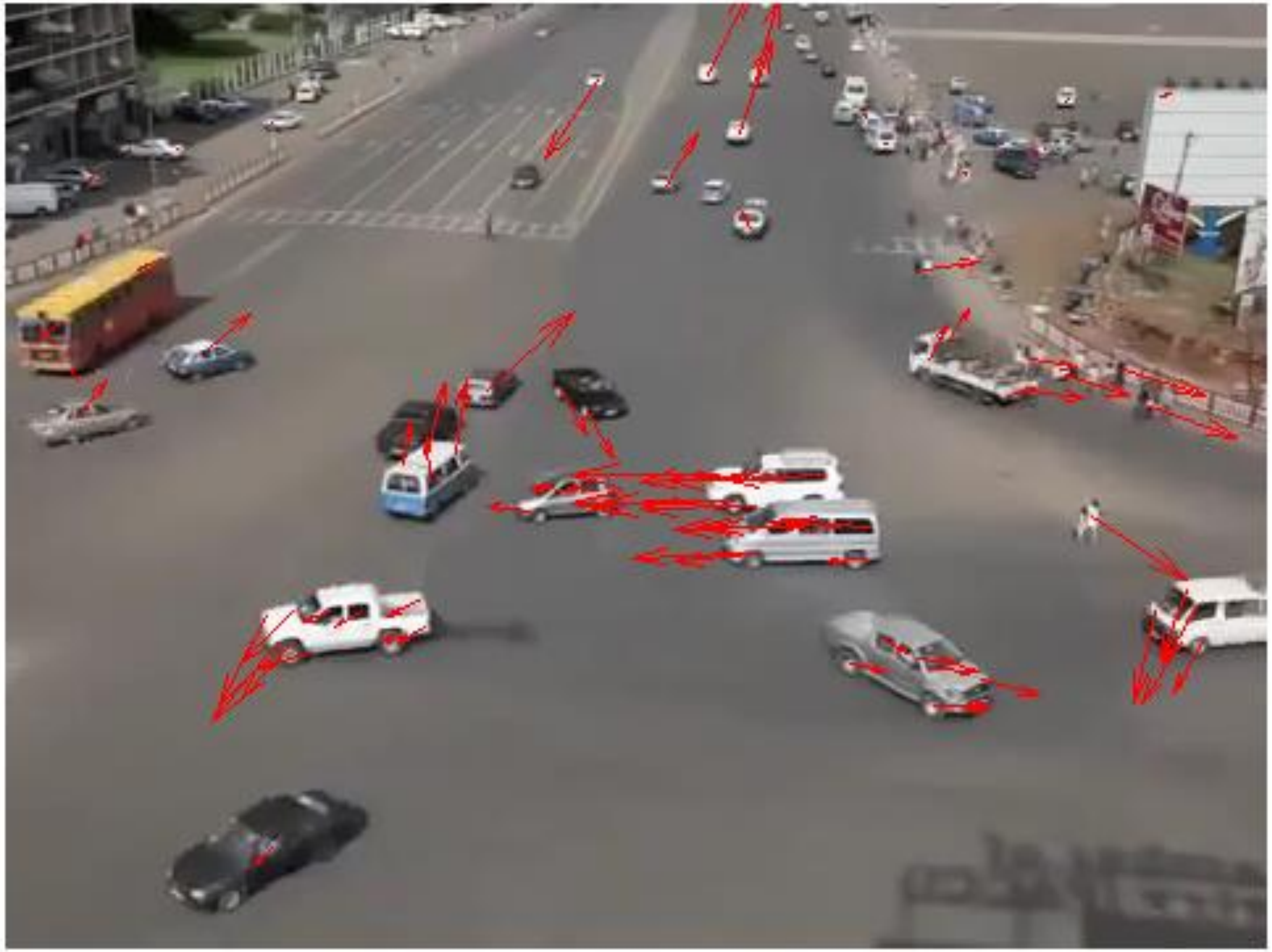


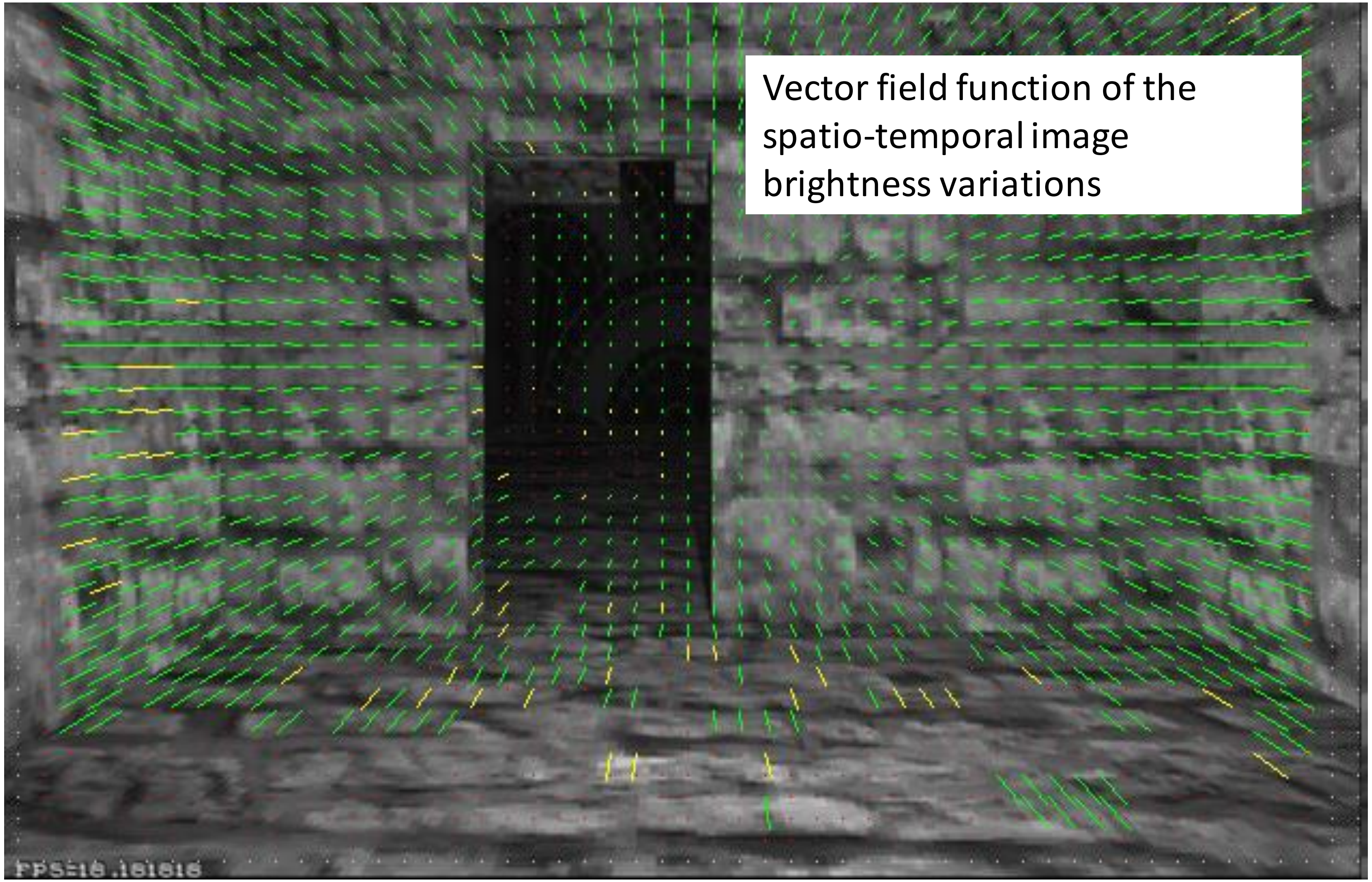
Optical flow

- Definition: optical flow is the *apparent* motion of brightness patterns in the image
- Note: apparent motion can be caused by lighting changes without any actual motion
  - Think of a uniform rotating sphere under fixed lighting vs. a stationary sphere under moving illumination

