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Subject - Machine Learning
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Type - Regular

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1 (b)

Animal	Height (mm)
A	555
B	450
C	160
D	410
E	300
F	240
	Total = 2115

(1) Mean = (Total height)/(No. of animals)

$= 2115/6$

$= 352.5$

(2) Median:

Sort in ascending order of height.

160, 240, 300, 410, 450, 555

No. of animals = 6 (Even)

So, Median = (element at) $\dfrac{\left(\frac{4}{2}\right)^{th} + \left(4/2 + 1\right)^{th}}{2}$

$= \dfrac{300 + 410}{2}$

$= 355.0$

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3) Variance:-

$$s^2 = \sum_{i=1}^{n} \frac{(x_i - \bar{x})^2}{n-1}$$

So

Animal	Height	$(x_i - \bar{x})$
A	555	202.5
B	450	97.5
C	160	-192.5
D	410	57.5
E	300	-52.5
f	240	-112.5

$$s^2 = \frac{(202.5)^2 + (97.5)^2 + (-192.5)^2 + (57.5)^2 + (-52.5)^2 + (-112.5)^2}{6-1}$$

$$s^2 = \frac{106287.5}{5}$$

$$s^2 = 21257.5$$

4) Standard Deviation :-

We got $s^2 = 21257.5$

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Now

Standard deviation (s) :

$= \sqrt{21257.3}$

$= 145.79952$

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2 (b)

EIGENVALUE :-

Eigenvalues are the special set of scalars associated with the system of linear equations. It is mostly used in matrix equations.

In simple words, eigenvalue is a scalar that is used to transform the eigenvector. The basic equation is

$$Ax = \lambda x$$

The number/scalar value 'λ' is an eigenvalue of A. for every real matrix, there is an eigen value. If the eigenvalue is negative, the direction of transformation is negative.

EIGEN VECTORS :-

(Non-zero) Vectors that don't change the direction when any linear transformation is applied. It changes by only a scalar factor.

In brief, we can say, if A is a linear transformation from a vector space V and x is a vector in V, which is not a zero vector, the V is an eigenvector of A if ~~A(x)~~ is A(X) is scalar multiple of x.

- There could be infinitely many Eigenvectors, corresponding to one eigenvalue.
- for distinct eigenvalues, the eigen vectors are linearly dependent.

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$$A = \begin{bmatrix} 8 & -6 & 2 \\ -6 & 7 & -4 \\ 2 & -4 & 3 \end{bmatrix}$$

Subtracting λ from diagonal entries of matrix A.

$$\begin{bmatrix} 8-\lambda & -6 & 2 \\ -6 & 7-\lambda & -4 \\ 2 & -4 & 3-\lambda \end{bmatrix}$$

$$C_1 = C_1 - \left(4 - \frac{\lambda}{2}\right) C_3$$

$$\begin{bmatrix} 0 & -6 & 2 \\ 10-2\lambda & 7-\lambda & -4 \\ -\frac{(\lambda-8)(\lambda-3)}{2}+2 & -4 & 3-\lambda \end{bmatrix}$$

$$C_2 = C_2 + 3C_3$$

$$\begin{bmatrix} 0 & 0 & 2 \\ 10-2\lambda & -\lambda-5 & -4 \\ -\frac{(\lambda-8)(\lambda-3)}{2}+2 & 5-3\lambda & 3-\lambda \end{bmatrix}$$

Expand along row 1.

$$\Rightarrow 0(-1)^{1+1} \begin{vmatrix} -\lambda-5 & -4 \\ 5-3\lambda & 3-\lambda \end{vmatrix}$$

$$+ 0(-1)^{1+2} \begin{vmatrix} 10-2\lambda & -4 \\ -\frac{(\lambda-8)(\lambda-3)}{2}+2 & 3-\lambda \end{vmatrix}$$

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$$+ 2(-1)^{1+3} \begin{vmatrix} 10-2\lambda & -\lambda-5 \\ -\frac{(\lambda-8)(\lambda-3)}{2}+2 & 5-3\lambda \end{vmatrix}$$

$$\Rightarrow \quad 2 \begin{vmatrix} 10-2\lambda & -\lambda-5 \\ -\frac{(\lambda-8)(\lambda-3)}{2}+2 & 5-3\lambda \end{vmatrix}$$

$$\Rightarrow \quad 2\left[(10-2\lambda)\cdot(5-3\lambda) - (-\lambda-5)\cdot\left(-\frac{(\lambda-8)(\lambda-3)}{2}+2\right)\right]$$

$$\Rightarrow \quad 2\left[\frac{-\lambda^3}{2} + 9\lambda^2 - \frac{45\lambda}{2}\right]$$

$$\Rightarrow \quad -\lambda(\lambda-15)(\lambda-3)$$

for $\lambda = 15$

Put $\lambda = 15$ in eq ① we get.

$$\begin{bmatrix} -7 & -6 & 2 \\ -6 & -8 & -4 \\ 2 & -4 & -12 \end{bmatrix}$$

$$R_1 = -R_1/7$$

$$\begin{bmatrix} 1 & 6/7 & -2/7 \\ -6 & -8 & -4 \\ 2 & -4 & -12 \end{bmatrix}$$

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$$R_2 = R_2 + 6R_1$$

$$\begin{bmatrix} 1 & 6/7 & -2/7 \\ 0 & -20/7 & -40/7 \\ 2 & -4 & -12 \end{bmatrix}$$

$$R_3 = R_3 - 2R_1$$

$$\begin{bmatrix} 1 & 6/7 & -2/7 \\ 0 & -20/7 & -40/7 \\ 0 & -40/7 & -80/7 \end{bmatrix}$$

$$R_2 = -\frac{7R_2}{20}$$

$$\begin{bmatrix} 1 & 6/7 & -2/7 \\ 0 & 1 & 2 \\ 0 & -40/7 & -80/7 \end{bmatrix}$$

$$R_1 = R_1 - \frac{6R_2}{7}$$

$$\begin{bmatrix} 1 & 0 & -2 \\ 0 & 1 & 2 \\ 0 & -40/7 & -80/7 \end{bmatrix}$$

$$R_3 = R_3 + \frac{40R_2}{7}$$

$$\begin{bmatrix} 1 & 0 & -2 \\ 0 & 1 & 2 \\ 0 & 0 & 0 \end{bmatrix}$$

Now,

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$$\begin{bmatrix} 1 & 0 & -2 \\ 0 & 1 & 2 \\ 0 & 0 & 0 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}$$

