

Q1 Team Name

0 Points

Goldfish

Q2 Commands

10 Points

List the commands used in the game to reach the ciphertext

go,enter,pick,c,back,give,back,back,thrnxtzy,read

Q3 Analysis

50 Points

Give a detailed analysis of how you figured out the password? (Explain in less than 500 words)

After giving above commands , we got hints in form of group theory equations, which we solved in following ways to get the password.

prime modulus $p = 455470209427676832372575348833$

we had three equation in form of $q * g^a$, where q is password, g is a element of the group and a is an integer

Given

$$q * g^{429} = 431955503618234519808008749742 \bmod(p) \text{ -----eqn(1)}$$

$$q * g^{1973} = 176325509039323911968355873643 \bmod(p) \text{ -----}$$

eqn(2)

$$q * g^{7596} = 98486971404861992487294722613 \bmod(p) \text{ -----}$$

eqn(3)

we will refer $q * g^a$ term in each equation as x_1, x_2, x_3 respectively

Dividing eqn(2) by eqn(1):

$$g^{1544} = x_2 * x_1^{-1} \pmod{p} \text{ -----eqn(4)}$$

Dividing eqn(3) by eqn(2):

$$g^{5623} = x_3 * x_2^{-1} \pmod{p} \text{ -----eqn(5)}$$

Dividing eqn(3) by eqn(1):

$$g^{7167} = x_3 * x_1^{-1} \pmod{p} \text{ -----eqn(6)}$$

To perform above operation we need to get the multiplicative inverses of x_1, x_2 . as g is member of the multiplicative group, so x_1, x_2 must also be members of the group and since p is prime x_1, x_2 are coprime to p . Hence their multiplicative inverse also exist.

let y_1 be the inverse of x_1

$$\text{Then, } (x_1 * y_1) \bmod p = 1$$

since p is a prime number, we can use Fermat little theorem to get the inverse

$$x_1^{p-1} \equiv 1 \pmod{p}$$

multiplying both side by x_1^{-1}

$$x_1^{p-2} \equiv x_1^{-1} \pmod{p} =$$

$$70749996790223471732904681640 \pmod{p}$$

from eqn(4)

$$g^{1544} = 70749996790223471732904681640 \pmod{p} \text{ -----eqn(7)}$$

similarly by solving eqn(5) and eqn(6) we get

$$g^{5623} = 420413074251022028027270785553 \pmod{p} \text{ -----eqn(8)}$$

$$g^{7167} = 110411376670918912626907526185 \pmod{p} \text{ -----eqn(9)}$$

Multiplying both side of eqn(9) by inverse of $(g^{1544})^3$

$$g^{7167} * ((g^{1544})^4)^{-1} \pmod{p} \equiv g^{991} =$$

$$161798558270556961732424822635 \pmod{p}$$

Similarly

$$(g^{991})^2 * ((g^{1544})^4)^{-1} \pmod{p} \equiv g^{438} =$$

$$327597482298082119695568192760 \pmod{p}$$

$$(g^{991}) * ((g^{438})^2)^{-1} \pmod{p} \equiv g^{115} =$$

$$212427760325417336316893638262 \pmod{p}$$

$$(g^{115})^4 * ((g^{438})^1)^{-1} \pmod{p} \equiv g^{22} =$$

$$62875864560156876567783127811 \pmod{p}$$

$$(g^{22})^{21} * ((g^{115})^4)^{-1} \pmod{p} \equiv g^2 =$$

$$108044907665466013935627786069(\text{mod } p)$$

$$(g^{115})^1 * ((g^{22})^5)^{-1}(\text{mod } p) \equiv g^5 = 254662155980870723273334022569(\text{mod } p)$$

$$(g^5)^1 * ((g^2)^2)^{-1}(\text{mod } p) \equiv g = 52565085417963311027694339(\text{mod } p)$$

It is mentioned in the hints that g is 5__50__4____31____94__9 , which actually matches with our answer.

