Analysis of multimachine system Padiyar chapter 9

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1 Introduction

To analyse critical mode of oscillations in large power system.

- To carry out load flow analysis
- To identify weak buses
- Load flow analysis after adequate support is provided to the bus.

1.1 simplified model

Assumptions

- transmission losses are neglected.
- voltage magnitude at buses are constant.
- loads are static. As voltage magnitude is assumed constant only the real power is analysed. The real power injected in the j^{th} node is

  $$P_j = \Sigma \frac{V_j V_i sin(\delta_i - \delta_j)}{x_{ij}} + \Sigma \frac{V_j E_k sin(\delta_j - \delta_k)}{x'_{dk}} \tag{1}$$

  V_i are the load buses connected to the jth bus.E_k internal voltage of generator buses connected to the jth bus. x'_{dk} is the internal transient reactance, also saliency is ignored.

Linearizing eqn 1. the deviation in power flow can be shown as

$$\Delta P_{ij} = \frac{V_i V_j cos(\delta_{ij}) \Delta\delta_{ij}}{x_{ij}} \tag{2}$$

Eqn 2, assuming ΔP as current and $\Delta\delta$ as voltage difference we have

$$\Delta P_{ij} = g_{ij} \Delta\delta_{ij} \tag{3}$$

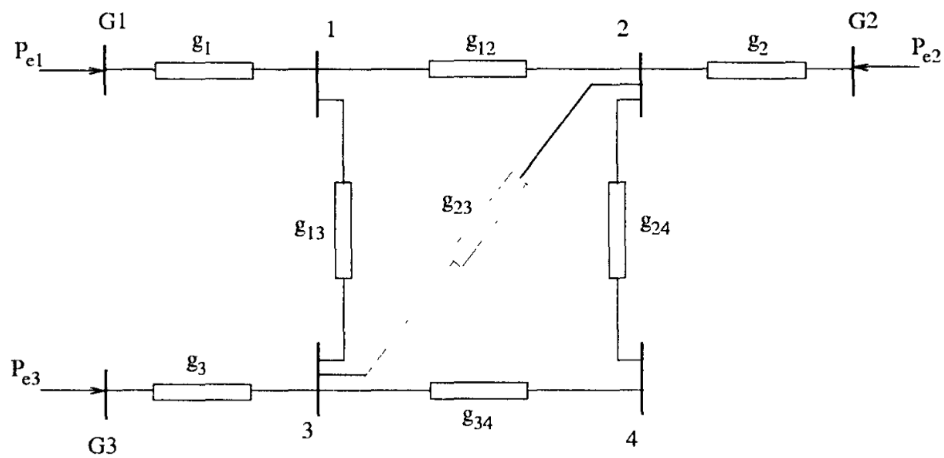

Figure 1: 3 gen 7 bus representation

Which leads to

$$[G]\begin{bmatrix}\Delta\delta_g\\ \Delta\delta_l\end{bmatrix}=\begin{bmatrix}\Delta P_e\\ 0\end{bmatrix}$$

$\Delta\delta_g$ is a vector of rotor angle deviation corresponding to m (number of generators) internal buses for which power is given as

$$\Delta P_e=\Sigma\frac{V_jE_ksin(\delta_j-\delta_k)}{x'_{dk}} \tag{4}$$

$\Delta\delta_l$ represents deviations in other bus angles. ΔP_e is the vector of deviations in the electrical power outputs of generators. The second entry on R.H.S. is zero as the active loads are assumed to be constant on account of constant voltages at load buses.

- Analouge- $\Delta\delta_{ij}$ as voltage and ΔP_{ek} as injected current.
- An admittance matrix is formulated similar to Y bus matrix with admittance = g_{ij} from equation (3).

