# Kundur small signal stability chapter 12

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14 July 2020

## 1 Fundamentals

- Equilibrium points (singular points) where $\dot{x} = f(x, u, t) = 0$, i.e. all derivatives are zero.

### 1.1 Stability of Dynamic System

- Local Stability- about equilibrium points if the after a small perturbation the system restore to original state it is local stability.
- Finite Stability- if the system is stable for a small region in state space corresponding to R.
- Global Stability- System is stable for entire space.

### 1.2 Linearization

$$\Delta\dot{x} = A\Delta x + B\Delta u \tag{1}$$

$$\Delta y = C\Delta x + D\Delta u \tag{2}$$

The eigen values of [A], are the pole of the system and hence provide stability information.

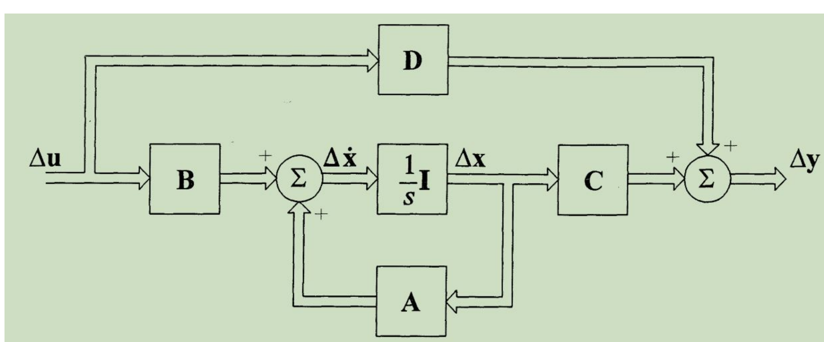


Figure 1: Block diagram state space

$$\mathbf{A} = \begin{bmatrix} \frac{\partial f_1}{\partial x_1} & \cdots & \frac{\partial f_1}{\partial x_n} \\ \cdots & \cdots & \cdots \\ \frac{\partial f_n}{\partial x_1} & \cdots & \frac{\partial f_n}{\partial x_n} \end{bmatrix} \qquad \mathbf{B} = \begin{bmatrix} \frac{\partial f_1}{\partial u_1} & \cdots & \frac{\partial f_1}{\partial u_r} \\ \cdots & \cdots & \cdots \\ \frac{\partial f_n}{\partial u_1} & \cdots & \frac{\partial f_n}{\partial u_r} \end{bmatrix}$$

$$\mathbf{C} = \begin{bmatrix} \frac{\partial g_1}{\partial x_1} & \cdots & \frac{\partial g_1}{\partial x_n} \\ \cdots & \cdots & \cdots \\ \frac{\partial g_m}{\partial x_1} & \cdots & \frac{\partial g_m}{\partial x_n} \end{bmatrix} \qquad \mathbf{D} = \begin{bmatrix} \frac{\partial g_1}{\partial u_1} & \cdots & \frac{\partial g_1}{\partial u_r} \\ \cdots & \cdots & \cdots \\ \frac{\partial g_m}{\partial u_1} & \cdots & \frac{\partial g_m}{\partial u_r} \end{bmatrix}$$

Figure 2: A= state, B= input, C= output, D= feedforward

## 1.3 Analysis of stability

### 1.3.1 Lyapunov first method

Assumption small signal stability of system.

1. negative real part of eigen value means system is aysmptotically stable at operating point.
2. positive real part means unstable.
3. zero real part uncertain to decide on the basis of first approx.

Large stability is decided by the explicit solution of the equations. However, by Lyapunov second method or Direct method stability can be analysed.

### 1.3.2 Lyapunov Direct Method

Important- Stability is determined by using suitable "functions" example bowl shape function. The sign of lyapunov function and the sign of lyapunov function time derivative wrt to system state are considered.
$\Delta\dot{x} = f(x,u)$ is stable if there exists positive definite function $V(x_1, x_2...., x_n)$ such that its total derivative wrt $\Delta\dot{x} = f(x,u)$ is not positive. * Definite in domain D means that function has same sign for all x in D. Example $V(x_1, x_2, x_3)=$ $x_1^2 + x_2^2 + x_3^2$ is positive deifinite. * *Semi- definite means the function has same sign in domain D or is 0 for all x in D. Example $V(x_1, x_2, x_3)= (x_1 - x_2)^2 + x_3^2$*

Lyapunov function example

$$\dot{x} = f(x,y) = -y - x^3 \tag{3}$$

$$\dot{y} = g(x,y) = x - y^3 \tag{4}$$

$$V(x,y) = y^2 + x^2 \tag{5}$$

Now we find total derivative of $V(x,y)$ wrt $\Delta\dot{x} = f(x,y)$

$$\dot{V}(x,y) = \frac{dV(x,y)}{dt} = \frac{\delta V(x,y)}{\delta x}\dot{x} + \frac{\delta V(x,y)}{\delta y}\dot{y} \tag{6}$$

$$\dot{V}(x,y) = -2(x^4 + y^4) <= 0 \tag{7}$$

So the system is lyapunov stable.

# 2 Eigen properties of the matrix

Small signal stability of system is defined by eigen values of A.

## 2.1 Eigenvalues

Eigen value ($\Lambda$) for non trivial solution is obtained as

$$A\Phi = \lambda\Phi \tag{8}$$

## 2.2 Eigen Vectors

From equation (8), $\Phi$ is the right eigen vector for given eigen value $\lambda$.

$$A\Phi_i = \lambda_i\Phi_i \tag{9}$$

$\Phi$ is a column vector.

$$\Phi_i = [\phi_{1i}, \phi_{2i}, ..., \phi_{ni}]^T \tag{10}$$

Left eigen vector, which is a row vector is found by

$$\Psi_i A = \lambda_i \Psi_i \tag{11}$$

$$\Psi_i = [\psi_{i1}, \psi_{i2}, ..., \psi_{in}] \tag{12}$$

Now important properties:

$$\Psi_j\Phi_i = 0$$

$$\Psi_i\Phi_i = Constant$$

This constant can be normalized as

$$\Psi_i\Phi_i = 1$$

## 2.3 Modal Matrices

To explain eigen properties following column vectors and matrices are introduced

$$\Phi = [\Phi_1, \Phi_2, ..\Phi_i., \Phi_n] \tag{13}$$

where $\Phi_i$ is represented by equation 10.

$$\Psi = [\Psi_1^T, \Psi_2^T, ..., \Psi_n^T] \tag{14}$$

$$\Lambda = diag[\lambda_1, \lambda_2...] \tag{15}$$

$$\Psi\Phi = I \tag{16}$$

## 2.4 Free Motion of Dynamic system

$$\Delta\dot{x} = A\Delta x \tag{17}$$

Now we go for diagonalization, to remove dependency of one state on another by introducing new variable $z$.

$$\Delta x = \Phi z \tag{18}$$

$$\Phi\dot{z} = A\Phi z \tag{19}$$

$$\dot{z} = \Phi^{-1}A\Phi z \tag{20}$$

$$\dot{z} = \Lambda z \tag{21}$$

Since $\Lambda$ is diagonal there is no cross coupling between states.
Above transformation leads to simplified state space equation as

$$\dot{z_i} = \lambda_i z_i \tag{22}$$

And the solution is given as

$$z(t) = z(0)e^{\lambda_i t} \tag{23}$$

Thus

$$\Delta x(t) = \Phi z(t) = [\phi_1, \phi_2, ..., \phi_n]z(t) \tag{24}$$

$$\Delta x(t) = \sum_{i=1}^{n} \Phi_i z_i(0)e^{\lambda_i t} \tag{25}$$

This implies

$$z(t) = \Phi^{-1}\Delta x(t) = \Psi\Delta x(t) \tag{26}$$

$$z_i(0) = \Psi_i \Delta x(0) \tag{27}$$

From equation (25) and (27)

$$\Delta x(t) = \sum_{i=1}^{n} \Phi_i \Psi_i \Delta x_i(0)e^{\lambda_i t} \tag{28}$$

The general solution is then given by

$$\Delta x_i(t) = \phi_{i1}\psi_{i1}\Delta x_i(0)e^{\lambda_1 t} + \phi_{i2}\psi_{i2}\Delta x_i(0)e^{\lambda_2 t} + .... \tag{29}$$

The above equation gives solution in terms of eigen values, left eigen vector and right eigen vector. $\Psi_i \Delta x(0)$ shows how much $i^{th}$ mode (eigen value) is excited (magnitude of excitation) due to initial conditions $\Delta x_(0)$.

