Week 2: Risk and Reward: Modeling High Uncertainty Settings

Session 1

- High-Uncertainty Settings: Stock Price Example
- Probability Distributions: Scenario Approach
- Parameters of the Probability Distributions: Expected Value, Variance, Standard Deviation
- Uncertainty and Risk

Week 2: Risk and Reward: Modeling High Uncertainty Settings

- Common Scenarios for Multiple Random Variables
- Risk Reduction Example: Investing in a Pair of Stocks
- Calculating and Interpreting Correlation Values

Session 2

- Using Scenarios for Optimizing Under High Uncertainty: Portfolio Selection Problem
- Sensitivity Analysis and Efficient Frontier

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Session 3

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Low-Uncertainty vs. High Uncertainty Settings

- In our first example in Week 1, we have looked at a company (Hudson Readers Inc.) faced with a decision of how to allocate its advertising budget for a new product
- All of the parameters in that example were assumed to take **deterministic** values
- For example, the sales response to advertising the Standard version in India is assumed to be 0.05, rather than, say, having 50%-50% chance of being either 0.03 or 0.07
- Ignoring randomness in the data (for example, by replacing random quantities by their expected values) dramatically simplifies the process of finding the best solution

High-Uncertainty Setting: A Stock Price Example

- Consider a set of daily closing prices for a hypothetical stock A for a period of 40 consecutive trading days (Stock A.xlsx)
- “Closing price” is the last price at which a stock was traded on a particular day

Trading Day	Closing Price for Stock A (in $)
1	35.79
2	36.96
3	36.15
…	…
37	43.37
38	43.43
39	43.21
40	43.70

Closing price on Day 1 = $35.79

Closing price on Day 2 = $36.96

Closing price on Day 40 = $43.70

High-Uncertainty Setting: A Stock Price Example

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Trading Day	Closing Price for Stock A (in $)
1	35.79
2	36.96
3	36.15
…	…
37	43.37
38	43.43
39	43.21
40	43.70

- Historical values for closing stock prices are available, for example, at Yahoo Finance (http://finance.yahoo.com/q/hp?s=YHOO)

High-Uncertainty Setting: A Stock Price Example

- Analysis of randomness is often focused on stock “returns”
- The “return” on a particular trading day is the relative (percentage) change between the closing price on that trading day and the closing price on the previous trading day

Trading Day	Closing Price for Stock A (in $)
1	35.79
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High-Uncertainty Setting: A Stock Price Example

- Analysis of randomness is often focused on stock “returns”

- The “return” on a particular trading day is the relative (percentage) change between the closing price on that trading day and the closing price on the previous trading day

Return on Day 2 = ($36.96-$35.79)/$35.79 ≈ 0.03269

Trading Day	Closing Price for Stock A (in $)
1	35.79
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3	36.15
…	…
37	43.37
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High-Uncertainty Setting: A Stock Price Example

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Trading Day	Closing Price for Stock A (in $)
1	35.79
2	36.96
3	36.15
…	…
37	43.37
38	43.43
39	43.21
40	43.70

Return on Day 3 = ($36.15-$36.96)/$36.96 ≈ - 0.02191

High-Uncertainty Setting: A Stock Price Example

- Analysis of randomness is often focused on stock “returns”
- The “return” on a particular trading day is the relative (percentage) change between the closing price on that trading day and the closing price on the previous trading day

Trading Day	Closing Price for Stock A (in $)
1	35.79
2	36.96
3	36.15
…	…
37	43.37
38	43.43
39	43.21
40	43.70

Return on Day 40 = ($43.70-$43.21)/$43.21 ≈ 0.01134

Investing in Stock A: Modeling Future Value

- Consider an investor that purchases a number of shares of stock A at the closing price on day 40

Trading Day	Closing Price for Stock A (in $)
1	35.79
2	36.96
3	36.15
...	...
37	43.37
38	43.43
39	43.21
40	43.70

- What value will this investment have at the closing of trading on the next day?

- This value depends on the return on stock A on the next day, R

- How do we model the value of R?

