Exam preparation exercises

Monday, 10. February 2020 19:31

8 $G_0(s) = \frac{1}{s} f(s)$ ($\because$ $G_0(s)$ has integrating behaviour

- controller is statically stable, $\because$ it has integrating behaviour.

FVT (Final Value Theorem)

- Theorem to relate frequency domain to time domain behaviour when $t \to \infty$ i.e $s \to 0$

$$\lim_{t\to\infty} f(t) = \lim_{s\to 0} s\,F(s)$$

Now, For closed loop

$$G_2(s) = \frac{G_0(s)}{1+G_0(s)}$$

$$= \frac{f(s)}{s + f(s)}$$

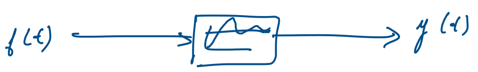
$$\boxed{\lim_{s\to 0} G_2(s) = \frac{\lim_{s\to 0} f(s)}{\lim_{s\to 0} f(s)} = 1}$$

9 Mass damper system :

$$\ddot{y} + 2D\omega_0 \dot{y} + \omega_0^2 y = \omega_0^2 f(t)$$

$$Y(s)\{s^2 + (2D\omega_0)s + \omega_0^2\} = \omega_0^2 f(t)$$

$$G(s) = \frac{Y(s)}{U(s)} = \frac{\omega_0^2}{s^2 + (2D\omega_0)s + \omega_0^2}$$



D, ω_0, K

10 $u(t) = 4\sin(0.1)t + 3\cos(2t)$

* Calculating $F(s)$ for $f(t)$ as below.

$$f(t) = e^{j\omega t} = \cos\omega t + j\sin\omega t$$

$$F(s) = \int_0^\infty f(t)\, e^{-st}\cdot dt$$

$$= \int_0^\infty e^{-(s-j\omega)t}\cdot dt$$

$$= \left.\frac{e^{-(s-j\omega)t}}{(j\omega - s)}\right|_0^\infty$$

$$= \frac{0-1}{-(-j\omega+s)} = \frac{1}{s-j\omega}$$

$$= \frac{s+j\omega}{s^2+\omega^2}$$

$$\therefore \quad \mathcal{L}(\cos \omega t) = \frac{s}{s^2+\omega^2}$$

$$\mathcal{L}(\sin \omega t) = \frac{\omega}{s^2+\omega^2}$$

$$\therefore \quad U(s) = 4\left(\frac{0.1}{s^2+0.01}\right) + 3\left(\frac{s}{s^2+4}\right)$$

Now

$$Y(s) = G(s)\,U(s)$$

$$= \underbrace{\frac{0.8}{(s^2+0.01)(1+5s)}}_{Y_1(s)} + \underbrace{\frac{6s}{(s^2+4)(1+5s)}}_{Y_2(s)}$$

For $Y_1(s)$ for Partial fraction

$$\{A(s^2+0.01) + (Bs+C)(1+5s)\} = 1$$

$$\Rightarrow \quad s^2(A+5B) + s(B+5C) + (0.01A + C) = 1$$

$$\therefore \quad A = -5B \qquad B = -5C$$

$$\Rightarrow A = 25C$$

Now, $0.01\,25C + C = 1$

$$1.25\,C = 1$$

$$C = \frac{1}{1.25} = \boxed{\frac{4}{5} = C}$$

$$\boxed{B = -4}$$
$$\boxed{A = 20}$$

$$\therefore Y_1(s) = 0.8\left\{\frac{20}{1+5s} + \frac{-4s}{s^2+0.01} + \frac{4}{5(s^2+0.01)}\right\}$$

$$\therefore \quad = \frac{16}{1+5s} - \frac{3.2}{(0.1)^2}\,\frac{s(0.1)^2}{s^2+(0.1)^2} + \frac{3.2}{5(0.1)^2}\,\frac{(0.1)^2}{s^2+(0.1)^2}$$

$$\therefore \quad y_1(t) = \frac{16}{5}e^{-t/5} - 3.2(\cos 0.1t) + 0.064\,\sin(0.1t)$$

$$Y_2(s) = \frac{6s}{(s^2+4)(1+5s)}$$

Partial fraction of selected side

$$\frac{s}{(s^2+4)(1+5s)} = \frac{A(s^2+4) + (Bs+C)(1+5s)}{(s^2+4)(1+5s)}$$

$$s^2(A+5B) + s(B+5C) + (4A+C) = 6s$$

$$\therefore \quad A = -5B \qquad B + 5C = 6 \qquad C = -4A$$

$$\Rightarrow C = 20B$$

$$\therefore \qquad \Rightarrow B + 20B = 6$$

$$A = \frac{-10}{7} \Leftarrow B = \frac{2}{7} \Rightarrow C = \frac{40}{7}$$

$$Y_2(s) = \frac{-10}{7}\,\frac{1}{\ldots} + \frac{2}{7}\left\{\frac{(2^2)\,s}{\ldots}\right\} + \frac{40}{7}\,\frac{2^2}{\ldots}$$

$$7 \quad (1+5s) \qquad 7(2)(s^2+2^2)) \qquad 7(2^2)(s^2+2^2)$$

$$y_2(t) = \frac{-10}{7}\,\frac{1}{5}\,e^{-t/5} + \frac{2}{7}\cos 2t + \frac{20}{7}\sin 2t$$

$$\therefore \quad y(t) = y_1(t) + y_2(t)$$