Week 4: Balancing Risk and Reward Using Simulation

- Modeling Uncertainty: From Scenarios to Continuous Distributions
- Example: Designing a New Apartment Building
- Connecting Random Inputs and Random Outputs in a Simulation
- Setting up and Running a Simulation in Excel
- Analyzing and Interpreting Simulation Output
- Evaluating Alternative Decisions using Simulation Results

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Session 1

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Session 3

- Evaluating Alternative Decisions using Simulation Results

Modeling Uncertainty: From Scenarios to Continuous Distributions

- In Week 2, we have used a relatively small number of scenarios to model all future outcomes of uncertain quantities

Scenario	Probability
-0.00024	0.05
0.01760	0.05
-0.02114	0.05
-0.01178	0.05
-0.01515	0.05
-0.00353	0.05
-0.01772	0.05
-0.02345	0.05
0.03562	0.05
0.03108	0.05
0.01557	0.05
0.00073	0.05
-0.02188	0.05
0.02063	0.05
0.03044	0.05
0.01276	0.05
0.01214	0.05
0.00138	0.05
-0.00507	0.05
0.01134	0.05

Scenario 1: Return R_1 = -0.00024, occurring with probability p_1 = 0.05

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Reward

Expected Value of R:

$$E(R) = p_1 {*} R_1 + p_2 {*} R_2 + \ldots + p_{19} {*} R_{19} + p_{20} {*} R_{20}$$

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Risk

Standard Deviation of R:

$$SD(R) = \sqrt{p_1 * (R_1 - E(R))^2 + \ldots + p_{20} * (R_{20} - E(R))^2}$$

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Risk

$\text{Prob}(R<0) = 9 * 0.05 = 0.45$

- When choosing between alternative options, a decision maker can select the one with highest value of the reward that also satisfies the constraints on acceptable levels of risk

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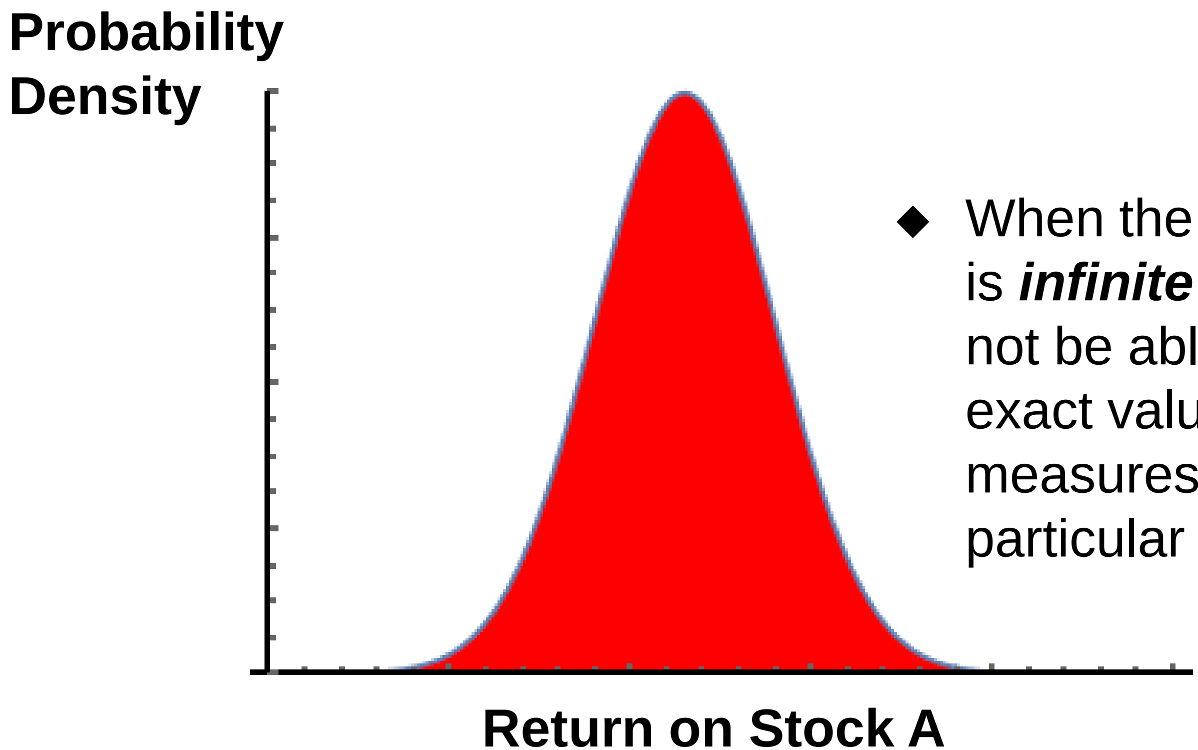
Risk