GOAL: Recover image motion at each pixel from optical flow

Source: Silvio Savarese

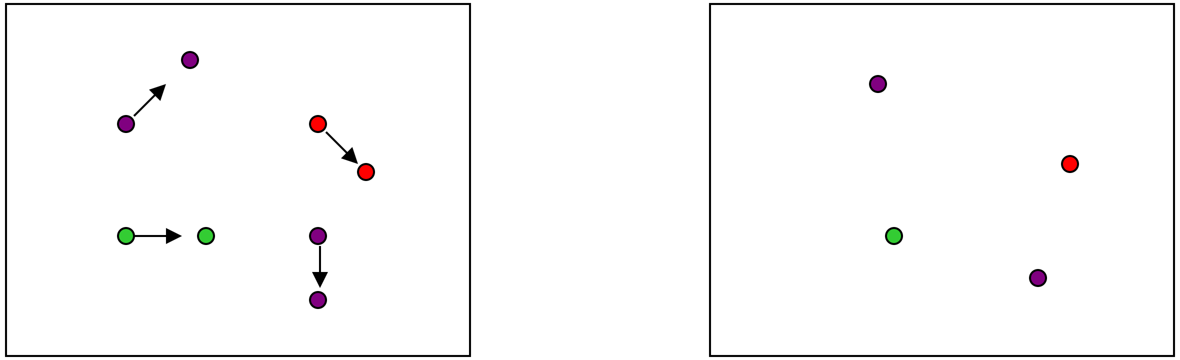
Optical flow



Vector field function of the spatio-temporal image brightness variations

Picture courtesy of Selim Temizer - Learning and Intelligent Systems (LIS) Group, MIT

