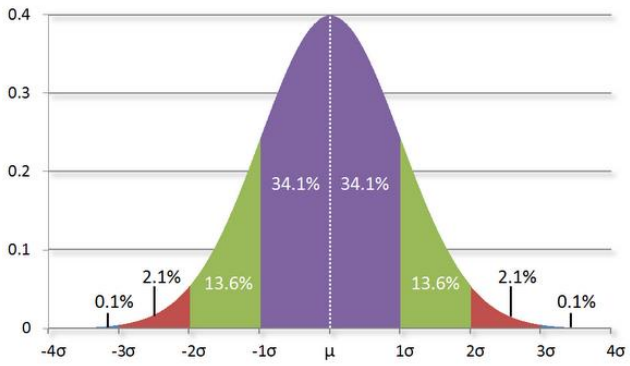


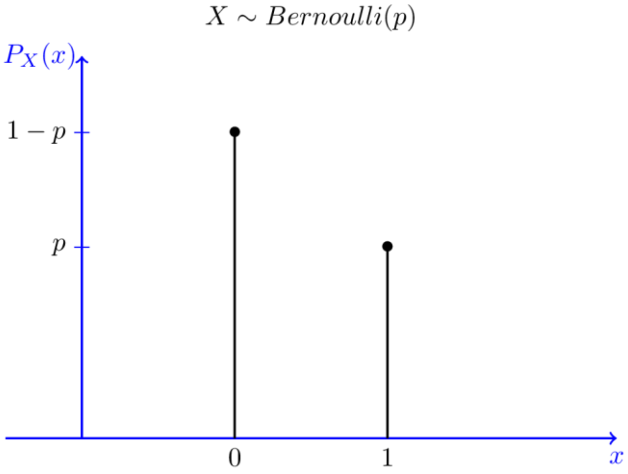
Distribution

Bernouli Distribution

Bernouli

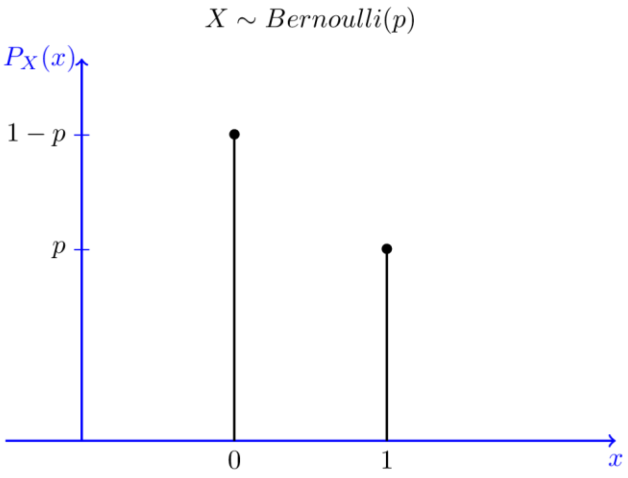
- There are two possibilities (pass or fail) with probability *p* of success and

  $q = 1-p$ of failure..



Expectation : p
Variance : pq

Bernouli



$$Expectation, E(x) = \sum x_i \, P(x_i)$$

$= 1 * p + 0 * q$

$= p$

$$Variance, Var = \sum (x_i - \mu)^2 \; P(x_i)$$

$= (1 - p)^2 * p + (0 - p)^2 * (1 - p)$

$= p\,(1 - p)$

$= pq$

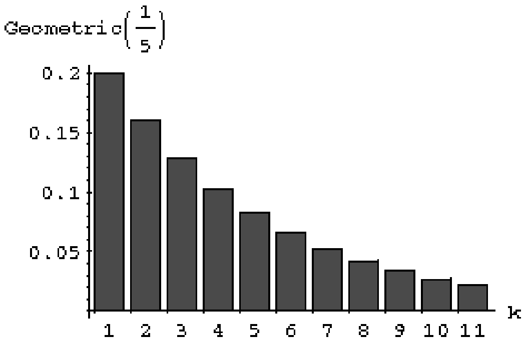
Geometric Distribution

Number of independent and identical Bernoulli trails needed to get ONE success.

Eg. Number of attempts before I pass the exam

Geometric Distribution

- You run a series of independent trail.
- There can be either a success or failure for each trail, and the probability of success is the same for each trail.
- How many trails are needed in order to get the first successful outcome.



