# Week 4: Balancing Risk and Reward Using Simulation

- Modeling Uncertainty: From Scenarios to Continuous Distributions
- Example: Designing a New Apartment Building
- Connecting Random Inputs and Random Outputs in a Simulation
- Setting up and Running a Simulation in Excel
- Analyzing and Interpreting Simulation Output
- Evaluating Alternative Decisions using Simulation Results

# Week 4: Balancing Risk and Reward Using Simulation

- Modeling Uncertainty: From Scenarios to Continuous Distributions
- Example: Designing a New Apartment Building **Session 1**
- Connecting Random Inputs and Random Outputs in a Simulation
- Setting up and Running a Simulation in Excel
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- Evaluating Alternative Decisions using Simulation Results

# Week 4: Balancing Risk and Reward Using Simulation

- Modeling Uncertainty: From Scenarios to Continuous Distributions
- Example: Designing a New Apartment Building
- Connecting Random Inputs and Random Outputs in a Simulation
- Setting up and Running a Simulation in Excel **Session 2**
- Analyzing and Interpreting Simulation Output
- Evaluating Alternative Decisions using Simulation Results

# Week 4: Balancing Risk and Reward Using Simulation

- Modeling Uncertainty: From Scenarios to Continuous Distributions
- Example: Designing a New Apartment Building
- Connecting Random Inputs and Random Outputs in a Simulation
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- Analyzing and Interpreting Simulation Output **Session 3**
- Evaluating Alternative Decisions using Simulation Results

# Simulation: Results for n=10 Simulation Runs

- The profit (in $):

  $\Pi = 500{,}000*\min(D_R,R) + 900{,}000*\min(D_L,L) + 100{,}000*(R-\min(D_R,R)) + 150{,}000*(L-\min(D_L,L))$

- Stargrove.xlsx

**Stargrove.xlsx**
**Modeling Risk and Realities**

| Apartments | | Regular | | Luxury |
|---|---|---|---|---|
| Profit during the year | $ | 500,000.00 | $ | 900,000.00 |
| Salvage Profit | $ | 100,000.00 | $ | 150,000.00 |
| Profit Threshold | $ | 45,000,000.00 | | |

| | Regular | Luxury |
|---|---|---|
| Numbers of Apartments | 96 | 12 |
| Demand Parameters | | |
| Expected | 90 | 10 |
| St. Dev. | 25 | 3 |

| Reward | $ | 50,325,000.00 |
|---|---|---|
| Risk | | 0.2 |

| Demand Realizations | | RV for Regular Demand | RV for Luxury Demand | DR | DL | Profit from Regular apartments | Profit from Luxury apartments | Total Profit | Profit below Threshold? |
|---|---|---|---|---|---|---|---|---|---|
| | 1 | 34.65997588 | 6.536523668 | 34 | 6 | $ 23,200,000.00 | $ 6,300,000.00 | $ 29,500,000.00 | 1 |
| | 2 | 95.14134513 | 2.548728187 | 95 | 2 | $ 47,600,000.00 | $ 3,300,000.00 | $ 50,900,000.00 | 0 |
| | 3 | 103.4140237 | 9.157716957 | 103 | 9 | $ 48,000,000.00 | $ 8,550,000.00 | $ 56,550,000.00 | 0 |
| | 4 | 83.66079351 | 8.136827344 | 83 | 8 | $ 42,800,000.00 | $ 7,800,000.00 | $ 50,600,000.00 | 0 |
| | 5 | 146.1676643 | 11.60464879 | 146 | 11 | $ 48,000,000.00 | $ 10,050,000.00 | $ 58,050,000.00 | 0 |
| | 6 | 57.91029844 | 7.837553565 | 57 | 7 | $ 32,400,000.00 | $ 7,050,000.00 | $ 39,450,000.00 | 1 |
| | 7 | 128.539838 | 9.745067498 | 128 | 9 | $ 48,000,000.00 | $ 8,550,000.00 | $ 56,550,000.00 | 0 |
| | 8 | 159.5054041 | 12.00413297 | 159 | 12 | $ 48,000,000.00 | $ 10,800,000.00 | $ 58,800,000.00 | 0 |
| | 9 | 76.73592927 | 6.531155375 | 76 | 6 | $ 40,000,000.00 | $ 6,300,000.00 | $ 46,300,000.00 | 0 |
| | 10 | 116.6149982 | 9.286600314 | 116 | 9 | $ 48,000,000.00 | $ 8,550,000.00 | $ 56,550,000.00 | 0 |

