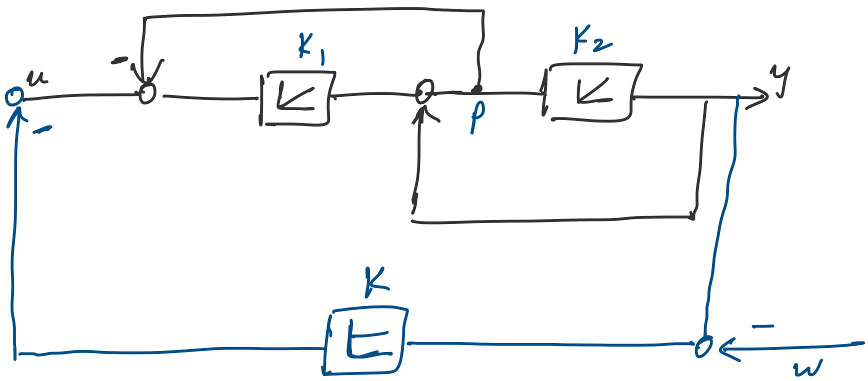
Exam 03-Control Engineering

Monday, 10. February 2020 16:40

1)

a)



consider, $K_1 = K_2 = 1$

b $$\left[\left[\{-K(Y-W) - P\}\frac{1}{s}\right] + Y\right]\frac{1}{s} = Y$$

$$\{-K(Y-W) - P\}\frac{1}{s} + Y = sY$$

$$Y\left(s - 1 + \frac{K}{s}\right) = \frac{K}{s}W - \frac{P}{s}$$

also, $Y = \frac{1}{s}P \Rightarrow P = sY$

$$\Rightarrow Y\left(s \cancel{-1} + \frac{K}{s} \cancel{+1}\right) = \frac{K}{s}W$$

$$Y\left(s + \frac{K}{s}\right) = \frac{K}{s}W$$

$$Y(s^2 + K) = KW$$

$$\Rightarrow \ddot{y} + Ky = Kw$$

$\therefore$ $K = 1$ (compare w/ given eq.)

c) $w = 0$

$\therefore$ $\ddot{y} + y = 0$

$\lambda^2 + 1 = 0$

$\lambda = \pm j$

$\therefore \quad y(t) = c_1 (\cos t) + c_2 (\sin t)$

$\dot{y}(t) = -c_1 \sin t + c_2 \cos t$

$\ddot{y}(t) = -(c_1 \cos t + c_2 \sin t)$

$y(0) = c_1 = 1$

$\dot{y}(0) = c_2 = 1$

$\therefore \boxed{y(t) = \cos t + \sin t}$ ✓

2.

a) $K = 4$ ✓

$\frac{1}{T_1} = 0.5 \ sec^{-1} \Rightarrow T_1 = 2 \ sec$ ✓

b) At $\omega = \frac{1}{T_2}$, $\phi_2(j\omega) = -90^\circ$

Now, $\phi_p = \phi_1 + \phi_2$.

$\phi_2 = -90^\circ \Rightarrow \phi_p = \phi_1 - 90^\circ$

i.e $90 = \phi_1 - \phi_p$

or $90 = \phi_T - \phi_p$

so, measure this difference at Bode plot, that would be $\omega = \frac{1}{T_2} = 2 \ sec^{-1}$

$\Rightarrow \boxed{T_2 = 0.5 \ sec}$ ✓

c) $G_c = K_c(1 + 0.1 j\omega)$

Now from G_o using $K_c = 1$

we have,

Value of $A_R = \frac{1}{\ldots}$

$G_0(\omega_\pi)$
@ $K_c = 1$

For $G_0\big|_{@ K_c = 1}$

Using scale,

$$\frac{29 \text{ units}}{24 \text{ units}} = \frac{\log 10 - \log 1}{\log x - \log 0.1}$$

