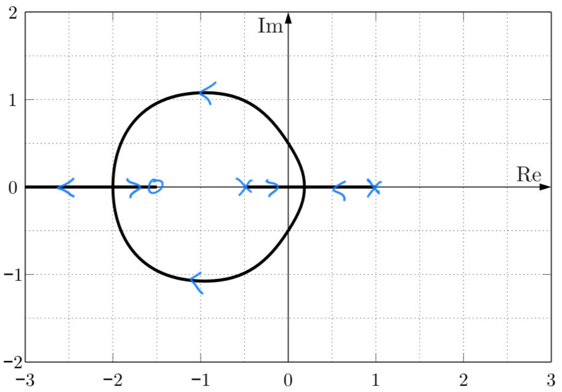
Exam 02-Control Engineering

Monday, 10. February 2020 15:45

1.

a



b From figure,

$s_{N_1} = -1.5$

$s_{P_1} = -0.5 \qquad s_{P_2} = 1$

Now,

$$G_0(s) = 1 \cdot \frac{(s+1.5)}{(s+0.5)\,(s-1)}$$

$$= \frac{1}{2}\left\{\frac{2s+3}{(s+0.5)\,(s-1)}\right\}$$

$$= \frac{1}{2}\,\frac{(s+0.5)+(s-1)+3.5}{(s+0.5)\,(s-1)}$$

$$= \frac{1}{2}\left\{\frac{1}{s-1}+\frac{1}{s+0.5}+\frac{3.5}{(s+0.5)(s-1)}\right\}$$

$$= \frac{1}{2}\left\{\frac{-1}{1-s}+\frac{2}{1+2s}+\frac{-7}{(1+2s)(1-s)}\right\}$$

$$g(t) = \mathcal{L}^{-1}\{G_0(s)\}(t)$$

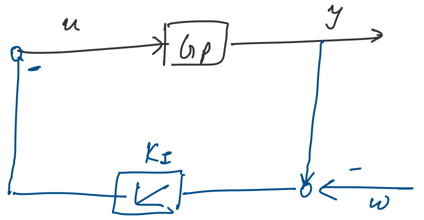
$$= \frac{1}{2}\left\{+1\,e^{t}+\frac{2}{2}e^{-t/2}+\frac{-7}{3}\left(e^{-t/2}-e^{t}\right)\right\}$$

$$= \frac{1}{2}\left\{e^{t}\left(\frac{10}{3}\right)+e^{-t/2}\left(\frac{-4}{3}\right)\right\}$$

$$g(t) = \frac{5}{3}e^{t} - \frac{2}{3}e^{-t/2}$$

2.

a



b The $G_I(j\omega)$ controller always show integrating behaviour. ∴ It is preferable for stationary accuracy

From Bode plot:

[illegible]

$\therefore$ $G_{PD}(j\omega)$ is preferable w.r.t phase margin

c) $\because$ General form of $G_p = \dfrac{1}{T_1 T_2 s^2 + (T_2 + T_1)s + 1}$

with slope changes of -1 at $\frac{1}{T_1}$ and $\frac{1}{T_2}$

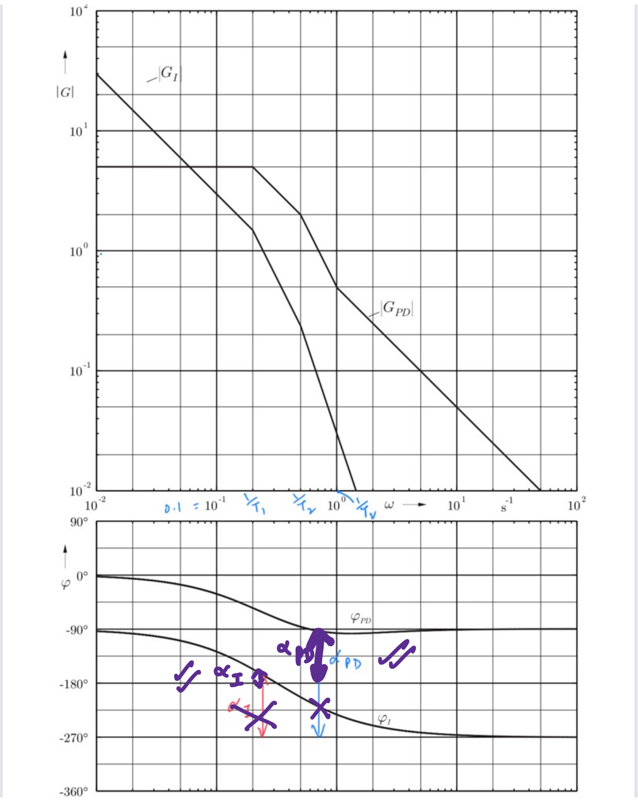
From BODE Plot and comparing it w/ given $G_p(j\omega)$