*Comments:

- If the initial conditions lie across $j^{th}$ eigen vector then $\Psi_i \Delta x(0)$ for all i $\neq$ j is zero. Therefore only $j^{th}$ mode is excited.
- If initial condition is not along any eigen vector, then a linear combination of only some eigen vectors that represents initial condition are excited and other modes are not excited.

*

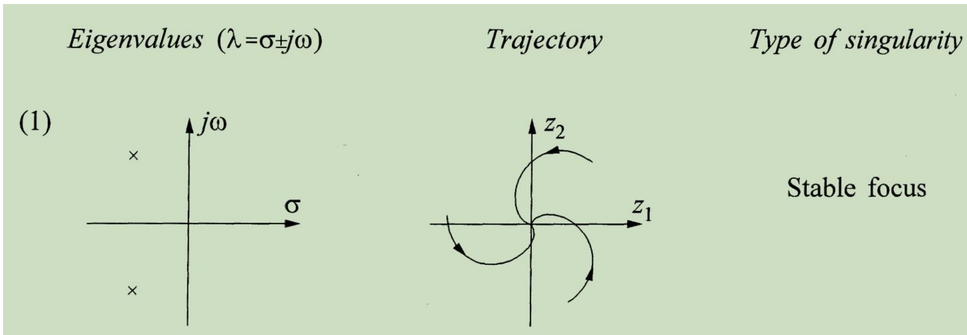


Figure 3: Mode Shapes

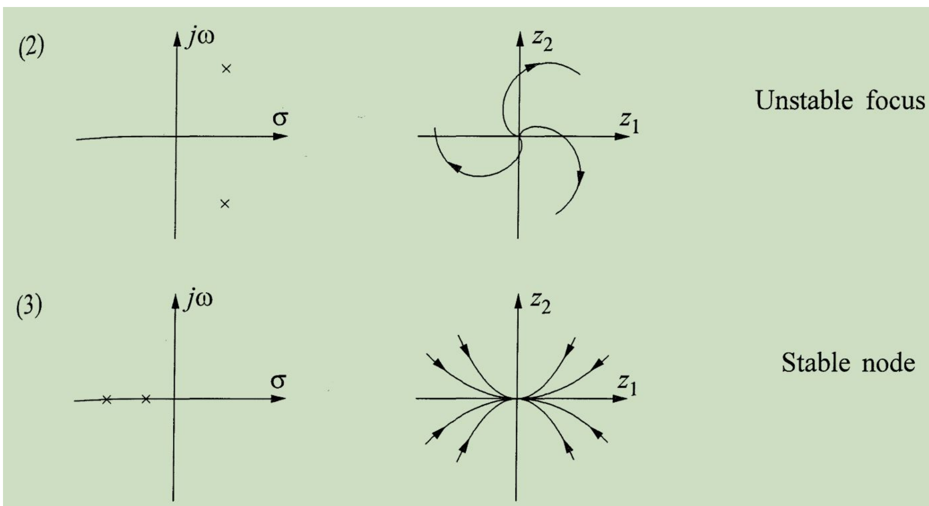


Figure 4: Mode Shapes

### 2.5 Eigen value and Stability

Solution

- if eigen value real and distinct, $x(t) = Ae^{(at)} + Be^{(bt)}$
- if eigen value complex, $x(t) = real(Ae^{(bt)} + Be^{(-bt)}$
- if eigen value real and similar, $x(t) = (A + Bt)e^{(-bt)}$

- Right eigen vector- under observation state variable, with $i^{th}$ mode excited.
- Left eigen vector- under observation mode, which states are displayed in the $i^{th}$ mode of excitation.

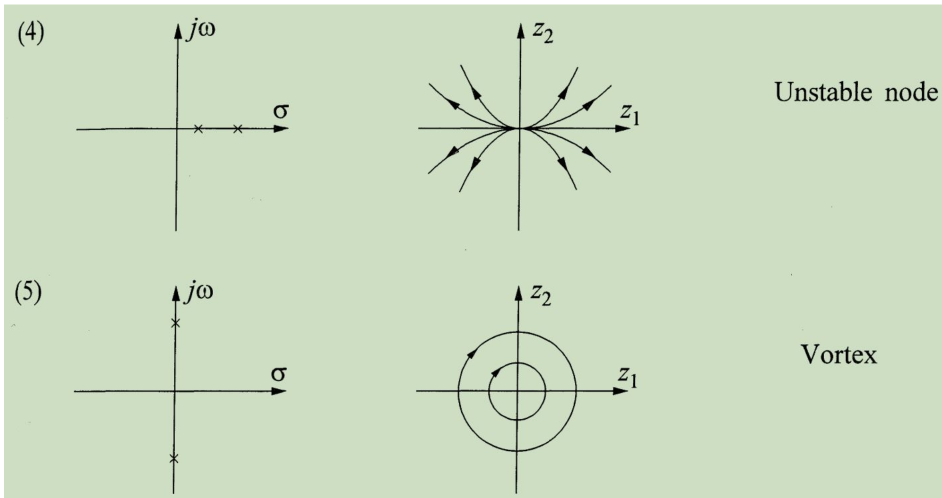


Figure 5: Mode Shapes

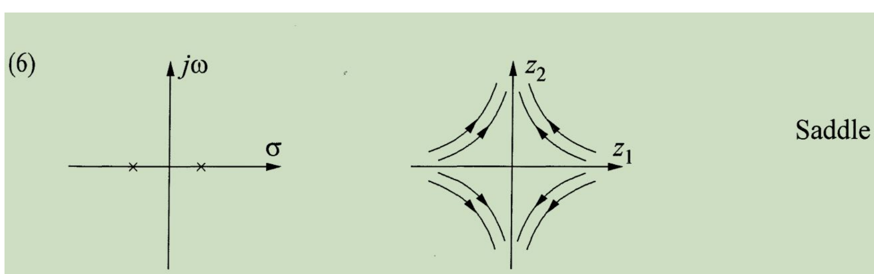


Figure 6: Mode Shapes

## 2.6 Eigen Value Sensitivity

$$A\Phi_i = \lambda_i \Phi_i \tag{30}$$

Differentiating, wrt $a_{kj}$ (element in kth row and jth column in A matrix) yields

$$\frac{\delta A}{\delta a_{kj}}\Phi_i + A\frac{\delta \Phi_i}{\delta a_{kj}} = \frac{\delta \lambda_i}{\delta a_{kj}}\Phi_i + \lambda_i \frac{\delta \Phi_i}{\delta a_{kj}} \tag{31}$$

Multiplying both sides with $\Psi_i$ to both sides.

$$\Psi_i \frac{\delta A}{\delta a_{kj}}\Phi_i + \Psi_i A\frac{\delta \Phi_i}{\delta a_{kj}} = \Psi_i \Phi_i \frac{\delta \lambda_i}{\delta a_{kj}} + \Psi_i \lambda_i \frac{\delta \Phi_i}{\delta a_{kj}} \tag{32}$$

$\Psi_i \Phi_i = 1$ and $\Psi(A - \lambda_i I = 0)$

$$\Psi_i \frac{\delta A}{\delta a_{kj}}\Phi_i = \frac{\delta \lambda_i}{\delta a_{kj}} \tag{33}$$

All elements of $frac \delta A \delta a_{kj}$ are 0 except for kth row and jth column which are 1. Thus equation becomes

$$\psi_{ik}\phi_{ji} = \frac{\delta \lambda_i}{\delta a_{kj}} \tag{34}$$

So sensitivity of the $\lambda_i$ to an element $a_k j$ is given as product of left eigen vector $\psi_{ik}$ and right eigen vector $\phi_{ji}$.

