Value-at-risk and expected shortfall

QUANTITATIVE RISK MANAGEMENT IN R



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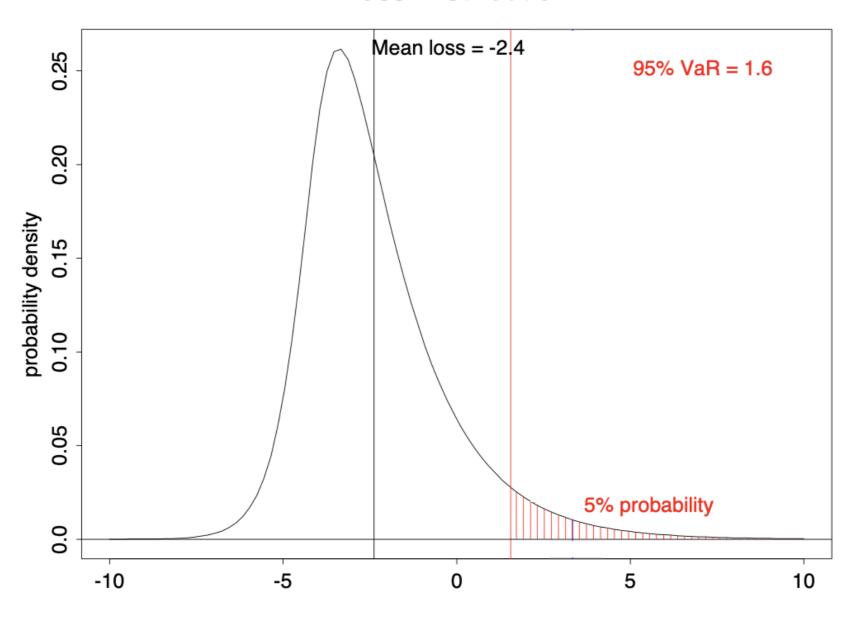


Value-at-risk (VaR)

- Consider the distribution of losses over a fixed time period (day, week, etc.)
- lpha-VaR is the lpha-quantile of the loss distribution
- α known as **confidence level** (e.g. 95%, 99%)
- ullet Should lose no more than lpha-VaR with probability lpha

95% VaR illustrated

Loss Distribution

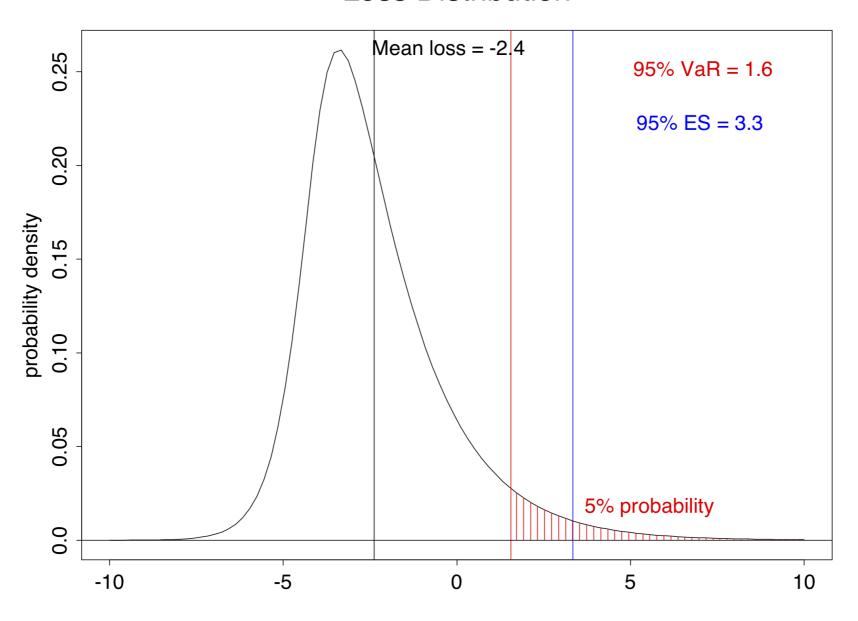


Expected shortfall (ES)

- Increasingly important in banking regulation
- Tail VaR (TVaR), conditional VaR (CVaR) or expected shortfall (ES)
- lpha-ES is expected loss given that loss exceeds lpha-VaR
- Expectation of tail of distribution

95% of ES illustrated

Loss Distribution



Let's practice!

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International equity portfolio

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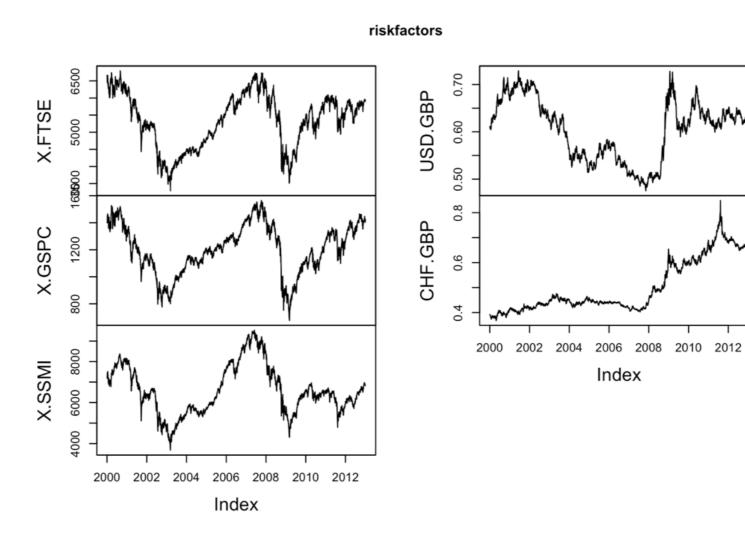
International equity portfolio

