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POSITION AND ATTITUDE CONTROL BY ROTOR TILT AND ROTOR SPEED SYNCHRONIZATION FOR SINGLE AXIS TILTING-ROTOR QUADCOPTER

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ABSTRACT

In this paper, the control maneuvering, and performance analysis of a tilting-rotor quadcopter during autonomous flight is presented. Unlike traditional quadcopters, a tilting-rotor quadcopter provides additional actuated controls as the propeller motors are actuated for tilt which can be utilized to improve efficiency of the aerial vehicle during flight. The tilting-rotor quadcopter design is accomplished by using an additional servo motor for each rotor that enables the rotor to tilt about the axis of quadcopter arm. Here, a detailed control strategy has been discussed to use the propeller tilts for position and orientation control during completely autonomous flights of the quadcopter. In conventional quadcopters, the variation in rotational speeds of the four propellers is utilized for maneuvering. This work incorporates use of varying propeller rotational speeds along with tilting of the propellers for maneuvering the quadcopter during flight. A PD controller is developed to achieve various modes of flight and numerical simulation results are presented demonstrating the performance of the controller. Furthermore, the performance of the tilt-rotor design is compared with respect to the conventional quadcopter in the presence of wind disturbances and uncertainties in the system.

1 Introduction

In recent years, Unmanned Aerial Vehicles (UAVs) have evolved by their numerous applications in various engineering and non-engineering domains. They can be classified in terms of their sizes, number of moving parts and applications. In rotary UAV class, quadcopters are the most popular which can be attributed to their simplicity of design, easy handling, minimal operational cost and several applications. Quadcopters are popular in both military and civilian applications. In the past, they have been utilized for reconnaissance, surveillance and exploration of disasters (such as fire, earthquake, flood), search and rescue operations, data collection for remote land surveying, tunnel-mapping and agriculture [1] [2]. The applications of quadcopter UAV are increasing rapidly as FAA is setting up new regulations for UAV flights. Radmanesh *et al.* in [3] showed UAV flight formation in presence of moving obstacles using fast-dynamic mixed integer linear programming. Similarly, several companies are trying to use them for package delivery and transporting goods from one location to the other. Swarming and cooperative control to accomplish complex missions are more sophisticated scenarios where these aircrafts can play a key role. Swarms of quadcopters can be deployed for remote sensing, land and tunnel mapping. Recently, there was a demonstration from ETH Zurich, where a group of quadcopters constructed a temporary cable bridge, such systems can be used during rescue operations in future [4].

In plus (+) configuration conventional quadcopters, attitude and position is controlled by varying the rotational speed of each motor. The quadcopter possesses one pair of propellers rotating in clockwise direction, while the other pair rotates in counter-clockwise direction [1] [5]. This configuration balances the yawing moments created by rotating propellers. The roll angle, roll rate and lateral position of the quadcopter is controlled by varying the rotational speeds of left and right propellers. Whereas the pitch angle, pitch rate and forward movement is achieved by changing the rotational speeds of front and back propellers. By increasing or decreasing the speeds of all four propellers simultaneously, the collective thrust is generated for the UAV [2]. The on-board autopilot on quadcopters enable semi-autonomous and complete autonomous capabilities to assist the pilot during flight. The development and testing of a flight test-bed and on board PID flight controllers for quadcopters has been discussed in [6]. The quadcopters are inherently unstable and essentially under-actuated system as the four propellers are used to control all six degrees of freedom. It arises the need of more independent control inputs for the system [7].

In previous works, different type of flight controllers have been developed for quadcopters covering all aspects of control theory. The tilting-rotor quadcopter is a structural advancement in existing design of the conventional quadcopter. It adds more complexity in design as the propellers are allowed to tilt about the axes connecting them to the main body frame [2] [8]. It provides advantage in terms of additional independent control inputs, thus providing full control over position and orientation of the UAV [7]. Nemati *et al.* presented the linear PD control, non-linear control by feedback linearization and fault tolerant control for the tilting-rotor quadcopter in [2], [9] and [10]. Ryll *et al.* in [8] presented the controllability analysis of tilt-rotor quadcopter along with non-linear controller to follow desired trajectories asymptotically. Ryll *et al.* further extended their work in [11] by illustrating the control implementation and trajectory tracking performance of a real prototype tilt-rotor quadcopter developed by the group. They reported several experimental results for different flight conditions. The hovering control using rotor tilt has been presented in [2] [12] where the quadcopter can hover by maintaining a specific roll or pitch angle. G. Scholz [13] presented model based control approach for tilting-rotor quadcopter by implementing nonlinear inverse dynamics and pseudo control hedging (PCH) which increases the controller performance by driving the system to its limits. Experimental results for attitude transition for pitch angles from 0° to 90° have been presented in [12] highlighting extreme maneuverable capabilities of tilt-rotor quadcopter.