Modeling Future Values

- Modeling future values is a complex task that can combine statistical analysis of historical data and subjective inputs, such as expert opinions
- Experience with making decisions in a particular business context can be a major factor in determining how historical data are to be used and how to combine historical data with subjective inputs
- Testing alternative plausible models of the future may be necessary to increase confidence in the recommended decisions

Scenario Approach to Modeling Future Realizations of A Random Quantity

- We are going to base our analysis of the future price of stock A on the following **modeling assumption**: the daily return on stock A is a random value that can take each of 20 values observed in the past 20 trading days, with equal probability (1/20)

- In other words, we are making an assumption that the last 20 values of the return on stock A completely describe all the possible values of tomorrow's return, and that each of those 20 values is equally likely to be repeated tomorrow

- The term **"scenario"** is used to describe each of the past realizations of the random quantity – and modeling the future using a number of scenarios is called **"scenario approach"**

Scenario Approach to Modeling Future Realizations of A Random Quantity

- We have chosen the number of scenarios to consider – 20 – arbitrarily. In general, one should try to vary the number of used scenarios to test the robustness of model predictions

- This choice of scenario approach, however, underlines two implicit assumptions:
  - 1) historical return values observed beyond the last 20 days are not likely to be relevant for predicting tomorrow's return, and
  - 2) each of the values observed in the past 20 days is equally likely to be observed tomorrow

- Stock A.xlsx

Scenario Approach to Modeling the Value of R: Complete Probability Distribution

Scenario	Probability
-0.00024	0.05
0.01760	0.05
-0.02114	0.05
-0.01178	0.05
-0.01515	0.05
-0.00353	0.05
-0.01772	0.05
-0.02345	0.05
0.03562	0.05
0.03108	0.05
0.01557	0.05
0.00073	0.05
-0.02188	0.05
0.02063	0.05
0.03044	0.05
0.01276	0.05
0.01214	0.05
0.00138	0.05
-0.00507	0.05
0.01134	0.05

Scenario 1: Return R_1 = -0.00024, occurring with probability p_1 = 0.05

- Stock A.xlsx

Scenario Approach to Modeling the Value of R: Complete Probability Distribution

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-0.00024	0.05
0.01760	0.05
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-0.01772	0.05
-0.02345	0.05
0.03562	0.05
0.03108	0.05
0.01557	0.05
0.00073	0.05
-0.02188	0.05
0.02063	0.05
0.03044	0.05
0.01276	0.05
0.01214	0.05
0.00138	0.05
-0.00507	0.05
0.01134	0.05

Scenario 20: Return R_{20} = 0.01134, occurring with probability p_1 = 0.05

- 40 parameters provide ***complete description*** of this distribution

Scenario Approach to Modeling the Value of R: Complete Probability Distribution

Scenario	Probability
-0.00024	0.05
0.01760	0.05
-0.02114	0.05
-0.01178	0.05
-0.01515	0.05
-0.00353	0.05
-0.01772	0.05
-0.02345	0.05
0.03562	0.05
0.03108	0.05
0.01557	0.05
0.00073	0.05
-0.02188	0.05
0.02063	0.05
0.03044	0.05
0.01276	0.05
0.01214	0.05
0.00138	0.05
-0.00507	0.05
0.01134	0.05

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-0.02345	0.05
0.03562	0.05
0.03108	0.05
0.01557	0.05
0.00073	0.05
-0.02188	0.05
0.02063	0.05
0.03044	0.05
0.01276	0.05
0.01214	0.05
0.00138	0.05
-0.00507	0.05
0.01134	0.05

Expected Value of R:

$$E(R) = p_1 * R_1 + p_2 * R_2 + \ldots + p_{19} * R_{19} + p_{20} * R_{20}$$

- Expected value tells you what you will get if you average the values of the infinite number of independent random “draws” from a distribution

Scenario Approach to Modeling the Value of R: Complete Probability Distribution

Scenario	Probability
-0.00024	0.05
0.01760	0.05
-0.02114	0.05
-0.01178	0.05
-0.01515	0.05
-0.00353	0.05
-0.01772	0.05
-0.02345	0.05
0.03562	0.05
0.03108	0.05
0.01557	0.05
0.00073	0.05
-0.02188	0.05
0.02063	0.05
0.03044	0.05
0.01276	0.05
0.01214	0.05
0.00138	0.05
-0.00507	0.05
0.01134	0.05

Expected Value of R:

$$E(R) = p_1{*}R_1 + p_2{*}R_2 + \ldots + p_{19}{*}R_{19} + p_{20}{*}R_{20}$$

- In Excel, you can use the =SUMPRODUCT() function to calculate the expected value of R

Scenario Approach to Modeling the Value of R: Complete Probability Distribution

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0.00073	0.05
-0.02188	0.05
0.02063	0.05
0.03044	0.05
0.01276	0.05
0.01214	0.05
0.00138	0.05
-0.00507	0.05
0.01134	0.05