# Simulation: Results for n=10 Simulation Runs

- Sample of random variables from the normal distribution with mean 90 and standard deviation of 25 and its descriptive statistics

| Demand Realizations | RV for Regular Demand |
|---|---|
| 1 | 34.65997588 |
| 2 | 95.14134513 |
| 3 | 103.4140237 |
| 4 | 83.66079351 |
| 5 | 146.1676643 |
| 6 | 57.91029844 |
| 7 | 128.539838 |
| 8 | 159.5054041 |
| 9 | 76.73592927 |
| 10 | 116.6149982 |

| *RV for Regular Demand* | |
|---|---|
| Mean | 100.235027 |
| Standard Error | 12.31380659 |
| Median | 99.2776844 |
| Mode | #N/A |
| Standard Deviation | 38.9396755 |
| Sample Variance | 1516.298328 |
| Kurtosis | -0.581372514 |
| Skewness | -0.101693197 |
| Range | 124.8454282 |
| Minimum | 34.65997588 |
| Maximum | 159.5054041 |
| Sum | 1002.35027 |
| Count | 10 |
| Confidence Level(95.0%) | 27.85576579 |

- Sample mean is based on a small sample of n=10 instances of the underlying random variable. It is just an approximation to the true expected value of the random variable being simulated

# Simulation: Results for n=10 Simulation Runs

- Sample of random variables from the normal distribution with mean **90** and standard deviation of 25 and its descriptive statistics

| Demand Realizations | RV for Regular Demand |
|---|---|
| 1 | 34.65997588 |
| 2 | 95.14134513 |
| 3 | 103.4140237 |
| 4 | 83.66079351 |
| 5 | 146.1676643 |
| 6 | 57.91029844 |
| 7 | 128.539838 |
| 8 | 159.5054041 |
| 9 | 76.73592927 |
| 10 | 116.6149982 |

| *RV for Regular Demand* | |
|---|---|
| Mean | 100.235027 |
| Standard Error | 2.31380659 |
| Median | 99.2776844 |
| Mode | #N |
| Standard Deviation | 38.9396755 |
| Sample Variance | 516.298328 |
| Kurtosis | .581372514 |
| Skewness | .101693197 |
| Range | 24.8454282 |
| Minimum | 4.65997588 |
| Maximum | 59.5054041 |
| Sum | 1002.35027 |
| Count | 10 |
| Confidence Level(95.0%) | 7.85576579 |

- **95% confidence level** identifies the "95% confidence interval" for the true expected value of the simulated random variable: based on the results of this simulation, we are 95% confident that the true expected value is in the interval = sample mean +/- 95% confidence level ≈ 100.24 +/- 27.86 = **[72.38, 128.10]**

# Simulation: Results for n=10 Simulation Runs

- With a simulation that samples the random input variables only 10 times, the reliability of the estimates for the mean and the standard deviation for any random quantity involved may be limited

| | Total Profit |
|---|---|
| $ | 29,500,000.00 |
| $ | 50,900,000.00 |
| $ | 56,550,000.00 |
| $ | 50,600,000.00 |
| $ | 58,050,000.00 |
| $ | 39,450,000.00 |
| $ | 56,550,000.00 |
| $ | 58,800,000.00 |
| $ | 46,300,000.00 |
| $ | 56,550,000.00 |

| | | |
|---|---|---|
| Reward | $ | 50,325,000.00 |
| Risk | | 0.2 |

# Simulation: Results for n=10 Simulation Runs

- With a simulation that samples the random input variables only 10 times, the reliability of the estimates for the mean and the standard deviation for any random quantity involved may be limited

| | Total Profit |
|---|---|
| $ | 29,500,000.00 |
| $ | 50,900,000.00 |
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| $ | 58,050,000.00 |
| $ | 39,450,000.00 |
| $ | 56,550,000.00 |
| $ | 58,800,000.00 |
| $ | 46,300,000.00 |
| $ | 56,550,000.00 |

| | | |
|---|---|---|
| **Reward** | **$** | **50,325,000.00** |
| **Risk** | | **0.2** |

| *Profit* | |
|---|---|
| Mean | 50325000 |
| Standard Error | 3011655.599 |
| Median | 53725000 |
| Mode | 56550000 |
| Standard Deviation | 9523691.22 |
| Sample Variance | 9.07007E+13 |
| Kurtosis | 1.379387771 |
| Skewness | -1.381025738 |
| Range | 29300000 |
| Minimum | 29500000 |
| Maximum | 58800000 |
| Sum | 503250000 |
| Count | 10 |
| Confidence Level(95.0%) | 6812838.284 |

