FUNDAMENTALS OF QUANTITATIVE MODELING

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Module 1: Introduction and core modeling math

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Course goals

- Goals
  - Exposure to the language of modeling
  - See a variety of quantitative business models and applications
  - Learn the process of modeling and how to critique models
  - Associate business process characteristics with appropriate models
  - Understand the value and limitations of quantitative models
  - Provide the foundational material for the other three courses in the Specialization

Resources

- Software used in this Specialization
  - Excel
  - Google sheets
  - R – an open source modeling platform
- Math review
  - E-book: Business math: essential mathematics for business modeling

Module 1 content

- Examples and uses of models
- Keys steps in the modeling process
- A vocabulary for modeling
- Mathematical functions
  - Linear
  - Power
  - Exponential
  - Log

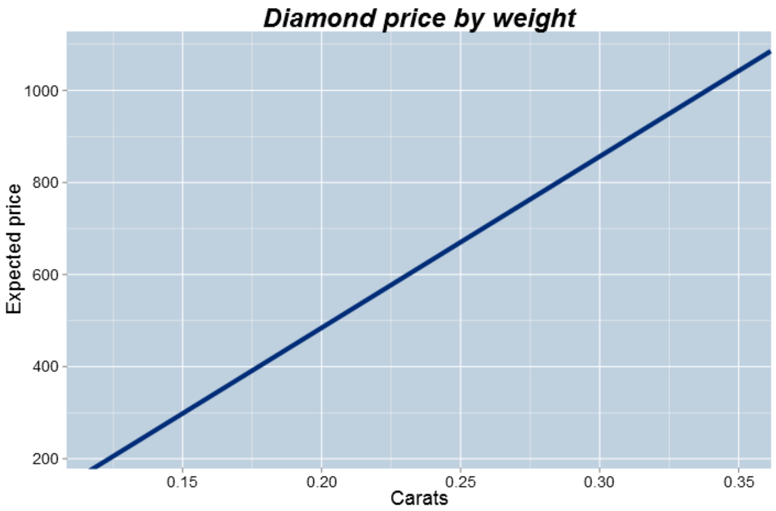
What is a model?

- A formal **description** of a business process
- It typically involves **mathematical** equations and/or random variables
- It is almost always a **simplification** of a more complex structure
- It typically relies upon a set of **assumptions**
- It is usually implemented in a computer program or using a spreadsheet

Examples of models

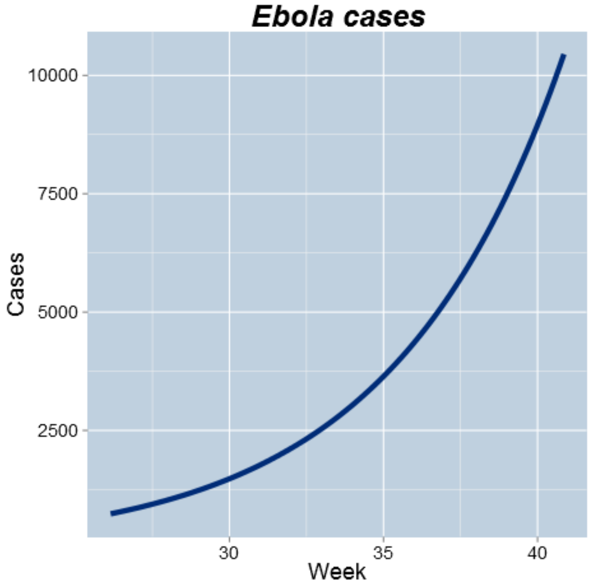
- The price of a diamond as a function of its weight
- The spread of an epidemic over time
- The relationship between demand for, and price of, a product
- The uptake of a new product in a market

