1.
Suppose a certain birth defect occurs independently at random with probability p=0.02 in any live birth. Use a Chernoff bound to bound the probability that more than 4% of the 1 million children born in a given large city have this birth defect.

Solution:

For i= 1,…..,10^6, Compute μ

$$\mu = E[X] = \sum_{i=1}^{1000000} E[X_i] = \sum_{i=1}^{1000000} 0.02 = 20000$$

4% of 1 million children would be = 0.04 * 1000000 = 40000

By Chernoff bounds, for δ = 1, upper bound is

$$Pr(X \geq (1+\delta)\mu) = P(X \geq 40000) < [e / (1 +)^{(1+)}] = [e / 4]^{20000}$$

2. Consider a modification of the Fisher-Yates random shuffling algorithm where we replace the call to random(k+1) with random (n), and take the for-loop down to 0, so that the algorithm now swaps each element with another element in the array, with each cell in the array having an equal likelihood of being the swap location. Show that this algorithm does not generate every permutation with equal probability.
Hint: Consider the case when n=3

Solution:
Not every permutation is produced by this method with an equal probability. For instance, the permutation (1, 2, 3) is twice as likely to be created when n = 3 compared to the permutation (1, 3, 2). The method will switch each element with another element in the array when n is equal to three. But not every cell in the array has an equal chance of being the swap site. The likelihood of generating, for instance, the permutation (1, 2, 3) is double that of the permutation (1, 3, 2). This is due to the fact that the permutation (1, 2, 3) may be created by either switching the first and second or first and third elements.

The permutation (1, 3, 2), on the other hand, can only be created by switching the first and third elements. Because it is more likely to produce permutations in which the components are near their original places, the method will not have every permutation with an equal chance. Because the former can be produced by swapping the first element with either the second element or the third element, the permutation (1, 2, 3) is twice as likely to be generated when n = 3 as the permutation

(1, 3, 2) is. The permutation (1, 3, 2), on the other hand, can only be created by switching the first and third elements.

3.

Solution:

a)

For implementing probabilistic packet marking strategy, a router R with some probability p ≤ ½ .

The probability of the packet received by the recipient i.e. marked by the i^{th} $(1 \leq i \leq d)$ router along the attack path is

= p(1 - p) raised to d - i where d is the number of routers.

b)

Above problem is the same as the coupon collector problem. The recipient has to collect d routers by visiting a series of routers.

Let X be the random variable representing the number of times to visit for *d* routers:

X can be written as

$X = X_1 + X_2 + X_3 + \ldots\ldots X_d$

Let X_i be the number of trips the recipient has to made in order to go from having *i-1* distinct routers. Got *i-1* distinct coupons, the probability of getting a new router will be

P_i = *d - (i - 1)) / d

Since there are *d* routers, and *d-(i-1)* we don't have. By the linearity of expectation

$E[X] = E[X_1] + E[X_2] + E[X_3] + \ldots\ldots E[X_d]$

$= 1/ \quad + 1/ \quad + 1/ + \ldots\ldots + 1/$

$= dH_d$

where H_d is the harmonic number and can be approximated as $\ln d < H_d < \ln d+1$

Now according to tail estimate, recipient has to make more than ***d ln d*** traceback to get all *d* routers.