Assignment no 4

Name: AKSHAY PRADEEP PATADE CWID: 20009092

Q1 Use the limit definition of the derivative to exactly evaluate the derivative

a. $f(x) = \sqrt{x+4}$

Formula to calculate the derivative of a function is given by

$$f'(x) = \lim_{h\to 0} \frac{f(x+h) - f(x)}{h} = \lim_{h\to 0} \frac{\sqrt{x+h+4} - \sqrt{x+4}}{h}$$

Mutiplying Denominator & Numerator by $\sqrt{x+h+4} + \sqrt{x+4}$ we get

$$f'(x) = \lim_{h\to 0} \frac{\left(\sqrt{x+h+4} - \sqrt{x+4}\right)\left(\sqrt{x+h+4} + \sqrt{x+4}\right)}{h\left(\sqrt{x+h+4} + \sqrt{x+4}\right)}$$

$$= \lim_{h\to 0} \frac{x+h+4-x-4}{h\left(\sqrt{x+h+4} + \sqrt{x+4}\right)} = \lim_{h\to 0} \frac{h}{h\left(\sqrt{x+h+4} + \sqrt{x+4}\right)}$$

$$= \lim_{h\to 0} \frac{1}{\sqrt{x+h+4} + \sqrt{x+4}}$$

Removing the limit by Substituting $h=0$ we get

$$f'(x) = \frac{1}{\sqrt{x+0+4} + \sqrt{x+4}} \qquad \therefore f'(x) = \frac{1}{2\sqrt{x+4}}$$

b. $f(x) = \frac{3}{x}$

Formula to calculate the derivative of a function is given by

$$f'(x) = \lim_{h\to 0} \frac{f(x+h) - f(x)}{h} = \lim_{h\to 0} \frac{\frac{3}{(x+h)} - \frac{3}{x}}{h}$$

$$= \lim_{h\to 0} \frac{3x - 3(x+h)}{hx(x+h)} = \lim_{h\to 0} \frac{-3h}{hx(x+h)}$$

$$= \lim_{h\to 0} \frac{-3}{x(x+h)}$$

Removing the limits by substituting $h=0$ we get

$$f'(x) = \frac{-3}{x^2}$$

Q2 Find the Derivatives of the following functions

a. $f(x) = 3x^3 - \frac{4}{x^2}$ i.e $f(x) = 3x^3 - 4x^{-2}$

Using the Power Rule and sum rule we get

$f'(x) = 9x^2 + 8x^{-3}$ i.e $f'(x) = 9x^2 + \frac{8}{x^3}$

b. $f(x) = (4-x^2)^3$

Using the Chain Rule we get

$$f'(x) = 3(4-x^2)^2 \cdot -2x$$

$$\therefore f'(x) = -6x(4-x^2)^2$$

c) $f(x) = e^{\sin(x)}$

Using the Chain Rule we get

$f'(x) = e^{\sin x} \cdot \cos x$

$\therefore f'(x) = \cos x \, e^{\sin x}$

d) $f(x) = \log(x+2)$

$\therefore$ Using the Chain Rule we get

$f'(x) = \dfrac{1}{(x+2)}$

e) $f(x) = x^2 \cos x + x \tan x$

Using the Chain Rule & the Product Rule we get

$f'(x) = 2x\cos x - x^2 \sin x + \tan x + x \sec^2 x$

f) $f(x) = \sqrt{3x^2+2}$

Using the chain rule we get

$f'(x) = \dfrac{1}{2\sqrt{3x^2+2}} \, 6x$

$\therefore f'(x) = \dfrac{3x}{\sqrt{3x^2+2}}$

g) $f(x) = \dfrac{x}{4} \sin^{-1} x$

i.e $f(x) = \dfrac{x}{4 \sin x}$

Using the Quotient Rule we get

$$f'(x) = \frac{1}{4}\left[\frac{\sin x - x\cos x}{(\sin x)^2}\right]$$

~~h) $x^2y = (y+2) + xy(\sin$~~

~~h) $x^2y = (y+2) + xy\sin(x)$~~

~~Differentiating with respect to x we get. Also we~~ will be using the product rule we get

h) $x^2y = (y+2) + xy\sin(x)$

$$x^2y - xy\sin(x) - y = 2$$

Taking y common

$$y(x^2 - x\sin x - 1) = 2$$

$$y = \frac{2}{(x^2 - x\sin x - 1)}$$

Using the Chain rule we get

$$\frac{dy}{dx} = \frac{-2}{(x^2 - x\sin x - 1)^2}\left[2x - \sin x - x\cos x\right]$$

Q3

a) we will make use of central difference formula which is used to calculate the derivative of a function.

It is given by

$f'(a) = \frac{f(a+h) - f(a-h)}{2h}$ here $h = 10$ & $a = 39$

$\therefore f'(39) = \frac{f(39+10) - f(39-10)}{2h} = \frac{f(49) - f(29)}{20}$

$= \frac{145 - 115}{20} = \frac{30}{20} = 1.5$

The value of the first order derivative is positive meaning that the value of the function is increasing. At 39 hour, the wind speed is increasing at the rate of 1.5 mph/hr

b) we will make use of central difference formula which is used to calculate the derivative of a function

It is given by

$f'(a) = \frac{f(a+h) - f(a-h)}{2h}$ here $h = 2$ & $a = 83$

$\therefore f'(83) = \frac{f(83+2) - f(83-2)}{2 \times 2} = \frac{f(85) - f(81)}{4}$

$= \frac{95 - 125}{4} = \frac{-30}{4} = -7.5$

The value of the first order derivative is negative meaning that the value of the function is negative. At 83 hour, the wind speed is decreasing at the rate of 7.5 mph/hr.