Week 2: Risk and Reward: Modeling High Uncertainty Settings

- High-Uncertainty Settings: Stock Price Example
- Probability Distributions: Scenario Approach
- Parameters of the Probability Distributions: Expected Value, Variance, Standard Deviation
- Uncertainty and Risk

Session 1

Week 2: Risk and Reward: Modeling High Uncertainty Settings

- Common Scenarios for Multiple Random Variables
- Risk Reduction Example: Investing in a Pair of Stocks
- Calculating and Interpreting Correlation Values

Session 2

- Using Scenarios for Optimizing Under High Uncertainty: Portfolio Selection Problem
- Sensitivity Analysis and Efficient Frontier

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Session 3

Portfolio of Two Stocks

- An investor considers putting $100,000 in two stocks: stock A and stock B “today”

Scenario	Return on Stock A	Return on Stock B	Probability
1	-0.00024	0.0482	0.05
2	0.01760	-0.0047	0.05
3	-0.02114	0.0003	0.05
4	-0.01178	0.0022	0.05
5	-0.01515	0.0022	0.05
6	-0.00353	-0.0115	0.05
7	-0.01772	0.0462	0.05
8	-0.02345	0.0191	0.05
9	0.03562	-0.0168	0.05
10	0.03108	0.0251	0.05
11	0.01557	0.0278	0.05
12	0.00073	0.0067	0.05
13	-0.02188	0.0274	0.05
14	0.02063	0.0176	0.05
15	0.03044	-0.0122	0.05
16	0.01276	-0.0277	0.05
17	0.01214	-0.0634	0.05
18	0.00138	0.0100	0.05
19	-0.00507	-0.0379	0.05
20	0.01134	-0.0393	0.05

- For two stocks, random daily returns “tomorrow” are described by 20 equally probable scenarios

- Two Stocks.xlsx

Stocks A and B: Measures of Reward, Risk, and Correlation

- An investor considers putting $100,000 in two stocks: stock A and stock B “today”

	Return on Stock A, R_A	**Return on Stock B, R_B**
Expected Value	0.003467	0.0009641
Standard Deviation	0.018078	0.028095
Correlation	-0.275752	

- Stock B offers lower reward than Stock A, as measured by the expected return values, and higher risk than Stock A, as measured by the standard deviation of the returns

- But, stock B is negatively correlated with stock A, so it is conceivable that the investor may want to bring them together into a portfolio if she wants to achieve a reduction in risk

Portfolio of Two Stocks: Building an Analytical Model

- Components of a model: **decisions**, objective, and constraints

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  - X_A = how much to invest in Stock A, in $
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- Components of a model: **decisions**, **objective**, and constraints
- Decisions:
  X_A = how much to invest in Stock A, in $
  X_B = how much to invest in Stock B, in $
- An investor would like to have as much profit as possible "tomorrow"
- But, for each pair of decision variables X_A and X_B, tomorrow's profit, $\Pi = R_A * X_A + R_B * X_B$, is a random variable, since the returns on stocks A and B, R_A and R_B, are random

Building an Analytical Model: Risk and Reward

- An investor can build a model to **maximize the reward** why **controlling the risk at an acceptable level**

- **Objective** to be maximized: expected profit value, i.e., expected value of the random variable $\Pi = R_A * \mathbf{X_A} + R_B * \mathbf{X_B}$

- **Constraint** on the level of risk: standard deviation of Π, SD(Π), must not exceed a level tolerable for the investor, $\overline{SD}$