## 2.7 Participation factor

The relationship between modes and states are indexed by left and right eigen vectors. The difficulty with these parameters are that they are subject to scale and can have different units. So participation factors are introduced as

$$p_i = [p_{1i}, p_{2i}, .., p_{ni}]^T = [\phi_1 i \psi_i 1, \phi_2 i \psi_i 2, ..., \phi_n i \psi_i n]^T \tag{35}$$

$\phi_k i \psi_i k$, denotes the relative participation of kth state in ith mode and vice versa.

- $\sum_{k=1}^{n} p_{ki}$=1 or $\sum_{i=1}^{n} p_{ki}$=1
- $p_{ki}$ is the sensitivity of the eigen value $\lambda_i$ to the diagonal element $a_{kk}$ of A matrix.

$$p_{ki} = \frac{\delta \lambda_i}{\delta a_{kk}} \tag{36}$$

## 2.8 Controlability Observability

The state equations in normal form is given as equation 20. which can be written as

$$\dot{z} = \Lambda z + B'\Delta u$$
$$\Delta y = C'z + D\Delta u$$

where, $B' = \Phi^{-1}B$, $C' = C\Phi$.

- If $i^{th}$ row of the B' is zero inputs have no effect on $i^{th}$ mode in such case ith mode is uncontrollable.
- If $i^{th}$ column of the C' is zero then corresponding state $z_i$ has no effect on output thus unobservable.

## 2.9 Eigen properties and Transfer functions

$$\dot{\Delta}x = A\Delta x + B\Delta u$$
$$G(s) = \Delta y = C\Delta x$$

transfer function is given as

$$\frac{\Delta y}{\Delta u} = C(sI - A)^{-1}B$$

G(s) can be witten as transfer function with poles and zeros as

$$G(s) = \frac{R_1}{s+p_1} + \frac{R_2}{s+p_2} + ..\frac{R_n}{s+p_n}$$

where $R_i$ the residue of the pole $p_i$. For expressing transfer function in term of eigen values and eigen vectors we choose normal form $\Delta x = z$ then G(s) is given as

$$G(s) = C\Phi(sI - \Lambda)^{-1}\Psi B$$
$$G(s) = \sum_{i=1}^{n} \frac{R_i}{s - \lambda_i}$$
$$R_i = C\Phi_i \Psi_i B$$

**Comments : Residue method

$$f(x) = \sum_i \left(\frac{R_{i1}}{x - x_i} + \frac{R_{i2}}{(x - x_i)^2} + \frac{R_{i3}}{(x - x_i)^3} + .. + \frac{R_{ik_i}}{(x - x_i)^{K_i}}\right)$$

Let $g_{ij}(x)$= $(x - x_i)^{j-1}f(x)$,

$$R_{ij} = \frac{1}{(k_i - j)!}\lim_{x\to+x_i}\frac{d^{k_i-j}}{dx^{k_i-j}}(x - x_i)^{k_i}f(x)$$

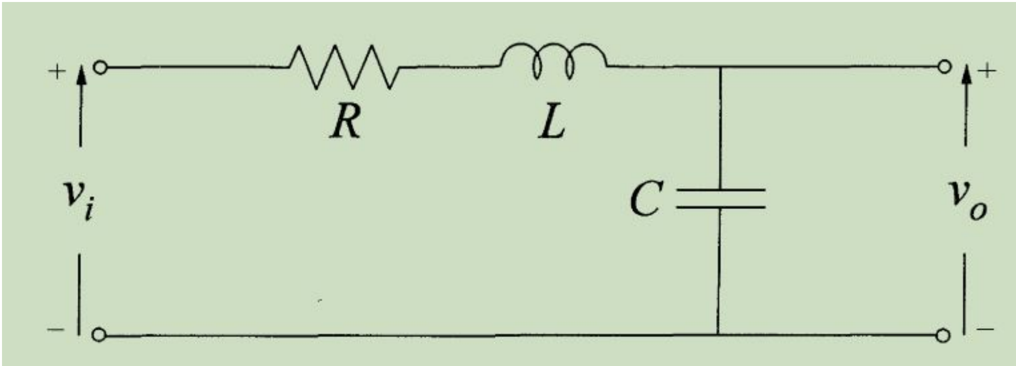


Figure 7: RLC circuit

$$f(x) = \sum_i \left(\frac{R_1}{x-x_1} + \frac{R_2}{(x-x_2)} + \frac{R_3}{(x-x_3)} + .. + \frac{R_k}{(x-x_K)}\right)$$

$$R_i = \lim_{x \to +x_i} (x-x_i) f(x)$$

**

# 3 Example: RLC circuit

$$LC\frac{d^2 v_o}{dt^2} + RC\frac{dv_o}{dt} + v_o = v_i \tag{37}$$

$$\frac{d^2 v_o}{dt^2} + (2\epsilon\omega_n)\frac{dv_o}{dt} + (\omega_n)^2 v_o = (\omega_n)^2 v_i \tag{38}$$

where:

$$\omega_n = \frac{1}{\sqrt{LC}}$$

$$\epsilon = \frac{R/2}{\sqrt{L/C}}$$

$$x_1 = v_o$$

$$x_2 = \dot{v}_o$$

$$u = v_i$$

$$y = v_o = x_1$$

$$\dot{x}_1 = x_2$$

$$\dot{x}_2 = -\omega^2 x_1 - (2\epsilon\omega_n) x_2 + \omega^2 u$$

$$\begin{bmatrix} \dot{x}_1 \\ \dot{x}_2 \end{bmatrix} = \begin{bmatrix} 0 & 1 \\ -\omega^2 & -2\epsilon\omega_n \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \end{bmatrix} + \begin{bmatrix} 0 \\ 1 \end{bmatrix} \begin{bmatrix} u \end{bmatrix}$$

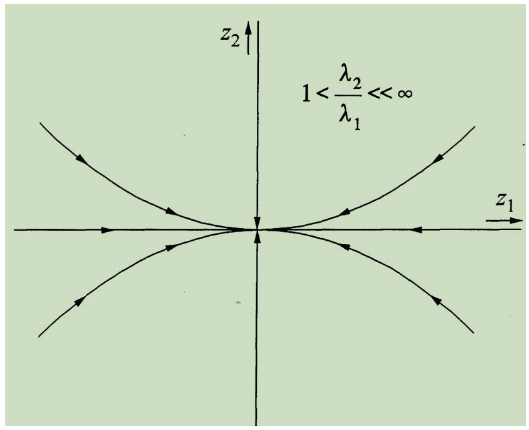


Figure 8: Stability and mode shape

On solving eigen values are given as

$$\lambda_i = -\epsilon\omega_n \pm \omega_n\sqrt{\epsilon^2 - 1} \tag{39}$$

The eigen vectors are evaluated as $\begin{bmatrix} -\lambda_i & 1 \\ -\omega^2 & -2\epsilon\omega_n - \lambda_i \end{bmatrix} \begin{bmatrix} \phi_1 i \\ \phi_2 i \end{bmatrix}$ =0. Upon further solving we find that the vectors are not independent. So we fix $\phi_{1i} = 1$ then $\phi_{2i} = \lambda_i$.

