

Week 1: Modeling Business Decisions in Low-Uncertainty Settings

- ◆ The First Example: Advertising Campaign at Hudson Readers Inc.
 - ◆ How to Build an Optimization Model: Decisions Variables, Objective Function, Constraints
- Session 1**
- ◆ Optimizing with Solver
 - ◆ Alternative Data Inputs
 - ◆ Bringing in Risk Considerations: Managing Investments at Epsilon Delta Capital

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Session 2

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Session 3

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Advertising Campaign at Hudson Readers Inc.

- ◆ Hudson Readers Inc. (HRI) is a company specializing in the design and manufacturing of a popular line of e-readers
- ◆ Its new product is available in two different configurations: Standard (S), and Enhanced (E)
- ◆ In designing its advertising campaign in Asia, HRI is considering two main advertising markets: India (I) and China (C)
- ◆ The total advertising budget that HRI has allocated to this campaign is \$195,000,000

Advertising Campaign at Hudson Readers Inc.

- ◆ In order to predict the effectiveness of its advertising campaign, HRI uses the results of advertising campaigns for previous versions of its products, as well as the opinions of its marketing experts
- ◆ HRI's management estimates that the sales response to advertising for this campaign (expected extra sales amount, in equivalent of US\$, per each US\$ spent on advertising, net of advertising cost) is proportional to advertising amount and can be expressed as follows:

Sales Response	India	China
Standard	0.05	0.04
Enhanced	0.02	0.03

- ◆ For example, \$1,000,000 spent promoting the Standard version of the product in India result in the net sales increase of $\$1,000,000 \times 0.05 = \$50,000$.

Advertising Campaign at Hudson Readers Inc.

- ◆ HRI estimates that the sales response will be additive across different versions and markets; e.g., spending \$1,000,000 promoting the Enhanced version in China and, in addition, spending \$2,000,000 promoting the Standard version in India will result in the total net sales increase of

$$\begin{aligned} & \$1,000,000 \times 0.03 + \$2,000,000 \times 0.05 \\ & = \$30,000 + \$100,000 = \$130,000. \end{aligned}$$

Advertising Campaign at Hudson Readers Inc.

- ◆ HRI's management wants to determine how much to spend for each version/market combination in order to maximize the total net sales increase while not exceeding the advertising budget of \$195,000,000
- ◆ In doing so, the management would like to follow several guidelines:
 - The Indian market must achieve at least \$3,000,000 in the total net sales increase
 - The Chinese market must achieve at least \$4,000,000 in the total net sales increase
 - The Enhanced version should achieve at least 80% of the net sales increase realized for the Standard version

Low-Uncertainty Assumptions

- ◆ The HRI assumes that all parameters that may influence its budget allocation decision are **deterministic** – they are all known, non-random quantities
- ◆ For example, the sales response of advertising the Standard version in India is assumed to be 0.05, rather than, say, having 50%-50% chance of being either 0.03 or 0.07
- ◆ Using deterministic values for problem parameters eliminates uncertainty about the impact of any potential decision: HRI assumes that if it spends \$10,000,000 advertising the Standard version in India, it will generate exactly \$500,000 in net sales increase, rather than, say, having a 50%-50% chance of generating \$300,000 or \$700,000

Low-Uncertainty Assumptions

- ◆ Imagine that the actual sales response of advertising the Standard version is uncertain, and can take two values of 0.03 or 0.07 with the probability of 0.5 each
- ◆ The expected value of this random quantity is $0.5 \cdot 0.03 + 0.5 \cdot 0.07 = 0.05$
- ◆ Assuming away uncertainty (for example, by replacing random quantities with their expected values) makes the task of finding the best decision much easier
- ◆ In particular, it allows us to efficiently solve problems with large numbers of decisions (e.g., product version/market combinations) and large numbers of business requirements (e.g., advertising guidelines)

Low-Uncertainty Assumptions: Pitfalls

- ◆ The more significant is the uncertainty in parameter values that go into decision making, the less reliable are the recommendations based on low-uncertainty models
- ◆ In Weeks 2-4 of our course we will look at how to model uncertainty and how to make decisions in the face of significant uncertainty

