SSC **MARCH 2022**

MATHEMATICS
ALGEBRA – PART I

Time allowed: 2 hours **Maximum marks: 40**

General Instructions:

(i) All questions are compulsory.
(ii) Use of calculator is not allowed.
(iii) The numbers to the right of the questions indicate full marks.
(iv) In case MCQ's Q. No. 1(A) only the first attempt will be evaluated and will be given credit.
(v) For every MCQ, the correct alternative (A), (B), (C) or (D) of answers with sub question number is:

1. (A) For every sub question four alternative answers are given. Choose the correct answer and write the alphabet of it: **[4]**

(i) Which one is the quadratic equation?

A) $\frac{5}{x} - 3 = x^2$ B) $x(x + 5) = 2$
C) $n - 1 = 2n$ D) $\frac{1}{x^2}(x + 2) = x$

Answer: B) $x(x + 5) = 2$
Solution:
The general form of a quadratic equation is $\mathrm{ax^2 + bx + c = 0}$.
Option A: $\frac{5}{x} - 3 = x^2 \Rightarrow x^3 + 3x - 5 = 0$
We can see it is not in the form of $\mathrm{ax^2 + bx + c = 0}$.
Hence, it is not a quadratic equation.
Option B: $x(x + 5) = 2 \Rightarrow x^2 + 5x - 2 = 0$
We can see it is in the form of $\mathrm{ax^2 + bx + c = 0}$, with $a = 1, b = 5$, and $c = -2$.
Hence, it is a quadratic equation.
Option C: $n - 1 = 2n \Rightarrow n + 1 = 0$
We can see it is not in the form of $\mathrm{ax^2 + bx + c = 0}$.
Hence, it is not a quadratic equation.
Option D: $\frac{1}{x^2}(x + 2) = x \Rightarrow x^3 - x - 2 = 0$
We can see it is not in the form of $\mathrm{ax^2 + bx + c = 0}$.
Hence, it is not a quadratic equation.

(ii) First four terms of an A.P. are ………, whose first term is – 2 and common difference is -2.

A) $-2, 0, 2, 4$ B) $-2, 4, -8, 16$
C) $-2, -4, -6, -8$ D) $-2, -4, -8, -16$

Answer: C) -2, -4, -6, -8
Solution:
Let the first four terms be $a, a + d, a + 2d$ and $a + 3d$.
Given, first term $a = -2$ and common difference, $d = -2$, then AP would be:
$a, a + d, a + 2d$ and $a + 3d$
$\Rightarrow -2, -2 + (-2), 2 + 2 \times (-2), 2 + 3(-2)$
$\Rightarrow -2, -4, -6, -8$

(iii) For simultaneous equations in variable x and y, $D_x = 49, D_y = -63$, and D = 7, then what is the value of y?

A) 9
B) 7
C) -7
D) -9

Answer: D) -9
Solution:
Given, $D_y = -63$, and $D = 7$
We know that,
$y = \frac{D_y}{D} = \frac{-63}{7} = -9$

(iv) Which number cannot represent probability?

A) 1.5
B) $\frac{2}{3}$
C) 15%
D) 0.7

Answer: A) 1.5
Solution:
$\frac{2}{3} = 0.67$, and $15\% = 0.15$
We know that $0 \leq$ Probability of an event ≤ 1.
So, among 1.5, 0.67, 0.15, and 0.7, 1.5 cannot represent probability.

(B) Solve the following subquestions: **[4]**

(i) To draw a graph of $4x + 5y = 19$, find y when $x = 1$.

Solution:
Given,
Equation of the graph $4x + 5y = 19$
Considering the value of x to be 1,
$\Rightarrow 4 \times 1 + 5y = 19$
$\Rightarrow 4 + 5y = 19$
$\Rightarrow 5y = 19 - 4$
$\Rightarrow y = \frac{15}{5} = 3$
$\therefore y = 3$

(ii) Determine whether 2 is a root of quadratic equation $2m^2 - 5m = 0$.

Solution:
Given quadratic equation,
$2m^2 - 5m = 0$
Substituting $m = 2$, in $2m^2 - 5m = 0$
$\Rightarrow 2(2)^2 - 5(2) = 0$
$\Rightarrow 8 - 10 = 0$
$\Rightarrow -2 \neq 0$
∴ We can observe 2 is not a root of the equation.

(iii) Write second and third term of an A.P. whose first term is 6 and common difference is -3.

Solution:
Given,
First term, $a = 6$
Common difference, $d = -3$
We know that, Second term = $a + d = 6 + -3 = 6 - 3 = 3$
Third term = $a + 2d = 6 + 2 \times -3 = 6 + (-6) = 6 - 6 = 0$
So,
Second term = 3
Third term = 0

(iv) Two coins are tossed simultaneously. Write the sample space 'S'.

Solution:
Since 2 coins are tossed the sample space
∴ S = {HH,HT, TH, TT}

2. (A) Complete and write any two activities from the following: [4]

(i) Complete the activity to find the value of the determinant.

$$\begin{vmatrix} 2\sqrt{3} & 9 \\ 2 & 3\sqrt{3} \end{vmatrix}$$
$= 2\sqrt{3} \times \_\_\_\_ - 9 \times \_\_\_$
$= \_\_\_ - 18$
$= 0$

Solution:
Activity:

$$\begin{vmatrix} 2\sqrt{3} & 9 \\ 2 & 3\sqrt{3} \end{vmatrix}$$
$= 2\sqrt{3} \times 3\sqrt{3} - 9 \times 2$
= 18 -18
= 0

(ii) Complete the activity to find the $19th$ term of an A.P.: 7, 13, 19, 25.