Geometric Distribution

- PMF* = P(X = r) = $q^{r-1}p$
- $P(X > r) = q^r$
- *CDF***, $P(X \leq r) = 1 - q^r$
- $E(X) = \frac{1}{p}$ $var(x) = \frac{q}{p^r}$

PMF : Probaility Mass Funciton
CDF : Cumulative Distribution Function

Binomial Distribution

Binomial Experiment

- The process consists of a sequence of n trials.
- Only two exclusive outcomes are possible in each trail. One outcome is called "Success" and other a "failure".
- The probability of a success denotes p , does not change from trail to trail. The probability of failure is 1-p and is also fixed from trail to trail.
- The trails are independent; the outcome of previous trail not influence future trail.

Binomial Variable

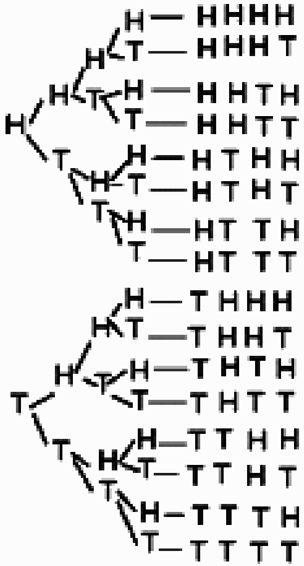
- Let's consider a coin $P(H) = 0.6$
- $P(T) = 0.4$
- $X = \#$ of heads after 10 flip of my coin
- Made up of independent trails
- Each trail can be classified as either success or failure
- Probability of success on each trail is constant

Binomial distribution

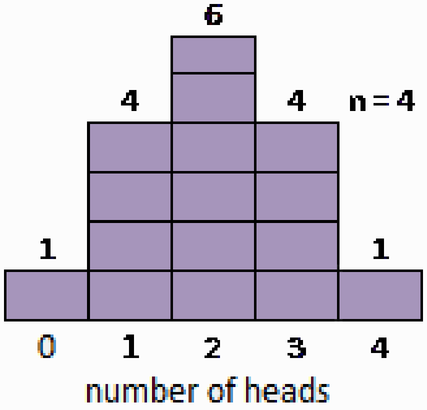
- X = # of heads after flipping coin 4 times
- Possible outcomes = 2 * 2 * 2 * 2 = 16
- P(X = 0) = $\frac{C_0^4}{32} = \frac{1}{32}$
- P(X = 1) = $\frac{C_1^4}{32} = \frac{5}{32}$
- P(X = 2) = $\frac{C_2^4}{32} = \frac{6}{32}$

Binomial distribution

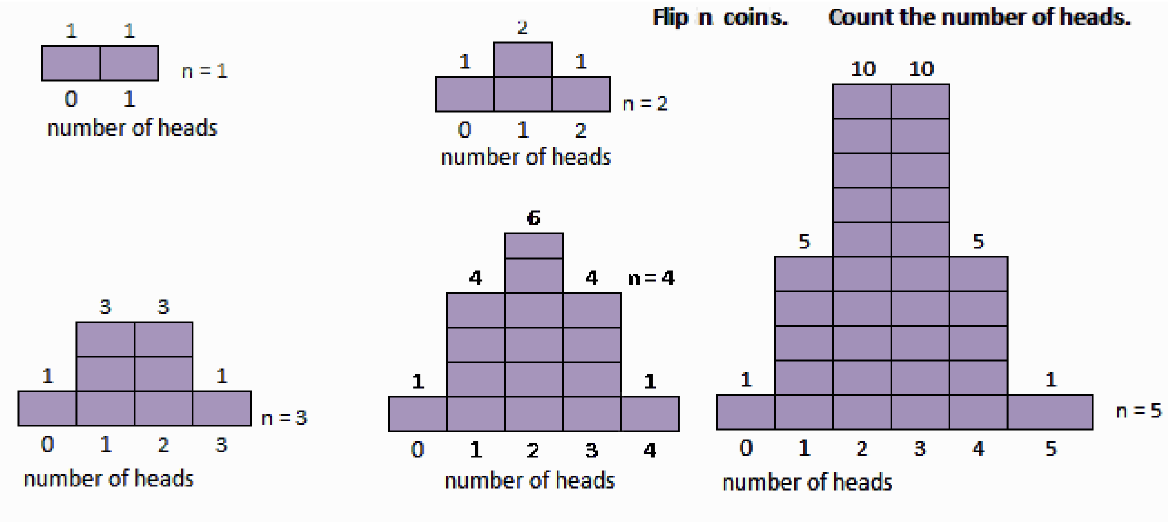
Flip a coin four times. Count the number of heads.