Expected Value of R:
$E(R) = p_1 * R_1 + p_2 * R_2 + \ldots + p_{19} * R_{19} + p_{20} * R_{20} \approx \boxed{0.003467}$

- While, on average, R's value is 0.003467, on any particular random "draw", the actual value of R can be as low as **-0.02345** or as high as **0.03562**

Parameters Summarizing the Properties of a Distribution: Variance and Standard Deviation

Scenario	Probability
-0.00024	0.05
0.01760	0.05
-0.02114	0.05
-0.01178	0.05
-0.01515	0.05
-0.00353	0.05
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0.03562	0.05
0.03108	0.05
0.01557	0.05
0.00073	0.05
-0.02188	0.05
0.02063	0.05
0.03044	0.05
0.01276	0.05
0.01214	0.05
0.00138	0.05
-0.00507	0.05
0.01134	0.05

- ***Variance*** and ***standard deviation*** indicate how “far away”, on average, a random value of R is from its expected value E(R) = 0.003467

Parameters Summarizing the Properties of a Distribution: Variance and Standard Deviation

Scenario	Probability
-0.00024	0.05
0.01760	0.05
-0.02114	0.05
-0.01178	0.05
-0.01515	0.05
-0.00353	0.05
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0.03562	0.05
0.03108	0.05
0.01557	0.05
0.00073	0.05
-0.02188	0.05
0.02063	0.05
0.03044	0.05
0.01276	0.05
0.01214	0.05
0.00138	0.05
-0.00507	0.05
0.01134	0.05

Variance of R:

$$Var(R) = p_1*(R_1 - E(R))^2 + p_2*(R_2 - E(R))^2 + \ldots + p_{19}*(R_{19} - E(R))^2 + p_{20}*(R_{20} - E(R))^2$$

- Variance and standard deviation indicate how "far away", on average, a random value of R is from its expected value E(R) = 0.003467

Parameters Summarizing the Properties of a Distribution: Variance and Standard Deviation

Scenario	Probability
-0.00024	0.05
0.01760	0.05
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0.00073	0.05
-0.02188	0.05
0.02063	0.05
0.03044	0.05
0.01276	0.05
0.01214	0.05
0.00138	0.05
-0.00507	0.05
0.01134	0.05

Variance of R:

$$Var(R) = p_1*(R_1 - E(R))^2 + p_2*(R_2 - E(R))^2 + \ldots + p_{19}*(R_{19} - E(R))^2 + p_{20}*(R_{20} - E(R))^2$$

Standard Deviation of R:

$$SD(R) = \sqrt{Var(R)}$$

- Variance and standard deviation indicate how "far away", on average, a random value of R is from its expected value E(R) = 0.003467

Parameters Summarizing the Properties of a Distribution: Variance and Standard Deviation

Scenario	Probability
-0.00024	0.05
0.01760	0.05
-0.02114	0.05
-0.01178	0.05
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0.01557	0.05
0.00073	0.05
-0.02188	0.05
0.02063	0.05
0.03044	0.05
0.01276	0.05
0.01214	0.05
0.00138	0.05
-0.00507	0.05
0.01134	0.05

Variance of R:

$$\mathrm{Var}(R) = p_1*(R_1 - E(R))^2 + p_2*(R_2 - E(R))^2 + \ldots + p_{19}*(R_{19} - E(R))^2 + p_{20}*(R_{20} - E(R))^2$$

Standard Deviation of R:

$$SD(R) = \sqrt{\mathrm{Var}(R)}$$

- In Excel, variance can be computed by first evaluating, for each scenario, the squared deviation from the expected value, and then using =SUMPRODUCT()

Parameters Summarizing the Properties of a Distribution: Variance and Standard Deviation

Scenario	Probability
-0.00024	0.05
0.01760	0.05
-0.02114	0.05
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-0.01772	0.05
-0.02345	0.05
0.03562	0.05
0.03108	0.05
0.01557	0.05
0.00073	0.05
-0.02188	0.05
0.02063	0.05
0.03044	0.05
0.01276	0.05
0.01214	0.05
0.00138	0.05
-0.00507	0.05
0.01134	0.05

Variance of R:

$$Var(R) = p_1*(R_1 - E(R))^2 + p_2*(R_2 - E(R))^2 + \ldots + p_{19}*(R_{19} - E(R))^2 + p_{20}*(R_{20} - E(R))^2 \approx \boxed{0.000327}$$