Prob(R<0) = 9∗0.05 = 0.45

- When the number of scenarios is ***finite***, Excel can calculate the exact values of reward and risk measures for any decision, and Solver can select the optimal decision

Modeling Uncertainty: From Scenarios to Continuous Distributions

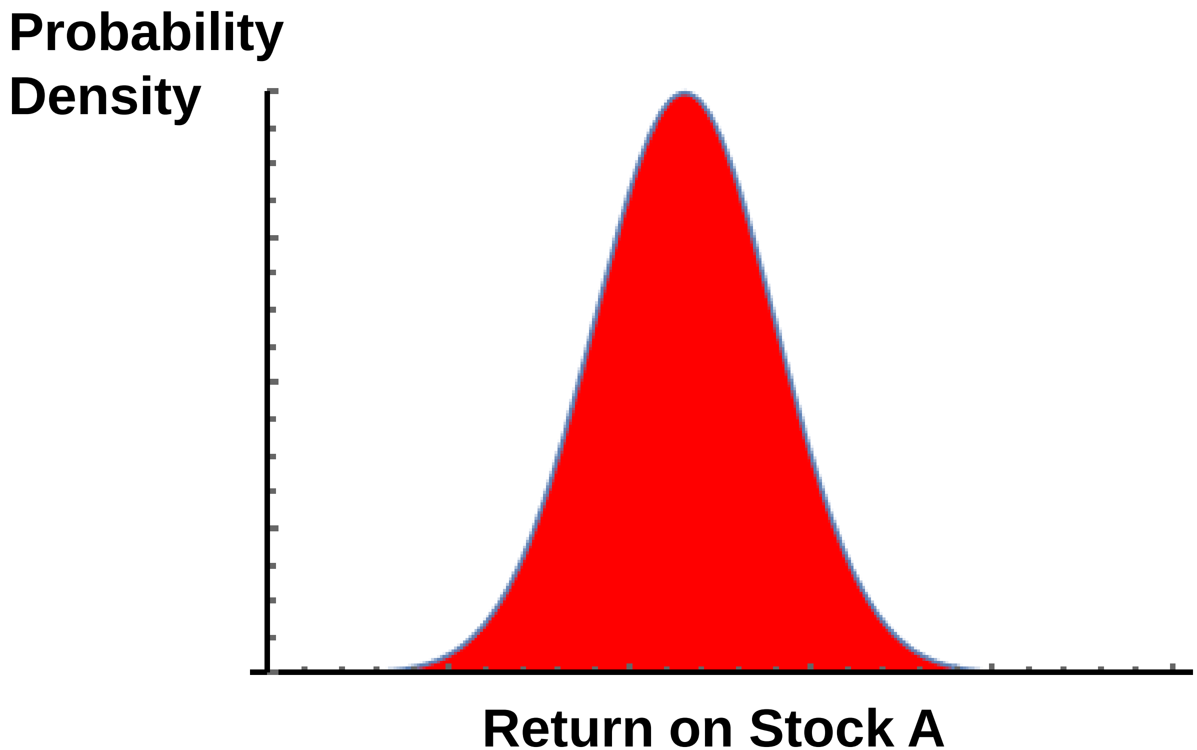
- What if the number of possible values of the random variable (scenarios) is ***infinite***, and the future is described by a continuous distribution?