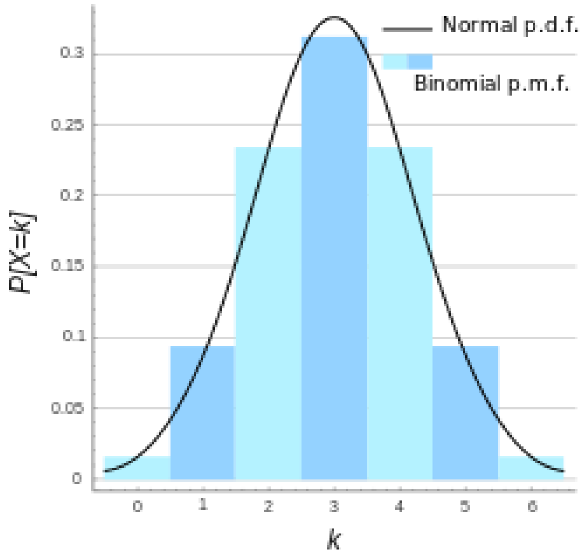
x	P(x) = p	
0	0.0625	1 /16
1	0.25	4 /16
2	0.375	6 / 16
3	0.25	4 /16
4	0.0625	1/16



Binomial distribution



Binomial distribution



Binomial Probability example

P(Score) = 70 % or 0.7
P(miss) = 30 % or 0.3

P(Exactly 2 scores in 6 attempts) =

$$= C_2^6\,(0.7)^2\,(0.3)^4$$

P(Exactly k scores in n attempts) =

$$= C_k^n\,(p)^k\,(1-p)^{n-k}$$

Question

In this area it is expected that 9 cars are passing in a hours. What is the probability that there are exactly 6 cars are passing in this hour

$E(X) = \lambda = n * p = 9 \text{ cars/hr}$
Since , 1 hours = 60 minutes
So, n = 60
K = 6 (number of expected cars in hour)

$$P(x = k) = C_k^n (p)^k (1 - p)^{n-k}$$

$$p = \lambda/n = 9 / 60$$

Question

In this area it is expected that 9 cars are passing in a hours. What is the probability that there are exactly 6 cars are passing in this hour

$p \quad = \quad \lambda / n = 9 / 60$
$1 - p = 51 / 60$

Therefore, probability of exactly 6 cars are passing in an hours is

$P(x = 6) = C_6^{60} \left(\frac{9}{60}\right)^6 \left(\frac{51}{60}\right)^{54}$

$n = 3600$ (no. of seconds in an hour)

Poisson Distribution

Poisson

E(X) = $\lambda \quad = n \ * p$

P(X = k) $= \lim_{n\to\infty} C_k^n \, (p)^k \ \ (1 \ - p)^{n-k}$

$$= \quad \frac{\lambda^{\boldsymbol{k}}}{\boldsymbol{k}\,!}\,\boldsymbol{e}^{\,-\lambda}$$

λ = 9 cars pass

P(X = 2)

Question

Q. Probability that a cars will not pass in 6 min

$$= \frac{\lambda^{k}}{k\,!} e^{-\lambda}$$

For 1st second:

P (x = 0) = $\frac{\lambda^{0}}{0!} e^{-\lambda} = e^{-\lambda}$ (for 1 sec)

For *n* second:

Probability that a cars will pass for n sec.

$$= e^{-n\lambda}$$

Exponential Distribution

Q. Probability that a cars will pass in n sec

$$1 - e^{-n\lambda}$$

CDF = $1 - e^{-n\lambda}$ $, n \geq 0$

PDF= $\lambda e^{-n\lambda}$ $, n \geq 0$

Exponential Distribution

- Poisson process
- Continuous analog of Geometric distribution

$$E(X) = \frac{1}{\lambda} \qquad\qquad var(X) = \frac{1}{\lambda^2}$$