Exam 01-Control Engineering

Friday, 24. January 2020 15:09

1

a)

$$\ddot{X} = -\dot{X}U + \cos(X)\,X + \frac{U^2}{4} = f(X, \dot{X}, U)$$

Let us, say the initial point is $P \equiv (X(0), \dot{X}(0), U(0))$

$$= \left.\frac{\partial f}{\partial X}\right|_P X + \left.\frac{\partial f}{\partial \dot{X}}\right|_P \dot{X} + \left.\frac{\partial f}{\partial U}\right|_P U$$

$$= \left.\left(\cos(X) - X\sin X\right)\right|_P X + \left.(-U)\right|_P \dot{X} + \left.\left(\frac{U}{2} - \dot{X}\right)\right|_P U$$

$$\ddot{X} = \{\cos X_0 - X_0 \sin X_0\} X + (-U_0)\dot{X} + \left(\frac{U_0}{2}\right) U$$

$$\Rightarrow \ddot{X} + (U_0)\dot{X} + (X_0 \sin X_0 - \cos X_0) X = \left(\frac{U_0}{2}\right) U$$

b.

$$\ddot{x} + 4\dot{x} + 4x = 2u$$

$$X(s)(s^2 + 4s + 4) = 2U(s)$$

For unit step response,

$$u(t) = \begin{cases} 1, & t > 0 \\ 0 & t < 0 \end{cases}$$

$$\therefore \quad U(s) = \frac{1}{s}$$

$$\therefore \quad X(s) = \frac{2}{s} \frac{1}{(s^2 + 4s + 4)}$$

$$= \frac{2}{s} \frac{1}{(s+2)^2}$$

Now,

$$x(t) = 2 \int \mathcal{L}^{-1}\left\{\frac{1}{(s+2)^2}\right\}(t) \cdot dt$$

$$= 2 \int t e^{-2t} \cdot dt$$

$$= 2\left[\frac{t e^{-2t}}{-2} - \int \frac{e^{-2t}}{-2} \cdot dt\right]$$

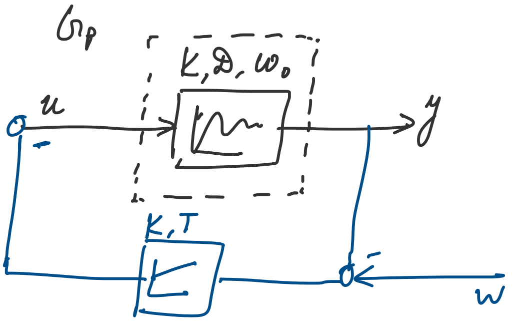
$$= 2\left[\frac{-t\,e}{2} + \frac{1}{2}\,\frac{e}{-2}\right] + c$$

$$= -t\,e^{-2t} - \frac{e^{-2t}}{2} + c$$

$$= -e^{-2t}\left(t + \frac{1}{2}\right) + c$$

2.

a)



b) $$G_c = K\left(1 + \frac{1}{Tj\omega}\right)$$

$$= K\left(\frac{1 + Tj\omega}{Tj\omega}\right)$$

phase, $\phi(\omega) = \tan^{-1}(T\omega) - \frac{\pi}{2}$

At eigenfrequency,

$$\omega_e = \frac{1}{T}$$

$$\phi(\omega_e) = \frac{\pi}{4} - \frac{\pi}{2} = -\frac{\pi}{4}$$

$\therefore$ $\omega = 10\ sec^{-1}$

or $T = 0.1$ sec.

at, point of A_R

$\phi_0(\omega) = -180°$

$\therefore$ $A_R = \left\{\frac{1}{G_0(\omega_\pi)}\right\} = \frac{1}{0.5} = 2$