- When the number of scenarios is ***infinite***, Excel, in general, will not be able to calculate the exact values of reward and risk measures corresponding to a particular decision

Simulation: Making Decisions when Randomness is Described by a Continuous Distribution

- For a number of probability distributions (for example, normal distribution) Excel can generate instances of a random variable coming from that distribution



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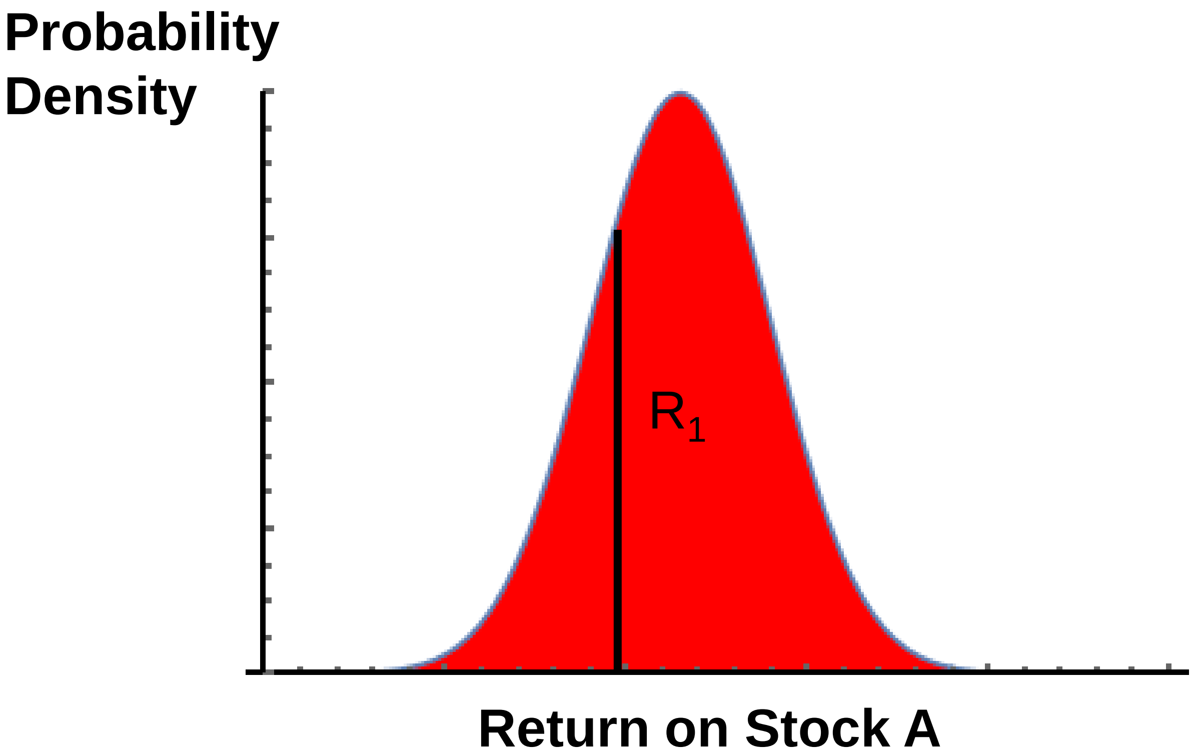