Estimating optical flow



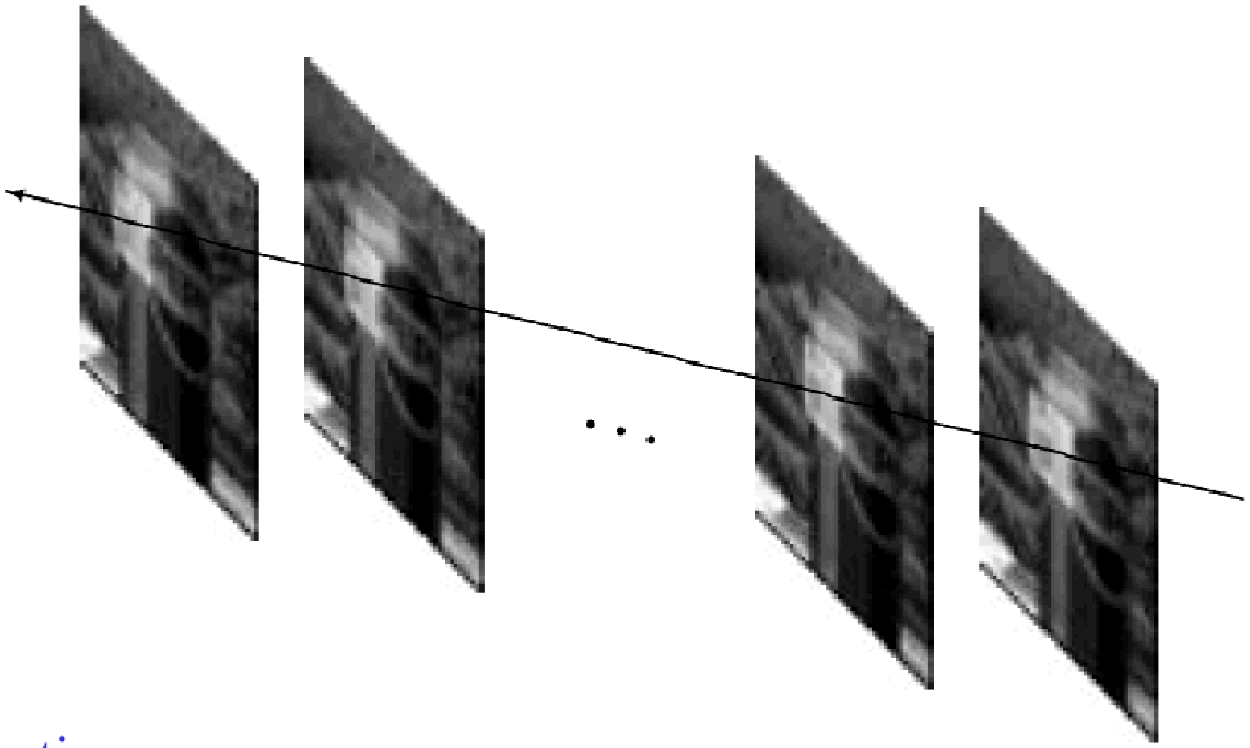
$I(x, y, t)$ $\qquad$ $I(x, y, t+1)$

- Given two subsequent frames, estimate the apparent motion field u(x,y), v(x,y) between them
- Key assumptions
  - **Brightness constancy**: projection of the same point looks the same in every frame
  - **Small motion**: points do not move very far
  - **Spatial coherence**: points move like their neighbors

Source: Silvio Savarese

Key Assumptions: small motions

Temporal Persistence

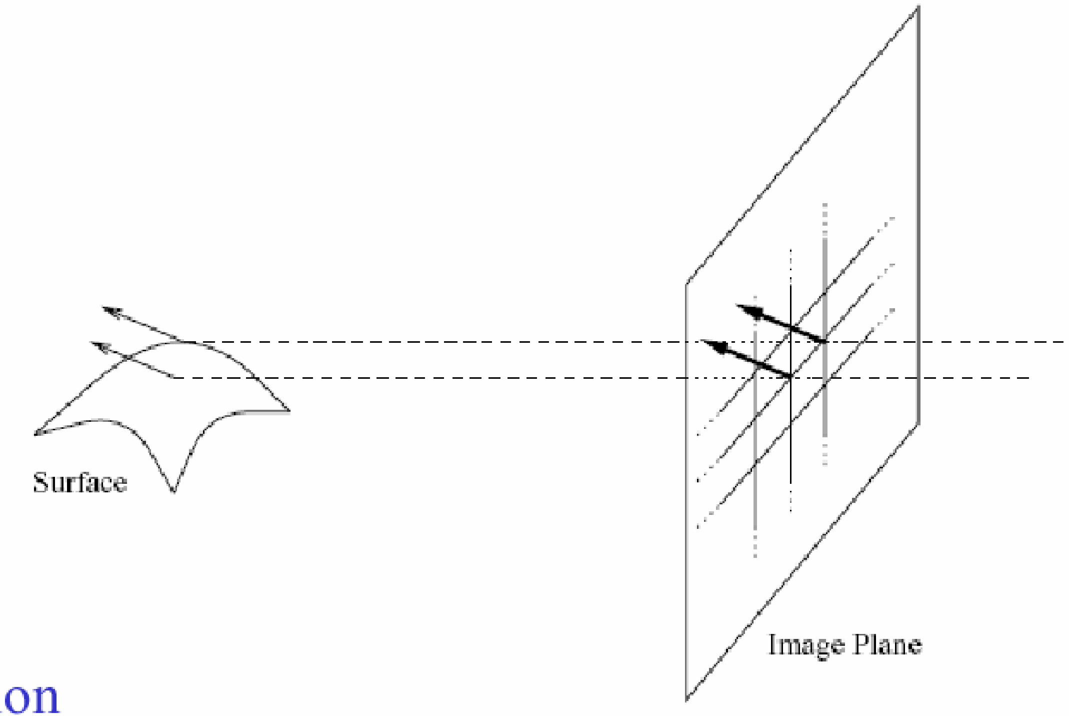


Assumption:

The image motion of a surface patch changes gradually over time.

