

# OM Homework Set 5

C.V. Jawahar

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## 1 Non-Linear Optimization

### 1.1

Prove the following result using Jensen's inequality

$$\frac{1}{x-1} + \frac{1}{x} + \frac{1}{x+1} > \frac{3}{x}$$

Use this result to show that  $\sum \frac{1}{1} + \frac{1}{2} + \frac{1}{3} + \cdots + \frac{1}{n} + \dots$  does not converge to a real number.

### 1.2

Prove the three chord lemma: if  $f : [a, b] \rightarrow \mathbb{R}$  is convex and  $a < x < b$ , then

$$\frac{f(x) - f(a)}{x - a} \leq \frac{f(b) - f(a)}{b - a} \leq \frac{f(b) - f(x)}{b - x}$$

### 1.3

If  $x, y, z$  are positive real number such that  $x + y + z = 1$ , show using Jensen's inequality that

$$(1 + \frac{1}{x})(1 + \frac{1}{y})(1 + \frac{1}{z}) \geq 64$$

### 1.4

Conduct 2 iterations of the secant method for

$$x^3 - 2x - 5 = 0$$

where  $x_0 = 2$  and  $x_1 = 3$ .

### 1.5

Conduct 2 iterations of the bisection method for

$$xe^x = 1$$

where  $x \in [0, 1]$ .

### 1.6

Find a real root of the equations  $x^2 - y^2 = 3$  and  $x^2 + y^2 = 13$  by doing 2 iterations of Newton's method with  $x_0 = y_0 = \sqrt{6.5}$

### 1.7

Do 2 iterations of the fixed point method with

$$2x - \cos x - 3 = 0$$

Take

$$g(x) = \frac{\cos x + 3}{2}, \quad x_0 = \pi/2$$

### 1.8

Write the pseudocode for the gradient ascent algorithm. Your code must be vectorized. Also, run the algorithm to find the maximum of  $6 - (x_1^2(x_1^2 - 16) + x_2^2(x_2^2 - 9))$  start from (0,0)

### 1.9

Many people prefer to use the Gradient Descent algorithm in batch mode. How and why do you think they do that?

### 1.10

Give a proof of the convergence of the gradient descent algorithm. Also, discuss how to choose and adjust the learning rate so that it converges.

## 2 Submission Instructions

Your submission is expected to be a **pdf** file with title **rollnumber-hw5.pdf** containing the in-order scanned images of your hand-written answers (and plots).

- Submission Deadline: April 27<sup>th</sup>, 2020, time: 23:55
- No deadline extensions will be provided.
- Final submission needs to be uploaded on Moodle.
- Any form of plagiarism will be penalized.