1.1 Contributions

Variation in angular speed of propeller motors is the primary maneuvering control so far in quadcopter research. In this paper, we consider a novel idea in which the variation in angular speed of propeller motors work in a synchronized combination with rotor tilts for controlling the position and orientation of the UAV. The operational bandwidth of propeller motors is different from rotor-tilt servo motors, but they are tuned to work in a synchronized manner. The rotor-tilt manipulates the direction of the force produced by a propeller by splitting it in horizontal and vertical components. This is identical to tip path plane tilting in helicopters. The same concept has been utilized to enhance the operational capabilities of the tilt-rotor quadcopter. The collective vertical force produced by all propellers help the UAV to stay airborne and the horizontal component of force can be utilized for controlling the motion of the UAV. The angular speed of propeller motors is increased or decreased by the on board controller to follow a desired altitude whereas the collective horizontal component of force produced by the rotor-tilts help to maneuver the quadcopter in longitudinal as well as lateral direction. The additional controls achieved via tilting of the rotors not only makes the quadcopter more maneuverable but it also increases the robustness of the UAV to external disturbances and uncertainties. This paper covers dynamic modeling and detailed discussion on the proposed control strategy. Further, numerical study is performed to prove functional and disturbance rejection capability of the proposed flight controller via simulating flight under normal conditions and flight with uncertainties in sensor readings and other environmental noises such as wind gusts. The results of robustness study are presented by comparing the performance of tilt-rotor quadcopter flight against traditional quadcopter flight in presence of same system uncertainties.

2 Tilting-Rotor Quadcopter Dynamic Model

This section presents the dynamic mathematical model of the tilt-rotor quadcopter which includes equation of motions governing the flight. The equations has been presented in matrix form for easy understanding to the reader. Mathematical representation of quadcopter subsystems is also presented. The world-frame (E) is the fixed frame on earth and all the quadcopter mo-

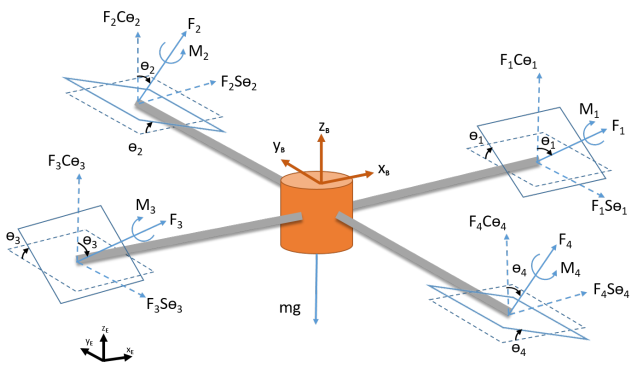

FIGURE 1: Free Body Diagram of Tilting-Rotor Quadcopter

tions can be referred w.r.t. this frame of reference. On the other hand, the body-fixed frame (B) is attached to the quadcopter body with origin located at center of mass of the vehicle. This frame of reference moves with the quadcopter in three dimensional space of world-frame. During quadcopter flight, there are forces and moment components for each rotor which arise from the rotational motion and tilt of the rotor. These forces and moments are first transformed from propeller frame of references to body frame of reference as discussed in [8]. This transformation is achieved by taking the components of forces and moments along body axes system. Further the forces and moments from body frame are transformed to world reference frame by using Euler angle transformation. Here we consider a plus (+) configuration quadcopter and rotors 1 and 3 rotate in counter-clockwise direction to produce a clockwise moment about ($-Z_B$) direction. Similarly, rotors 2 and 4 rotate in clockwise sense to produce counter clockwise moment about (Z_B) direction.

Euler angle transformations are defined by ψ, θ and ϕ which represent yaw, pitch and roll angles respectively. The orthogonal direction cosine matrix ($R_{E/B}$) relating world frame parameters to the body fixed frame is obtained by three successive rotations. The first rotation is about $Z-$axis, followed by another rotation about $Y-$axis and the last rotation is about $X-$axis. The transformation matrix ($R_{B/E}$) relating body fixed frame parameters to the world frame is the inverse of $R_{E/B}$. A more generalized transformation can be achieved by using quaternions in place of Euler angles which will avoid any kind of singularities in the system caused during aggressive maneuvers [5] [14].

$$R_{B/E} = \begin{bmatrix} c\psi c\theta & c\psi s\theta s\phi - s\psi c\phi & c\psi s\theta c\phi + s\psi s\phi \\ s\psi c\theta & s\psi s\theta s\phi + c\psi c\phi & s\psi s\theta c\phi - c\psi s\phi \\ -s\theta & c\theta s\phi & c\theta c\phi \end{bmatrix} \tag{1}$$

Where $c\psi$ and $s\psi$ denote $cos(\psi)$ and $sin(\psi)$ respectively, and similarly for θ and ϕ Euler angles. The equations of motion comprise of set of force equations in world frame. The body angular acceleration equations are obtained by simplifying the moment equations in body frame. Another set of equations relate the Euler angle rates with body angular rates of the quadcopter as discussed in [15]. Using the Newton-Euler method, the equations of motion in world-frame can be written as:

$$\begin{bmatrix} \ddot{x} \\ \ddot{y} \\ \ddot{z} \end{bmatrix} = \frac{R_{B/E}}{m} \begin{bmatrix} F_2 s\theta_2 + F_4 s\theta_4 \\ -F_1 s\theta_1 - F_3 s\theta_3 \\ F_1 c\theta_1 + F_2 c\theta_2 + F_3 c\theta_3 + F_4 c\theta_4 \end{bmatrix} - \begin{bmatrix} C_1 \dot{x} \\ C_2 \dot{y} \\ C_3 \dot{z} + g \end{bmatrix} \tag{2}$$

$$I \begin{bmatrix} \dot{p} \\ \dot{q} \\ \dot{r} \end{bmatrix} = \begin{bmatrix} l(F_2 c\theta_2 - F_4 c\theta_4 - C_1' \dot{\phi}) \\ +M_2 s\theta_2 + M_4 s\theta_4 + M_2{}' + M_4{}' \\ l(F_3 c\theta_3 - F_1 c\theta_1 - C_2' \dot{\theta}) \\ +M_3 s\theta_3 + M_1 s\theta_1 + M_1{}' + M_3{}' \\ l(-F_1 s\theta_1 - F_2 s\theta_2 + F_3 s\theta_3 + F_4 s\theta_4 - C_3' \dot{\psi}) \\ -M_1 c\theta_1 + M_2 c\theta_2 - M_3 c\theta_3 + M_4 c\theta_4 \end{bmatrix} - \begin{bmatrix} p \\ q \\ r \end{bmatrix} \times I \begin{bmatrix} p \\ q \\ r \end{bmatrix} \tag{3}$$