$x_3 = t$

Then, $x_1 = 2t$ & $x_2 = 2t$

$$\bar{x} = \begin{bmatrix} 2t \\ -2t \\ t \end{bmatrix} = \begin{bmatrix} 2 \\ -2 \\ 1 \end{bmatrix} t$$

for $\lambda = 3$

Put $\lambda = 3$ in eq ① we get.

$$\begin{bmatrix} 5 & -6 & 2 \\ -6 & 4 & -4 \\ 2 & -4 & 0 \end{bmatrix}$$

$R_1 = R_1/5$

$$\begin{bmatrix} 1 & -6/5 & 2/5 \\ -6 & 4 & -4 \\ 2 & -4 & 0 \end{bmatrix}$$

$R_2 \to R_2 + 6R_1$

$$\begin{bmatrix} 1 & -6/5 & 2/5 \\ 0 & -16/5 & -8/5 \\ 2 & -4 & 0 \end{bmatrix}$$

$R_3 \to R_3 - 2R_1$

$$\begin{bmatrix} 1 & -6/5 & 2/5 \\ 0 & -16/5 & -8/5 \\ 0 & -8/5 & -4/5 \end{bmatrix}$$

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$$R_2 = \frac{-5R_2}{16}$$

$$\begin{bmatrix} 1 & -6/5 & 2/5 \\ 0 & 1 & 1/2 \\ 0 & -8/5 & -4/5 \end{bmatrix}$$

$$R_1 = R_1 + \frac{6R_2}{5}$$

$$\begin{bmatrix} 1 & 0 & 1 \\ 0 & 1 & 1/2 \\ 0 & -8/5 & -4/5 \end{bmatrix}$$

$$R_3 = R_3 + \frac{8R_2}{5}$$

$$\begin{bmatrix} 1 & 0 & 1 \\ 0 & 1 & 1/2 \\ 0 & 0 & 0 \end{bmatrix}$$

Now $\begin{bmatrix} 1 & 0 & 1 \\ 0 & 1 & 1/2 \\ 0 & 0 & 0 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}$

$x = t$

then $x_1 = -t$ & $x_2 = -t/2$

$$\vec{x} = \begin{bmatrix} -t \\ -t/2 \\ t \end{bmatrix} = \begin{bmatrix} -1 \\ -1/2 \\ 1 \end{bmatrix} t$$

for $\lambda = 0$

Put $\lambda = 0$ in eq ① we get:

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$$\begin{bmatrix} 8 & -6 & 2 \\ -6 & 7 & -4 \\ 2 & -4 & 3 \end{bmatrix}$$

$$R_1 = R_1/8$$

$$\begin{bmatrix} 1 & -3/4 & 1/4 \\ -6 & 7 & -4 \\ 2 & 4 & 3 \end{bmatrix}$$

$$R_2 = R_2 + 6R_1$$

$$\begin{bmatrix} 1 & -3/4 & 1/4 \\ 0 & 5/2 & -5/2 \\ 2 & -4 & 3 \end{bmatrix}$$

$$R_3 = R_3 - 2R_1$$

$$\begin{bmatrix} 1 & -3/4 & 1/4 \\ 0 & 5/2 & -5/2 \\ 0 & -5/2 & 5/2 \end{bmatrix}$$

$$R_2 = \frac{2R_2}{5}$$

$$\begin{bmatrix} 1 & -3/4 & 1/4 \\ 0 & 1 & -1 \\ 0 & -5/2 & 5/2 \end{bmatrix}$$

$$R_1 = R_1 + \frac{3R_2}{4}$$

$$\begin{bmatrix} 1 & 0 & -1/2 \\ 0 & 1 & -1 \\ 0 & -5/2 & 5/2 \end{bmatrix}$$

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$R_3 \to R_3 + \frac{5R_2}{2}$

$$\begin{bmatrix} 1 & 0 & -1/2 \\ 0 & 1 & -1 \\ 0 & 0 & 0 \end{bmatrix}$$

Now

$$\begin{bmatrix} 1 & 0 & -1/2 \\ 0 & 1 & -1 \\ 0 & 0 & 0 \end{bmatrix}\begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}$$

If we take $x_3 = t$

Then, $x_1 = t/2$ & $x_2 = t$

$$\vec{x} = \begin{bmatrix} t/2 \\ t \\ t \end{bmatrix} = \begin{bmatrix} 1/2 \\ 1 \\ 1 \end{bmatrix} t$$

So

Eigenvalue : 15   Eigenvector : $\begin{bmatrix} 2 \\ -2 \\ 1 \end{bmatrix}$

Eigenvalue : 3   Eigenvector : $\begin{bmatrix} -1 \\ -0.5 \\ 1 \end{bmatrix}$

Eigenvalue : 0   Eigenvector : $\begin{bmatrix} 0.5 \\ 1 \\ 1 \end{bmatrix}$

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