Standard Deviation of R:

$$SD(R) = \sqrt{Var(R)} = \boxed{0.01808}$$

- In Excel, variance can be computed by first evaluating, for each scenario, the squared deviation from the expected value, and then using =SUMPRODUCT()

Uncertainty and Risk

Scenario	Probability
-0.00024	0.05
0.01760	0.05
-0.02114	0.05
-0.01178	0.05
-0.01515	0.05
-0.00353	0.05
-0.01772	0.05
-0.02345	0.05
0.03562	0.05
0.03108	0.05
0.01557	0.05
0.00073	0.05
-0.02188	0.05
0.02063	0.05
0.03044	0.05
0.01276	0.05
0.01214	0.05
0.00138	0.05
-0.00507	0.05
0.01134	0.05

- An investor buying shares of stock A, can use the expected value of 0.3467% as one indicator of what an actual return R can be
- At the same time, a standard deviation of 1.808% indicates that the actual value of R can be that far away, on average, from 0.3467%
- Standard deviation can serve as an indicator of the degree of ***uncertainty*** in the actual value of R
- Some decision makers may be averse to uncertainty, and, therefore, would prefer smaller values of standard deviation if they have a choice

Uncertainty and Risk

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-0.00024	0.05
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0.02063	0.05
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0.01276	0.05
0.01214	0.05
0.00138	0.05
-0.00507	0.05
0.01134	0.05

- Standard deviation indicates how far ***above*** or ***below*** the expected value the actual value can be, on average
- Some decision makers, however, would probably not mind having the actual return to be above its expected value
- In a similar way, they would likely be more concerned about the actual return to be below its expectation
- ***Risk*** can be defined as the likelihood and/or magnitude of ***undesirable*** outcome(s)

Uncertainty and Risk

Scenario	Probability
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0.01760	0.05
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0.02063	0.05
0.03044	0.05
0.01276	0.05
0.01214	0.05
0.00138	0.05
-0.00507	0.05
0.01134	0.05

- Risk and uncertainty may not always coincide
- Risk measures can come in different forms, and the same probability distribution can be used to evaluate multiple different risk measures
- Some decision makers may use the standard deviation as a risk measure they would like to control
- Others may prefer to focus on risk measures they associate with specific undesirable scenarios

Measures of Risk: Loss Probability

Scenario	Probability
-0.00024	0.05
0.01760	0.05
-0.02114	0.05
-0.01178	0.05
-0.01515	0.05
-0.00353	0.05
-0.01772	0.05
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0.03044	0.05
0.01276	0.05
0.01214	0.05
0.00138	0.05
-0.00507	0.05
0.01134	0.05

- For example, some decision makers may choose to focus on the likelihood of a loss

Measures of Risk: Loss Probability

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-0.02188	0.05
0.02063	0.05
0.03044	0.05
0.01276	0.05
0.01214	0.05
0.00138	0.05
-0.00507	0.05
0.01134	0.05

- For example, some decision makers may choose to focus on the likelihood of a loss
- In the distribution of R we use, the negative returns occur in 9 scenarios out of 20, with the total probability of a loss being 9*0.05=0.45

Measures of Risk: Probability of a “Substandard” Return

Scenario	Probability
-0.00024	0.05
0.01760	0.05
-0.02114	0.05
-0.01178	0.05
-0.01515	0.05
-0.00353	0.05
-0.01772	0.05
-0.02345	0.05
0.03562	0.05
0.03108	0.05
0.01557	0.05
0.00073	0.05
-0.02188	0.05
0.02063	0.05
0.03044	0.05
0.01276	0.05
0.01214	0.05
0.00138	0.05
-0.00507	0.05
0.01134	0.05

- Others would like to know the likelihood of generating a return that is below some threshold they consider acceptable, for example, a threshold of 1.5%

- In the distribution of R we use, the returns below 1.5% occur in 14 scenarios out of 20, with the total probability being 14*0.05=0.70

Reward and Risk

- The notions of “reward” and “risk” are often used to characterize decisions in high-uncertainty settings
- In the case of investing in stocks, the expected return can be used as a measure of “reward”: the higher is the expected return, all other things being equal, the more attractive is a particular investment choice
- “Risk” can be expressed in terms of a single quantity, such as standard deviation of returns, or probability of a loss, or multiple quantities used simultaneously
- The best alternative in high-uncertainty settings can then be identified by maximizing the reward while imposing constraints on the values of risk measures