Diamonds and weight



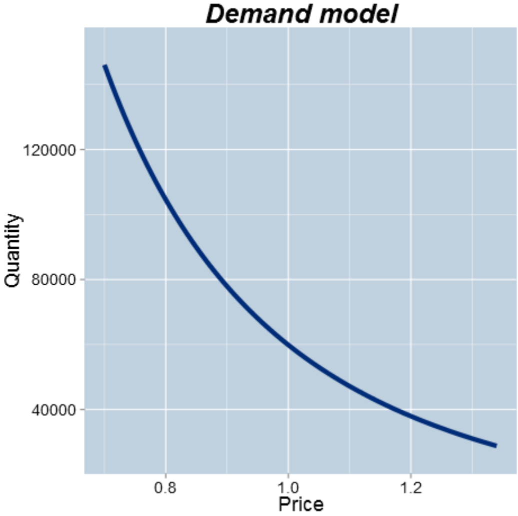
Model: $\text{Expected price} = -260 + 3721\ \text{Weight}$

Spread of an epidemic



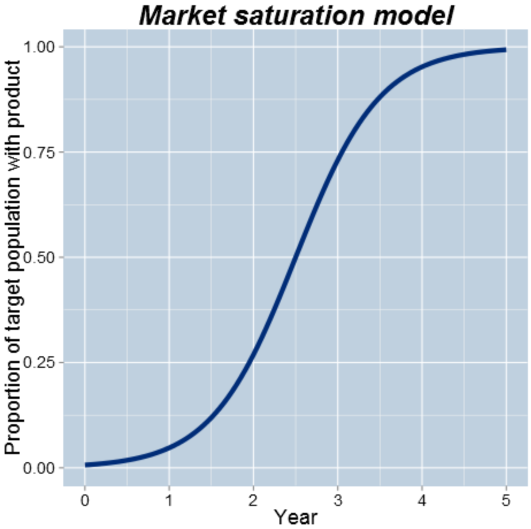
Model: $\text{Cases} = 6.69\, e^{0.18\,\text{Weeks}}$

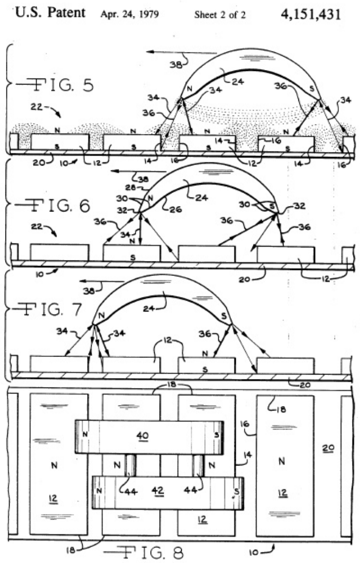
Demand models



$$\text{Model: Quantity} = 60{,}000\ \text{Price}^{-2.5}$$

The uptake of a new product





Model: Prop= $e^{2(Year-2.5)} / 1+ e^{2(Year-2.5)}$

How models are used in practice

- Prediction: calculating a single output
  - What's the expected price of a diamond ring that weighs 0.2 carats?
- Forecasting
  - How many people are expected to be infected in 6 weeks?
  - Scheduling – who is likely to turn up for their outpatient appointment?
- Optimization
  - What price maximizes profit?
- Ranking and targeting
  - Given limited resources, which potential diamonds for sale should be targeted first for potential purchase?