# Simulation: Results for n=10 Simulation Runs

- With a simulation that samples the random input variables only 10 times, the reliability of the estimates for the mean and the standard deviation for any random quantity involved may be limited

| | Total Profit |
|---|---|
| $ | 29,500,000.00 |
| $ | 50,900,000.00 |
| $ | 56,550,000.00 |
| $ | 50,600,000.00 |
| $ | 58,050,000.00 |
| $ | 39,450,000.00 |
| $ | 56,550,000.00 |
| $ | 58,800,000.00 |
| $ | 46,300,000.00 |
| $ | 56,550,000.00 |

| | |
|---|---|
| **Reward** | **$ 50,325,000.00** |
| **Risk** | **0.2** |

| *Profit* | |
|---|---|
| Mean | 50325000 |
| Standard Error | 3011655.599 |
| Median | 53725000 |
| Mode | 56550000 |
| Standard Deviation | 9523691.22 |
| Sample Variance | 9.07007E+13 |
| Kurtosis | 1.379387771 |
| Skewness | -1.381025738 |
| Range | 29300000 |
| Minimum | 29500000 |
| Maximum | 58800000 |
| Sum | 503250000 |
| Count | 10 |
| Confidence Level(95.0%) | 6812838.284 |

- Our estimate for the reward measure, ≈ **$50,325,000** may also be quite removed from the true expected profit value associated with the decision we consider

# Simulation: Results for n=10 Simulation Runs

- With a simulation that samples the random input variables only 10 times, the reliability of the estimates for the mean and the standard deviation for any random quantity involved may be limited

| | Total Profit |
|---|---|
| $ | 29,500,000.00 |
| $ | 50,900,000.00 |
| $ | 56,550,000.00 |
| $ | 50,600,000.00 |
| $ | 58,050,000.00 |
| $ | 39,450,000.00 |
| $ | 56,550,000.00 |
| $ | 58,800,000.00 |
| $ | 46,300,000.00 |
| $ | 56,550,000.00 |

| | |
|---|---|
| **Reward** | **$ 50,325,000.00** |
| **Risk** | **0.2** |

| *Profit* | |
|---|---|
| Mean | 50325000 |
| Standard Error | 3011655.599 |
| Median | 53725000 |
| Mode | 56550000 |
| Standard Deviation | 9523691.22 |
| Sample Variance | 9.07007E+13 |
| Kurtosis | 1.379387771 |
| Skewness | -1.381025738 |
| Range | 29300000 |
| Minimum | 29500000 |
| Maximum | 58800000 |
| Sum | 503250000 |
| Count | 10 |
| Confidence Level(95.0%) | 6812838.284 |

- Based on the results of this simulation, we can be 95% confident that the true expected profit under the decision we consider lies in the interval ≈ $50,325,000 +/- $6,812,838 = **[$43,512,162, $57,137,838]**

# Simulation: Results for n=1000 Simulation Runs

- Stargrove_1000.xlsx, seed 123 for the B column and seed 1234 for the C column

| Total Profit | |
|---|---|
| $ | 29,500,000.00 |
| $ | 50,900,000.00 |
| $ | 56,550,000.00 |
| $ | 50,600,000.00 |
| $ | 58,050,000.00 |
| $ | 39,450,000.00 |
| $ | 56,550,000.00 |
| $ | 58,800,000.00 |
| $ | 46,300,000.00 |
| $ | 56,550,000.00 |
| $ | 57,300,000.00 |
| $ | 34,900,000.00 |
| $ | 45,200,000.00 |
| $ | 39,900,000.00 |
| $ | 55,800,000.00 |
| $ | 55,750,000.00 |
| $ | 55,050,000.00 |

| Reward | $ | 51,733,050.00 |
|---|---|---|
| Risk | | 0.17 |

| *Profit* | |
|---|---|
| Mean | 51733050 |
| Standard Error | 208452.1948 |
| Median | 53800000 |
| Mode | 58800000 |
| Standard Deviation | 6591837.188 |
| Sample Variance | 4.34523E+13 |
| Kurtosis | 0.694035866 |
| Skewness | -1.110170488 |
| Range | 32950000 |
| Minimum | 25850000 |
| Maximum | 58800000 |
| Sum | 51733050000 |
| Count | 1000 |
| Confidence Level(95.0%) | 409054.3845 |