The vectors involved as:

$$\begin{bmatrix}\Delta\delta_{G1} & \Delta\delta_{G2} & \Delta\delta_{G3} & \Delta\delta_1 & \Delta\delta_{G2} & \Delta\delta_{G3} & \Delta\delta_{G4}\end{bmatrix}$$

The deviations in power due to internal bus angle deviations are given as ΔP_{ek} and at buses the power deviations are 0 as constant loads are assumed.

$$\begin{bmatrix}\Delta P_{e1} & \Delta P_{e2} & \Delta P_{e3} & 0 & 0 & 0\end{bmatrix}$$

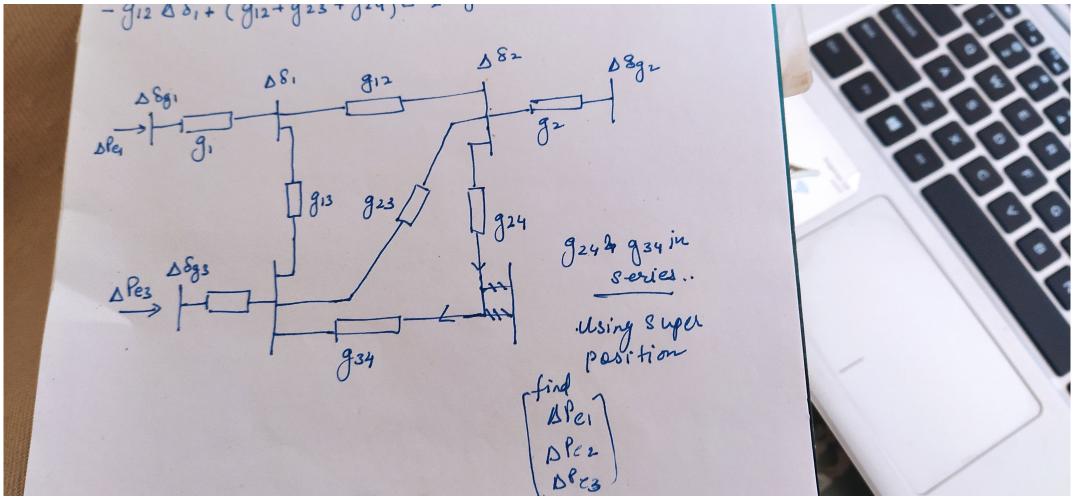

Figure 2: Reduced Network

The connecting matrix is given as:

$$\begin{bmatrix} g_1 & 0 & 0 & -g_1 & 0 & 0 & 0 \\ 0 & g_2 & 0 & 0 & -g_2 & 0 & 0 \\ 0 & 0 & g_3 & 0 & 0 & -g_3 & 0 \\ -g_1 & 0 & 0 & g_1+g_{12}+g_{13} & -g_{12} & -g_{13} & 0 \\ 0 & -g_2 & 0 & -g_{12} & g_{12}+g_{23}+g_{24}+g_2 & -g_{23} & -g_{24} \\ 0 & 0 & -g_3 & -g_{13} & -g_{23} & g_3+g_{13}+g_{23}+g_{24} & -g_{34} \\ 0 & 0 & 0 & 0 & -g_{24} & -g_{34} & g_{24}+g_{34} \end{bmatrix}$$

This is a singular matrix so the inverse of the matrix is not possible. Now we look for a reduced matrix $[G]^R$ in which we can remove bus 4.

Thus, we can derive reduced order matrix from two methods:

- Using linear equations from full order matrix
- By superposition theorem, putting ΔP_{ek} one at a time and considering power flow from $\Delta\delta_k$ to next j bus only.

In this case reduced bus is given as:

$$\begin{bmatrix} g_{12}+g_{13} & -g_{12} & -g_{13} \\ -g_{12} & g_{12}+g_{23}+g_{24}(1-k) & -(g_{23}+g_{24}(1-k)) \\ -g_{13} & -(g_{23}+g_{34}k) & g_{13}+g_{23}+g_{34}k \end{bmatrix}$$