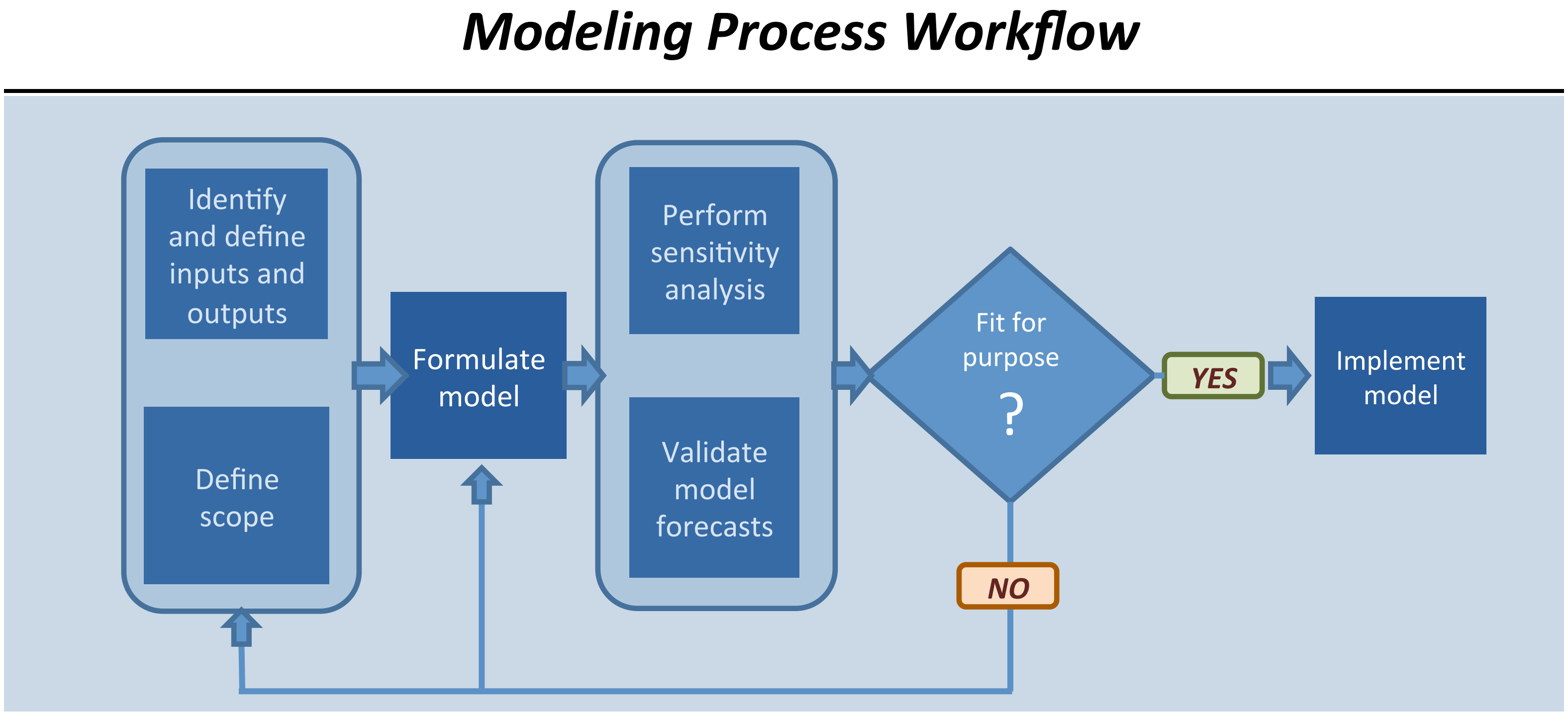
How models are used in practice

- Exploring what-if scenarios
  - If the growth rate of the epidemic increased to 20% each week, then how many infections would we expect in the next 10 weeks?
- Interpreting coefficients in model
  - What do we learn from the coefficient -2.5 in the price/demand model?
- Assessing how sensitive the model is to key assumptions

Benefits of modeling

- Identify gaps in current understanding
- Make assumptions explicit
- Have a well-defined description of the business process
- Create an institutional memory
- Used as a decision ***support*** tool
- Serendipitous insight generator

Key steps in the modeling process



What if the model doesn't always work?

- When the observed outcome differs greatly from the model's prediction, then there is the possibility of learning from this event if we can understand why the difference occurs
- Modeling is a continuous and evolutionary process
- We identify the weaknesses and limitations and ***iterate*** the modeling process to overcome them

A modeling lexicon

Discrete

Data driven

Stochastic

Dynamic

Theory driven

Deterministic

Continuous

Static

Data driven v. theory driven

Empirical ⟷ Theoretical

- Theory: given a set of assumptions and relationships, then what are the logical consequences?
  - Example: if we assume that markets are efficient then what should the price of a stock option be?
- Data: given a set of observations, how can we approximate the underlying process that generated them?
  - Example: I've separated out my profitable customers from the unprofitable ones. Now, what features are able to differentiate them?

