Reactive power current oscillations related to Active power is transmission line notes

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1 equation

- np = no. of branches connected to l^{th} node.
- pl= branch connecting node at the other end of l^{th} node.

$$i_{rl} = -b_l^s h V_l + \Sigma_{pl}^{np}(\frac{V_l - V_{pl}cos(\phi_{pl})}{x_{pl}}) \tag{1}$$

$$\frac{di_{rl}}{dt} = \frac{\delta i_{rl}}{\delta V_l}\frac{dV_l}{dt} + \Sigma_{pl}^{np}(\frac{\delta i_{rl}}{\delta \phi}\frac{d\phi}{dt} + \frac{\delta i_{rl}}{\delta V_{pl}}\frac{dV_{pl}}{dt}) \tag{2}$$

$$\frac{di_{rl}}{dt} = (-b_l^s h + \Sigma_{pl=1}^{np}\frac{1}{x_{pl}})\frac{dV_l}{dt} + (\Sigma\frac{V_{pl}sin(\phi)}{x_{pl}}\frac{d\phi}{dt}) + (\Sigma\frac{-V_{pl}cos(\phi)}{x_{pl}}\frac{dV_{pl}}{dt}) \tag{3}$$

Similarly,

$$\frac{\Delta di_{rl}}{dt} = (-b_l^s h + \Sigma_{pl=1}^{np}\frac{1}{x_{pl}})\frac{d\Delta V_l}{dt} + (\Sigma\frac{V_{pl}sin(\phi)}{x_{pl}}\frac{d\Delta\phi}{dt}) + (\Sigma\frac{-V_{pl}cos(\phi)}{x_{pl}}\frac{d\Delta V_{pl}}{dt}) \tag{4}$$

multiplying both sides with ΔV_l

$$\Delta V_l\frac{\Delta di_{rl}}{dt} = (-b_l^s h + \Sigma_{pl=1}^{np}\frac{1}{x_{pl}})\Delta V_l\frac{d\Delta V_l}{dt} + (\Sigma\frac{V_{pl}\Delta V_l sin(\phi)}{x_{pl}}\frac{d\Delta\phi}{dt}) + (\Sigma\frac{-V_{pl}cos(\phi)}{x_{pl}}\Delta V_l\frac{d\Delta V_{pl}}{dt}) \tag{5}$$

$$P_{pl} = \frac{V_{pl}V_l sin(\phi_{pl})}{x} \tag{6}$$

$$(\frac{V_{pl}sin(\phi)}{x_{pl}}) = \frac{\delta P_{pl}}{\delta V_l} \tag{7}$$

$$\Sigma_{l=1}^{n}\Sigma_{pl}^{np}(\frac{\delta i_{rl}}{\delta\phi}\frac{d\phi}{dt}\Delta V_l) = (\Sigma_{k=1}^{n_b}(\frac{\delta P_k \Delta V_{tk}}{\delta V_{tk}}) + (\frac{\delta P_k \Delta V_{fk}}{\delta V_{fk}}))\frac{d\Delta\phi_k}{dt} \tag{8}$$

$$\Sigma\Delta P_k\frac{d\Delta\phi_k}{dt} = \frac{\delta P_k}{\delta\phi_k}\Delta\phi_k\frac{d\Delta\phi_k}{dt} + \frac{\delta P_k}{\delta V_{fk}}\Delta V_{fk}\frac{d\Delta\phi_k}{dt} + \frac{\delta P_k}{\delta V_{tk}}\Delta V_{tk}\frac{d\Delta\phi_k}{dt} \tag{9}$$

From 8 and 9

$$\frac{\delta P_k}{\delta V_{fk}}\Delta V_{fk}\frac{d\Delta\phi_k}{dt}+\frac{\delta P_k}{\delta V_{tk}}\Delta V_{tk}\frac{d\Delta\phi_k}{dt}=\Sigma(-\frac{\delta i_{rl}}{\delta V_l}\frac{dV_l}{dt}\Delta V_l+\frac{d\Delta i_{rl}}{dt}\Delta V_l+(\frac{-V_{pl}cos(\phi)}{x_{pl}}\Delta V_l\frac{d\Delta V_{pl}}{dt}) \tag{10}$$

Also,

$$\frac{-V_{pl}cos(\phi)}{x_{pl}}\Delta V_l\frac{d\Delta V_{pl}}{dt}=-\Sigma\frac{\delta(\frac{P_k}{V_{fk}V_{tk}})}{\delta\phi_k}\frac{d\Delta V_{fk}\Delta V_{tk}}{dt} \tag{11}$$

This equation is not proved how to prove this.

The average power loss for damping is given as given as

$$Av(\Sigma P_k\frac{d\Delta\phi_k}{dt})=Av(\Sigma\frac{d\Delta i_{rl}}{dt}\Delta V_l) \tag{12}$$

As all other terms average comes to zero.

2 showing node voltage deviation in terms of reactive current

$$\Delta V_l=L^{th}\Delta i_{rl}+\Delta V_l^{th} \tag{13}$$

$\begin{vmatrix} -\Delta P_l \\ \Delta i_{rl} \end{vmatrix} = \begin{vmatrix} A_{21} & A_{22} & A_{23} \\ A_{31} & A_{32} & A_{33} \end{vmatrix} \begin{vmatrix} \Delta\delta \\ \Delta\phi \\ \Delta V_l \end{vmatrix}$ where P_l is load demand at bus. A matrix is jacobian, now find ΔV_l from wrt above two equations. Here ΔV_l^{th}, is the function of rotor angle alone. Substituting, eqn. 13 in eqn 12

$$Av(\Sigma P_k\frac{d\Delta\phi_k}{dt})=Av(\Sigma\frac{d\Delta i_{rl}}{dt}\Delta V_l^{th}) \tag{14}$$

As average $\Delta i_{rl}\frac{d\Delta i_{rl}}{dt}$ is zero (irl is sinusoid).

Now say we want to damp out Ω mode. By taking control action, as if ΔV_l^{th} increaes reactive current injection Δi_{rl} is reduced as

$$\Delta I_{rl}=-k_l\frac{\Delta V_l^{th}}{dt} \tag{15}$$

Thus from equation 14 in frequency domain

$$P^{loss}=\Omega^2(\Sigma|\Delta V_l^{th}(j\Omega)|^2) \tag{16}$$

A new factor V_{lm} is introduced which will tell where to place controller.

$$V_{lm}=\frac{dP^{loss}}{dk_l} \tag{17}$$

Higher value of factor a bus means controller to be placed at that particular bus.