$$\begin{bmatrix} \dot{\phi} \\ \dot{\theta} \\ \dot{\psi} \end{bmatrix} = \begin{bmatrix} 1 & s_\phi \frac{s_\theta}{c_\theta} & c_\phi \frac{s_\theta}{c_\theta} \\ 0 & c_\phi & -s_\phi \\ 0 & \frac{s_\phi}{c_\theta} & \frac{c_\phi}{c_\theta} \end{bmatrix} \begin{bmatrix} p \\ q \\ r \end{bmatrix} \tag{4}$$

In figure (1), θ_i, $\forall i \in \{1,2,3,4\}$ is the parameter representing the tilted angle of corresponding rotors. The planes shown with dashed lines are the original planes of rotation with zero tilt angles whereas the planes shown with the rigid lines are the planes after exercising tilt. The propeller thrust forces are perpendicular to these respective tilted planes. In equation (2), m is the total mass of quadcopter, g is the acceleration due to gravity, x, y and z are quadcopter position in world frame coordinate, C_1, C_2 and C_3 are drag coefficients which is negligible at low speed. F_i, $\forall i \in \{1,2,3,4\}$ are forces produced by the four rotors as given by the following equation:

$$F_i = k_f \omega_i^2 \tag{5}$$

where ω_i is the angular velocity of i^{th} rotor and k_f is a proportional constant [6]. In equation (3), p, q, r represent the roll rate, pitch rate and yaw rate terms in quadcopter body frame respectively. M_i',$\forall i \in \{1,2,3,4\}$ are the tilting moments created by servo motors to tilt the rotors. I matrix contains terms of moment of inertia about x, y and z axes. C_1' , C_2' and C_3' are rotational drag coefficients and M_i, $\forall i \in \{1,2,3,4\}$ are rotors moment produced by angular velocity ω_i of rotors such that:

$$M_i = k_m \omega_i^2 \tag{6}$$

where, k_m is a proportional constant [6]. It should be noted that the linear acceleration terms of the quadcopter along XYZ-axes are dependent on orientation of the quadcopter and rotor tilt angles as shown in equation (2). It means that tilting motion of rotors can be used to control the position of the UAV in XY-plane. On the other hand, equation (3) shows that even if the rotors are tilted, the system can attain a balanced moment about yaw-axis, but the tilting motion of the rotors leads to a residual angular acceleration about roll and pitch axes which may lead to system instability. The flight controller should account for these residual angular accelerations and should be able to minimize them by orientation control. The servo motors used for rotor tilt is represented mathematically by the following first order transfer function that relates the motor angular velocity (ω_s) of motor to input voltage (V) as:

$$\frac{\omega_s(s)}{V(s)} = \frac{K}{\tau s + 1} \tag{7}$$

where τ represents the time constant of the system, and K represents the steady state gain value. The angular position of the servo motor can be obtained by integrating the motor angular velocity. The transfer function relating the angular position (θ_s) and input voltage (V) can be obtained as:

$$\frac{\theta_s(s)}{V(s)} = \frac{K}{s(\tau s + 1)} = \frac{K}{\tau s^2 + s} \tag{8}$$

This is identical to a second order actuation system. Such systems exhibit a transient response when they are subjected to external inputs or environmental disturbances. Here, we model the behavior of servo motor by means of this transfer function. This transient response is an important factor in system design for the proposed quadcopter that affects the maneuverability and responsiveness of the quadcopter to external disturbances. The transient response accounts for time response of the servo motor towards desired change in tilt angle of the respective rotor.

3 Controller Design

In this section, the controller development strategy is presented for syncing variable angular speed of propeller motors with tilt angle of rotors to follow a desired trajectory in three dimension. The primary objective is to have full control over the position and orientation of the quadcopter and ensure disturbance rejection towards uncertainties during flight.

There are eight control inputs available in this system comprising of four independent rotational speeds of propeller motors and four rotor tilt inputs about the axes of quadcopter arms. By referring figure (1), it is assumed that $\theta_1 = \theta_3$ and $\theta_2 = \theta_4$. Now, six control inputs are utilized in the quadcopter to control six degrees of freedom. The dynamic model of the tilting-rotor quadcopter, described in equations (1) to (8), has been used to design the PD controller for complete control of the system.

3.1 Complete Position and Attitude Control

Here, we present the method to track a desired trajectory in three dimensional space using rotor speeds and rotor tilt as control inputs. The position errors (e_x, e_y, e_z) and velocity errors ($\dot{e_x}, \dot{e_y}, \dot{e_z}$) along $X,Y,Z-$axes are computed from the relative difference between feedback and desired values. Then, these errors are utilized by the PD controller in equation (9) to compute desired accelerations along X,Y,Z-axes.

$$\ddot{r}_i^{des} = k_{p_i} e_i + k_{d_i} \dot{e}_i; \quad \forall i \in \{x,y,z\} \tag{9}$$

Here, k_{pi}, $\forall i \in \{x,y,z\}$ are the proportional gain and k_{di}, $\forall i \in \{x,y,z\}$ represent the derivative gains of the position controller for propeller motors.