- Based on the results of this longer simulation, we can now be 95% confident that the true expected profit under the decision we consider lies in the interval ≈ $51,733,050 +/- $409,054 = **[$51,323,996, $52,142,104]**

# Simulation: Results for n=1000 Simulation Runs

- Stargrove_1000.xlsx

| | Total Profit |
|---|---|
| $ | 29,500,000.00 |
| $ | 50,900,000.00 |
| $ | 56,550,000.00 |
| $ | 50,600,000.00 |
| $ | 58,050,000.00 |
| $ | 39,450,000.00 |
| $ | 56,550,000.00 |
| $ | 58,800,000.00 |
| $ | 46,300,000.00 |
| $ | 56,550,000.00 |
| $ | 57,300,000.00 |
| $ | 34,900,000.00 |
| $ | 45,200,000.00 |
| $ | 39,900,000.00 |
| $ | 55,800,000.00 |
| $ | 55,750,000.00 |
| $ | 55,050,000.00 |

| | | |
|---|---|---|
| **Reward** | **$** | **51,733,050.00** |
| **Risk** | | **0.17** |

*below Threshold?*

| | |
|---|---|
| Mean | 0.17 |
| Standard Error | 0.011884496 |
| Median | 0 |
| Mode | 0 |
| Standard Deviation | 0.375820757 |
| Sample Variance | 0.141241241 |
| Kurtosis | 1.098651798 |
| Skewness | 1.759675679 |
| Range | 1 |
| Minimum | 0 |
| Maximum | 1 |
| Sum | 170 |
| Count | 1000 |
| Confidence Level(95.0%) | 0.023321439 |

- Based on the results of this longer simulation, we can be 95% confident that the true value of the risk measure under the decision we consider lies in the interval ≈ 0.17+/- 0.023 = **[0.147, 0.193]**

# Simulation: Comparing Two Alternatives

- Suppose that Stargrove would like to compare the decision of building 12 regular floors and 3 luxury floors (R=96 and L=12) with the decision of building 11 regular floors and 4 luxury floors (R=88 and L=16)

- We can use 1000 random values we have already generated for the demand for regular apartments and 1000 random values we have already generated for the demand for luxury apartments to estimate the reward and the risk associated with the decision of R=88 and L=16

- We can then compare reward and risk estimates for the two decisions

# Simulation: Comparing Two Alternatives

- Stargrove_1000_TwoDecisions.xlsx

| *Profit for R=96, L=12* | | *Profit for R=96, L=12 below Threshold?* | |
|---|---|---|---|
| Mean | 51733050 | Mean | 0.17 |
| Standard Error | 208452.1948 | Standard Error | 0.011884496 |
| Median | 53800000 | Median | 0 |
| Mode | 58800000 | Mode | 0 |
| Standard Deviation | 6591837.188 | Standard Deviation | 0.375820757 |
| Sample Variance | 4.34523E+13 | Sample Variance | 0.141241241 |
| Kurtosis | 0.694035866 | Kurtosis | 1.098651798 |
| Skewness | -1.110170488 | Skewness | 1.759675679 |
| Range | 32950000 | Range | 1 |
| Minimum | 25850000 | Minimum | 0 |
| Maximum | 58800000 | Maximum | 1 |
| Sum | 51733050000 | Sum | 170 |
| Count | 1000 | Count | 1000 |
| Confidence Level(95.0%) | 409054.3845 | Confidence Level(95.0%) | 0.023321439 |

| *Profit for R=88, L=16* | | *Profit for R=88, L=16 below Threshold?* | |
|---|---|---|---|
| Mean | 50203250 | Mean | 0.168 |
| Standard Error | 178926.0363 | Standard Error | 0.011829 |
| Median | 51650000 | Median | 0 |
| Mode | 53900000 | Mode | 0 |
| Standard Deviation | 5658138.074 | Standard Deviation | 0.374053 |
| Sample Variance | 3.20145E+13 | Sample Variance | 0.139916 |
| Kurtosis | 1.657913101 | Kurtosis | 1.16612 |
| Skewness | -1.322835317 | Skewness | 1.778705 |
| Range | 32750000 | Range | 1 |
| Minimum | 25650000 | Minimum | 0 |
| Maximum | 58400000 | Maximum | 1 |
| Sum | 50203250000 | Sum | 168 |
| Count | 1000 | Count | 1000 |
| Confidence Level(95.0%) | 351113.9795 | Confidence Level(95.0%) | 0.023212 |