* Slide from Michael Black, CS143 2003

Key Assumptions: spatial coherence

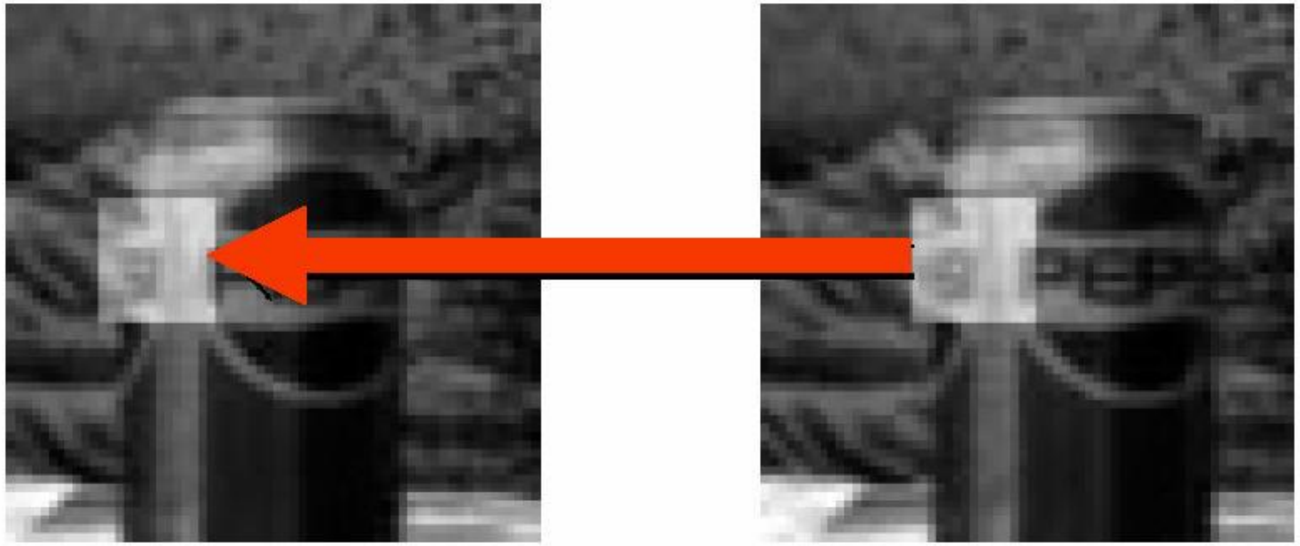


Assumption

* Neighboring points in the scene typically belong to the same surface and hence typically have similar motions.
* Since they also project to nearby points in the image, we expect spatial coherence in image flow.

* Slide from Michael Black, CS143 2003

Key Assumptions: brightness Constancy



Assumption

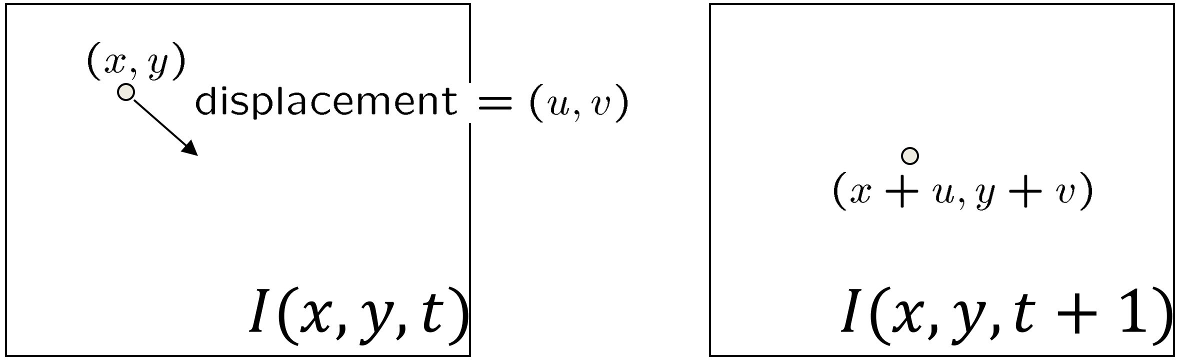
Image measurements (e.g. brightness) in a small region remain the same although their location may change.

$$I(x, y, t) = I(x + u(x, y), y + v(x, y), t + 1)$$

(assumption)

* Slide from Michael Black, CS143 2003

The brightness constancy constraint



- Brightness Constancy Equation:

$$I(x, y, t) = I(x + u, y + v, t + 1)$$

Linearizing the right side using Taylor expansion:

$$I(x + u, y + v, t + 1) \approx I(x, y, t) + \overset{\text{Image derivative along } x}{\boxed{I_x}} \cdot u + I_y \cdot v + \overset{\text{Image derivative along } t}{\boxed{I_t}}$$

$$I(x + u, y + v, t + 1) - I(x, y, t) \approx I_x \cdot u + I_y \cdot v + I_t$$

Hence, $I_x \cdot u + I_y \cdot v + I_t \approx 0 \quad \rightarrow \quad \nabla I \cdot [u \quad v]^T + I_t = 0$

Source: Silvio Savarese

Filters used to find the derivatives

$$\begin{bmatrix} -1 & 1 \\ -1 & 1 \end{bmatrix} \text{first image} \qquad \begin{bmatrix} -1 & -1 \\ 1 & 1 \end{bmatrix} \text{first image} \qquad \begin{bmatrix} -1 & -1 \\ -1 & -1 \end{bmatrix} \text{first image}$$

$$\begin{bmatrix} -1 & 1 \\ -1 & 1 \end{bmatrix} \text{second image} \qquad \begin{bmatrix} -1 & -1 \\ 1 & 1 \end{bmatrix} \text{second image} \qquad \begin{bmatrix} 1 & 1 \\ 1 & 1 \end{bmatrix} \text{second image}$$

$$I_x \qquad\qquad I_y \qquad\qquad I_t$$

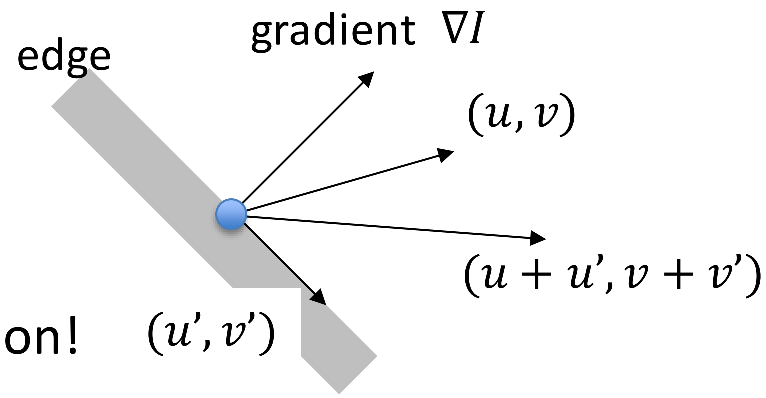
The brightness constancy constraint

- Can we use this equation to recover image motion (u, v) at each pixel?