The angular speed required for individual propeller motors necessary for hovering (ω_h) is given by the following equation:

$$\omega_h = \sqrt{\frac{mg}{2k_f(c\theta_1 + c\theta_2)}} \tag{10}$$

The change angular speed for individual propeller motors necessary for motion along the $Z-$axis is given by:

$$\Delta\omega_f = \frac{m\ddot{r}_z^{des}}{4k_f\omega_h(c\theta_1 + c\theta_2)} \tag{11}$$

The desired accelerations obtained in equation (9) are used to compute the desired orientation angles (θ^{des}, ϕ^{des}) of the UAV given by the relationships in [6] [9]:

$$\phi^{des} = \frac{\ddot{r}_x^{des} sin\psi^{des} - \ddot{r}_y^{des} cos\psi^{des}}{g} \tag{12}$$

$$\theta^{des} = \frac{\ddot{r}_x^{des} cos\psi^{des} - \ddot{r}_y^{des} sin\psi^{des}}{g} \tag{13}$$

where, ψ^{des} represents the desired yaw angle for the quadcopter. The errors in roll, pitch and yaw angles and angular rates are computed from the relative difference between feedback and desired values of these states. Further, these errors are utilized by the PD controller to compute the desired rotor speeds and desired tilt angles to achieve a complete control over the orientation. The rotor speed signals are sent to individual electronic speed controllers of propeller motors and tilt angle signals are sent to tilt servo motors. On the other hand, the position errors (e_x, e_y, e_z) and velocity errors $(\dot{e_x}, \dot{e_y}, \dot{e_z})$ along $X, Y, Z-$axes are utilized by the PD controller to compute the respective tilt angles of individual rotors for trajectory control. Tilt motion of rotors 1 and 3 are responsible for $Y-$position and roll angle control whereas tilt motion of rotors 2 and 4 are utilized for $X-$position and pitch angle control. The primary objective of this PD controller is to guide the quadcopter along the desired trajectory by minimizing the $X-Y$ position and velocity errors to zero by maintaining the UAV in desired orientation.

$$\begin{aligned}
\Delta\omega_\phi &= k_{p,\phi}(\phi^{des}-\phi)+k_{d,\phi}(p^{des}-p) \\
\Delta\omega_\theta &= k_{p,\theta}(\theta^{des}-\theta)+k_{d,\theta}(q^{des}-q) \\
\Delta\omega_\psi &= k_{p,\psi}(\psi^{des}-\psi)+k_{d,\psi}(r^{des}-r) \\
\Delta\theta_{T_x} &= k_{p,x_T}e_x+k_{d,x_T}\dot{e_x} \\
\Delta\theta_{T_y} &= k_{p,y_T}e_y+k_{d,y_T}\dot{e_y} \\
\Delta\theta_{T_\phi} &= k_{p,\phi_T}(\phi^{des}-\phi)+k_{d,\phi_T}(p^{des}-p) \\
\Delta\theta_{T_\theta} &= k_{p,\theta_T}(\theta^{des}-\theta)+k_{d,\theta_T}(q^{des}-q)
\end{aligned} \tag{14}$$

$\Delta\omega_i$, $\forall i \in \{\phi, \theta, \psi\}$ represents the change in rotor angular speeds required for orientation control. $\Delta\theta_{T_i}$, $\forall i \in \{x, y, \phi, \theta\}$ is the change in rotor tilt required for position and orientation control. The vector representing complete set of tilting-rotor quadcopter controls can be written as a linear combination of terms shown in equation (10), (11) and (14) as:

$$\begin{bmatrix} \omega_1^{des} \\ \omega_2^{des} \\ \omega_3^{des} \\ \omega_4^{des} \\ \theta_1 \\ \theta_2 \\ \theta_3 \\ \theta_4 \end{bmatrix} = \begin{bmatrix} 1 & 0 & -1 & -1 & 0 & 0 & 0 & 0 \\ 1 & 1 & 0 & 1 & 0 & 0 & 0 & 0 \\ 1 & 0 & 1 & -1 & 0 & 0 & 0 & 0 \\ 1 & -1 & 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 & 0 & 1 \\ 0 & 0 & 0 & 0 & 1 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 & 0 & 1 \end{bmatrix} \begin{bmatrix} \omega_h + \Delta\omega_f \\ \Delta\omega_\phi \\ \Delta\omega_\theta \\ \Delta\omega_\psi \\ \Delta\theta_{T_y} \\ \Delta\theta_{T_x} \\ \Delta\theta_{T_\phi} \\ \Delta\theta_{T_\theta} \end{bmatrix} \tag{15}$$

The outputs resulting from equation (15) are regarded as the controller outputs. The $\omega_i{}^{des}$, $\forall i \in \{1,2,3,4\}$ terms are rotor speed signals which are sent to individual electronic speed controllers of the propeller motors. The force and moment variation in propellers is governed by equation (5) and (6). The θ_i, $\forall i \in \{1,2,3,4\}$ terms are the rotor tilt angle signals sent to servo motors. The transient behavior of the servo motor is governed by the second order actuator model as shown in equation (8). This system represents the complete control of tilt-rotor quadcopter with the designed PD controller. It should be noted that this PD controller synchronizes the control action of propeller rotational speeds and propeller tilts for controlling the position and orientation of the UAV as shown in equation (15). A complete control architecture of the PD based flight controller has been shown in figure (2).