$$\Rightarrow \quad \log_{10} 10x = \frac{24}{29}$$

$$10x = 10^{24/29} \Rightarrow x = \frac{10^{24/29}}{10}$$

$$G_0(\omega_\pi)\Big|_{K_c=1} = x = 0.672$$

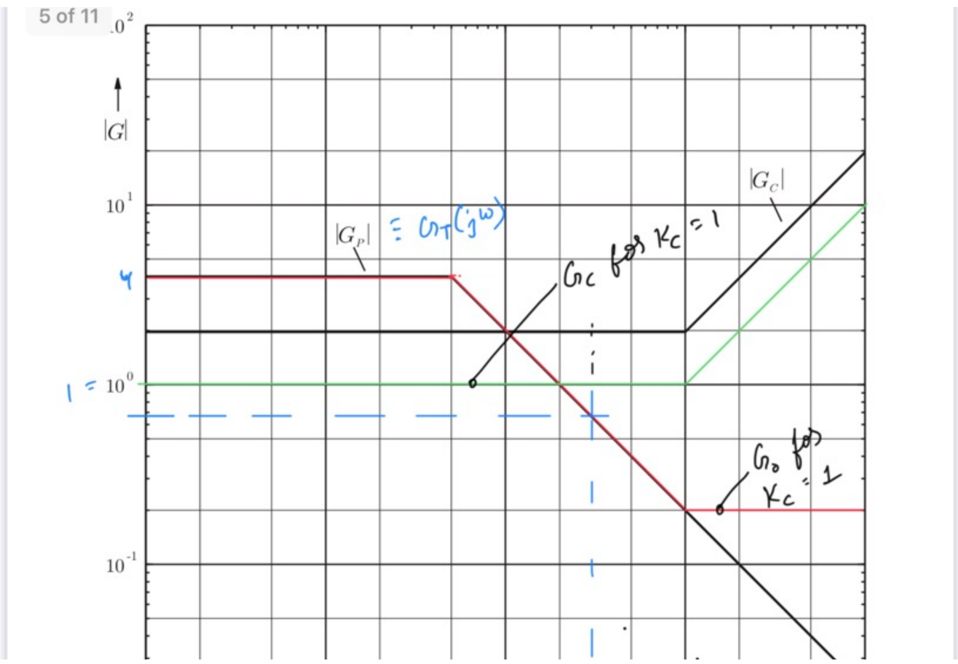
$$\therefore \quad A_R\Big|_{K_c=1} = 1.487$$

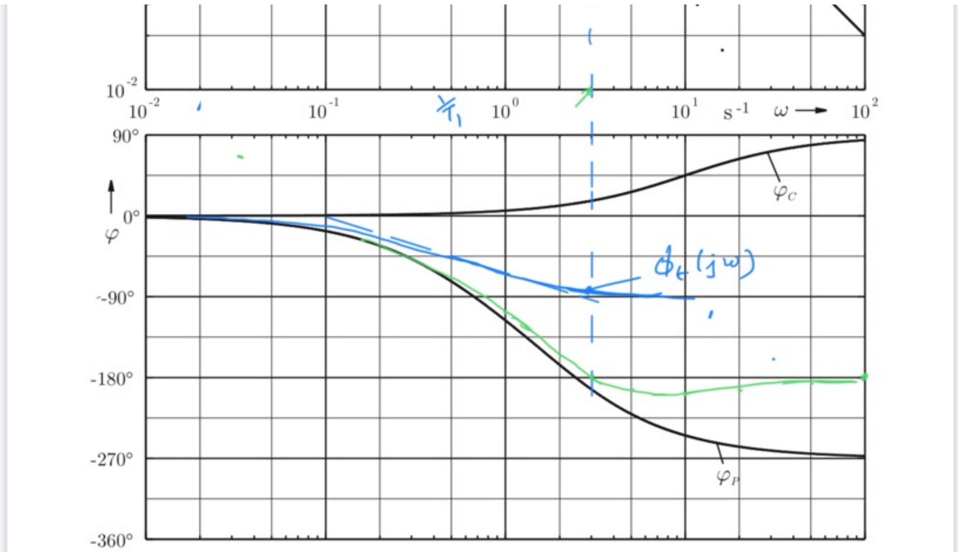
Now, for A_R to be, $A_R = 3$
we should have.

$$G_c(\omega_\pi) = \frac{1}{3}$$

$$\therefore \quad K_c = \frac{1}{0.672} \times \frac{1}{3}$$

$$\boxed{K_c = 0.496}$$





3. $\dddot{y} = u^3 - \ddot{y}(y+2) - 2\sin\dot{y} - y^2 u$
$= f(u, y, \dot{y}, \ddot{y})$

Operating point $P_0 \equiv (u_0, y_0, \dot{y}_0, \ddot{y}_0)$
$= (K, K, 0, 0)$

also, $\dddot{y}_0 = 0$

Now,
$$\dddot{y} = \left.\frac{\partial f}{\partial u}\right|_{P_0} u + \left.\frac{\partial f}{\partial y}\right|_{P_0} y + \left.\frac{\partial f}{\partial \dot{y}}\right|_{P_0} \dot{y} + \left.\frac{\partial f}{\partial \ddot{y}}\right|_{P_0} \ddot{y}$$

$$= (3K^2 - K^2)\,u + (0 - 2K^2)\,y + (-2)\,\dot{y} - (K+2)\,\ddot{y}$$

$$\therefore \quad \dddot{y} = u(2K^2) - y(2K^2) - 2\dot{y} - (K+2)\ddot{y}$$

b) $\dddot{y} + (K+2)\ddot{y} + 2\dot{y} + y(2K^2) = (2K^2)\,u$

$$\Rightarrow \quad \frac{Y(s)}{U(s)} = \frac{2K^2}{s^3 + (K+2)s^2 + 2s + (2K^2)}$$