1.2 Generator Equations and COI reference

We start with swing equations of the generators

$$M_k \frac{d^2\delta_{gk}}{dt^2} + D'_k \frac{d\delta_{gk}}{dt} = P_{mk} - P_{ek} \tag{5}$$

$$M_k = \frac{2H}{\omega_B}$$

,

$$D'_k = \frac{D_k}{\omega_B}$$

Here D_k is per unit damping and the damping is constant

$$\frac{D'_k}{M_k} = \alpha$$

,

$$D'_k = \alpha M_k$$

The center of inertia angle δ_o defined as

$$\delta_o = \frac{1}{M_T} \sum_{k=1}^{m} M_k \delta_k \tag{6}$$

here $M_T = M_1 + M_2 + M_3$ sum of inertia of all the buses. Basic Relations for swing equations

$$M_T \frac{d^2\delta_0}{dt^2} = M_1 \frac{d^2\delta_{g1}}{dt^2} + M_2 \frac{d^2\delta_{g2}}{dt^2} + M_3 \frac{d^2\delta_{g3}}{dt^2}$$

$$\alpha M_T \frac{d\delta_0}{dt} = D'_1 \frac{d\delta_{g1}}{dt} + D'_2 \frac{d\delta_{g2}}{dt} + D'_3 \frac{d\delta_{g3}}{dt}$$

The swing equation for COI is given as

$$M_T(p^2 + \alpha p)\delta_0 = \sum_{k=1}^{m} P_{mk} - \sum_{k=1}^{m} P_{ek} = P_{COI} \tag{7}$$

In power network $\sum_{k=1}^{m} P_{ek} = \sum_{j=1}^{n} P_{load_j} + P_{loss}$. **For lossless line and constant power loads P_{COI} is constant if mechanical input is constant.** Replacing eqn (7) in eqn(5) and due to linearity we have

$$M_k \frac{d^2\delta_k - \delta_0}{dt^2} + \alpha M_k \frac{d\delta_k - \delta_0}{dt} = P_{mk} - P_{ek} - \frac{M_k}{M_T} P_{COI} \tag{8}$$

For constant P_{COI}, Linearizing eqn. (8) we have

$$M_k \frac{d^2\Delta\delta_k - \Delta\delta_0}{dt^2} + \alpha M_k \frac{d\Delta\delta_k - \Delta\delta_0}{dt} = -\Delta P_{ek} \tag{9}$$

Important

$$\Delta P_{e1} = \sum_{j=1, j\neq k}^{m} -G^R_{kj}(\Delta\delta_k - \Delta\delta_j) \tag{10}$$

$$\Delta P_{e1} = -G_{12}(\Delta\delta_1 - \Delta\delta_2) - G_{13}(\Delta\delta_1 - \Delta\delta_2)$$

This result is similar to the reduced matrix obtained previously. So eqn.(10) can be given as

$$\Delta P_{e1} = \sum_{j=1, j\neq 1}^{m} -G_{1j}^{R}(\Delta\theta_1 - \Delta\theta_j) \tag{11}$$

If $\alpha = 0$, then

$$[M]p^2\Delta\theta = -[G]^R\Delta\theta \tag{12}$$

and for lossless lines, constant active power loads

$$M_T p^2 \Delta\delta_0 = \Delta P_{COI} = 0 \tag{13}$$

1.3 Linearized State Space equations

$$M_1\frac{d\Delta\omega_1}{dt} + D_1'\Delta\omega_1 = -\Delta P_{e1} \tag{14}$$

$$M_2\frac{d\Delta\omega_2}{dt} + D_2'\Delta\omega_2 = -\Delta P_{e2} \tag{15}$$

$$M_3\frac{d\Delta\omega_3}{dt} + D_3'\Delta\omega_3 = -\Delta P_{e3} \tag{16}$$

$$\frac{d\Delta\delta_{12}}{dt} = \Delta\omega_1 - \Delta\omega_2 \tag{17}$$

$$\frac{d\Delta\delta_{13}}{dt} = \Delta\omega_1 - \Delta\omega_3 \tag{18}$$

$$\Delta P_{e1} = G_{12}\delta_{12} + G_{12}\delta_{13} \tag{19}$$

$$\Delta P_{e2} = G_{21}\delta_{21} + G_{23}\delta_{23} \tag{20}$$

$$\Delta P_{e3} = G_{31}\delta_{31} + G_{32}\delta_{32} \tag{21}$$

2 Case: Load as constant impedence

This case is going to be simulated. The paper tuning performance of STATCOM considers load with constant impedence. Unlike constant power in previous example. Assumptions in this case:

- Loads as constant impedence.
- Generator modelled with field winding control.
- Transient saliency is neglected.
- Mech. input is constant.