4 Numerical Simulations

To validate the quadcopter dynamic model and PD controller in previous section, numerical simulation of the tilting-rotor quadcopter flight has been presented in this section. The simulation model of the UAV was developed using MATLAB software. The vehicle's initial position was set to origin of the world reference frame i.e. [0, 0] and the quadcopter is commanded to follow a set of predefined way points autonomously. The desired altitude during the way point navigation was set to five meters and desired yaw angle was set to zero degree. The equations of motion have been solved in a discrete manner using Euler integration. The simulation has been carried out for two cases: first without any system uncertainties and then with system uncertainties in the second case.

4.1 Case 1: Numerical Simulations Without System Uncertainties

The set of predefined way points are [6, 6; 6, 12; 10, 12; 14, 16; 18, 16; 18, 20; 20, 20] and as discussed earlier the quadcopter has to maintain an altitude of five meters during flight. The UAV must reach the desired height and then exercise maneuvers to the destination way points. During flight, the orientation of the vehicle should change according to reference inputs without losing the altitude. The time response of various flight parameters during the flight has been presented in the figures (3) to (8).

4.2 Case 2: Numerical Simulations With Uncertainties

Here, the simulation is performed simultaneously on both tilt-rotor quadcopter and the conventional quadcopter for performance comparison. The simulation is performed with uncertainties in orientation angles by introducing normally distributed random noise in ψ, θ and ϕ angles. Normally distributed random noise of mean zero and standard deviation 0.8 degree is introduced in θ and ϕ angles. Similarly, normally distributed random noise of mean zero and standard deviation 2 degree is introduced in ψ angle. This disturbance causes both the UAVs to drift from the desired trajectory. Such drifting response can be attributed to the presence of wind gusts or error in sensor readings of the UAV. It should be noted that the mass and moment of inertia properties of both the UAVs are considered same for this simulation. The controllers for rotational speed of propellers are also identical in both UAVs. The tilt-rotor quadcopter has extra control inputs in terms of rotor tilt for position and orientation control. In the simulation, both quadcopters have to start from the origin and reach the destination way point located at [15, 15] by maintaining an

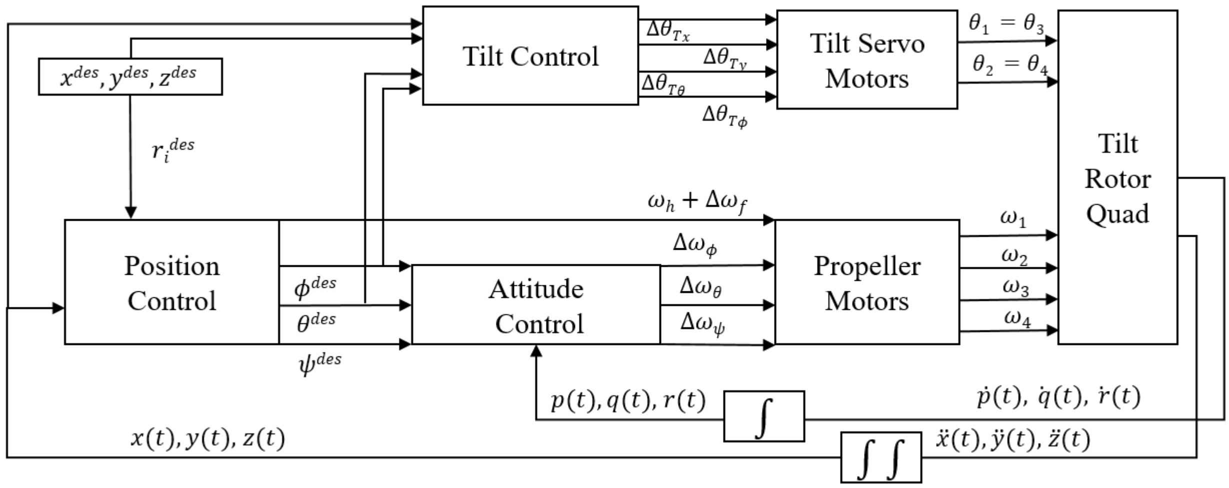

FIGURE 2: Control Architecture of Tilting-Rotor Quadcopter

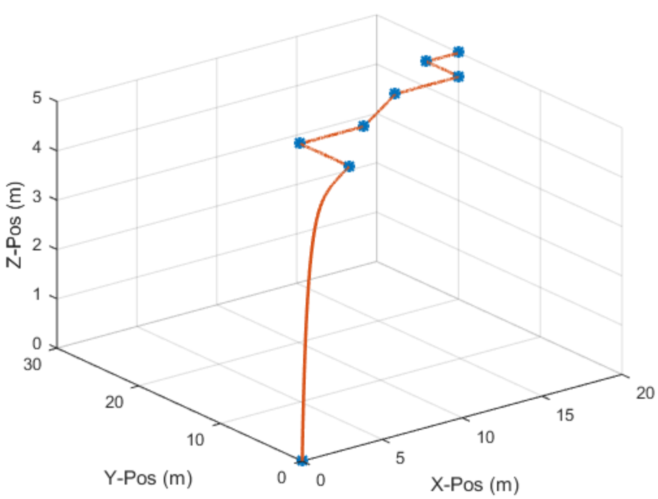

FIGURE 3: Three Dimensional Trajectory Plot (Case 1)