- Imagine a UK investor who has invested her wealth:
 - 30% FTSE, 40% S&P 500, 30% SMI
- 5 risk factors: FTSE, S&P 500 and SMI indexes, GBP/USD and GBP/CHF exchange rate

```
riskfactors <- merge(FTSE, SP500, SMI, USD_GBP, CHF_GBP, all = FALSE)["/2012-12-31", ]
```

Displaying the risk factors

plot.zoo(riskfactors)





Historical simulation

- Simple method that is widely used in financial industry
- Resample historical risk-factor returns and examine their effect on current portfolio
- Loss operator shows effect of different risk-factor returns on the portfolio
- Loss operator functions will be provided in the exercises

Empirical estimates of VaR and ES

```
losses <- rnorm(100)</pre>
losses_o <- sort(losses, decreasing = TRUE)</pre>
head(losses_o, n = 8)
1.836163 1.775163 1.745427 1.614479 1.602120 1.590034 1.483691 1.408354
quantile(losses, 0.95)
     95%
1.590638
qnorm(0.95)
1.644854
```



Empirical estimates of VaR and ES

mean(losses[losses > quantile(losses, 0.95)])

1.714671

ESnorm(0.95)

2.062713



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Option portfolio and Black Scholes

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European options and Black-Scholes

- European call option: gives right but not obligation to buy stock for price K at time T
- European put option: gives right but not obligation to sell stock for price K at T
- Value at time t < T depends on:
 - \circ Stock price S, time to maturity T-t, interest rate r, annualized volatility σ or sigma
 - Pricing by Black-Scholes formula

Pricing a first call option

```
K <- 50
T <- 2
t <- 0
S <- 40
r <- 0.005
sigma <- 0.25
Black_Scholes(t, S, r, sigma, K, T, "call")</pre>
```

2.619183

```
Black_Scholes(t, S, r, sigma*1.2, K, T, "call")
```

3.677901

- Price increases with volatility
- Option above is out-of-the-money

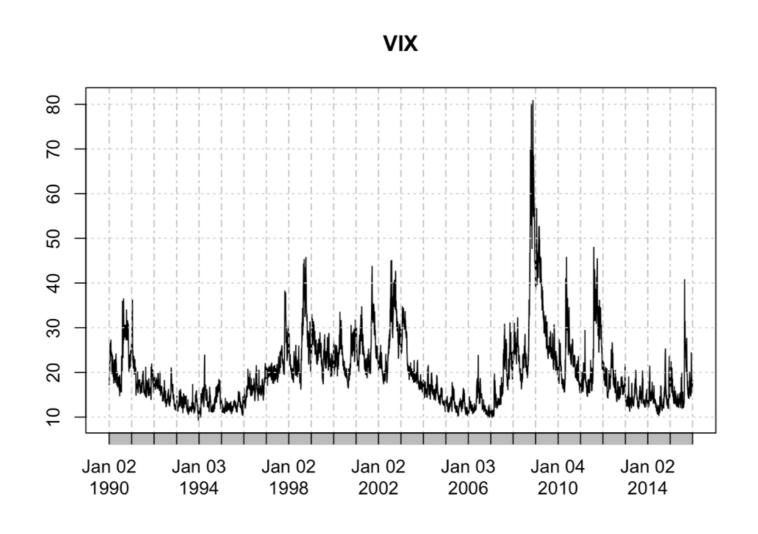


Implied volatility X needs change

- Volatility not directly observable
- Market participants use implied volatility, the value of volatility implied by quoted option price

The VIX index

plot(VIX)





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Historical simulation for the option example

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Historical simulation

- Portfolio: single European call option on equity index
- Consider losses and profits over one day
- ullet Changes to index value S, implied volatility σ and interest rate r affect value of portfolio
- We consider S and σ (and assume r stays constant)
- ullet Create loss operator taking S and σ as input and giving the loss or profit as output

Estimating VaR and ES

- Apply loss operator lossop() to historical log-returns of S&P 500 and VIX to get simulated losses
- Estimate VaR by sample quantile as before
- Estimate ES by average of losses exceeding VaR estimate

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Wrap-up QUANTITATIVE RISK MANAGEMENT IN R



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Not the end of the story...

Consider two things:

- 1. Can we improve risk sensitivity of VaR and ES estimates?
 - Filtered historical simulation
 - GARCH models
 - EWMA volatility filters
- 2. Can we improve simple empirical estimates of VaR and ES?
 - o Parametric tail models, heavy-tailed distributions, extreme value theory

Thanks for taking the course!

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