Deterministic v. probabilistic/stochastic

- Deterministic: given a fixed set of inputs, the model always gives the same output
  - Example: Invest $1000 at 4% annual compound interest for 2 years. After 2 years the initial $1000 will **always** be worth $1081.60.
- Probabilistic: Even with identical inputs, the model output can vary from instance to instance
  - Example: A person spends $1000 on lottery tickets. After the lottery is drawn how much they are worth depends on a random variable, whether or not they won the lottery.

Discrete v. continuous variables

- Watches can be digital or analog

- Likewise models can involve discrete or continuous variables
  - Discrete: characterized by jumps and distinct values
  - Continuous: a smooth process with an infinite number of potential values in any fixed interval

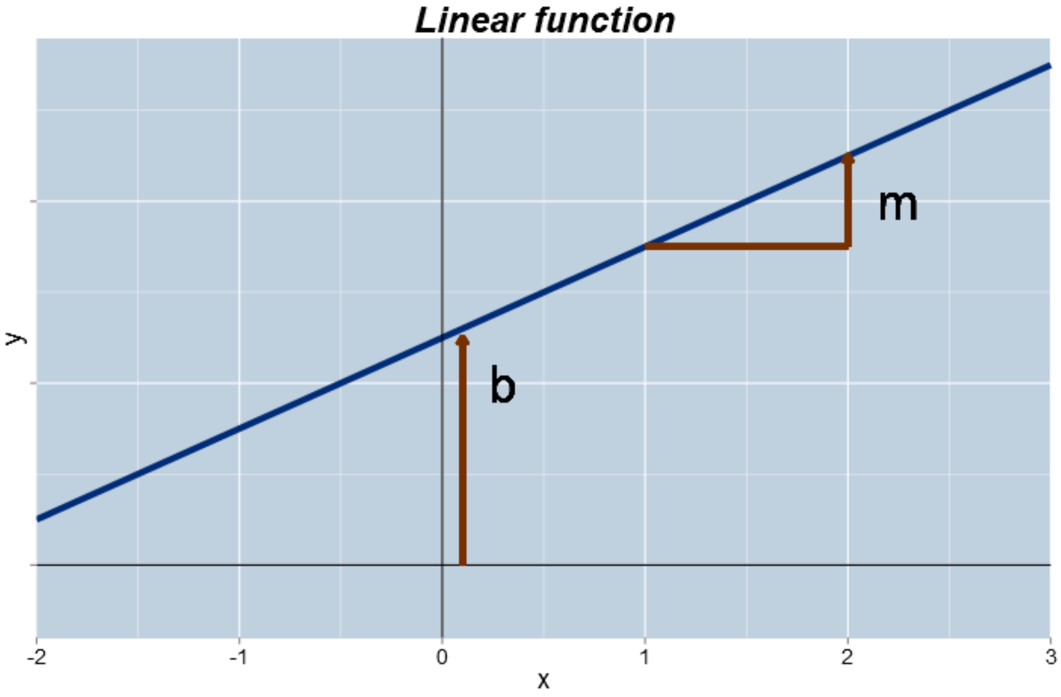
Static v. dynamic

- Static: the model captures a single snapshot of the business process
  - Given a website's installed software base, what are the chances that it is compromised today?
- Dynamic: the evolution of the process itself is of interest. The model describes the movement from state to state
  - Given a person's participation in a job training program, how long will it take until he/she finds a job and then, if they find one, for how long will they keep it?