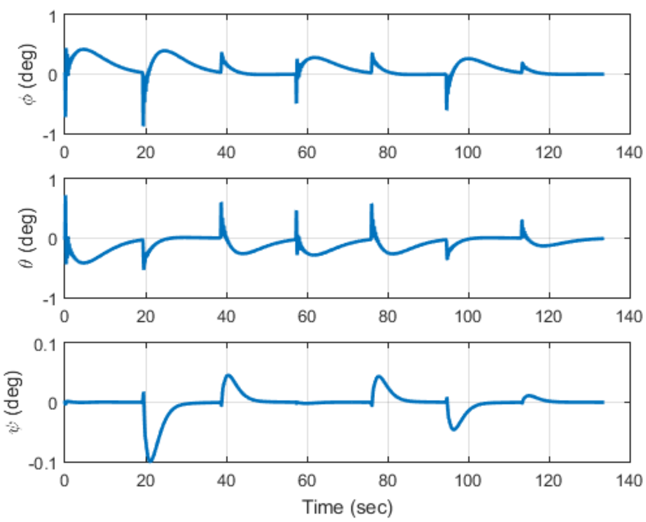

FIGURE 4: Variation of Euler Angles (Case 1)

height of five meters. Both UAVs take off together to follow this trajectory and encounter the uncertainty at the same time for t = 0 to 10 seconds. The time response during the flight has been presented in figures (9) to (14).

5 Results and Discussion

The time history plots of various flight parameters for numerical simulations without uncertainties are covered from figure (3) to (8) for the tilt-rotor quadcopter. Figure (3) and (7) show the 3-D and 2-D flight trajectory plots respectively. It should be noted that the UAV is able to track the desired trajectory without any cross track error. Figure (4) shows the variation of Euler angles (ϕ,θ,ψ) during flight which are in acceptable range and ensure stable flight characteristics. The variation of rotor tilt angles along the trajectory is shown in figure (6) and it can be inferred from combination of figure (5) and (6) that the rotor tilts vary to minimize of position errors in the XY-axes space. The UAV changes rotor tilt angles and rotor angular speeds in real time while changing way points for minimizing the errors in states as shown in figure (6) and (8), the UAV shows a very acceptable synchronization between the operation of rotor tilts and rotor speeds.

Similarly, the flight characteristics with uncertainty in ori-

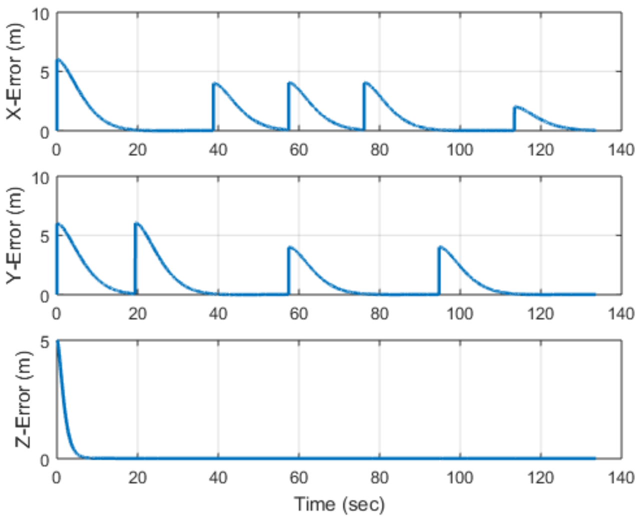

FIGURE 5: Position Errors in Way Point Navigation (Case 1)

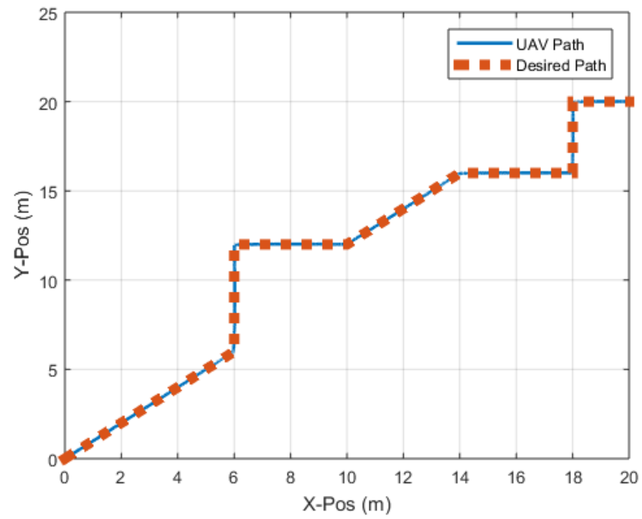

FIGURE 7: Two Dimensional Track Plot (Case 1)

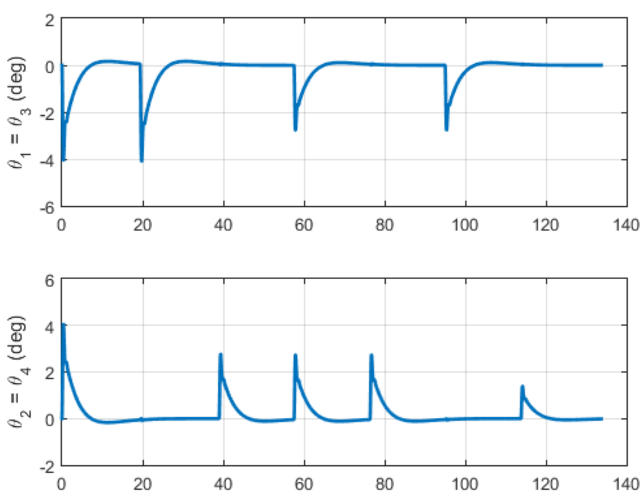

FIGURE 6: Variation in Rotor Tilt Angles (Case 1)