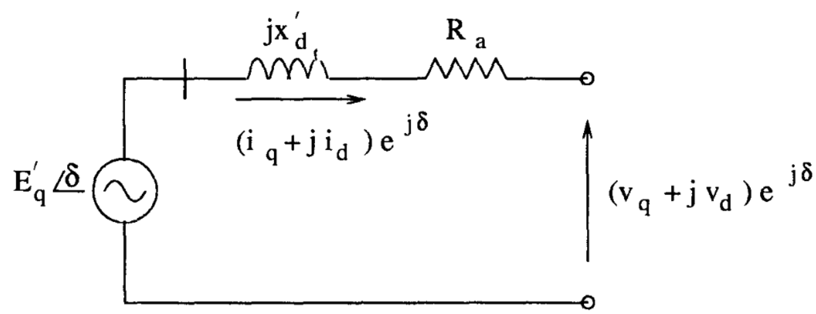

Figure 3: Stator equivalent model

The difference between classical model and this model is that E'_q is treated as state variable which varies with field excitation.

As the load are constant impedence we can reduce network with only generator buses and power flow between them. I am getting power flow equation as

$$GE'^2_{q1} - GE'_{q1}E'_{q2}cos(\delta_{12}) - BE'_{q1}E'_{q2}sin(\delta_{12}) \tag{22}$$

But book has given

$$GE'_{q1}E'_{q2}cos(\delta_{12}) + BE'_{q1}E'_{q2}sin(\delta_{12}) \tag{23}$$

2.1 Generator Equations

The effect of field winding is given as E_{fdk}.

$$pE'_{qk} = \frac{-E'_{qk} + (x_{dk} - x'_{dk})i_{dk} + E_{fdk}}{T'_{dok}} \tag{24}$$

$$p\delta_k = \omega_b(S_{mk} - S_{mk0}) \tag{25}$$

$$pS_{mk} = \frac{-D_k(S_{mk} - S_{mk0}) + P_{mk} - P_{ek})}{2H_k} \tag{26}$$

The derivation for i_{dk}

$$i_{dk} = \frac{-Q_{ek}}{E'_{qk}} \tag{27}$$

where Q_{ek} is given as

$$Q_{ek} = GE'_{q1}E'_{q2}sin(\delta_{12}) - BE'_{q1}E'_{q2}cos(\delta_{12}) \tag{28}$$

$$
\begin{aligned}
\frac{d\delta}{dt} &= \omega_B(S_m - S_{mo}) \\
\frac{dS_m}{dt} &= \frac{1}{2H}\left[-D(S_m - S_{mo}) + T_m - T_e\right] \\
\frac{dE'_q}{dt} &= \frac{1}{T'_{do}}\left[-E'_q + (x_d - x'_d)i_d + E_{fd}\right] \\
\frac{dE'_d}{dt} &= \frac{1}{T'_{qo}}\left[-E'_d - (x_q - x'_q)i_q\right]
\end{aligned}
$$

Figure 4: State equations SMIB

$$T_e = E'_d i_d + E'_q i_q + (x'_d - x'_q) i_d i_q$$

Figure 5: State equations SMIB

$$
\begin{bmatrix} (x'_d + z_I) & -(R_a + z_R) \\ -(R_a + z_R) & -(x'_q + z_I) \end{bmatrix}
\begin{bmatrix} i_d \\ i_q \end{bmatrix} =
\begin{bmatrix} f_1(\delta) - E'_q \\ f_2(\delta) - E'_d \end{bmatrix}
$$

$$
\begin{aligned}
f_1(\delta) &= h_1 E_b \cos\delta + h_2 E_b \sin\delta \\
f_2(\delta) &= h_2 E_b \cos\delta - h_1 E_b \sin\delta
\end{aligned}
$$

Figure 6: State equations SMIB

3 SVC representation

SVC represented as non linear load

$$P_L = P_{Lo}(\frac{V}{V_o})^{m_p} \tag{29}$$

$$Q_L = Q_{Lo}(\frac{V}{V_o})^{m_q} \tag{30}$$

4 Two Bus Example Voltage Stability