Key mathematical functions

- Math: the language of modeling
  - Four key mathematical functions provide the foundations for quantitative modeling
  - 1. Linear
  - 2. Power
  - 3. Exponential
  - 4. Log

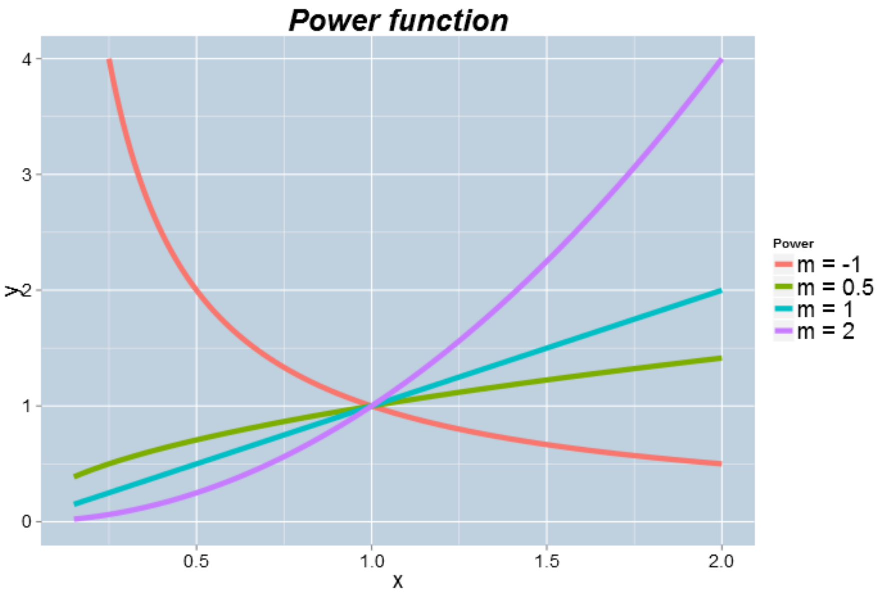
The linear function



The linear function

- $y = mx + b$
- x is the input, y is the output
- b is the intercept
- m is the slope
- Essential characteristic: the slope is constant
  - A one-unit change in x corresponds to an m-unit change in y.

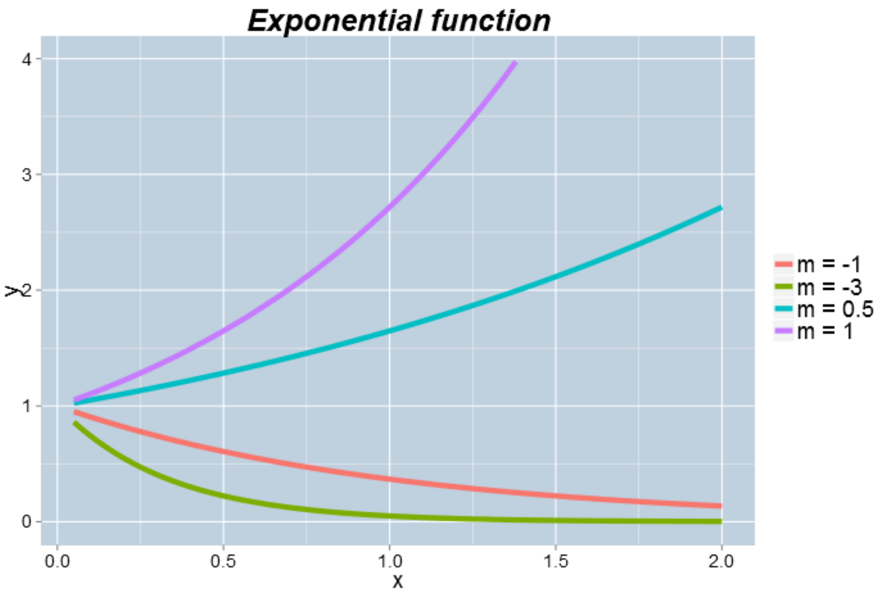
The power function for various powers of x



