jee2020-paper1

1. Let $\vec{k} = \hat{\imath} - 2\hat{\jmath} + \hat{k}$ and $\mathfrak{b} = \hat{\imath} - \hat{\jmath} + \hat{k}$ be two vectors. If $\vec{c}$ is a vector such that $\vec{b} \cdot \vec{c} = \vec{b} \cdot \vec{a}$ and $a_0 \neq 0$, then c_b is equal to:

   (a) $\frac{1}{2}$
   (b) -1
   (c) $-\frac{1}{2}$
   (d) $-\frac{3}{2}$

2. The area (in square units) of the region $\{(x, y) \in \mathbb{R}^2 \mid x^2 \leq y \leq 3 - 2x\}$ is:

   (a) $\frac{29}{3}$
   (b) $\frac{31}{3}$
   (c) $\frac{34}{3}$
   (d) $\frac{32}{3}$

3. The length of the perpendicular from the origin to the normal of the curve $x^2 + 2xy - 3y^2 = 0$ at the point $(2, 2)$ is:

   (a) $4\sqrt{2}$
   (b) $2\sqrt{2}$
   (c) 2
   (d) $\sqrt{2}$

4. If $I = \int_1^2 \frac{dx}{\sqrt{2x^3-9x^2+12x+4}}$, then:

   (a) $\frac{1}{9} < I^2 < \frac{1}{8}$
   (b) $\frac{1}{16} < I^2 < \frac{1}{9}$
   (c) $\frac{1}{6} < I^2 < \frac{1}{2}$
   (d) $\frac{1}{8} < I^2 < \frac{1}{4}$

5. If a line $y = mx + c$ is a tangent to the circle $(x - 3)^2 + y^2 = 1$ and it is perpendicular to a line L, where L is the tangent to the circle at the point $(\frac{1}{\sqrt{2}}, \frac{1}{\sqrt{2}})$,then

(a) $c^2 - 6c + 7 = 0$

(b) $c^2 + 6c + 7 = 0$

(c) $c^2 + 7c + 6 = 0$

(d) $c^2 - 7c + 6 = 0$

6. Let S be the set of all functions $f : [0, 1] \to \mathbb{R}$ which are continuous on $[0, 1]$ and differentiable on $(0, 1)$. Then for every $f \in S$, there exists a $c \in (0, 1)$, depending on f, such that:

   (a) $|f(c) - f(1)| < (1 - c)|f'(c)|$

   (b) $f(c) - f(1) = f'(c)$

   (c) $|f(c) + f(1)| > (1 + c)|f'(c)|$

   (d) $\frac{f(1)-f(c)}{1-c} = f'(c)$

7. Which of the following statements is a tautology?

   (a) $\neg(p \vee \neg q) \to (p \vee q)$

   (b) $\neg(p \wedge \neg q) \to (p \vee q)$

   (c) $\neg(p \vee \neg q) \to (p \wedge q)$

   (d) $(p \vee \neg q) \to (p \wedge q)$

8. If the 10^{th} term of an arithmetic progression is $\frac{1}{20}$ and its 20^{th} term is $\frac{1}{10}$, then the sum of its first 200 terms is:

   (a) $50\frac{1}{4}$

   (b) $100\frac{1}{2}$

   (c) 50

   (d) 100

9. Let $f : (1, 3) \to \mathbb{R}$ be a function defined by

$$f(x) = \frac{x - \lfloor x \rfloor}{1 + x^2}$$

   where $\lfloor x \rfloor$ denotes the greatest integer function. Then the range of f is:

   (a) $(\frac{3}{5}, \frac{4}{5})$

   (b) $(\frac{2}{5}, \frac{3}{5}) \cup (\frac{3}{4}, \frac{4}{5})$

   (c) $(\frac{2}{5}, \frac{4}{5})$

   (d) $(\frac{2}{5}, \frac{1}{2}) \cup (\frac{3}{5}, \frac{4}{5})$

10. The system of linear equations:

$$\lambda x + 2y + 2z = 5$$
$$bx + 3y + 5z = 8$$
$$4x + ky + 6z = 10$$

has:

(a) Infinitely many solutions when $\lambda = 2$
(b) A unique solution when $\lambda = -8$
(c) No solution when $\lambda = 8$
(d) No solution when $\lambda = 2$

11. If α and β be the coefficients of x and x^2 respectively in the expansion of $(x + \sqrt{x^2 - 1})^6 + (x - \sqrt{x^2 - 1})^6$ then given