$$\nabla I \cdot [u \quad v]^T + I_t = 0$$

- How many equations and unknowns per pixel?
  - One equation (this is a scalar equation!), two unknowns (u,v)

- The component of the flow perpendicular to the gradient (i.e., parallel to the edge) cannot be measured
  - If (u, v) satisfies the equation, then $\nabla I \cdot [u \quad v]^T + I_t = 0$.
  - Assume (u', v') is perpendicular to ∇I, then $\nabla I \cdot [u' \quad v']^T = 0$.
  - Therefore, $\nabla I \cdot [u + u' \quad v + v']^T + I_t = 0$, which means $(u + u', v + v')$ also satisfies the equation!

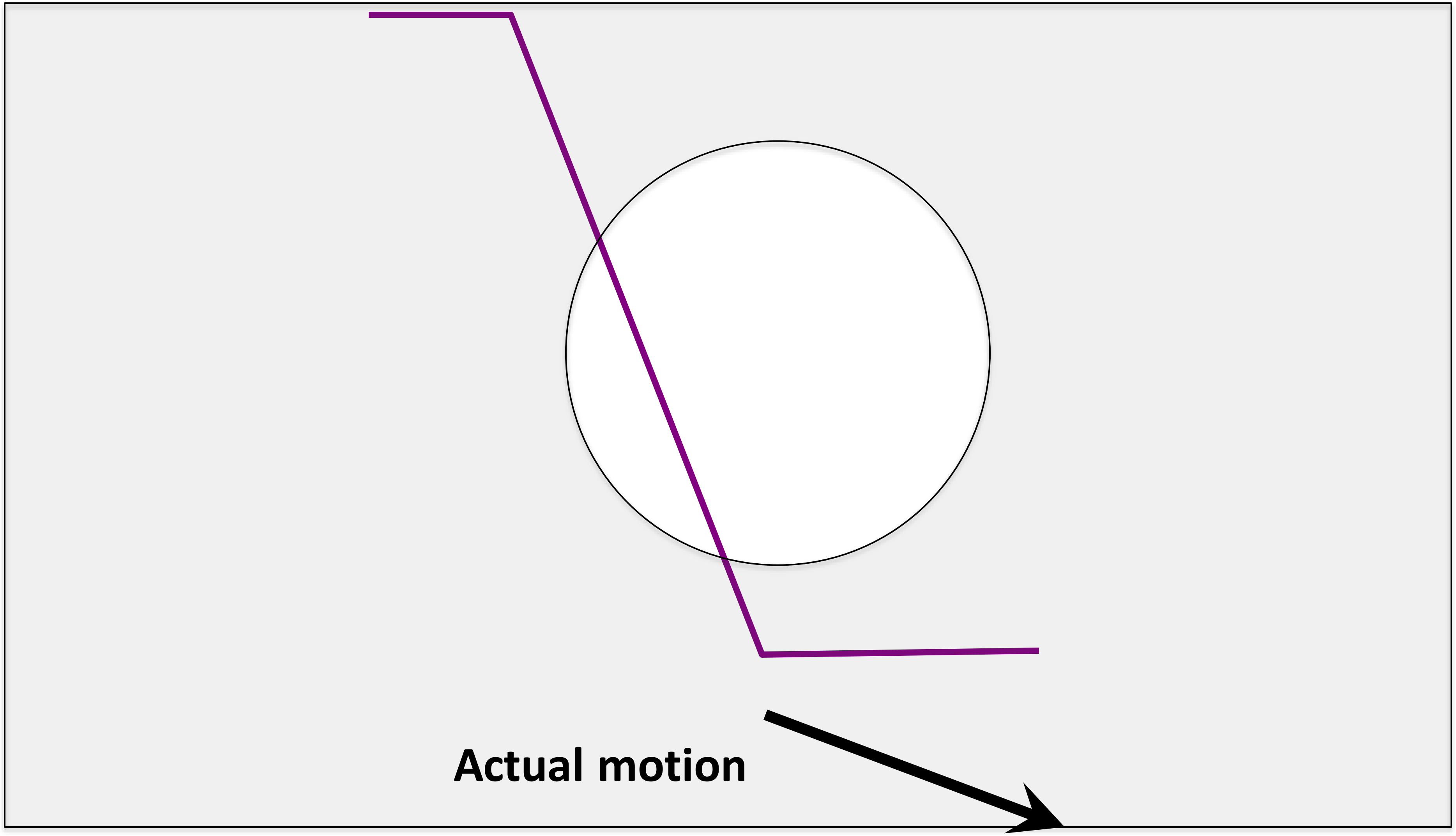


Source: Silvio Savarese

Motion

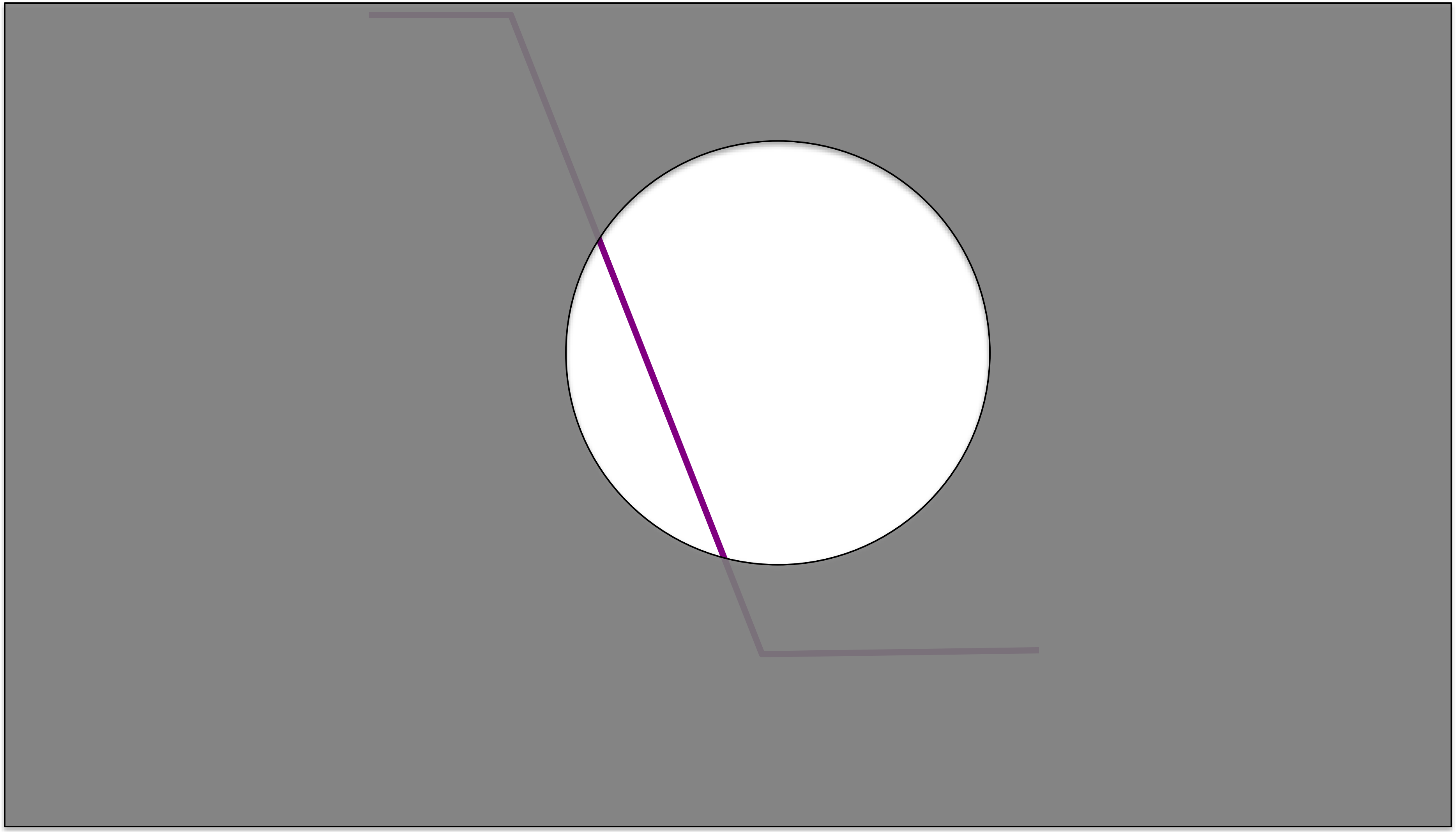
Optical flow

The aperture problem



Source: Silvio Savarese

The aperture problem



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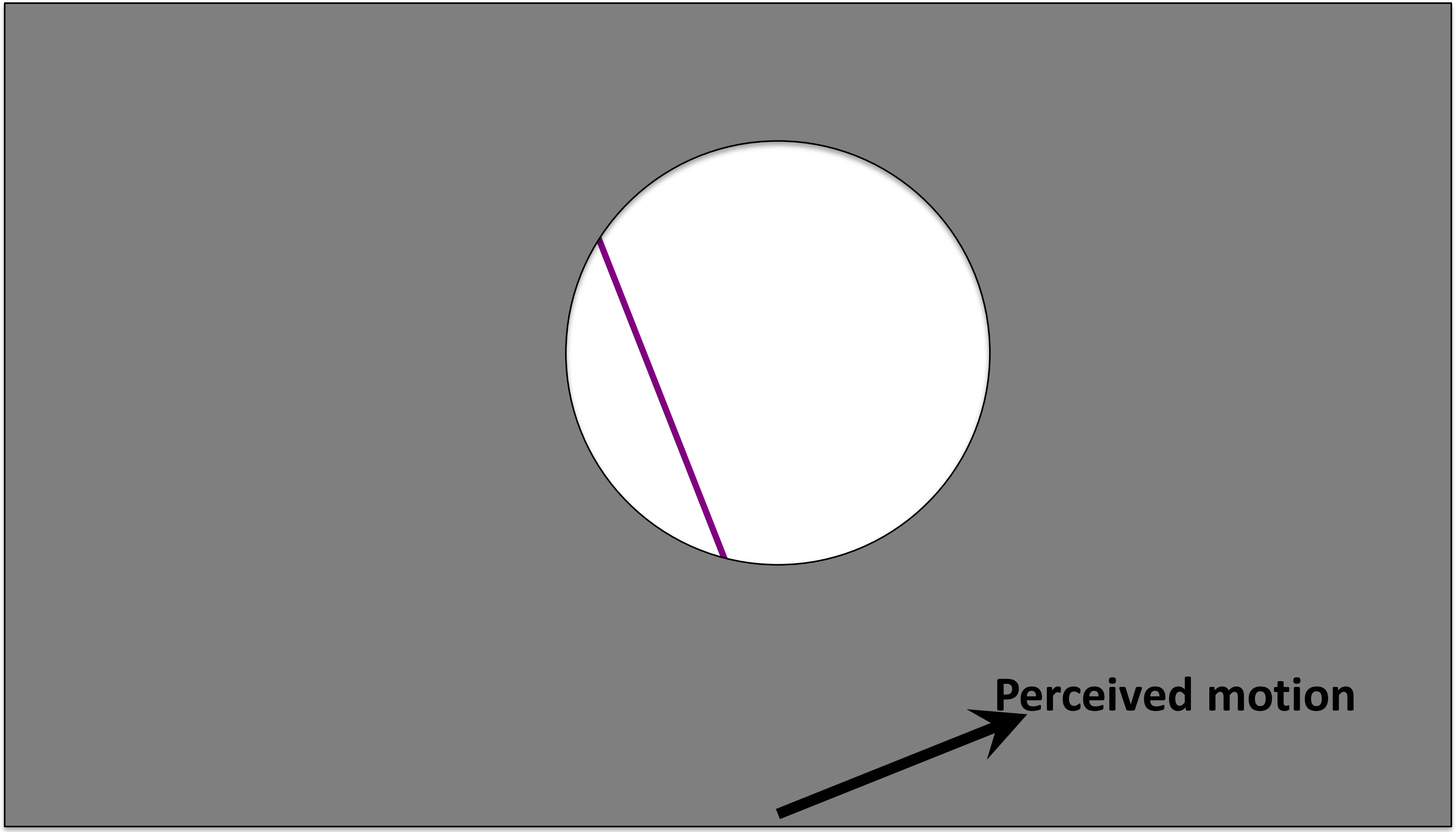