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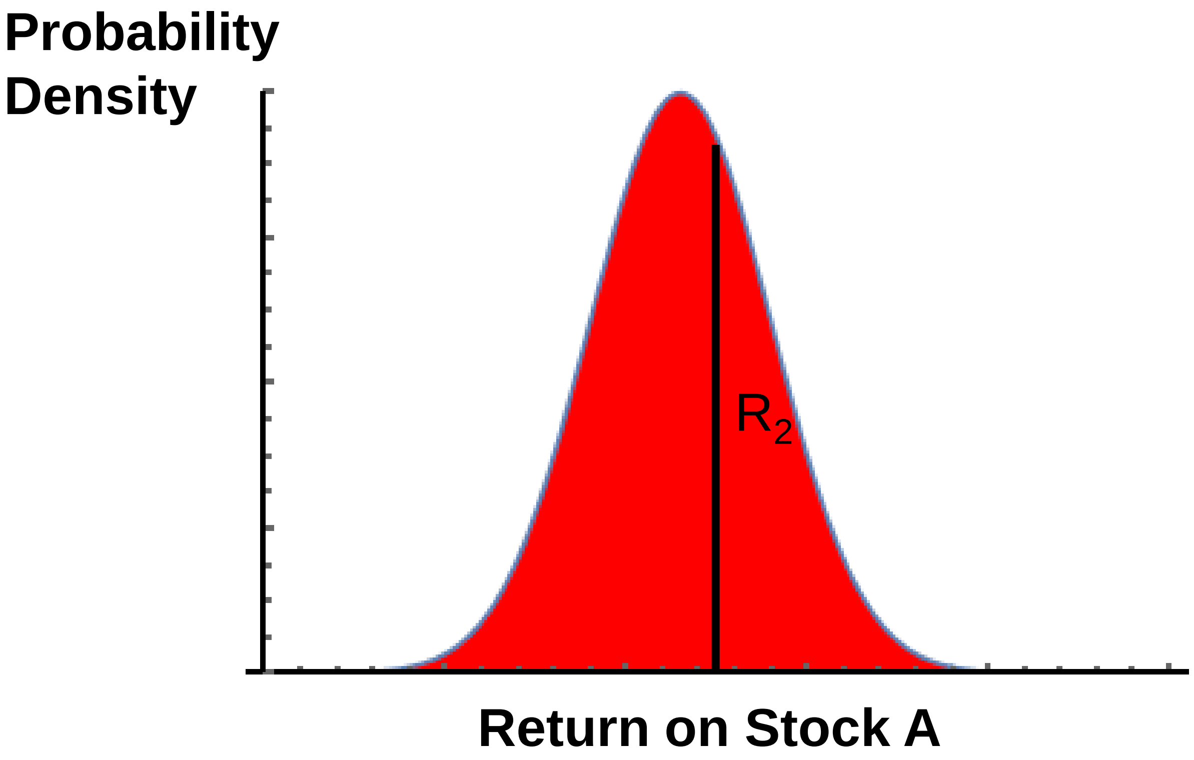

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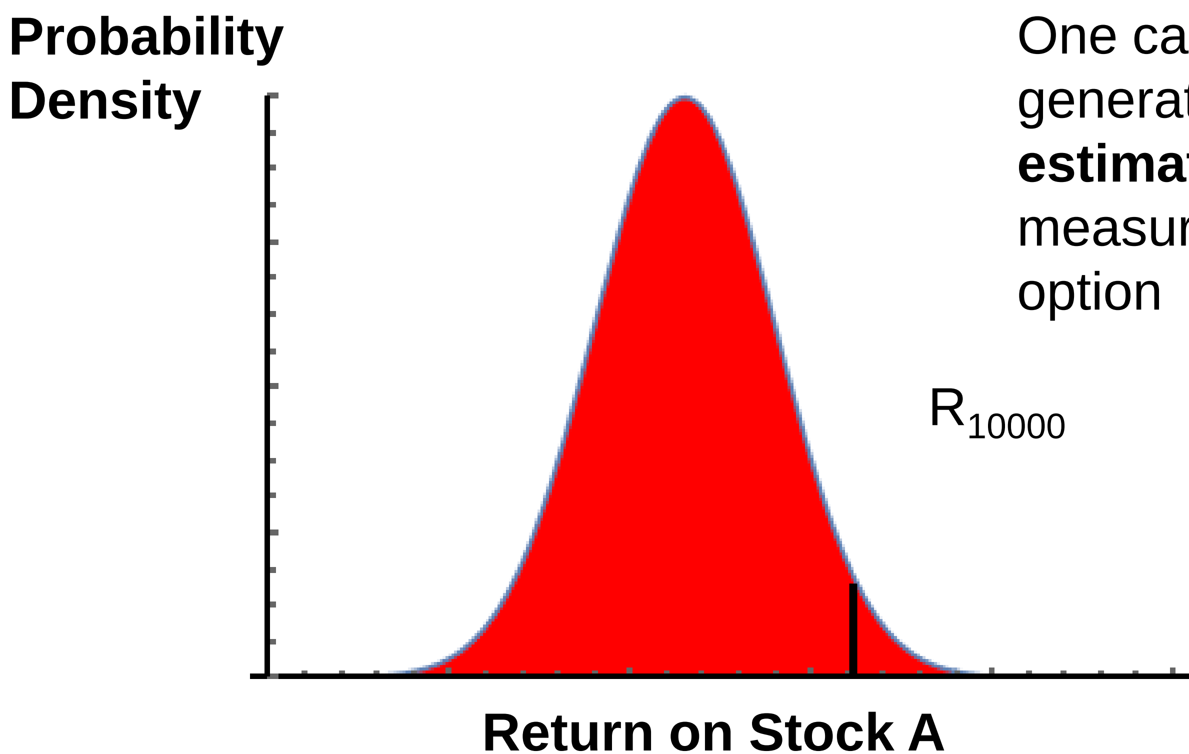