Advertising Spending Decision: Elements of a Model

- ◆ HRI's management wants to determine how much to spend for each version/market combination in order to maximize the total net sales increase while not exceeding the advertising budget of \$195,000,000
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 - The Indian market must achieve at least \$3,000,000 in the total net sales increase
 - The Chinese market must achieve at least \$4,000,000 in the total net sales increase
 - The Enhanced version should achieve at least 80% of the net sales increase realized for the Standard version
- ◆ We will convert this “verbal” problem description into an analytical model and then use Excel to find the best spending decision

Advertising Spending Decision: Elements of a Model

- ◆ HRI's management wants to determine *how much to spend for each version/market combination* in order to maximize the total net sales increase while not exceeding the advertising budget of \$195,000,000

Decision Variables

- ◆ In doing so, the management would like to follow several guidelines:
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Advertising Spending Decision: Elements of a Model

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Advertising Spending Decision: Elements of a Model

- ◆ HRI's management wants to determine how much to spend for each version/market combination in order to **maximize the total net sales increase** while not exceeding the advertising budget of \$195,000,000
Objective
- ◆ In doing so, the management would like to follow several guidelines:
 - The Indian market must achieve at least \$3,000,000 in the total net sales increase
 - The Chinese market must achieve at least \$4,000,000 in the total net sales increase
 - The Enhanced version should achieve at least 80% of the net sales increase realized for the Standard version

Advertising Spending Decision: Elements of a Model

- ◆ HRI's management wants to determine how much to spend for each version/market combination in order to maximize the total net sales increase while not exceeding the advertising budget of \$195,000,000
- ◆ In doing so, the management would like to follow several guidelines:
 - The Indian market must achieve at least \$3,000,000 in the total net sales increase
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Advertising Spending Decision: Elements of a Model

- ◆ HRI's management wants to determine how much to spend for each version/market combination in order to maximize the total net sales increase ***while not exceeding the advertising budget of \$195,000,000***
- ◆ In doing so, the management would like to follow several guidelines:
 - ***The Indian market must achieve at least \$3,000,000 in the total net sales increase***
 - ***The Chinese market must achieve at least \$4,000,000 in the total net sales increase***
 - ***The Enhanced version should achieve at least 80% of the net sales increase realized for the Standard version***

Constraints



Advertising Spending Decision: Elements of a Model

- ◆ HRI's management wants to determine **how much to spend for each version/market combination** in order to **maximize the total net sales increase** while not exceeding the advertising budget of \$195,000,000
- ◆ In doing so, the management would like to follow several guidelines:
 - *The Indian market must achieve at least \$3,000,000 in the total net sales increase*
 - *The Chinese market must achieve at least \$4,000,000 in the total net sales increase*
 - *The Enhanced version should achieve at least 80% of the net sales increase realized for the Standard version*
- ◆ HRI's task is to choose the **values of decision variables** to make the **objective value as high as possible**, while satisfying all **constraints**

Advertising Spending Model: Decision Variables

- ◆ HRI's management wants to determine *how much to spend for each version/market combination* in order to maximize the total net sales increase while not exceeding the advertising budget of \$195,000,000
- ◆ In doing so, the management would like to follow several guidelines:
 - The Indian market must achieve at least \$3,000,000 in the total net sales increase
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 - The Enhanced version should achieve at least 80% of the net sales increase realized for the Standard version

Advertising Spending Model: Decision Variables

- ◆ Two product versions (Standard and Enhanced) and two markets (India and China) – four decision variables:

A_{SI} = \$ millions to spend advertising the Standard version in India

A_{SC} = \$ millions to spend advertising the Standard version in China

A_{EI} = \$ millions to spend advertising the Enhanced version in India

A_{EC} = \$ millions to spend advertising the Enhanced version in China

Advertising Spending Model: Decision Variables

- ◆ Two product versions (Standard and Enhanced) and two markets (India and China) – four decision variables:

Spending	India	China
Standard	A_{SI}	A_{SC}
Enhanced	A_{EI}	A_{EC}

Advertising Spending Model: Decision Variables

- ◆ A combination of decision variable values is called a **solution** to the model. For example, $A_{SI} = 70$, $A_{SC} = 50$, $A_{EI} = 50$, $A_{EC} = 25$ is a solution.