d Now, $K \neq 1$

$\alpha_R = 45° = \phi(\omega) - (-180°)$

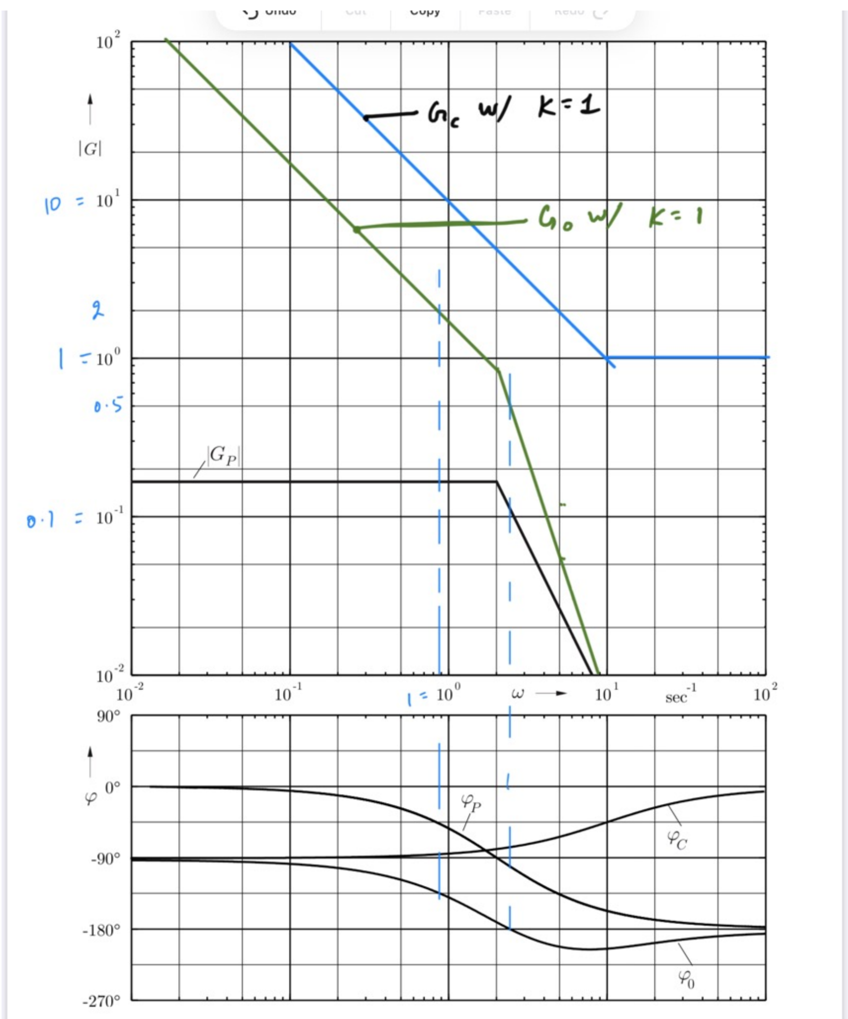
$\phi(\omega) = 45° - 180°$

$\phi(\omega) = -135^\circ$

$\therefore \quad G_o(j\omega) = 2$

Now, new value of K such that $G_o(j\omega) = 1$

is $\quad K = \dfrac{1}{G_o(j\omega)} = 0.5$



3

a. $\quad G_o = G_p G_c$

$$= \left(\frac{K_s}{1+T_s s}\right)\left(\frac{K_I}{s}\right)\left(\frac{K_c(1+T_n s)}{T_n s}\right)$$

$$= \frac{(K_s K_I K_c)}{T_n s^2} \frac{(1+T_n s)}{(1+T_s s)}$$

b $\quad G(s) = \dfrac{G_o(s)}{1+G_o(s)}$

$$= \frac{K_s K_I K_c (1+T_n s)}{T_n s^2 (1+T_s s) + K_s K_I K_c (1+T_n s)}$$

c) For closed loop to be stable,
apply Hurwitz criteria on $1 + G_o(s)$
For Numerator of $1 + G_o(s)$

$\Rightarrow Z(s) = s^3 (T_n T_s) + s^2 (T_n) + s(T_n K_s K_I K_c) + K_s K_I K_c$

Now,
1 st condition : All coefficients > 0

$T_n T_s > 0 \Rightarrow T_n > 0$

$T_n > 0$ ✓

$T_n K_s K_I K_c > 0$ ✓

$K_s K_I K_c > 0$ ✓

2nd condition

$$\begin{vmatrix} T_n & K_s K_I K_c \\ T_n T_s & T_n K_s K_I K_c \end{vmatrix} > 0$$

$$\Rightarrow K_s K_I K_c T_n^2 - K_s K_I K_c T_n T_s > 0$$

$$\boxed{T_n > T_s}$$ ✓

4 $$\underbrace{\begin{pmatrix} x_1 \\ x_2 \end{pmatrix} = \begin{pmatrix} y \\ \dot{y} \end{pmatrix}}_{\text{given}} = X \text{ (say)}$$