To get shape and stability we analyse normal form of state space

$$\dot{z} = \Lambda z \tag{40}$$

$$\Lambda = \begin{bmatrix} \lambda_1 & 0 \\ 0 & \lambda_2 \end{bmatrix}$$

Thus$\dot{z}_1 = \lambda_1 z_1$ and $\dot{z}_2 = \lambda_2 z_2$ Analysing in $z_1 - z_2$ plane $\frac{dz_2}{dz_1}$= $\frac{\lambda_2 z_2}{\lambda_1 z_1}$ So we get $z_2 = c{z_1}^{\lambda_2/\lambda_1}$. Since $\frac{\lambda_2}{\lambda_1} > 1$ we get following plot At t tending to infinity the plot moves towards the stable point along the arrows. This means the node is stable. This plot is then translated to actual states $(x_1, x_2)$ plane. As x= $\Phi z$. The new terms will be introduced $e^{\lambda_1 t}$ and $e^{\lambda_2 t}$. The magnitude of $\lambda_1 < \lambda_2$ so the time constant of $\lambda_2$ is faster than $\lambda_1$. Therefore the solution reaches the equilibrium point along $\lambda_1$ direction as shown in Fig. 9.

# 4 Small Signal Stability of Single Machine Infinite Bus

By using thevenin equivalent we reduced network Fig. 10 (a) as shown in Fig. 10 (b). The synchronous machine dynamics does not effect the infinite bus.

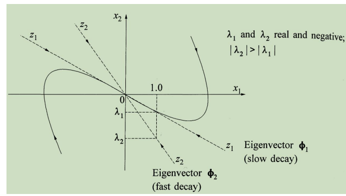


Figure 9: $x_1 - x_2$ plane solution

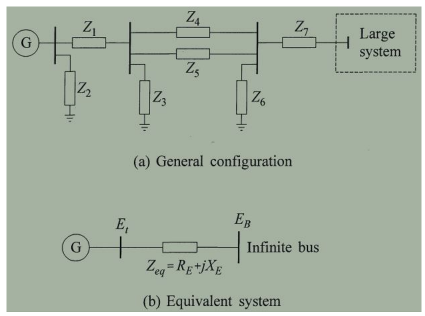


Figure 10: Single Machine Infinite Bus

- The synch. gen. dynamics are modeled as classical model.
- Increase detail like field circuit dynamics, excitation dynamics and damper winding.
- State space representation and modal analysis, torque angle relationship and transfer function model is carried out.

## 4.1 Classical Model of Synch. Machine

- The classical model is represented as Fig. 11.
- Internal voltage is given as E' which is assumed constant except for disturbance.
- $E_b$ is the infinite bus voltage which is assumed constant for synch. machine dynamics.
- Taking E' reference.
- $E'\angle 0 = E_b\angle -\delta + j(X'_d + X_E)I_t$
- $I_t = \frac{E'\angle 0 - E_b\angle -\delta}{jX_T}$
- The complex power at sending end is given as $S = P + jQ = E'I_t^*$
- $I_t^* = \frac{E'\angle 90 - E_b\angle(\delta+90)}{X_T} = \frac{jE' - E_b(-sin(\delta) + jcos(\delta))}{X_T}$

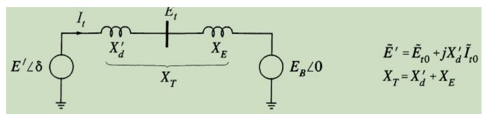


Figure 11: Classical Model of Synch. Machine

- S= $\frac{E' E_b sin(\ delta)}{X_T}$+ j$\frac{E'(E' - E_b cos(\delta))}{X_T}$
- Ignoring stator resistive loss, Air gap power ($P_e$)= terminal power (P). In p.u. system, Air gap torque = air gap power thus $T_e = \frac{E' E_b sin(\ delta)}{X_T}$
- Linearising, the above equation along $\delta$= $\delta_o$ we have $\frac{E' E_b}{X_T} cos(\delta_o)\Delta\delta$
- Rotational equations in pu is given as $p\Delta\omega_r = \frac{1}{2H}(T_m - T_e - K_D\Delta\omega_r)$ and $p\delta = \omega_o\Delta\omega_r$
- Linearizing we have

$$p\Delta\omega_r = \frac{1}{2H}(\Delta T_m - K_s\Delta\delta - K_D\Delta\omega_r)$$

$$p\Delta\delta = \omega_o\Delta\omega_r$$

From above equations state space vector matrix form is given as $\begin{bmatrix} \dot{\Delta}\omega_r \\ \dot{\Delta}\delta \end{bmatrix}$= $\begin{bmatrix} \frac{-K_D}{2H} & \frac{-K_s}{2H} \\ \omega_0 & 0 \end{bmatrix} \begin{bmatrix} \Delta\omega_r \\ \Delta\delta \end{bmatrix} + \begin{bmatrix} \frac{1}{2H} \\ 0 \end{bmatrix} T_m$ Now we look at block diagram model,

$$\Delta\delta(s) = \frac{\omega_o}{s}[\frac{1}{s2H}(-K_s\Delta\delta - K_D\Delta\omega_r + \Delta T_m)] \tag{41}$$

$$\Delta\delta(s) = \frac{\omega_o}{s}[\frac{1}{s2H}(-K_s\Delta\delta - K_D s\frac{\Delta\delta}{\omega_o} + \Delta T_m)] \tag{42}$$

Rearranging, we have

$$s^2(\Delta\delta) + \frac{K_D}{2H}s(\Delta\delta) + \frac{K_s}{2H}\omega_o(\Delta\delta) = \frac{\omega_o}{2H}\Delta T_m \tag{43}$$

The characteristic equation is given as

$$s^2 + \frac{K_D}{2H}s + \frac{K_s}{2H}\omega_o = 0 \tag{44}$$

This is of the general form

$$s^2 + 2\epsilon\omega_n s + \omega_n^2 = 0 \tag{45}$$

The natural frequency is given by

$$\omega_n = \sqrt{\frac{K_s\omega_o}{2H}}$$

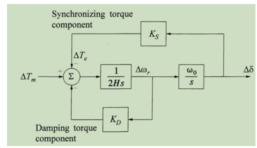


Figure 12: Classic model SMIB

The damping ratio is given as

$$\frac{K_D}{2\sqrt{K_s 2H\omega_o}}$$

Fig.12 shows the final block diagram of the classical model of the SMIB.