Spending	India	China
Standard	$A_{SI} = 70$	$A_{SC} = 50$
Enhanced	$A_{EI} = 50$	$A_{EC} = 25$

Advertising Spending Model: Objective

- ◆ HRI's management wants to determine how much to spend for each version/market combination in order to maximize **the total net sales increase** while not exceeding the advertising budget of \$195,000,000
- ◆ In doing so, the management would like to follow several guidelines:
 - The Indian market must achieve at least \$3,000,000 in the total net sales increase
 - The Chinese market must achieve at least \$4,000,000 in the total net sales increase
 - The Enhanced version should achieve at least 80% of the net sales increase realized for the Standard version

Advertising Spending Model: Objective

- ◆ So, how much of the total net sales increase will HRI achieve if it implements the solution $A_{SI} = 70$, $A_{SC} = 50$, $A_{EI} = 50$, $A_{EC} = 25$?

Advertising Spending Model: Objective

- ◆ So, how much of the total net sales increase will HRI achieve if it implements the solution $A_{SI} = 70$, $A_{SC} = 50$, $A_{EI} = 50$, $A_{EC} = 25$?

Spending	India	China
Standard	$A_{SI} = 70$	$A_{SC} = 50$
Enhanced	$A_{EI} = 50$	$A_{EC} = 25$

Sales Response	India	China
Standard	0.05	0.04
Enhanced	0.02	0.03

- ◆ Total net sales increase:

$$0.05*70 + 0.04*50 + 0.02*50 + 0.03*25 = 3.5 + 2 + 1 + 0.75 = 7.25$$

Advertising Spending Model: Objective

- ◆ So, how much of the total net sales increase will HRI achieve if it implements the solution A_{SI} , A_{SC} , A_{EI} , A_{EC} ?

Spending	India	China
Standard	A_{SI}	A_{SC}
Enhanced	A_{EI}	A_{EC}

Sales Response	India	China
Standard	0.05	0.04
Enhanced	0.02	0.03

- ◆ **Total net sales increase:**

$$0.05 * A_{SI} + 0.04 * A_{SC} + 0.02 * A_{EI} + 0.03 * A_{EC}$$

Advertising Spending Model: Constraints

- ◆ HRI's management wants to determine how much to spend for each version/market combination in order to maximize the total net sales increase *while not exceeding the advertising budget of \$195,000,000*
- ◆ In doing so, the management would like to follow several guidelines:
 - *The Indian market must achieve at least \$3,000,000 in the total net sales increase*
 - *The Chinese market must achieve at least \$4,000,000 in the total net sales increase*
 - *The Enhanced version should achieve at least 80% of the net sales increase realized for the Standard version*

Advertising Spending Model: Constraints

- ◆ So, if HRI decides to go with the spending plan $A_{SI} = 70$, $A_{SC} = 50$, $A_{EI} = 50$, $A_{EC} = 25$, will it be within the **advertising budget**?
- ◆ Yes: the total spending is $70+50+50+25 = 195$ (\$ millions)

Advertising Spending Model: Constraints

- ◆ What about the spending guidelines?
 - **“The Indian market must achieve at least \$3,000,000 in the total net sales increase”**

Spending	India	China
Standard	$A_{SI} = 70$	$A_{SC} = 50$
Enhanced	$A_{EI} = 50$	$A_{EC} = 25$

Sales Response	India	China
Standard	0.05	0.04
Enhanced	0.02	0.03

- The total net sales increase in the Indian market is $0.05 \cdot 70 + 0.02 \cdot 50 = 4.5$ (\$ million). The first guideline is satisfied.

Advertising Spending Model: Constraints

- ◆ What about the spending guidelines?
 - **“The Chinese market must achieve at least \$4,000,000 in the total net sales increase”**

Spending	India	China
Standard	$A_{SI} = 70$	$A_{SC} = 50$
Enhanced	$A_{EI} = 50$	$A_{EC} = 25$

Sales Response	India	China
Standard	0.05	0.04
Enhanced	0.02	0.03

- The total net sales increase in the Chinese market is $0.04 \cdot 50 + 0.03 \cdot 25 = 2.75$ (\$ million). The second guideline is violated.

