OM- Gradient based learning Problem set
(ILM-2)

Akshay Bankar
2019201011

PAGE No
DATE / /

Q.1 Function model

- To a spread of disease ~~like~~ such as COVID-19, a non-stationary time-series is a preferable model.
- Non-stationary time-series: Series whose statistical properties change over time
- Exponential models can be fit but these do not apply whenever rate of charge of quantity depends on quantity itself.
- ~~Reason for selecting non-stationary process:~~
(i) i.e, the more people are infected, the higher the number of newly infected people. But the trend can't go on forever as, once everyone is infected the growth rate must be zero. The extended model taking the population size into account is logistic growth model.
- The growth rate changes over time for following reasons-
(i) Pool of people to infect shrinks as more & more people get infected.
(ii) If the infection numbers become too large, actions & measures such as quarantine will be taken.
(iii) After a while, a vaccine which prevents infections may be found.

- The following model is adopted with a start window day x_0.

$$y = a(x_0) \cdot e^{b(x_0) \cdot x}$$

Linear relationship is obtained by taking log,

$$\log(y) = \log(a(x_0)) + b(x_0) \cdot x$$

The initial parameters $\log(a)$ & b are obtained by Moore-Penrose pseudo inverse (Least square fitting of input X, y)

- Non-linear least square ~~was~~ is used as loss function to obtain a least-square fit.

For a non-linear function with variable x & dependent parameters β $f(x_i;\beta)$, its least square is, (or residual)

$$LS, \text{or} \; r_i = \sum_i (y_i - f(x_i, \beta))^2$$

PAGE No
DATE / /

~~Hence~~ And it is minimized using the gradient

$$\frac{\partial LS}{\partial \beta} = 2 \sum_i (y - f(x_i;\beta)) \frac{\partial}{\partial \beta}(y - f(x_i,\beta))$$

In our case, the objective function is,

$$\min z = \sum_i (y_i - a \cdot e^{bx_i})^2$$

The parameters a, b will be learned iteratively. Using gradient descent ~~this~~ the update step in each iteration would be,

$$\frac{\partial z}{\partial a} = 2 \sum_i (y_i - a \cdot e^{bx_i}) \cdot (- e^{bx_i})$$

$$\frac{\partial z}{\partial b} = 2 \sum_i (y_i - a e^{bx_i}) (- a b e^{bx_i})$$

And update of a, b will be,

$$a_{k+1} = a_k - \alpha \cdot 2 \sum_i (y_i - a_k e^{bx_i})(- e^{bx_i})$$

$$b_{k+1} = b_k - \alpha \, 2 \sum_i (y_i - a e^{b_k x_i})(- e^{b_k x_i})$$

Using Newton's method,
The update step is given by,

$$\beta_{k+1} = \beta_k - \frac{f(x, \beta_k)}{f'(x, \beta_k)}$$

$$\therefore \quad a_{k+1} = a_k - \alpha \frac{\sum_i (y_i - f(x_i, \beta))}{2 \sum (y_i - a_k e^{bx_i})(- e^{bx_i})}$$

$$b_{k+1} = b_k - \alpha \frac{\sum_i (y_i - f(x_i, \beta))}{2 \sum (y_i - a e^{b_k x_i})(- e^{b_k x_i})}$$