One can use a scenarios generated in such a way to **estimate** the reward and risk measures for any decision option

Example: Designing a New Apartment Building

- The Stargrove Development Corporation is planning a new apartment building in Philadelphia
- The building will have two kinds of apartments, regular and luxury
- The building will have 15 floors, with lower floors housing regular apartments and higher floors – luxury apartments
- Each floor will contain only one kind of apartments – either regular or luxury
- Each floor can house either 8 regular apartments or 4 luxury apartments
- Stargrove needs to decide how many floors to allocate to regular apartments and how many floors to allocate to luxury apartments

Example: Designing a New Apartment Building

- Stargrove expects to complete construction within the next year, and during that period, it plans to sell apartments to prospective buyers

- Stargrove expects to obtain a profit of P_R = \$500,000 for each regular apartment it sells during the next year, and a profit of P_L = \$900,000 for each luxury apartment it sells during the next year

- If, at the end of the next year, there are unsold apartments, Stargrove will sell all of them to a real estate investment company at a “salvage” profit of S_R = \$100,000 for each remaining regular apartment and S_L = \$150,000 for each remaining luxury apartment

Example: Designing a New Apartment Building

- Stargrove analysts project that at the price levels that the company will charge for the apartments, the total demand for **regular apartments** over the next year will be **normally distributed with a mean of 90 and a standard deviation of 25**, and the total demand for **luxury apartments** over the next year will be **normally distributed with a mean of 10 and a standard deviation of 3**. The demands for two apartment types will be modeled as independent (non-correlated) random variables

- Normal random variables can take fractional and negative values, while the demand values must be positive and integer. The values used for estimating the reward and risk measures will take this into account

Example: Designing a New Apartment Building

- Stargrove knows that if the demand for either type of apartment exceeds the availability of the apartments, all those extra prospective buyers will be lost
- None of the prospective buyers looking for a regular apartment will pay for a luxury one
- None of the prospective buyers looking for a luxury apartment will want a regular one

Number of Sold Apartments

- So, if the number of regular apartments built is R, and the demand for regular apartments during the next year is D_R , then the number of regular apartments sold during the next year at the profit P_R = \$500,000 each will be min(R, D_R), i.e., the smallest of two numbers, R and D_R

- For example, if R = 96, and D_R = 90, the number of regular apartments sold during the year is min(R, D_R) = min(96, 90) = 90

- On the other hand, if R = 96, and D_R = 100, the number of regular apartments sold during the year is min(R, D_R) = min(96, 100) = 96

- The rest of the regular apartments, R - min(R, D_R) will be “salvaged” at the profit S_R = \$100,000 each

Number of Sold Apartments

- Similarly, if the number of luxury apartments built is L, and the demand for luxury apartments during the next year is D_L , then the number of luxury apartments sold during the next year at the profit P_L = \$900,000 each will be min(L, D_L)

- The number of luxury apartments “salvaged” at the profit S_L = \$150,000 each will be L - min(L, D_L)

Evaluating an Alternative

- Stargrove wants to evaluate the plan of having 12 regular floors (with $R = 12*8 = 96$ regular apartments) and 3 luxury floors (with $L = 3*4 = 12$ luxury apartments)

- The company needs to evaluate a distribution of profit Π it will earn under this plan

- In particular, Stargrove is interested in estimating the expected profit $E(\Pi)$ and the likelihood that the profit value will be below $45,000,000