$\frac{1}{T_1} = 0.2\ sec^{-1} \Rightarrow T_1 = 5\ sec$

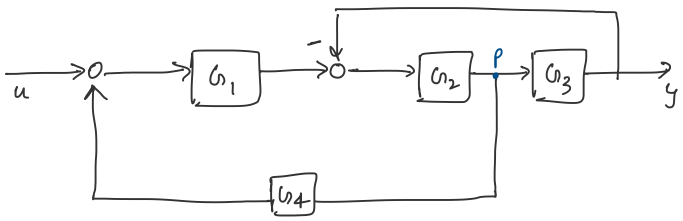
$\frac{1}{T_2} = 0.5\ sec^{-1} \Rightarrow T_2 = 2\ sec$

$\therefore$ $a_2 = T_1 T_2 = 10\ sec^2$ ✓

$a_1 = (T_1 + T_2) = 7\ sec$ ✓



3



$$\{(U + G_4 P) G_1 - Y\} G_2 G_3 = Y$$

also, $Y = G_3 P \Rightarrow P = \dfrac{Y}{G_3}$

Now, $\left\{\left(U + \dfrac{G_4}{G_3} Y\right) G_1 - Y\right\} G_2 G_3 = Y$

$((\quad G_3 \quad / \quad)$

$$Y(1 + G_2 G_3 - G_1 G_2 G_4) = U\, G_2 G_3 G_1$$

$$G(s) = \frac{Y(s)}{U(s)} = \frac{G_2 G_3 G_1}{1 + G_2 G_3 - G_1 G_2 G_4}$$

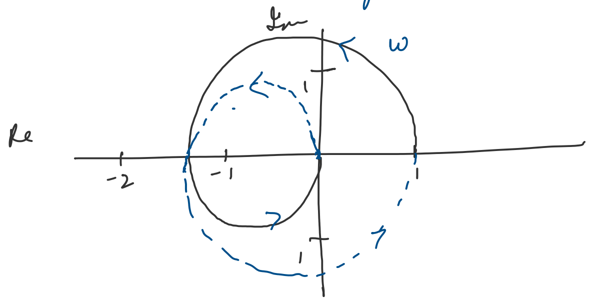
b. $$G(s) = \frac{G_2 G_3}{1 + G_2 G_3}$$

here, $G_0(s) = G_2 G_3$. (say)

Now,

$\because$ G_2 and G_3 both have one pole in right half plane.

$\therefore$ Total pole in right half plane $= 2 = p$



$\because$ 2 rounds anticlockwise about -1

$\therefore$ $m = -2$, no. of revolutions of c'' around -1 (opposite mathematically +ve direction i.e clockwise)

Now, $m = n - p$

$n = m + p = -2 + 2 = 0$

$\therefore$ there are 0 zeros in $N(s)$ inside C (right s halfplane)

or 0 no. of poles of $G_2(s)$ in right s half plane. or C''

and, $G_2(s)$ is stable if $n \to 0$

$\therefore$ In our case $G(s)$ is stable

4)

a) $|sI - A| = 0$ For characteristic eq^n

$$\Rightarrow \begin{vmatrix} s+1 & 5 \\ -1 & s-3 \end{vmatrix} = 0$$

$\Rightarrow s^2 + s - 3s - 3 + 5 = 0$

$s^2 - 2s + 2 = 0$

$\cancel{(s-1)^2 = 0}$ $\quad s = \frac{+2 \pm \sqrt{4-8}}{2} = +1 \pm i\,1$

$\cancel{s = 1}$

$\because$ poles lie in ~~left~~ right half plane

$\therefore$ system is ~~stable~~. unstable.

b) $s_1 = -1+j \qquad s_2 = -1-j$

Characteristic eq^n

$(s-s_1)\ (s-s_2) = 0$

$\{s-(-1+j)\}\ \{s+1+j\} = 0$

$\{s+1-j\}\ \{s+1+j\} = 0$

$(s+1)^2 - j^2 = 0$

$s^2+1+2s+1 = 0$

$s^2+2s+2 = 0$ ——— (1)

Now, For given value of $U = -Kx$

$\dot{X} = AX + B - Kx$

$= (A - BK)\ X$

$= \left\{\begin{pmatrix} -1 & -5 \\ 1 & 3 \end{pmatrix} - \begin{pmatrix} k_1 & k_2 \\ 0 & 0 \end{pmatrix}\right\} X$

$= \underbrace{\begin{pmatrix} -1-k_1 & -5-k_2 \\ 1 & 3 \end{pmatrix}}_{A_K} X$

Now, Characteristic eq^n

$|sI - A_K| = 0$

$\begin{vmatrix} s+1+k_1 & +5+k_2 \\ -1 & s-3 \end{vmatrix} = 0$

$s^2 + s(1+k_1) - 3s - 3(1+k_1) + (5+k_2) = 0$

$s^2 + s(k_1-2) + (2-3k_1+k_2) = 0$ ——— (2)

Comparing eq (1) and (2)

$2 = k_1 - 2$

$\boxed{4 = k_1}$

$2 - 3\cdot 4 + k_2 = 2$

$\boxed{k_2 = 12}$