Advertising Spending Model: Constraints

- ◆ What about the spending guidelines?
 - “**The Enhanced version should achieve at least 80% of the net sales increase realized for the Standard version**”

Spending	India	China
Standard	$A_{SI} = 70$	$A_{SC} = 50$
Enhanced	$A_{EI} = 50$	$A_{EC} = 25$

Sales Response	India	China
Standard	0.05	0.04
Enhanced	0.02	0.03

- The net sales increase for the Standard version is $0.05 \cdot 70 + 0.04 \cdot 50 = 5.5$ (\$ million). The net sales increase for the Enhanced version is $0.02 \cdot 50 + 0.03 \cdot 25 = 1.75$ (\$ million). 80% of the Standard value is \$4.4 million, the third guideline is violated.

Advertising Spending Model: Constraints

- ◆ A spending plan that satisfies all constraints is called ***feasible***
- ◆ If a spending plan violates at least one constraint, it is ***infeasible***
- ◆ The spending plan $A_{SI} = 70$, $A_{SC} = 50$, $A_{EI} = 50$, $A_{EC} = 25$ is infeasible
- ◆ So, what conditions must any feasible plan A_{SI} , A_{SC} , A_{EI} , A_{EC} satisfy?

Advertising Spending Model: Budget Constraint

- ◆ The spending plan A_{SI} , A_{SC} , A_{EI} , A_{EC} , must be within the **advertising budget**
- ◆ The total spending is $A_{SI} + A_{SC} + A_{EI} + A_{EC}$ must not exceed 195:

$$A_{SI} + A_{SC} + A_{EI} + A_{EC} \leq 195$$

Advertising Spending Model: Constraints

- ◆ What about the spending guidelines?
 - “The Indian market must achieve at least \$3,000,000 in the total net sales increase”

Spending	India	China
Standard	A_{SI}	A_{SC}
Enhanced	A_{EI}	A_{EC}

Sales Response	India	China
Standard	0.05	0.04
Enhanced	0.02	0.03

$$0.05 * A_{SI} + 0.02 * A_{EI} \geq 3$$

Advertising Spending Model: Constraints

- ◆ What about the spending guidelines?
 - “**The Chinese market must achieve at least \$4,000,000 in the total net sales increase**”

Spending	India	China
Standard	A_{SI}	A_{SC}
Enhanced	A_{EI}	A_{EC}

Sales Response	India	China
Standard	0.05	0.04
Enhanced	0.02	0.03

$$0.04 * A_{SC} + 0.03 * A_{EC} \geq 4$$

Advertising Spending Model: Constraints

- ◆ What about the spending guidelines?
 - “The Enhanced version should achieve at least 80% of the net sales increase realized for the Standard version”

Spending	India	China
Standard	A_{SI}	A_{SC}
Enhanced	A_{EI}	A_{EC}

Sales Response	India	China
Standard	0.05	0.04
Enhanced	0.02	0.03

$$0.02 * A_{EI} + 0.03 * A_{EC} \geq 0.8 * (0.05 * A_{SI} + 0.04 * A_{SC})$$

Searching for the Best Spending Plan: A Model

- ◆ Combining the **decision variables**, **objective function** and **constraints** together, we can express our model as

Maximize $0.05*A_{SI} + 0.04*A_{SC} + 0.02*A_{EI} + 0.03*A_{EC}$

subject to

$A_{SI} + A_{SC} + A_{EI} + A_{EC} \leq 195$ (advertising budget)

$0.05*A_{SI} + 0.02*A_{EI} \geq 3$ (net sales increase in India)

$0.04*A_{SC} + 0.03*A_{EC} \geq 4$ (net sales increase in China)

$0.02*A_{EI} + 0.03*A_{EC} \geq 0.8*(0.05*A_{SI} + 0.04*A_{SC})$

(Standard vs. Enhanced net sales increase)

$A_{SI}, A_{SC}, A_{EI}, A_{EC} \geq 0$ (non-negative advertising spending)

This constraint may become important when the model is passed on to Excel