MODULE $3state$

EXTENDS $Integers$, $FiniteSets$

CONSTANT $N$

ASSUME $N \in Nat \setminus \{0\}$

$Procs \triangleq 1 \,..\, N-1$

Dijkstra's stabilizing 3 state token ring with processes

```
--algorithm 3StateTokenRing{
  variable c = [k ∈ 0 .. N ↦ IF k = 0 THEN 1 ELSE 0];

   fair process ( j ∈ Procs )

    { J0: while ( TRUE )
        { either
            { await (c[self] + 1)%3 = c[(self − 1)];
              c[self] := c[(self − 1)];
            }
          or
          { await (c[self] + 1)%3 = c[(self + 1)];
            c[self] := c[(self + 1)]
          }
         }
    }
   fair process ( i ∈ {0} )
    { I0: while ( TRUE )
          { await ((c[self] + 1)%3 = c[1]);
            c[self] := (c[1] + 1)%3;
          }
    }
    fair process ( k ∈ {N} )

    { N0: while ( TRUE )

            { await c[(self − 1)] = c[0] ∧ c[self] ≠ (c[(N − 1)] + 1)%3;
              c[self] := (c[(self − 1)] + 1)%3;
            }
    }
}
```

**********

BEGIN TRANSLATION

VARIABLE $c$

$vars \triangleq \langle c \rangle$

$ProcSet \triangleq (Procs) \cup (\{0\}) \cup (\{N\})$

$Init \triangleq$ Global variables
$\quad \wedge c = [k \in 0 \,..\, N \mapsto \text{IF } k = 0 \text{ THEN } 1 \text{ ELSE } 0]$

$j(self) \triangleq \; \vee \wedge (c[self] + 1)\%3 = c[(self - 1)]$
$\quad\quad\quad\quad\quad\quad \wedge c' = [c \text{ EXCEPT } ![self] = c[(self - 1)]]$
$\quad\quad\quad\quad\quad \vee \wedge (c[self] + 1)\%3 = c[(self + 1)]$
$\quad\quad\quad\quad\quad\quad \wedge c' = [c \text{ EXCEPT } ![self] = c[(self + 1)]]$

$i(self) \triangleq \; \wedge ((c[self] + 1)\%3 = c[1])$
$\quad\quad\quad\quad \wedge c' = [c \text{ EXCEPT } ![self] = (c[1] + 1)\%3]$

$k(self) \triangleq \; \wedge c[(self - 1)] = c[0] \wedge c[self] \neq (c[(N - 1)] + 1)\%3$
$\quad\quad\quad\quad \wedge c' = [c \text{ EXCEPT } ![self] = (c[(self - 1)] + 1)\%3]$

$Next \triangleq \; (\exists\, self \in Procs : j(self))$
$\quad\quad\quad\quad \vee (\exists\, self \in \{0\} \; : i(self))$
$\quad\quad\quad\quad \vee (\exists\, self \in \{N\} : k(self))$

$Spec \triangleq \; \wedge Init \wedge \Box[Next]_{vars}$
$\quad\quad\quad\quad \wedge \forall\, self \in Procs : \mathrm{WF}_{vars}(j(self))$
$\quad\quad\quad\quad \wedge \forall\, self \in \{0\} \; : \mathrm{WF}_{vars}(i(self))$
$\quad\quad\quad\quad \wedge \forall\, self \in \{N\} : \mathrm{WF}_{vars}(k(self))$

END TRANSLATION

$Tokens \triangleq \; Cardinality(\{x \in Procs : (c[x] + 1)\%3 = c[(x - 1)] \vee (c[x] + 1)\%3 = c[(x + 1)]\})$
$\quad\quad\quad\quad + \text{IF } ((c[0] + 1)\%3 = c[1]) \text{ THEN } 1 \text{ ELSE } 0$
$\quad\quad\quad\quad + \text{IF } c[(N - 1)] = c[0] \wedge c[N] \neq (c[(N - 1)] + 1)\%3 \text{ THEN } 1 \text{ ELSE } 0$
$InvProp \triangleq \; Tokens = 1$
$Stabilization \triangleq \; \Diamond InvProp$
$LowerBound \triangleq \; Tokens \geq 1$
$NotIncrease \triangleq \; \Box[Tokens' \leq Tokens]_{vars}$
$Decrease \triangleq \; \forall\, m \in 1 \,..\, N + 1 : \Box\Diamond(Tokens \leq m)$
$TypeOK \triangleq \; \forall\, x \in 0 \,..\, N : c[x] < 3$

Dijkstra's stabilizing 3 State token ring algorithm. Made by
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