## 4.2 Example of SMIB

$$I_t = \frac{(P+jQ)^*}{E_t} = \frac{0.9 - j0.3}{1}$$
$$= 0.9 - j0.3$$

pu

$$E' = E_t + jX'_d I_t$$
$$= 1 + j0.3(0.9 - j0.3)$$
$$1.09 + j0.27 = 1.123\angle 13.92$$
$$E' leads E_b by 13.92 + 36 = 49.92$$
$$X_T = 0.3 + 0.5 + 0.15 = 0.95j$$

pu

$$K_s = synchcoeffecient = \frac{E'E_b}{X_T} cos\delta_o$$
$$0.757 putorque/rad$$

$$\begin{bmatrix} \dot{\Delta}\omega_r \\ \dot{\Delta}\delta \end{bmatrix} = \begin{bmatrix} \frac{-K_D}{2H} & \frac{-K_s}{2H} \\ \omega_0 & 0 \end{bmatrix} \begin{bmatrix} \Delta\omega_r \\ \Delta\delta \end{bmatrix} + \begin{bmatrix} \frac{1}{2H} \\ 0 \end{bmatrix} T_m$$
$$\begin{bmatrix} \dot{\Delta}\omega_r \\ \dot{\Delta}\delta \end{bmatrix} = \begin{bmatrix} -0.143K_D & -0.108 \\ 377 & 0 \end{bmatrix} \begin{bmatrix} \Delta\omega_r \\ \Delta\delta \end{bmatrix} + \begin{bmatrix} 0.143 \\ 0 \end{bmatrix} T_m$$

The eigen values of the state matrix is given by $\begin{bmatrix} -0.143K_D - \lambda & -0.108 \\ 377 & -\lambda \end{bmatrix}$ $= 0$.

$$\lambda^2 + 0.143K_D\lambda + 40.79 = 0$$
$$\omega_n = \sqrt{40.79} = 6.387$$

rad/s

$$\epsilon = 0.0112K_D$$

```
clear all
clc
Sb=4*555*1000000; % on lt of xmer
Vb= 24000;
% post fault
P=0.9;
Q=0.3;% over excited s= 0.9+j0.3; overexcited synch gen means
 inductive
Et= 1; % taking this as base
Eb= 0.995*(cos(-36*pi/180)+sin(-36*pi/180)*j);

inf_angle= 180*(atan(imag(Eb)/real(Eb)))/pi;
Xd=0.3;
H=3.5;% MWs/MVA 2220 MVA, 24 KV base
% Step 1
% find internal voltage
% for that we need current
% I= P-jQ/Et
It= conj(P+Q*j)/Et;
%-------------------------------------------
%step 2 find internal voltage
Eint= Et+ Xd*It*j;
int_angle= 180*(atan(imag(Eint)/real(Eint)))/pi;
%-------------------------------------------
%step 3 the dynamic equation is given as
% pdwr = 1/2h(dTm-dTe-Kd*dwr)
%dTe= Ksddelta
delta= int_angle-inf_angle;
X_T= 0.15+0.5+0.3;
% synch torque equation
Ks= (abs(Eint)*abs(Eb)/X_T)*cos(delta*pi/180);
% STATE MATRICES
% A= State matrix
omegao= 2*pi*60;
%Kd=0;
Kd=-10;
%Kd=10;
A=[-(Kd/2*H),-(Ks/2*H);omegao,0];
[vect,a]= eig(A);
lefteigvect= inv(vect);
```

*Published with MATLAB® R2018a*

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Figure 13: Code for SMIB eigen values and vectors

Eigen values are given as

$$\lambda_1, \lambda_2 = -\epsilon\omega_n \pm \omega_n\sqrt{1-\epsilon^2}$$

The damped frequency is given as

$$\omega_d = \omega_n\sqrt{1-\epsilon^2}$$

The right eigen vector is given as

$$(A - \lambda I)\Phi = 0$$

For $K_D = 10$ $\begin{bmatrix} -1.43-\lambda_i & -0.108 \\ 377 & -\lambda_i \end{bmatrix} \begin{bmatrix} \phi_{1i} \\ \phi_{2i} \end{bmatrix}$ =0 Time response is given as $\begin{bmatrix} \Delta\omega_r \\ \Delta\delta \end{bmatrix} = \begin{bmatrix} \phi_{11} & \phi_{12} \\ \phi_{21} & \phi_{22} \end{bmatrix} ( \begin{bmatrix} \psi_{11} & \psi_{12} \\ \psi_{21} & \psi_{22} \end{bmatrix} \begin{bmatrix} \Delta\omega_r(0) \\ \Delta\delta(0) \end{bmatrix} \begin{bmatrix} e^{\lambda_1 t} \\ e^{\lambda_2 t} \end{bmatrix})$

# 5 Impact of field circuit dynamics

- include effect of field flux variations
- field voltage is assumed constant.
- synch. machine state space model is developed then it is combined with network equations.

## 5.1 synch. machine equations

Take three stator windings ABC,. For rotor take 4 windings, 3 damper windings, and one field winding. Two damper winding along q axis and one field and one dapmer along d axis. To find inductance consider following flux linkage. (Lii= $\phi_i/i_i$, Lij= $\phi_i/i_j$)

- Flux linkages
- Stator winding- self inductance, mutual inductance between phase winding
- Stator to rotor flux linkage
- rototr to stator flux linkage
- rotor to winding flux linkage

### 5.1.1 Inductances

Phase winding A has current = ia. Let, Self inductance = $\phi_{self}/ia$, self inductance is constant = LAo. similarly, self inductance= LBo, LCo. Flux linkage between AB. To find inductance assume flux of A linking to B. Now this flux with pass through stator, air gap and rotor. So unexcite the rotor windings. If the rotor is cylindrical the flux linkages are constant hence the mutual inductances are constant. For salient pole First align stator flux linkage to rotor and then from rotor to other field winding.

Stator armature **Armature Phase ABC Self inductance: Assume armature on rotor** armature field sirf a phase excited hai. Two reaction theory se dq axis mei flux linkage resovle kar lo jo field se link honge

$$F_d = Facos(\theta) \tag{46}$$

$$F_q = -Fasin(\theta) \tag{47}$$

Flux hoga MMF/reluctance, let 1/reluctance = Permanance $\Lambda$, so dq axis flux per pole is

$$\phi_d = \Lambda_d F_{da} \tag{48}$$

$$\phi_q = \Lambda_q F_{qa} \tag{49}$$

Self inductance nikalne ke liye field winding se magnetic circuit close loop hota hai, to ab field se armature kitna link ho rha hai wo nikalna hai to uska equation

$$\phi_g = \phi_d cos(\theta) + \phi_q cos(90 + \theta) \tag{50}$$

$$\phi_g = \phi_d cos(\theta) - \phi_q sin(\theta) \tag{51}$$

$\phi_d$,$\phi_q$ replace kardo to second harmonic+ constant aa jayegi

$$\phi_g = \Lambda_d Fa * cos^2(\theta) + \Lambda_d Fa * sin^2(\theta) \tag{52}$$

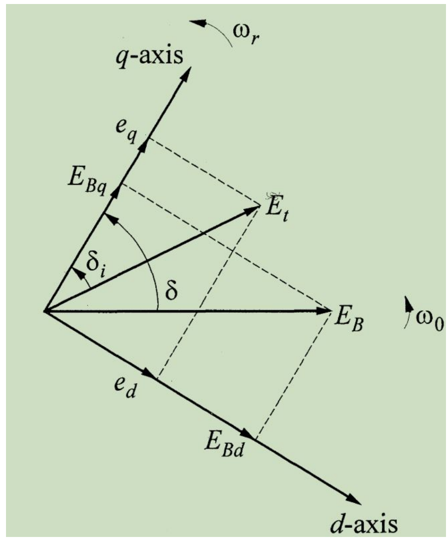


Figure 14: phasor of rotor and Eb

finally

$$Laa = Lago + Lga2cos(\theta) \tag{53}$$

**Mutual Armature Inductances**

Torque equations are given as:

$$p\Delta\omega_r = \frac{1}{2H}(T_m - T_e - K_D\Delta\omega_r)$$

$$p\delta = \omega_o\Delta\omega_r)$$

$$\omega_o = 2\pi f_o$$

- $E_b$ is taken as ref. so network equations can be applied.
- $E_t$ leads $E_b$ current from gen terminal to bus.
- rotor position is determined by q axis.