A Model: Maximizing Reward While Controlling Risk

Maximize $E(R_A * X_A + R_B * X_B)$ ← **Maximize the reward**

A Model: Maximizing Reward While Controlling Risk

Maximize $E(R_A*X_A + R_B*X_B)$

$SD(R_A*X_A + R_B*X_B) \leq \overline{SD}$ ← **Limit on risk**

A Model: Maximizing Reward While Controlling Risk

Maximize $E(R_A*X_A + R_B*X_B)$

$SD(R_A*X_A + R_B*X_B) \leq \overline{SD}$

$X_A + X_B = 100000$ ← **Size of the investment**

A Model: Maximizing Reward While Controlling Risk

Maximize $E(R_A*X_A + R_B*X_B)$

$SD(R_A*X_A + R_B*X_B) \leq \overline{SD}$

$X_A + X_B = 100000$

$X_A , X_B \geq 0$ ⟵ **Only non-negative investment amounts**

A Model: Maximizing Reward While Controlling Risk

Maximize $E(R_A*X_A + R_B*X_B)$

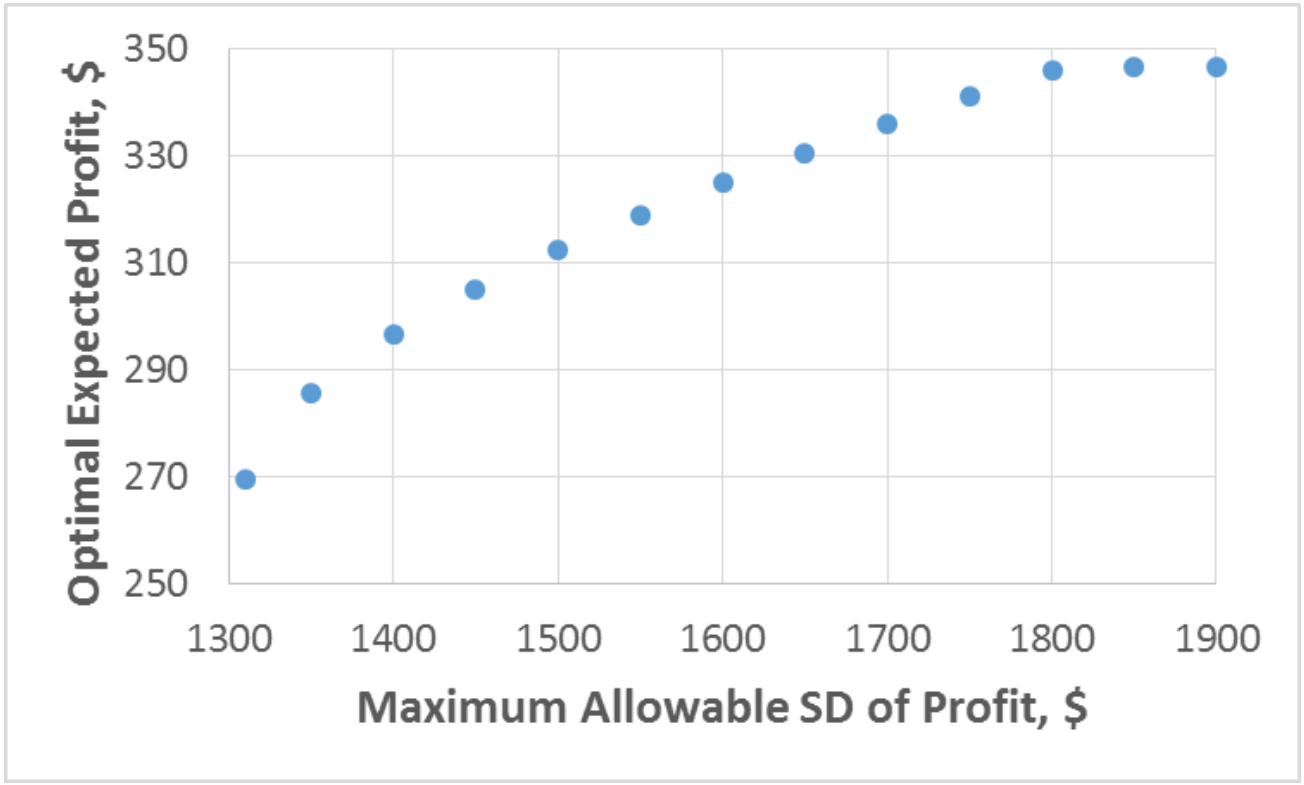
$SD(R_A*X_A + R_B*X_B) \leq \overline{SD}$