(a) $\alpha + \beta = 60$
(b) $\alpha + \beta = -30$
(c) $\alpha - \beta = -132$
(d) $\alpha - \beta = 60$

12. Evaluate the limit:

$$\lim_{x \to 0} \frac{\int_0^x t \sin(10t)\, dt}{x}$$

(a) 0
(b) $-\frac{1}{5}$
(c) $-\frac{1}{10}$
(d) $\frac{1}{10}$

13. If

$$A = \begin{bmatrix} 2 & 2 \\ 9 & 4 \end{bmatrix}, \quad I = r \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$$

then $10A$ is equal to:

(a) $41 - A$
(b) $A - 61$
(c) $61 - A$
(d) $A - 41$

14. The mean and variance of 20 observations are found to be 10 and 4, respectively. On rechecking, it was found that an observation 9 was incorrect and the correct observation was 11. Then the correct variance is:

Options:

(a) 3.99

(b) 3.98

(c) 4.02

(d) 4.01

15. If a hyperbola passes through the point $P(10, 16)$ and has vertices at $(\pm 6, 0)$, then the equation of the normal to it at P is:

    Options:

    (a) $x + 2y = 42$

    (b) $3x + 4y = 94$

    (c) $2x + 5y = 100$

    (d) $x + 3y = 58$

16. Let A and B be two events such that the probability that exactly one of them occurs is $\frac{2}{5}$ and the probability that A or B occurs is $\frac{1}{2}$, then the probability of both of them occurring together is:

    (a) 0.02

    (b) 0.01

    (c) 0.20

    (d) 0.10

17. The mirror image of the point $(1, 2, 3)$ in the plane $(-\frac{7}{3}, -\frac{4}{3}, -\frac{1}{3})$ is:

    Which of the following points lies on this plane?

    (a) $(-1, -1, -1)$

    (b) $(-1, -1, 1)$

    (c) $(1, 1, 1)$

    (d) $(1, -1, 1)$

18. Let S be the set of all real roots of the equation:

$$3x(3x - 1) + 2 = |3x - 1| + |3x - 2|.$$

    Then S:

    (a) is an empty set.

    (b) contains at least four elements.

    (c) contains exactly two elements.

    (d) is a singleton.

19. Let $\alpha = \frac{-1+i\sqrt{3}}{2}$ if $a = (1+\alpha)\sum_{k=0}^{100} \alpha^{2k} \cdot a_n \cdot db = \sum_{k=0}^{100} \alpha^{3k}$ and let a and b be the roots of the quadratic equation. Then:

   (a) $x^2 - 102x + 101 = 0$
   (b) $x^2 + 101x + 100 = 0$
   (c) $x^2 - 101x + 100 = 0$
   (d) $x^2 + 102x + 101 = 0$

20. The differential equation of the family of curves,

$$x^2 = 4b(y+b),$$

   where $b \in \mathbb{R}$, is:

   (a) $x(y')^2 - x + 2yy'$
   (b) $xy'' = y'$
   (c) $x(y')^2 = 2yy' - x$
   (d) $x(y')^2 = x - 2y$

21. Solve for α:

$$H \cdot \frac{\sqrt{2}\sin\alpha}{\sqrt{1+\cos 2\alpha}} = \frac{1}{7}$$

   and

$$\sqrt{\frac{1-\cos 2\beta}{2}} = \frac{1}{\sqrt{10}}$$

   where $\alpha \in (0, \frac{\pi}{2})$. Then $\tan(\alpha + 2\beta)$ is equal to ___________.

22. Let $f(x)$ be a polynomial of degree 3 such that $f(-1) = 10$, $f(1) = -6$. The function $f(x)$ has a critical point at $x = -1$, and $f''(x)$ has a critical point at $x = 1$. Then $f(x)$ has a local minimum at $x =$ ___________.

23. Let a line $y = mx$ $(m > 0)$ intersect the parabola $y^2 = 3x$ at a point P, other than the origin. Let the tangent to it at P meet the x-axis at the point Q. If area$(\triangle OPQ) = 4$ square units, then m is equal to ___________.

24. Evaluate the sum:

$$\sum_{n=1}^{7} \frac{n(n+1)(2n+1)}{4}$$

   which is equal to ___________.

25. The number of 4-letter words (with or without meaning) that can be formed from the eleven letters of the word **′EXAMINATION′** is ___________