For linear system to be stable.

cond^n 1

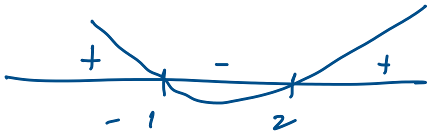
$K > -2$

cond^n 2

$$\begin{vmatrix} K+2 & 2K^2 \end{vmatrix} > 0$$

$1 \quad 1 \qquad 2 \quad 1$

$$2K + 4 - 2K^2 > 0$$
$$2K^2 - 2K - 4 > 0$$
$$K^2 - K - 2 < 0$$
$$(K+1)(K-2) < 0$$



$\therefore \boxed{-1 < K < 2}$

4)

a. No, because canonical form is following.

$$\dot{x} = \underbrace{\begin{bmatrix} 0 & 1 \\ a_1 & a_2 \end{bmatrix}}_{A} x + \underbrace{\begin{pmatrix} 0 \\ 1 \end{pmatrix}}_{b} u.$$

In our case $b = \begin{pmatrix} 1 \\ 1 \end{pmatrix} \neq \begin{pmatrix} 0 \\ 1 \end{pmatrix}$ $\therefore$ Not canonical

b. $|sI - A| = 0$ characteristic eq^n

$$\begin{vmatrix} s & -1 \\ +5 & s+2 \end{vmatrix} = 0$$

$$s^2 + 2s + 5 = 0$$

$$s = \frac{-2 \pm \sqrt{4 - 20}}{2}$$

$$= \frac{-2 \pm i4}{2}$$

$$s = -1 \pm i2$$

$s_{p_1} = -1 - i2 \qquad s_{p_2} = -1 + i2$

For, Damping critical, $\lambda = \frac{1}{\sqrt{2}}$ i.e angle $= 45°$

$\therefore$ poles lie outside region of sufficient damping value ∴ Damping is not sufficiently good.

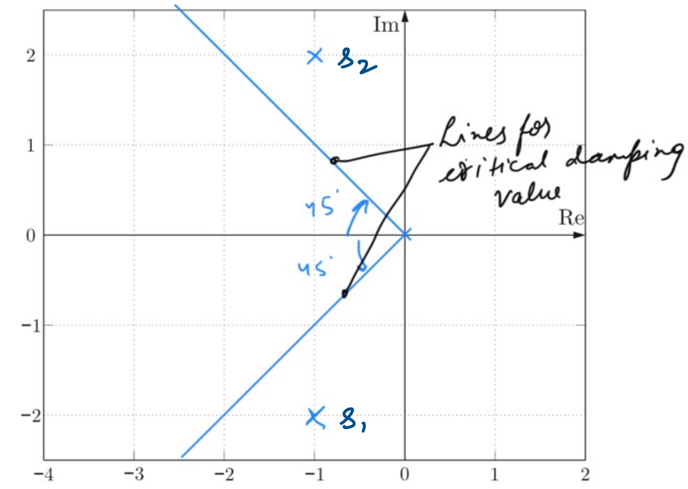


Figure 1: Poles of the system for task b).

c) $s_{p_1} = -1 \qquad s_{p_2} = -3$

∴ Characteristic eq^n

$$(s+1)(s+3) = 0$$
$$s^2 + 4s + 3 = 0$$

Now, characteristic eq^n

$$|sI - A_K| = 0$$

$$A_K = A - BK$$
$$= \begin{pmatrix} 0 & 1 \\ -5 & -2 \end{pmatrix} - \begin{pmatrix} k_1 & k_2 \\ k_1 & k_2 \end{pmatrix}$$
$$= \begin{pmatrix} -k_1 & 1-k_2 \\ -5-k_1 & -2-k_2 \end{pmatrix}$$

Now,

$$|sI - A_K| = 0$$
$$\begin{vmatrix} s+k_1 & k_2-1 \\ k_1+5 & s+2+k_2 \end{vmatrix} = 0$$

$$s^2 + s(k_1 + k_2 + 2) + k_1(2+k_2) - (k_2-1)(k_1+5) = 0$$
$$s^2 + s(k_1 + k_2 + 2) + 2k_1 + \cancel{k_1k_2} - \cancel{k_1k_2} + k_1 - 5k_2 + 5 = 0$$

$s^2 + s(k_1 + k_2 + 2) + (3k_1 - 5k_2 + 5) = 0$

$\therefore \quad k_1 + k_2 + 2 = 4$ and $3k_1 - 5k_2 + 5 = +3$

$k_1 + k_2 = 2$ $\Rightarrow 3k_1 - 5k_2 = -2$

$k_1 = 2 - k_2$

$\Rightarrow$

$6 - 3k_2 - 5k_2 = -2$

$8 = 8k_2$

$\boxed{1 = k_2}$ ✓

$k_1 = 2 - 1 = \boxed{1 = k_1}$ ✓