The field flux dynamics is given as

$$p\Psi_{fd} = \frac{R_{fd}\omega_o}{L_{fd}}E_{fd} - \omega_o R_{fd} i_{fd}$$

The new state variables are

$$\Delta\omega_r, \delta, \Psi_{fd}$$

. To express $T_e$ and $i_f d$ in terms of state variables equivalent circuit of flux linkage and current is shown in Fig. 15. The stator and rotor flux linkage are given as

$$\Psi_d = -L_l i_d + L_{ads}(-i_d + i_{fd}) \tag{54}$$

$$\Psi_q = -L_l i_q + L_{aqs}(-i_q) \tag{55}$$

$$\Psi_{fd} = L_{fd} i_{fd} + L_{ads}(-i_d + i_{fd}) \tag{56}$$

From above equations, $L_{ads}(-i_d + i_{fd}) = \Psi_{ad}$ and $L_{ads}(-i_q) = \Psi_{aq}$ where $\Phi_{ad}$ and $\Phi_{aq}$ are air gap mutual flux linkages between d-q axis stator and rotor coils and rotor field circuit in d axis.

$$i_{fd} = -L_l i_d + L_{ads}(-i_d + i_{fd}) \tag{57}$$

Then

$$\Psi_{ad} = (\frac{L_{fd} L_{ads}}{L_{fd} + L_{ads}})(-i_d + \Phi_{fd})$$

$$\Psi_{aq} = -L_{aqs} i_q$$

Air gap torque is given as

$$T_e = \Psi_{ad} i_q - \Psi_{aq} i_d \tag{58}$$

So we have established $i_{fd}$ and $T_e$ in terms of $\Psi_{fd}, i_d, i_q, \Psi_{ad}, \Psi_{aq}$. Before we go into network equations we look into emf equations. Neglecting p$\Psi$ and speed voltage terms

$$e_d = -R_a i_d - \Psi_q \tag{59}$$

$$e_q = -R_a i_q - \Psi_d \tag{60}$$

where $\Psi_q = L_l i_q - \Psi_{aq}$ and $\Psi_d = L_l i_d - \Psi_{ad}$

## 5.2 Network Equations

Terminal voltage in steady state along d-q axis is given as

$$E_t = e_d + j e_q \tag{61}$$

$$E_b = e_{bd} + j e_{bq} \tag{62}$$

$$E_t = E_b + (R_e + jX_e) I_t \tag{63}$$

$$e_d + j e_q = e_{bd} + j e_{bq} + (R_e + jX_e)(i_d + j i_q) \tag{64}$$

$$e_{bd} + j e_{bq} = E_b sin\delta + j E_b cos\delta \tag{65}$$

$$e_d = e_{bd} + R_e i_d - X_e i_q \tag{66}$$

$$e_q = e_{bq} + R_e i_q + X_e i_d \tag{67}$$

$$i_d(R_e + R_a) - i_q(X_e + L_l + L_{aqs}) + e_{bd} = 0 \tag{68}$$

$$i_q = \frac{1}{R_e + R_a}[-i_d(X_e + L_l + L'_{ads}) - e_{bq} + L'_{ads}\frac{\Psi_{fd}}{L_{fd}}] \tag{69}$$

$$X_e + L_l = X_T, L_{fd} + L_{ads} = L_T, R_e + R_a = R_T$$

$$i_q = R_T \frac{[L_{ads}\Psi_{fd} + X_T + \frac{L_{fd}L_{ads}}{L_T} e_{bd} - R_T L_T e_{bq}]}{R_T^2 L_T + (X_T^2 L_T + L_{aqs}L_T X_T + X_T L_{fd} L_{ads} + L_{aqs} L_{ads} L_{fd})} \tag{70}$$

$$i_d = (X_T + L_{aqs}) \frac{[L_{ads}\Psi_{fd} + X_T + \frac{L_{fd}L_{ads}}{L_T} e_{bd} - R_T L_T e_{bq}]}{R_T^2 L_T + (X_T^2 L_T + L_{aqs}L_T X_T + X_T L_{fd} L_{ads} + L_{aqs} L_{ads} L_{fd})} - \frac{e_{bd}}{R_T} \tag{71}$$

In book this is given as

$$i_d = \frac{X_{Tq}[\Psi_{fd}(\frac{L_{ads}}{L_{ads}+L_{fd}} - E_b cos\delta)] - R_T E_b sin\delta}{D} \tag{72}$$

$$i_q = \frac{R_T[\Psi_{fd}(\frac{L_{ads}}{L_{ads}+L_{fd}} - E_b cos\delta)] + X_{Td} E_b sin\delta}{D} \tag{73}$$

where

$R_T = R_a + R_e, X_{Tq} = X_e + (L_{aqs} + L_l), X_{Td} = X_e + (L'_{ads} + L_l), D = R_T^2 + X_{Tq} X_{Td}$

### 5.2.1 Linearized equations

$$\Delta i_d = m_1 \Delta\delta + m_2 \Delta\Psi_{fd}$$

$$\Delta i_q = m_3 \Delta\delta + m_4 \Delta\Psi_{fd}$$

$$m_1 = \frac{E_b(X_{Tq} sin\delta_o - R_T cos\delta_o)}{D}$$

$$m_2 = \frac{X_{Tq} L_{ads}}{D(L_{ads} + L_{fd})}$$

$$m_3 = \frac{E_b(R_T sin\delta_o + X_{Td} cos\delta_o)}{D}$$

$$m_4 = \frac{R_T L_{ads}}{D(L_{ads} + L_{fd})}$$

The deviations in air gap flux between windings are given as

$$\Delta\Psi_{ad} = L'_{ads}(\frac{\Delta\Psi_{fd}}{L_{fd}} - \Delta i_d)$$

$$\Delta\Psi_{aq} = L_{aqs}(-\Delta i_q)$$

$$\Delta i_{fd} = \frac{\Delta\Psi_{fd} - \Delta\Psi_{ad}}{L_{fd}}$$

Replace $i_d$ and $i_q$ in the flux and field currents.

$$T_e = K_1 \Delta + K_2 \Psi_{fd} \tag{74}$$

where

$$K_1 = m_3(\Psi_{ado} + L_{aqs} i_{do}) - m_1(\Psi_{aqo} + L'_{ads} i_{qo})$$

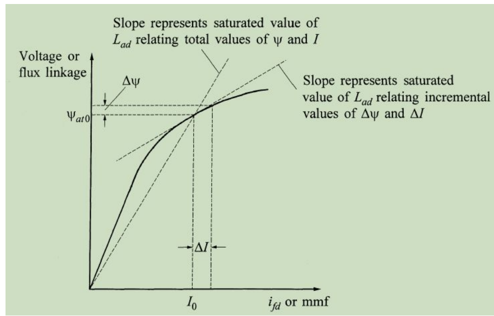


Figure 15: Saturation

$$K_2 = m_4(\Psi_{ado} + L_{aqs}i_{do}) - m_2(\Psi_{aqo} + L'_{ads}i_{qo}) + \frac{L'_{ads}}{L_{fd}}i_{qo}$$

$$\begin{bmatrix} \dot{\Delta}\omega_r \\ \dot{\Delta}\delta \\ \dot{\Delta}\Psi_{fd} \end{bmatrix} = \begin{bmatrix} a_{11} & a_{12} & a_{13} \\ a_{21} & 0 & 0 \\ 0 & a_{32} & a_{33} \end{bmatrix} \begin{bmatrix} \Delta\omega_r \\ \Delta\delta \\ \Delta\Psi_{fd} \end{bmatrix} + \begin{bmatrix} b_{11} & 0 \\ 0 & 0 \\ 0 & b_{32} \end{bmatrix} \begin{bmatrix} \Delta T_m \\ \Delta E_{fd} \end{bmatrix}$$