The power function

- $y = x^m$
- x is the **base**
- m is the **exponent**
- Essential characteristic:
  - A one **percent** (proportionate) change in x corresponds to an approximate m **percent** (proportionate) change in y.
- Facts
  1. $x^m x^n = x^{m+n}$
  2. $x^{-m} = 1/x^m$

The exponential function for various values of m



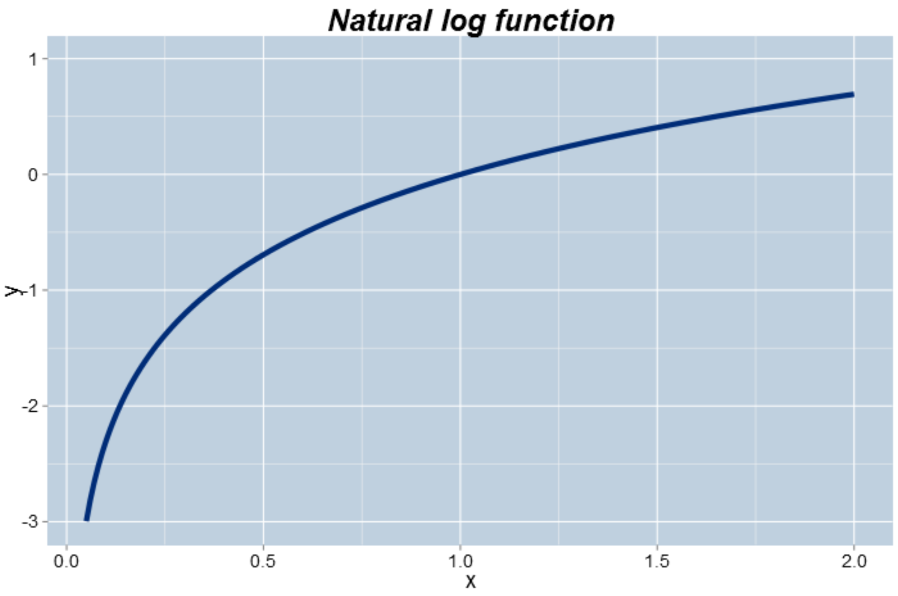
The exponential function

- $y = e^{mx}$
- e is the mathematical constant: 2.71828…
- Notice that as compared to a power function, x is in the exponent of the function and not the base

The exponential function

- Essential characteristic:
  - the rate of change of y is proportional to y itself
- Interpretation of m for small values of m (say $-0.2 \leq m \leq 0.2$):
  - For every one-unit change in x, there is an approximate 100m **%** (proportionate) change in y
  - Example: if m = 0.05, then a one-**unit** increase in x is associated with an approximate 5**%** increase in y