Now we determine q (password) by putting value of g in eqn(1)

$$\text{which is } q * g^{429} = 431955503618234519808008749742 \text{mod}(p)$$

We multiply both side of eqn(1) by $(g^{429})^{-1}$,

$$q = ((g^{429})^{-1} * 431955503618234519808008749742) \text{mod}(p)$$

The multiplicative inverse of (g^{429}) is 442956820316148690889301696615

That gives us value of q = password as 134721542097659029845273957

Hence the password is 134721542097659029845273957

Q4 Password

10 Points

What was the final command used to clear this level?



Q5 Codes

0 Points

Upload any code that you have used to solve this level

▼ Assignment_3.ipynb

Download

```
In [25]: def gcd(x,y):  
         if(x==0):  
             return y  
         return gcd(y%x,x)
```

```
In [26]: def po(a,b,m):  
         if(b==0):  
             return 1  
  
         p = po(a,b // 2,m)%m  
         p = (p*p)%m  
  
         if(b % 2 == 0):  
             return p  
         else:  
             return ((a*p) % m)
```

```
In [32]: def mI(a , m):  
         g = gcd(a, m)
```

```

if(g == 1):
    y = po(a, m-2 ,m)
    print("The modular inverse is ",y)
    return y
else:
    print("Does not exist")

```

In [33]:

```

y1 = 431955503618234519808008749742
y2 = 176325509039323911968355873643
y3 = 98486971404861992487294722613
p = 455470209427676832372575348833

```

In [34]:

```

y1_inverse = mI(y1,p)
y2_inverse = mI(y2,p)

```

```

The modular inverse is 70749996790223471732904681640
The modular inverse is 228947149478752602606353685125

```

In [35]:

```

g_1544 = (y2 * y1_inverse)%p
g_1544

```

Out [35]: 111590994894663139264552154672

In [36]:

```

g_7167 = (y3 * y1_inverse)%p
g_7167

```

Out [36]: 110411376670918912626907526185

In [37]:

```

g_5623 = (y3 * y2_inverse)%p
g_5623

```

Out [37]: 420413074251022028027270785553

In [38]:
$$g_{991} = (g_{7167} * mI(po(g_{1544}, 4, p), p)) \% p$$
$$g_{991}$$

The modular inverse is 304296090672599420401986286302

Out [38]: 161798558270556961732424822635

In [39]:
$$g_{438} = (po(g_{991}, 2, p) * mI(g_{1544}, p)) \% p$$
$$g_{438}$$

The modular inverse is 218173384175465437436454958180

Out [39]: 327597482298082119695568192760

In [40]:
$$g_{115} = (mI(po(g_{438}, 2, p), p) * g_{991}) \% p$$
$$g_{115}$$

The modular inverse is 297374246948059676278983181681

Out [40]: 212427760325417336316893638262

In [41]:
$$g_{22} = (mI(g_{438}, p) * po(g_{115}, 4, p)) \% p$$
$$g_{22}$$

The modular inverse is 453530656410176241507046342872

Out [41]: 62875864560156876567783127811

In [42]:
$$g_2 = (mI(po(g_{115}, 4, p), p) * po(g_{22}, 21, p)) \% p$$
$$g_2$$

The modular inverse is 105600931401330644523752113862

Out [42]: 108044907665466013935627786069

In [43]:
$$g_5 = (mI(po(g_{22}, 5, p), p) * g_{115}) \% p$$
$$g_5$$

The modular inverse is 100551735247242729663164535176

Out [43]: 254662155980870723273334022569

In [44]:
$$g = (mI(po(g_2, 2, p), p) * g_5) \% p$$
$$g$$

The modular inverse is 254950689434017345885415339945

Out [44]: 52565085417963311027694339

In [45]:
$$\text{password} = (y1 * mI(po(g, 429, p), p)) \% p$$
$$\text{password}$$

The modular inverse is 442956820316148690889301696615

Out [45]: 134721542097659029845273957