- $a_{11} = \frac{-K_D}{2H}$
- $a_{12} = \frac{-K_1}{2H}$
- $a_{13} = \frac{-K_2}{2H}$
- $a_{21} = 2\pi f_o$
- $a_{32} = \frac{\omega_o R_{fd}}{L_{fd}}[m_1 L'_{ads}]$
- $a_{33} = \frac{\omega_o R_{fd}}{L_{fd}}[1 + m_2 L'_{ads} - \frac{L'_{ads}}{L_{fd}}]$
- $b_{11} = \frac{1}{2H}$
- $b_{32} = \frac{\omega_o R_{fd}}{L_{adu}}$

**Representattion of Saturation in small signal study of synch. machine**

- Total saturation- Associated with the total values of the fluxes and currents
- Incremental saturation- Associated with perturbed value of flux linkage and current

$$L_{ads,incr} = K_{sd,incr} L_{adu} \tag{75}$$

$$K_{sd,incr} = \frac{1}{1 + B_{sta} A_{sat} e^{B_{sat}(\Psi_{ato} - \Psi_{T1})}} \tag{76}$$

**Summary of the procedure**

- Steady state operating points, machine and network parameters are known

| | | | | | | |
|---|---|---|---|---|---|---|
| P | Q | $E_t$ | $R_e$ | $X_e$ | $L_d$ | $L_q$ |
| $L_l$ | $R_a$ | $L_{fd}$ | $A_{sat}$ | $B_{sat}$ | $R_{fd}$ | $\Psi_{tl}$ |

- Compute initial steady state values of system $I_t, \phi, K_{sd}, K_{sq}, L_{ds}, L_{qs}, e_{do}, e_{qo}, i_{do}, i_{qo}, e_{bdo}, e_{bqo}, \delta_o, E_b, i_{fdo}, E_{fdo} = L_{adu}i_{fdo}, \Psi_{ado}, \Psi_{aqo}, \delta_i = tan^{-1}\frac{I_tX_{qs}cos(\phi)-I_tR_asin(\phi)}{E_t+I_tR_acos(\phi)+I_tX_{qs}sin(\phi)}$ , $\delta_o = tan^{-1}\frac{e_{bdo}}{e_{bqo}}$
- Compute incremental saturation factors and later $L_{ads}, L_{aqs}, L'_{ads}, R_T, X_T, D, m_1, m_2, m_3, m_4, K_1, K_2$
- Find A matrix elements

# 6 Effects of excitation system

$$E_t = e_d + je_q$$
$$E_t^2 = (e_d^2 + e_q^2)$$
$$(E_t + \Delta E_t)^2 = ((e_d + \Delta e_d)^2 + (e_q + \Delta e_q)^2)$$
$$(E_t + \Delta E_t)^2 = ((e_d + \Delta e_d)^2 + (e_q + \Delta e_q)^2)$$
$$E_{to}\Delta E_t = ((e_{do}\Delta e_d) + (e_{qo}\Delta e_q))$$

The equations from circuit coils pov can be written as

$$\delta e_d = -i_dR_a + L_l\Delta i_q - \Psi_{aq}$$

$$\delta e_q = -i_qR_a - L_l\Delta i_d - \Psi_{ad}$$

Then $E_t$ is also a function on $\delta$ and $\Psi_{fd}$ as

$$\Delta E_t = K_5\Delta\delta + K_6\Delta\Psi_{fd}$$

$$K_5 = \frac{e_{do}}{E_{to}}[-R_am_1 + L_1m_3 + L_{aqs}m_3] + \frac{e_{qo}}{E_{to}}[-R_am_3 - L_1m_1 - L'_{ads}m_1] \quad (77)$$

$$K_6 = \frac{e_{do}}{E_{to}}[-R_am_2 + L_1m_4 + L_{aqs}m_4] + \frac{e_{qo}}{E_{to}}[-R_am_4 - L_1m_2 + L'_{ads}(\frac{1}{L_{fd} - m_2})] \quad (78)$$

Excitation model is given as

$$p\Delta v_1 = \frac{1}{T_R}(\Delta E_t - \Delta v_1)$$

$$p\Delta v_1 = \frac{K_5}{T_R}\Delta\delta + \frac{K_6}{T_R}\Delta\Psi_{fd} - \frac{1}{T_R}\Delta v_1$$

In terms of $\Delta v_1$ $\delta E_{fd}$ is given as

$$\Delta E_{fd} = K_A\Delta - v_1$$

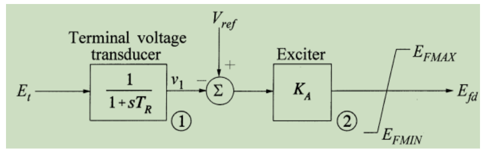


Figure 16: Excitation circuit model

Thus field circuit dynamic flux equation becomes

$$p\Delta\Psi_{fd} = a_{31}\Delta\omega_r + a_{32}\Delta\delta + a_{33}\Delta\Psi_{fd} + a_{34}\Delta v_1$$

In previous case $\Delta E_{fd}$ was the control input but in the exciter case it is included as a state equation thus the state matrix is transformed as.

$$\begin{bmatrix} \dot{\Delta}\omega_r \\ \dot{\Delta}\delta \\ \dot{\Delta}\Psi_{fd} \\ \dot{\Delta}v_1 \end{bmatrix} = \begin{bmatrix} a_{11} & a_{12} & a_{13} & 0 \\ a_{21} & 0 & 0 & 0 \\ 0 & a_{32} & a_{33} & a_{34} \\ 0 & a_{42} & a_{43} & a_{44} \end{bmatrix} \begin{bmatrix} \Delta\omega_r \\ \Delta\delta \\ \Delta\Psi_{fd} \\ \Delta v_1 \end{bmatrix} + \begin{bmatrix} b_1 \\ 0 \\ 0 \\ 0 \end{bmatrix} \begin{bmatrix} \Delta T_m \end{bmatrix} \text{ where}$$

- $a_{34} = \frac{-\omega_o R_{fd}}{L_{adu}} K_A$
- $a_{42} = \frac{K_5}{T_R}$
- $a_{43} = \frac{K_6}{T_R}$ $a_{44} = \frac{-1}{T_R}$

# 7 Multi machine analysis

For stability studies machine stator transients and network are neglected. The dynamics of rotor circuit, excitation systems, prime mover and other devices are represented by differential equations. Each machine is represented by it's own d-q reference frame. There is need for common reference (R-I) frame for all the machines which rotates at synchronous speed as shown in below Fig. 17. R axis of common reference frame is used as reference for measuring machine rotor angle.

- In **Detailed Model** including rotor circuit dynamics and machine the rotor angle $\delta$ is defined as the angle by which q- axis leads the R axis.
- In **Classical Model** the rotor angle $\delta$ is defined as the angle by which internal voltage E' leads the R axis.