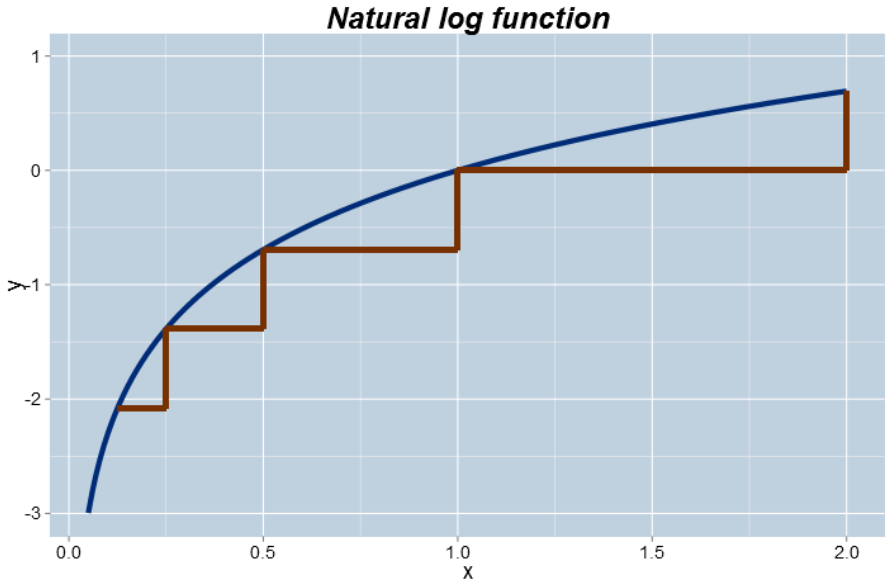
The log function



The log function

- The log function is very useful for modeling processes that exhibit ***diminishing returns to scale***
- These are processes that increase but at a decreasing rate
- Essential characteristic:
  - A constant proportionate change in x is associated with the same absolute change in y

The log function

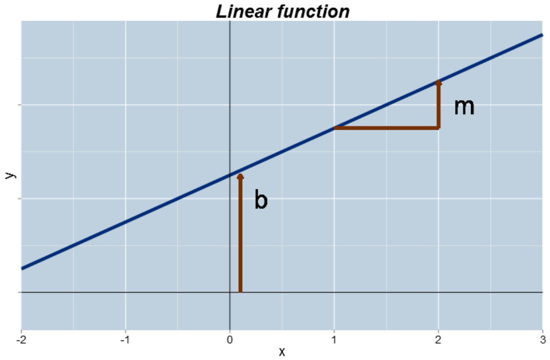


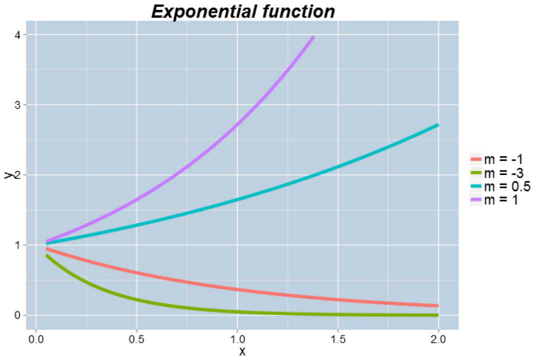
- Proportionate change in x is associated with constant change in y

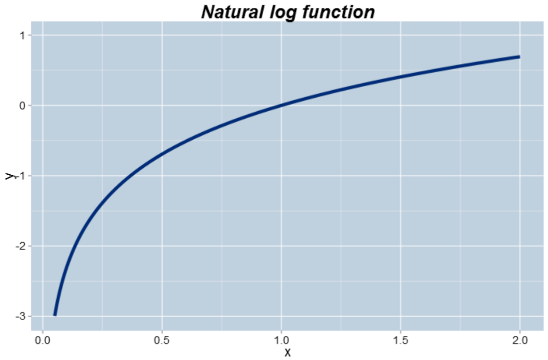
The log function

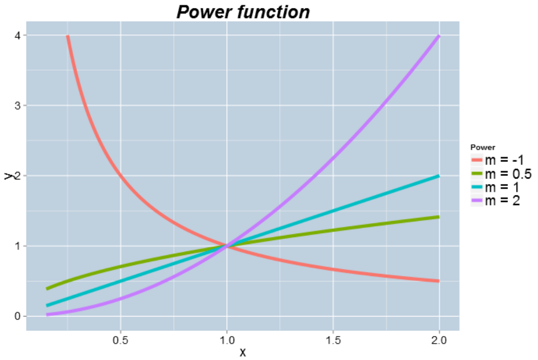
- $y = \log_b(x)$
- b is called the base of the logarithm
- The most frequently used base is the number “e” and the logarithm is called the “natural log”
- The log undoes (is the inverse of) the exponential function:
  - $\log_e e^x = x$
  - $e^{\log_e x} = x$
- $\log(xy) = \log(x) + \log(y)$
- In this course we will always use the natural log and write it simply as log(x)

The four functions









Module summary

- Uses for models
- Steps in the modeling process
  - It is an iterative process and model validation is key
- Discussed various types of models, discrete v. continuous etc.
- Reviewed essential mathematical functions that form the foundation of quantitative models

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