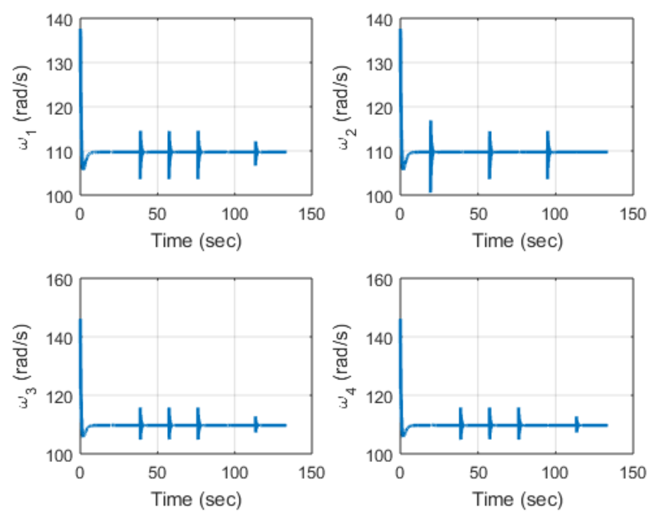

FIGURE 8: Variation in Propeller Speeds (Case 1)

entation parameters are presented from figures (9) to (14). This analysis includes a performance comparison of tilt-rotor quadcopter with respect to a conventional quadcopter. As discussed earlier, mass and moment of inertia properties of both UAVs are assumed to be same. The controllers of rotational speed of propellers are also identical for both the UAVs. The tilt-rotor quadcopter has additional control inputs as rotor tilts for position and orientation control. Figure (9) shows the three dimensional trajectory for both the UAVs. Similarly, figure (10) shows the variation of Euler angles during flight, it can be inferred that both quadcopters encountered uncertainties from t = 0 to 10 seconds. Similarly, figure (11) shows the variation in rotors speed subjected to uncertainties. Both the UAVs start flight at the same time to reach point [15, 15] with a height of 5 meters, but it should be noted from figure (12) and (14) that the tilt-rotor quadcopter is more efficient because of extra control inputs. Both UAVs start their flight together at time t = 0 seconds, the tilt-rotor quadcopter has reached close to point [15, 15] much earlier (approximately 12 sec) as compared to the conventional quadcopter. Both UAVs encounter normally distributed random noise of same

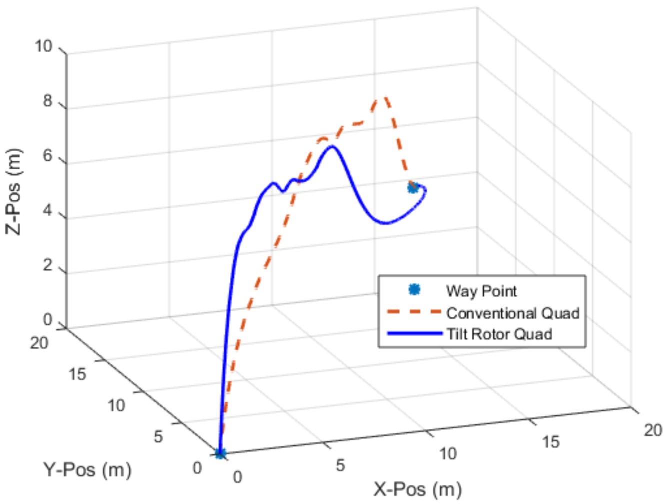

FIGURE 9: Three Dimensional Trajectory Plot (Case 2)

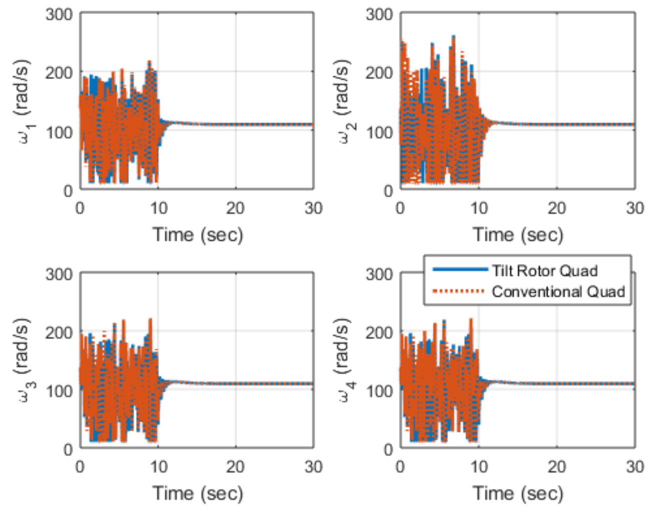

FIGURE 11: Variation of Rotor Speeds (Case 2)

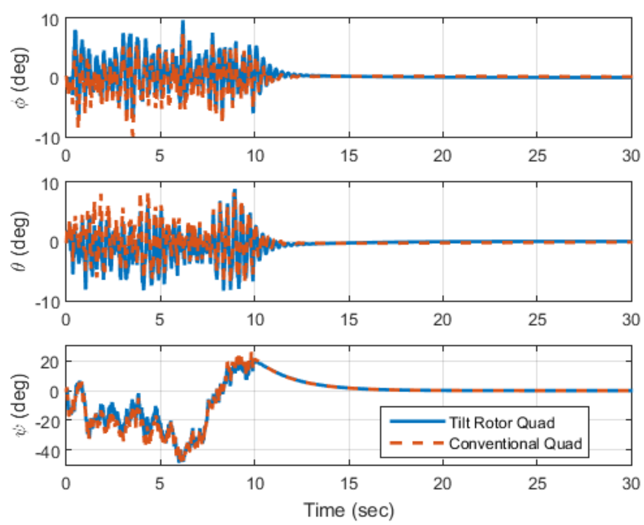

FIGURE 10: Variation of Euler Angles (Case 2)