Source: Silvio Savarese

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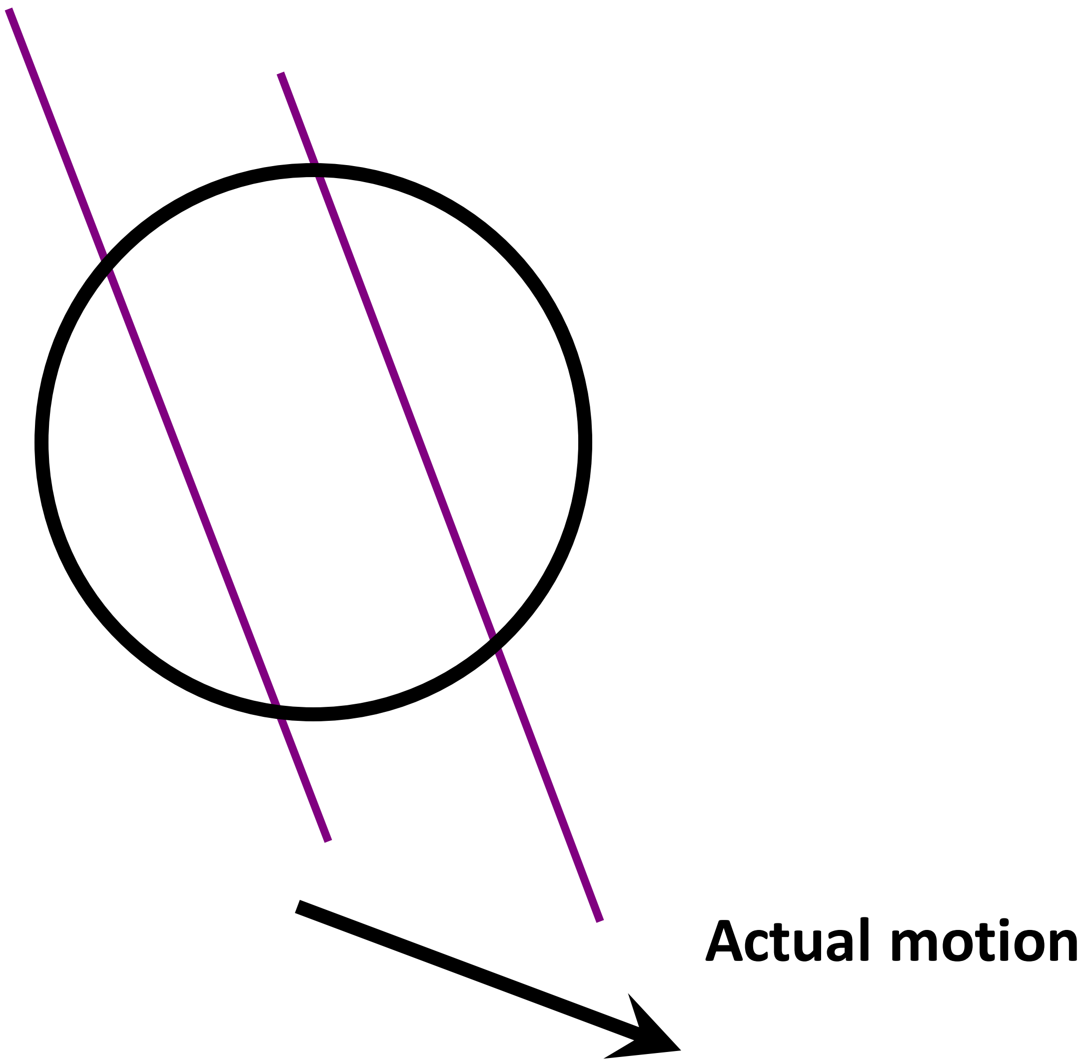
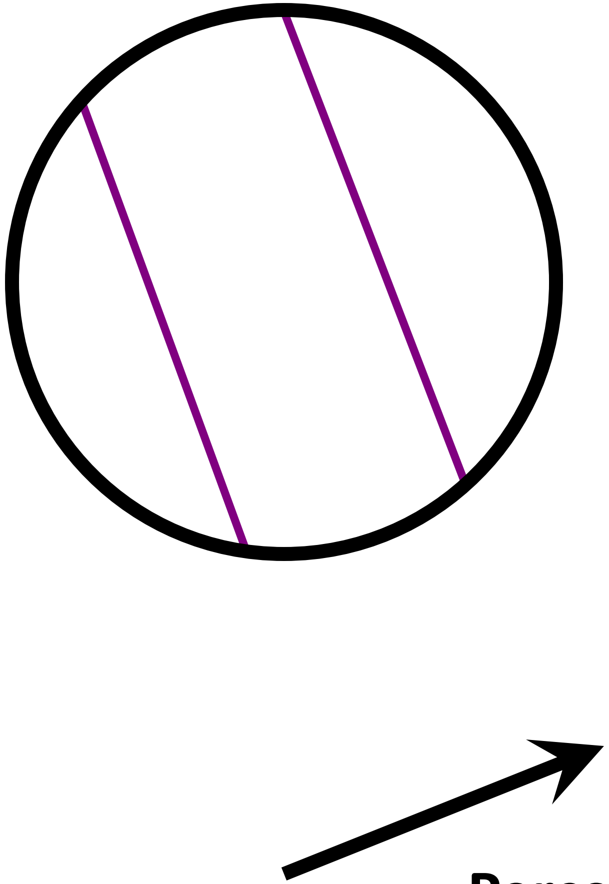


