Lecture 13: Visual Bag of Words

Naive Bayes

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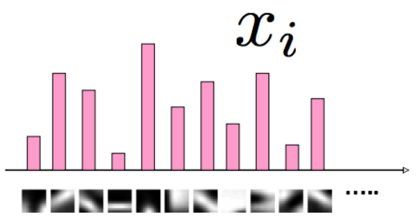
CS131 Computer Vision: Foundations and Applications

What will we learn today?

- Naive Bayes classification algorithm

Naive Bayes

- Classify image using histograms of occurrences on visual words:

$\mathbf{x} =$ 

- where:
  - x_i is the event of visual word v_i appearing in the image,
  - $N(i)$ the number of times word v_i occurs in the image,
  - m is the number of words in our vocabulary.

Csurka Bray, Dance & Fan, 2004

Naive Bayes - classification

- Our goal is to classify the image represented $\boldsymbol{x}$ as belonging to the class that has the highest *posterior* probability:

$$c^{*} = arg\max_{c} P(c \mid \boldsymbol{x})$$

Naive Bayes – conditional independence

- Naïve Bayes classifier assumes that visual words are conditionally independent given object class.

- Therefore, we can multiply the probability of each visual word to obtain the joint probability.

- Model for image x under object class c:

$$P(x \mid c) = \prod_{i=1}^{m} P(x_i \mid c)$$

- How do we compute $P(v_i \mid c)$?

Csurka Bray, Dance & Fan, 2004

Naive Bayes – prior

- Class priors $P(c)$ encode how likely we are to see one class versus others.
- Note that:

$$\sum_c P(c) = 1$$

Csurka Bray, Dance & Fan, 2004

Naive Bayes - posterior

- With the equations from the previous slides, we can now calculate the probability that an image represented by $\boldsymbol{x}$ belongs to class category c.

$$P(c \mid \boldsymbol{x}) = \frac{P(c)\, P(\boldsymbol{x} \mid c)}{\sum_{c'} P(c')\, P(\boldsymbol{x} \mid c')}$$

Bayes Theorem

Naive Bayes – posterior

- With the equations from the previous slides, we can now calculate the probability that an image represented by $\boldsymbol{x}$ belongs to class category c.

$$P(c \mid \boldsymbol{x}) = \frac{P(c)\, P(\boldsymbol{x} \mid c)}{\sum_{c'} P(c')\, P(\boldsymbol{x} \mid c')}$$

$$P(c \mid \boldsymbol{x}) = \frac{P(c) \prod_{i=1}^{m} P(x_i \mid c)}{\sum_{c'} P(c') \prod_{i=1}^{m} P(x_i \mid c')}$$

Naive Bayes - classification

- We can now classify that the image represented by $\boldsymbol{x}$ is belongs class that has the highest probability:

$$c^* = arg \max_c P(c \mid \boldsymbol{x})$$

$$c^* = arg \max_c \log P(c \mid \boldsymbol{x})$$

Let’s break down the posterior

The probability that $\boldsymbol{x}$ belongs to class c_1:

$$P(c_1 \mid \boldsymbol{x}) = \frac{P(c_1)\prod_{i=1}^{m} P(x_i \mid c_1)}{\sum_{c'} P(c')\prod_{i=1}^{m} P(x_i \mid c')}$$

And the probability that $\boldsymbol{x}$ belongs to class c_2:

$$P(c_2 \mid \boldsymbol{x}) = \frac{P(c_2)\prod_{i=1}^{m} P(x_i \mid c_2)}{\sum_{c'} P(c')\prod_{i=1}^{m} P(x_i \mid c')}$$

Both denominators are the same

The probability that $\boldsymbol{x}$ belongs to class c_1:

$$P(c_1 \mid \boldsymbol{x}) = \frac{P(c_1) \prod_{i=1}^{m} P(x_i \mid c_1)}{\sum_{c'} P(c') \prod_{i=1}^{m} P(x_i \mid c')}$$

And the probability that $\boldsymbol{x}$ belongs to class c_2:

$$P(c_2 \mid \boldsymbol{x}) = \frac{P(c_2) \prod_{i=1}^{m} P(x_i \mid c_2)}{\sum_{c'} P(c') \prod_{i=1}^{m} P(x_i \mid c')}$$

Both denominators are the same

- Since we only want the max, we can ignore the denominator:

$$P(c_1 \mid \boldsymbol{x}) \propto P(c_1) \prod_{i=1}^{m} P(x_i \mid c_1)$$

$$P(c_2 \mid \boldsymbol{x}) \propto P(c_2) \prod_{i=1}^{m} P(x_i \mid c_2)$$

For the general class c,

$$P(c \mid \boldsymbol{x}) \propto P(c)\prod_{i=1}^{m} P(x_i \mid c)$$

For the general class c,

$$P(c \mid \boldsymbol{x}) \propto P(c) \prod_{i=1}^{m} P(x_i \mid c)$$

We can take the log:

$$\log P(c \mid \boldsymbol{x}) \propto logP(c) + \sum_{i=1}^{m} logP(x_i \mid c)$$

Naive Bayes - classification

- So, the following classification becomes:

$$c^* = arg\max_c P(c \mid \boldsymbol{x})$$
$$c^* = arg\max_c \log P(c \mid \boldsymbol{x})$$

$$c^* = arg\max_c logP(c) + \sum_{i=1}^{m} logP(x_i \mid c)$$

Summary

- Naive Bayes classification algorithm