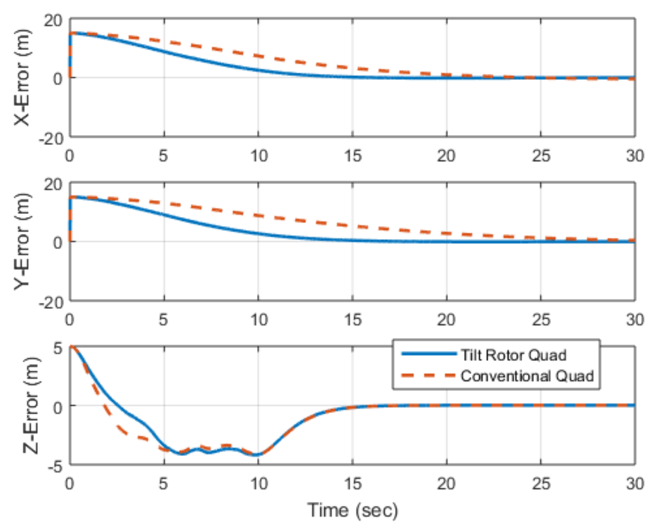

FIGURE 12: Position Errors in Way Point Navigation (Case 2)

magnitude at the same time for equal duration of time and it can be seen from figure (14) that both aircrafts drift from the desired trajectory. But, the tilt-rotor quadcopter resists the drifting motion by changing the rotor tilts in real time as shown in figure (13) and it drifts very little from the desired trajectory. On the other hand, the conventional quadcopter being an under actuated system drifts maximum to [11.13, 9.09]. It shows the advantage in terms of system performance and stability for the tilt-rotor quadcopter when compared against the conventional quadcopter. Figure (12) shows the position errors during flight and it can be seen that the tilt-rotor quadcopter reaches the destination way point very early as compared to the conventional quadcopter. This robust behavior of the tilt-rotor quadcopter towards random noise can be attributed to the presence of rotor tilt inputs as the UAV is capable of changing the rotor tilts for position and orientation control in real time to reject the external noise whereas the conventional quadcopter design lacks this capability. But, it should be noted that the comparative study discussed here was carried out by keeping the PD gain parameters for rotor speed control of both the quadcopters to be the same. A more accurate compar-

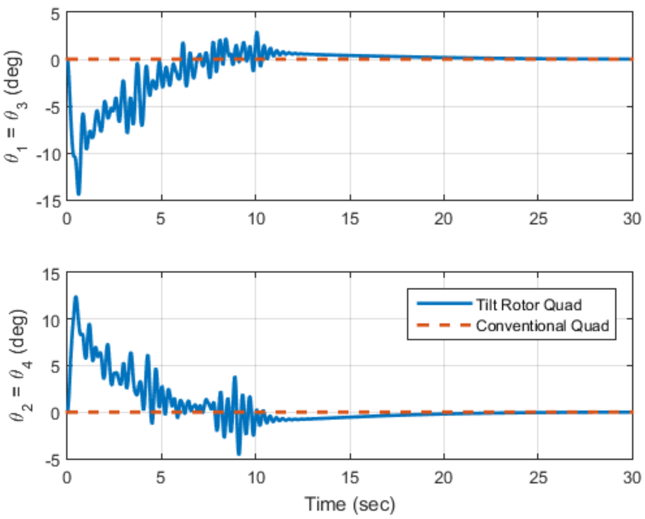

FIGURE 13: Variation in Rotor Tilt Angles (Case 2)

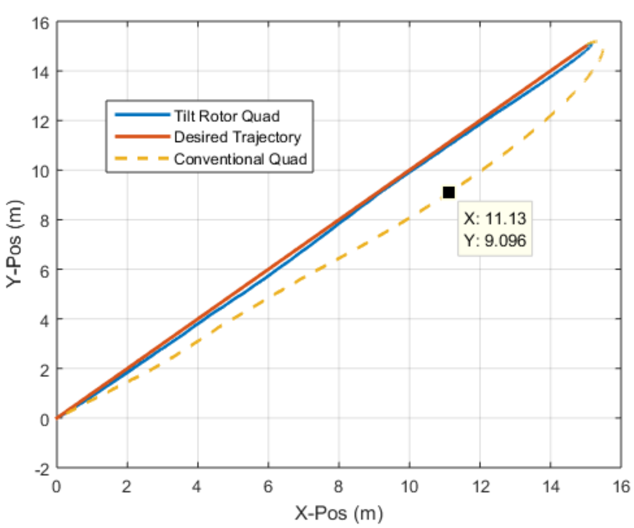

FIGURE 14: Two Dimensional Trajectory Plot (Case 2)

ison could be carried out by determining the optimal controller parameters for both the designs of the quadcopter and then comparing their performance.

6 Conclusion

In this paper, the dynamic model and control strategy for the tilting-rotor quadcopter was presented. The conventional quadcopter PD controller was enhanced by implementing position and orientation control achieved via rotor tilt. This work showed how conventional quadcopter control can work in a synchronized manner with tilting of rotors and this feature can be utilized to enhance the performance of the traditional design in terms of maneuverability and robustness to uncertainties. The numerical simulations were done to carry out the performance evaluation for cases with and without uncertainties. The proposed system was compared against a conventional quadcopter for understanding their relative performances by introducing uncertainties in flight parameters. The tilt-rotor quadcopter design proved to be more robust towards external disturbances during flight as compared to conventional quadcopter. Future work will involve experimental evaluation of the tilt-rotor quadcopter with the proposed PD controller for position and orientation control.

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