Simulation: Random Inputs and Random Outputs

- In Stargrove's case, the profit Π is a function of the demand values for regular and luxury apartments, D_L and D_R

- Since the demand values are random, the profit may also be random

- In algebraic terms, the profit value (in $) can be expressed as

$$\Pi = 500{,}000 * \min(D_R, R) + 900{,}000 * \min(D_L, L)$$
$$+ 100{,}000 * (R - \min(D_R, R)) + 150{,}000 * (L - \min(D_L, L))$$

- In a simulation, the demand values for regular and luxury apartments are **random inputs** and the profit value is a **random output**

- Simulation generates instances of random inputs and calculates the corresponding instances of random outputs

Simulation: Random Inputs and Random Outputs

- In algebraic terms, the profit is

  $$\Pi = 500{,}000 * \min(D_R, R) + 900{,}000 * \min(D_L, L)$$
  $$+ 100{,}000 * (R - \min(D_R, R)) + 150{,}000 * (L - \min(D_L, L))$$

- What is the distribution of Π?
- What is the expected value of Π?
- What is the probability that Π is less than 45,000,000?

Expected Value of Π?

- In algebraic terms, the profit is

  $\Pi = 500{,}000*\min(D_R,R) + 900{,}000*\min(D_L,L)$
  $+ 100{,}000*(R-\min(D_R,R)) + 150{,}000*(L-\min(D_L,L))$

- Expected value of D_R is 90, and the expected value of D_L is 10

- For R=96, and L=12, can't we just plug in the expected values of D_R and D_L into the above formula and get the expected value of Π?

- In general, we **do not** get the correct value for the expected random output that way

Expected Value of Π?

- In algebraic terms, the profit is

  Π = 500,000*min(D_R,R) + 900,000*min(D_L,L)
  + 100,000*(R-min(D_R,R)) + 150,000*(L-min(D_L,L))

- Example: D_R takes values of 65 and 115, each with probability 0.5, and D_L takes the values of 7 and 13, each with probability 0.5. D_R and D_L take these values in independent manner

- The expected value of D_R is 90, and the standard deviation of D_R is 25, the expected value of D_L is 10, and the standard deviation of D_L is 3

- If we "plug" the expected values of D_R (90) and D_L (10) into the formula above, we get, for R=96 and L=12, the value of 500,000*90 + 900,000*10 + 100,000*6 + 150,000*2 = 54,900,000

Expected Value of Π?

- The correct expected value of Π in this case can be calculated as follows

- If $D_R = 65$ and $D_L = 7$ (this happens with probability 0.5*0.5 = 0.25), then the value of profit Π is

  500,000*min(65,96) + 900,000*min(7,12)

  + 100,000*(96-min(65,96)) + 150,000*(12-min(7,12)))

  = 500,000*65 + 900,000*7+ 100,000*31 + 150,000*5 = 42,650,000

- If $D_R = 115$ and $D_L = 7$ (this also happens with probability 0.5*0.5 = 0.25), then the value of profit Π is

  500,000*min(115,96) + 900,000*min(7,12)

  + 100,000*(96-min(115,96)) + 150,000*(12-min(7,12)))

  = 500,000*96 + 900,000*7+ 100,000*0 + 150,000*5 = 55,050,000

Expected Value of Π?

- If D_R = 65 and D_L = 13 (this happens with probability 0.5*0.5 = 0.25), then the value of profit Π is

  500,000*min(65,96) + 900,000*min(13,12)

  + 100,000*(96-min(65,96)) + 150,000*(12-min(13,12)))

  = 500,000*65 + 900,000*12+ 100,000*31 + 150,000*0 = 46,400,000

- Finally, if D_R = 115 and D_L = 13 (this also happens with probability 0.5*0.5 = 0.25), then the value of profit Π is

  500,000*min(115,96) + 900,000*min(13,12)

  + 100,000*(96-min(115,96)) + 150,000*(12-min(13,12)))

  = 500,000*96 + 900,000*12+ 100,000*0 + 150,000*0 = 58,800,000

- The expected value Π is
  0.25*(42,650,000)+0.25*(55,050,000)+0.25*(46,400,000)

  +0.25*(58,800,000) = 50,725,000 – much lower value than 54,900,000