**Formulation of state equations** The formulation of state equations of multi machine analysis is similar to the smib system but large transmission network, machine dynamic equations, static and dynamic loads, various excitation systems and prime mover model makes it difficult for analysis. A step by step approach is taken for modeling.

$$\dot{x}_i = A_i x_i + B_i \Delta v \tag{79}$$

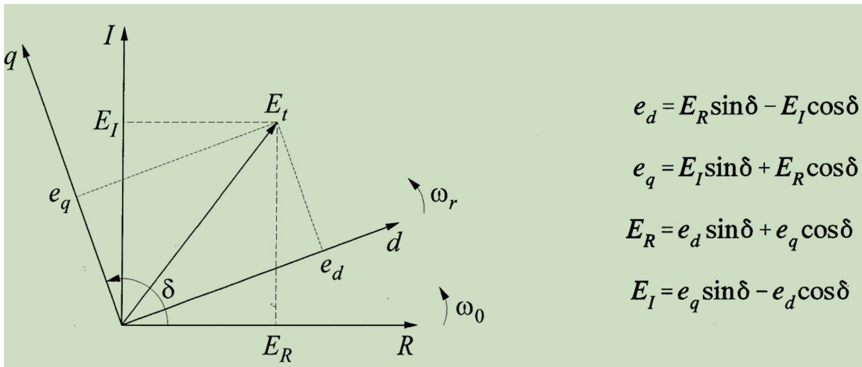


Figure 17: d-q to RI reference frame

$$\Delta i = C_i x_i - Y_i \Delta v \tag{80}$$

where $x_i$ is the perturbed values of individual device states. $i$ is the current injection in the network by a device. $v$ is the vector of network bus voltages to which the device is connected.

The above state-space equation is meant for individual device, when all the devices are clubbed together the state- space equation are given as

$$\dot{x} = A_D x + B_D \Delta v \tag{81}$$

$$\Delta i = C_D x_i - Y_D \Delta v \tag{82}$$

Here x is the state vector of entire system, $A_D, C_D$ are the diagonal matrices of $(A_i, C_i)$ associated with individual devices.
The interconnected transmission network is shown by node equation: $\Delta i = Y_N \Delta V$ where $Y_N$ includes the effect of the non linear static loads. Replacing, $\Delta i$ in equation 74 we have

$$\Delta v = (Y_N + Y_D)^{-1} C_D x$$

Replacing $(\Delta v)$ in equation 73.

$$\dot{x} = A_D x + B_D (Y_N + Y_D)^{-1} C_D x = Ax$$

$A_i, B_i, C_i, Y_i$ are calculated as per the previous sections.
**Computer programs for multi machine stability analysis**

- MASS (Multi area small signal stability) This uses general approach as above for formulating state matrix. This computes eigen values of the matrix by using QR transformative method. This method does not exploit sparsity, so it is not used for large systems.
- AESOPS algorithm its an frequency response based method to calculate eigen values based on rotor angle modes.

- Selective Modal analysis selects crucial states relevant for selected modes and constructs reduced order models.
- PEALS it uses two methods modified arnoldi and AESOPS algortihm.

**Representation of static Loads** *(a) Constant impedence linear load* Shunt admittance to ground representing the load is computed as

$$G_L = \frac{P_{Lo}}{V_o^2}$$

$$B_L = \frac{-Q_{Lo}}{V_o^2}$$

*(b) Non linear load* The voltage dependent load characteristic of load can be represented as:

$$P_L = P_{Lo}(\frac{V}{V_o})^m$$

$$Q_L = Q_{Lo}(\frac{V}{V_o})^n$$

where V is the bus magnitude given as

$$V = \sqrt{V_R^2 + V_I^2 +}$$

The R -I load current are given as

$$i_R = P_L\frac{V_R}{V^2} + Q_L\frac{V_I}{V^2}$$

$$i_I = P_L\frac{V_I}{V^2} - Q_L\frac{V_R}{V^2}$$

Linearizing above equations we find

$$\Delta i_R = \frac{P_o}{V_o^2}(\Delta V_R)+\frac{Q_o}{V_o^2}(\Delta V_I)+\frac{V_{Ro}}{V_o^2}(\Delta P_L)+\frac{V_{Io}}{V_o^2}(\Delta Q_L)+\frac{-2}{V_o^3}(P_{Lo}V_{Ro}+Q_{Lo}V_{Io})\Delta V$$

$$\Delta i_I = \frac{-Q_o}{V_o^2}(\Delta V_R)+\frac{P_o}{V_o^2}(\Delta V_I)+\frac{V_{Io}}{V_o^2}(\Delta P_L)+\frac{-V_{Ro}}{V_o^2}(\Delta Q_L)+\frac{-2}{V_o^3}(P_{Lo}V_{Io}-Q_{Lo}V_{Ro})\Delta V$$

where $\Delta V = \frac{V_{Ro}}{V_o}\Delta V_R + \frac{V_{Io}}{V_o}\Delta V_I$ $\Delta P = m\frac{P_o}{V_o}\Delta V$ $\Delta Q = n\frac{Q_o}{V_o}\Delta V$

Thus the $\Delta i_R, \Delta i_I$ can be represented as algebaric equations as $\begin{bmatrix}\Delta i_R \\ \Delta i_I\end{bmatrix} = \begin{bmatrix}G_{RR} & B_{RI} \\ -B_{IR} & G_{II}\end{bmatrix}\begin{bmatrix}\Delta V_R \\ \Delta V_I\end{bmatrix}$ The conductance and admittance values are given in fig.18.

**Redundant States** With state matrix without considereing infinite bus will have one or two zero eigen values. Reason- Lack of uniqueness of absolute rotor angle i.e if rotor angle of all the machines are changed by same value system stability is not effected. To counter this one of the machine is chosen as reference and its rotor value is kept constant i.e $p\Delta\delta_r = 0$. For other machines $p\Delta\delta_i = \Delta\omega_i - \Delta\omega_r ef$

The second zero eigen value exist because torque equations are considered independent of machine angular speed. This can also be avoided by refering to speed deviations to a reference machine.

$$G_{RR} = \frac{P_{L0}}{V_0^2}\left((m-2)\frac{v_{R0}^2}{V_0^2}+1\right)+\frac{Q_{L0}}{V_0^2}\left((n-2)\frac{v_{R0}v_{I0}}{V_0^2}\right)$$

$$B_{RI} = \frac{Q_{L0}}{V_0^2}\left((n-2)\frac{v_{I0}^2}{V_0^2}+1\right)+\frac{P_{L0}}{V_0^2}\left((m-2)\frac{v_{R0}v_{I0}}{V_0^2}\right)$$

$$B_{IR} = \frac{Q_{L0}}{V_0^2}\left((n-2)\frac{v_{R0}^2}{V_0^2}+1\right)-\frac{P_{L0}}{V_0^2}\left((m-2)\frac{v_{R0}v_{I0}}{V_0^2}\right)$$

$$G_{II} = \frac{P_{L0}}{V_0^2}\left((m-2)\frac{v_{I0}^2}{V_0^2}+1\right)-\frac{Q_{L0}}{V_0^2}\left((n-2)\frac{v_{R0}v_{I0}}{V_0^2}\right)$$

Figure 18: Conductance and admittance values

## 7.1 AESOPS Algorithm

- This algorithm computes modes associated only with the rotor angles.
- An external torque is applied to rotor of generator under observation which consists of complex waveform and resonant frequency is observed.
- If generator has several modes of rotor oscillations then eigen value depends on initial value assumed.

The AESOPS algorithm is derived form linearized rotor motion as

$$2H\frac{d\Delta\omega_r}{dt} = T_m - T_e = T_m - (K_s\Delta\delta + K_D\Delta\omega_r)$$

Taking laplace

$$2Hs\Delta\omega_r = T_m(s) - T_e(s) = T_m(s) - (K_s(s)\Delta\frac{\omega_r}{s} + K_D(s)\Delta\omega_r)$$

$$\Delta T_m = [2Hs + K_D(s) + K_s(s)/s]\Delta\omega_r$$