Now, $\dot{X} = \begin{pmatrix} \dot{x}_1 \\ \dot{x}_2 \end{pmatrix} = \begin{pmatrix} \dot{y} \\ \ddot{y} \end{pmatrix} = AX + BU$ ——— ①

And, $y = CX + DU$ ——— ②

We have $U = u$

comparing it with

$$\ddot{y} + l_1 \dot{y} + 2 l_2 y = u$$

we have,

$$\dot{X} = \underset{2\times1}{\begin{pmatrix} \dot{y} \\ \ddot{y} \end{pmatrix}} = \underset{2\times2}{\begin{pmatrix} 0 & 1 \\ -2l_2 & -l_1 \end{pmatrix}} \underset{2\times1}{\begin{pmatrix} y \\ \dot{y} \end{pmatrix}} + \underset{2\times1}{\begin{pmatrix} 0 \\ 1 \end{pmatrix}} u$$ ——— ③

Also, $y = \begin{pmatrix} 1 & 0 \end{pmatrix} \begin{pmatrix} y \\ \dot{y} \end{pmatrix}_{2\times 1} + \begin{pmatrix} 0 \end{pmatrix} u$ ——— (4)

Now, comparing eq 3 and 4 with 1 and 2

$$A = \begin{pmatrix} 0 & 1 \\ -2l_2 & -l_1 \end{pmatrix} \qquad B = \begin{pmatrix} 0 \\ 1 \end{pmatrix}$$

$$C = \begin{pmatrix} 1 & 0 \end{pmatrix} \qquad D = 0$$

b $\quad u = -Kx = -\begin{pmatrix} k_1 & k_2 \end{pmatrix} x$

Now,
$$\dot{X} = AX + BU$$
$$= (AX - BKX)$$
$$\dot{X} = (A - BK)X \quad \text{———} \quad (5)$$

Now, $\because$ 2 poles are located at -3

i.e $\quad s_{p_1} = s_{p_2} = -3$

$\therefore$ characteristics equation becomes

$$(\lambda - s_{p_1})(\lambda - s_{p_2}) = 0$$
$$(\lambda+3)^2 = 0$$
$$\lambda^2 + 6x + 9 = 0 \quad \text{———} \quad 6$$

From equation (5) characteristics eq. becomes.

$$|sI - (A - BK)| = 0$$

We have, $A - BK$

$$= \begin{pmatrix} 0 & 1 \\ -2 & 3 \end{pmatrix}_{2\times 2} - \begin{pmatrix} 0 \\ 1 \end{pmatrix}_{2\times 1} \begin{pmatrix} k_1 & k_2 \end{pmatrix}_{1\times 2}$$

$$= \begin{pmatrix} 0 & 1 \\ -2 & 3 \end{pmatrix} - \begin{pmatrix} 0 & 0 \\ k_1 & k_2 \end{pmatrix}$$

$$= \begin{pmatrix} 0 & 1 \\ -2-k_1 & 3-k_2 \end{pmatrix}$$

Now, $|sI - A| = 0$

$$\begin{vmatrix} s & -1 \\ 2+k_1 & s+k_2-3 \end{vmatrix} = 0$$

$$s(s+k_2-3) + (2+k_1) = 0$$

$$\Rightarrow s^2 + s(k_2-3) + (2+k_1) = 0 \quad \text{—— } (7)$$

Compare eq. 6 and 7

$k_2 - 3 = 6 \qquad k_1 + 2 = 9$

$k_2 = 9 \qquad k_1 = 7$

$\therefore K = [7 \quad 9]$

c) $u = -2x_1 - 5x_2$

$u = -2y - 5\dot{y}$

In time discrete values,

$$u_k = -2y_k - 5\left(\frac{y_k - y_{k-1}}{T}\right)$$

$$= -2y_k - 50(y_k - y_{k-1})$$

$$\boxed{u_k = y_k(-52) + 50\, y_{k-1}}$$