$X_A + X_B = 100000$

$X_A , X_B \geq 0$

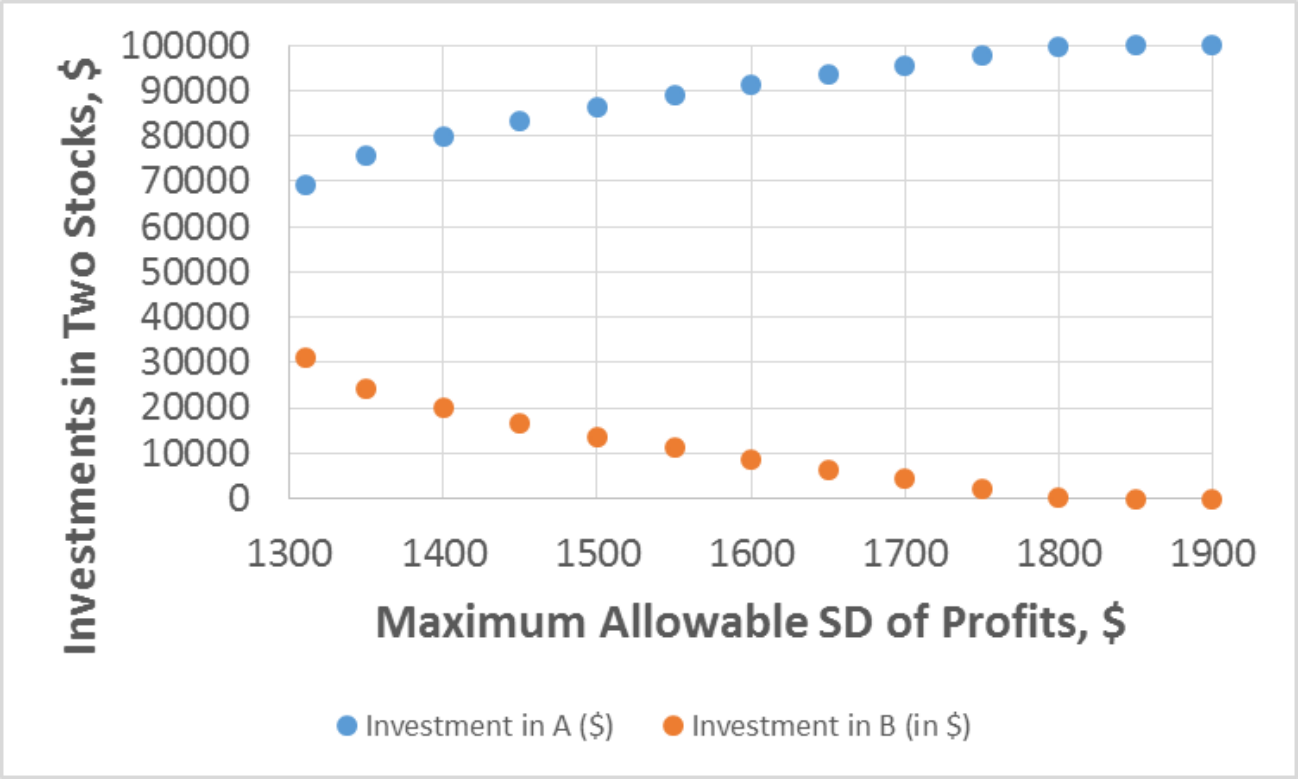
- We will use Excel to set up this model and Solver to find the best portfolio for different values of investor's tolerance for risk
- Two Stocks.xlsx

Risk-Reward Trade-Off



- Achieved reward level increases with the level of risk the investor is willing to tolerate
- Sheet “Results” in the file TwoStocks_Solved.xlsx

Optimal Investment Allocation



- As the risk tolerance of the investor increases, the higher rewards are obtained because of the increased the share of stock A
- Sheet “Results” in the file TwoStocks_Solved.xlsx