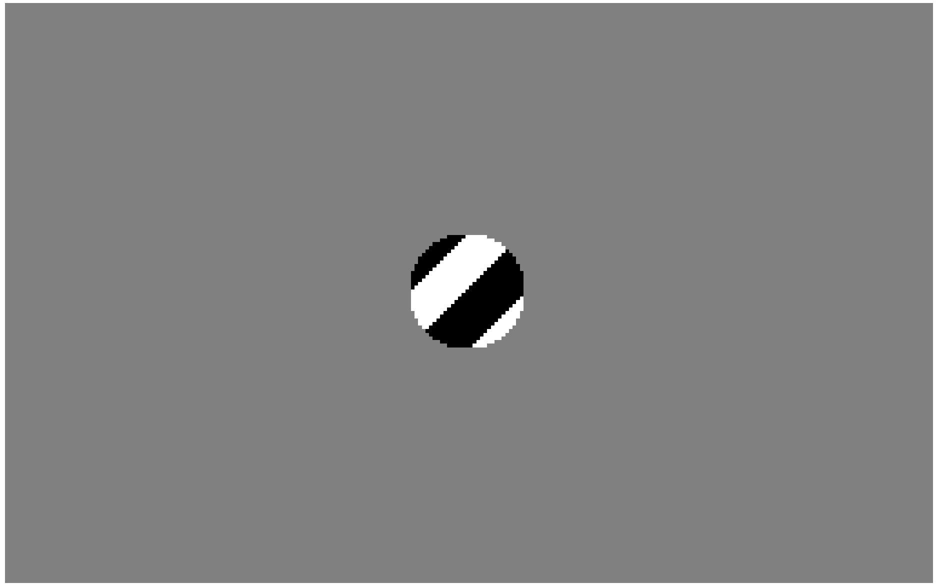
The aperture problem



Perceived motion

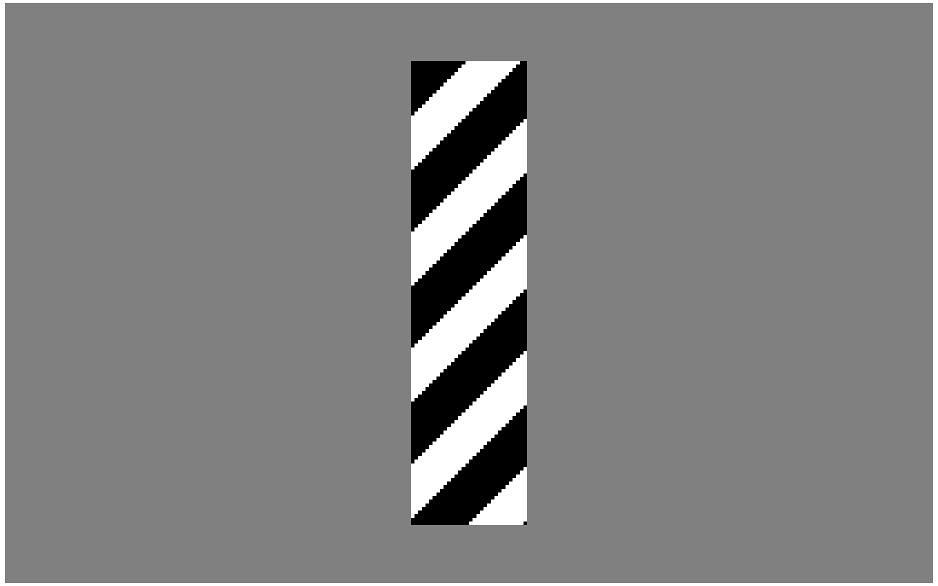
Source: Silvio Savarese

The barber pole illusion



http://en.wikipedia.org/wiki/Barberpole_illusion

The barber pole illusion



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