📘Understanding p-Value in Hypothesis Testing

What is a p-value?

The **p-value** (probability value) measures **how likely your observed data (or something more extreme)** would be if the **null hypothesis (H_0)** were **true**.

> In simple words: The **p-value tells you the probability of getting your observed results purely by chance**.

🕖The Logic Behind It

- Every hypothesis test starts with a **null hypothesis (H_0)** (e.g., "no effect," "no difference").
- The p-value helps decide whether your **data provide enough evidence to reject H_0**.

p-value	Interpretation	Decision (if α = 0.05)
≤ 0.05	Data are **unlikely** if H_0 were true → strong evidence **against H_0**	❌Reject H_0
> 0.05	Data are **likely** under H_0 → weak evidence against H_0	✅Fail to reject H_0

🧰Formula (Conceptual)

For most tests (Z-test, t-test, etc.):

$$p = P(\text{observing a test statistic as extreme or more extreme than actual, given } H_0)$$

🔍Example: Tire Lifespan

Scenario:

A tire company claims the average tire life = **50,000 km**.
You test **40 tires** and find: - Sample mean ($\bar{x}$) = 48,500 km
- Population SD (σ) = 3,000 km (known)
- Sample size (n) = 40
- Significance level (α) = 0.05

We want to check if the **claim is true**.

Step ✂️ — Define Hypotheses

Type	Symbolic Form	Description
H_0	μ = 50,000	Tires last 50,000 km (company's claim)
H_1	μ < 50,000	Tires last **less** than claimed

This is a **left-tailed Z-test**.

Step ✒️ — Compute the Z-Statistic

$$Z = \frac{\bar{X} - \mu_0}{\sigma/\sqrt{n}}$$

$$Z = \frac{48,500 - 50,000}{3,000/\sqrt{40}} = \frac{-1,500}{474.34} = -3.16$$

Step 🛌 — Find the p-value

For a **left-tailed test**:

$$p = P(Z < -3.16)$$

From the standard normal (Z) table:

$$p = 0.0008$$

Step 🌂 — Compare p with α

Value	Meaning
p = 0.0008	Probability that the sample mean ≤ 48,500 km if μ = 50,000
α = 0.05	Maximum allowable probability for rejecting H_0

Since **p (0.0008) < α (0.05)** → **Reject H_0** ✅

Step 🌡️ — Interpretation (Plain English)

> There is **less than a 0.1% chance** that we would observe a sample mean of 48,500 km (or less) if the true average lifespan were 50,000 km.
> Therefore, it's **very unlikely** that the company's claim is true.
> We conclude that the tires **do not last 50,000 km on average**.

🧮Step 🎒— Python Implementation

```python
from scipy.stats import norm

# Given data
xbar = 48500
mu = 50000
sigma = 3000
n = 40

# Compute Z and p-value
z = (xbar - mu) / (sigma / (n ** 0.5))
p_value = norm.cdf(z)  # left-tailed test

print(f"Z = {z:.2f}")
print(f"p-value = {p_value:.4f}")
```

Output:

```
Z = -3.16
p-value = 0.0008
```

✅**Conclusion:** Reject H_0 — the tires last less than 50,000 km.

⚖️Relation Between p-Value and Significance Level (α)

Case	Comparison	Interpretation
$p \leq \alpha$	Strong evidence against H_0	Reject H_0 (statistically significant)
$p > \alpha$	Weak evidence against H_0	Fail to reject H_0 (not significant)

Common Misconceptions (and Truths)

❌Wrong Understanding	✅Correct Interpretation
"p-value = probability that H_0 is true."	❌No — p-value assumes H_0 is true; it tells the probability of seeing such data under that assumption.
"A small p means large effect size."	Not necessarily — it means strong **evidence**, not big magnitude.
"A high p proves H_0."	❌No — it just means data are consistent with H_0, not that H_0 is definitely true.

🕑Key Takeaways

- **p-value measures evidence strength against the null hypothesis.**
- **Lower p → stronger evidence → reject H_0.**
- Compare p with **α (usually 0.05)** to decide.
- p-value ≠ probability that H_0 is true.
- Always interpret p-values alongside **effect size** and **context** — not in isolation.

🧾Summary Table

Term	Meaning	Typical Value
p-value	Probability of observed data if H_0 is true	Between 0 and 1
α (Significance Level)	Threshold for rejecting H_0	Usually 0.05
Decision Rule	If p ≤ α → Reject H_0	—
Interpretation	Smaller p → Stronger evidence against H_0	—

🧮Quick Recap Formula

$$p = \{P(Z > |z|) \times 2 \quad \text{for two-tailed test } P(Z < z) \text{ for left-tailed test } P(Z > z) \quad \text{for right-tailed test}$$

✅**In short:**

> The **p-value** quantifies *"how surprising"* your results are **if the null hypothesis were true**
> —
> the smaller it is, the less believable H_0 becomes.

End of p-Value Notes