# Simulation: Comparing Two Alternatives

- Stargrove_1000_TwoDecisions.xlsx

| *Profit for R=96, L=12* | | *Profit for R=96, L=12 below Threshold?* | |
|---|---|---|---|
| Mean | 51733050 | Mean | 0.17 |
| Standard Error | 208452.1948 | Standard Error | 0.011884496 |
| Median | 53800000 | Median | 0 |
| Mode | 58800000 | Mode | 0 |
| Standard Deviation | 6591837.188 | Standard Deviation | 0.375820757 |
| Sample Variance | 4.34523E+13 | Sample Variance | 0.141241241 |
| Kurtosis | 0.694035866 | Kurtosis | 1.098651798 |
| Skewness | -1.110170488 | Skewness | 1.759675679 |
| Range | 32950000 | Range | 1 |
| Minimum | 25850000 | Minimum | 0 |
| Maximum | 58800000 | Maximum | 1 |
| Sum | 51733050000 | Sum | 170 |
| Count | 1000 | Count | 1000 |
| Confidence Level(95.0%) | 409054.3845 | Confidence Level(95.0%) | 0.023321439 |

| *Profit for R=88, L=16* | | *Profit for R=88, L=16 below Threshold?* | |
|---|---|---|---|
| Mean | 50203250 | Mean | 0.168 |
| Standard Error | 178926.0363 | Standard Error | 0.011829 |
| Median | 51650000 | Median | 0 |
| Mode | 53900000 | Mode | 0 |
| Standard Deviation | 5658138.074 | Standard Deviation | 0.374053 |
| Sample Variance | 3.20145E+13 | Sample Variance | 0.139916 |
| Kurtosis | 1.657913101 | Kurtosis | 1.16612 |
| Skewness | -1.322835317 | Skewness | 1.778705 |
| Range | 32750000 | Range | 1 |
| Minimum | 25650000 | Minimum | 0 |
| Maximum | 58400000 | Maximum | 1 |
| Sum | 50203250000 | Sum | 168 |
| Count | 1000 | Count | 1000 |
| Confidence Level(95.0%) | 351113.9795 | Confidence Level(95.0%) | 0.023212 |

Reward and risk measures for two policies

# Simulation: Comparing Two Alternatives

- Stargrove_1000_TwoDecisions.xlsx

| **Decision** | **R=96, L=12** | **R=88, L=16** |
|---|---|---|
| 95% Confidence Interval for Reward, in $ millions | [51.3, 52.1] | [49.9, 50.6] |
| 95% Confidence Interval for Risk | [0.147, 0.193] | [0.145, 0.191] |

- Based on the results of the simulation with n=1000 runs, we are 95% confident that the expected profit under the decision R=96, L=12 is higher than the expected profit under the decision R=88, L=16

- The results of this simulation do not allow us to distinguish between the levels of risk associated with those two decisions at the same level of confidence

# Simulation: Comparing Two Alternatives

- We can add other reasonable decisions to our comparison set
- If two decisions cannot be distinguished on the basis of the results of a particular simulation, we can also run longer simulations to obtain more narrow confidence intervals for reward and risk measures

# Simulation: Comparing Two Alternatives

- We can add other reasonable decisions to our comparison set
- If two decisions cannot be distinguished on the basis of the results of a particular simulation, we can also run longer simulations to obtain more narrow confidence intervals for reward and risk measures
- Ultimately, the goals are to 1) limit the consideration set to decisions that result in risk measures limited by the tolerance level of a decision maker, and 2) among the decisions that satisfy constraint(s) on acceptable risk level(s), choose one that generates highest reward, at the selected confidence level
- Simulation provides “imperfect” estimates of reward and risk, but the notion of confidence intervals enables a decision maker to compare alternatives even using those imperfect estimates