Activity:
Given A.P.: 7, 13, 19, 25, ……………….
Here first term $a = 7; t_{19} = ?$
$t_n = a + (\_\_\_\_\_\_)d$………………(formula)
$\therefore t_{19} = 7 + (19 - 1)\_\_$
$\therefore t_{19} = 7 + \_\_\_\_\_$
$\therefore t_{19} = \_\_\_\_\_$

Solution:
Activity:
Given A.P.: 7, 13, 19, 25, ……………….
Here first term $a = 7; t_{19} = ?$
$t_n = a + (n - 1)d$………………(formula)
$\therefore t_{19} = 7 + (19 - 1)6$
$\therefore t_{19} = 7 + 108$
$\therefore t_{19} = 115$

(iii) If one die is rolled, then to find the probability of an event to get prime number on upper face, complete the following activity.

Activity:
One die is rolled.
'S' is the sample space.
S = { ______________ }
$\therefore n$(S) = 6
Event A : Prime number on the upper face.
A = { ________ }
$\therefore n$ (A) = 3
$P(A) = \frac{\_\_}{n(S)}$…………(formula)
$\therefore P(A) = \_\_\_$

Solution:
Activity:
One die is rolled.
'S' is the sample space.
S = { 1, 2, 3, 4, 5, 6 }
$\therefore n$(S) = 6
Event A : Prime number on the upper face.
A = { 2, 3, 5 }
$\therefore n$ (A) = 3
$P(A) = \frac{3}{n(S)}$…………(formula)
$\therefore P(A) = \frac{1}{2}$

(B) Solve any four subquestions from the following: [8]

(i) To solve the following simultaneous equations by Cramer's rule, find the values of D_x and D_y.
$3x + 5y = 26, x + 5y = 22$

Solution:
By Cramer's rule.

$D_x =$
$\begin{vmatrix} 2\sqrt{3} & 9 \\ 2 & 3\sqrt{3} \end{vmatrix}$
$= 26 \times 5 - 22 \times 5 = 130 - 110 = 20$

$D_y =$
$\begin{vmatrix} 2\sqrt{3} & 9 \\ 2 & 3\sqrt{3} \end{vmatrix}$
$= 3 \times 22 - 1 \times 26 = 66 - 26 = 40$

(ii) A box contains 5 red, 8 blue and 3 green pens. Rutuja wants to pick a pen at random. What is the probability that the pen is blue?

Solution:
Total number of pens = 5 + 8 + 3 = 16
So, the sample space, n(S) = 16
Let A be the event Rutuja picks a blue pen.
Number of blue pens = 8
So, the number of favourable outcomes, (A) = 8
Probability of the pen picked randomly to be blue,

$$P(A) = \frac{n(S)}{n(A)} = \frac{8}{16} = \frac{1}{2}$$

(iii) Find the sum of first 'n' even natural numbers.

Solution:
First n even natural numbers are 2, 4, 6, ………., 2n.
$t_1 = first\ term = 2$
$t_n = last\ term = 2n$
$S_n = \frac{n}{2}(t_1 + t_n) = \frac{n}{2}(2 + 2n)$
$= \frac{n}{2} \times 2 \times (1 + n)$
$= n \times (1 + n)$

(iv) Solve the following quadratic equation by factorisation method:
$x^2 + x - 20 = 0$

Solution:
$x^2 + x - 20 = 0$

$\Rightarrow x^2 + 5x - 4x - 20 = 0$
$\Rightarrow x(x + 5) - 4(x + 5) = 0$
$\Rightarrow (x - 4)(x + 5) = 0$
$\Rightarrow (x - 4) = 0, (x + 5) = 0$
$\therefore x = 4, x = -5$

(v) Find the values of $(\boldsymbol{x} + \boldsymbol{y})$ and $(\boldsymbol{x} - \boldsymbol{y})$ of the following simultaneous equations:
$\mathbf{49}\boldsymbol{x} - \mathbf{57}\boldsymbol{y} = \mathbf{172}$, $\mathbf{57}\boldsymbol{x} - \mathbf{49}\boldsymbol{y} = \mathbf{252}$

Solution:
Adding the given equations, we get
$106x - 106y = 424$
$106(x - y) = 424$
$(x + y) = \frac{424}{106}$
$\therefore x - y = 4$

Subtracting the given equations, we get
$-8x - 8y = -80$
$-8\ x + y = -80$
$(x + y) = \frac{-80}{-8}$
$\therefore x + y = 10$

3. (A) Complete the following activity and rewrite it (any one): [3]

(i) One of the roots of equation $\boldsymbol{kx^2 - 10x + 3 = 0}$ is $\mathbf{3}$. Complete the following activity to find the value of $\boldsymbol{k}$:

Activity:
One of the roots of equation $kx^2 - 10x + 3 = 0$ is **3**
Putting x = __ in the above equation
$\therefore k(\_\_)^2 - 10 \times \_\_ + 3 = 0$
$\therefore$ ____ $- 30 + 3 = 0$
$\therefore 9k$ = ___
$\therefore k$ = ___

Solution:
Activity:
One of the roots of equation $kx^2 - 10x + 3 = 0$ is **3**
Putting $x = 3$ in the above equation
$\therefore k(3)^2 - 10 \times 3 + 3 = 0$
$\therefore 9\text{k} - 30 + 3 = 0$
$\therefore 9k = 27$
$\therefore k = 3$

(ii) A card is drawn at random from a pack of well shuffled 52 playing cards.
Complete the following activity to find the probability tha the card drawn is –
Event A: The card drawn is an ace.
Event B: The card drawn is a spade.

Activity:
'S' is the sample space.
$\therefore n(S) = 52$
Event A: The card drawn is an ace.
$\therefore n(\text{A})$ = __
$\therefore P(A) =$ ___.........(formula)

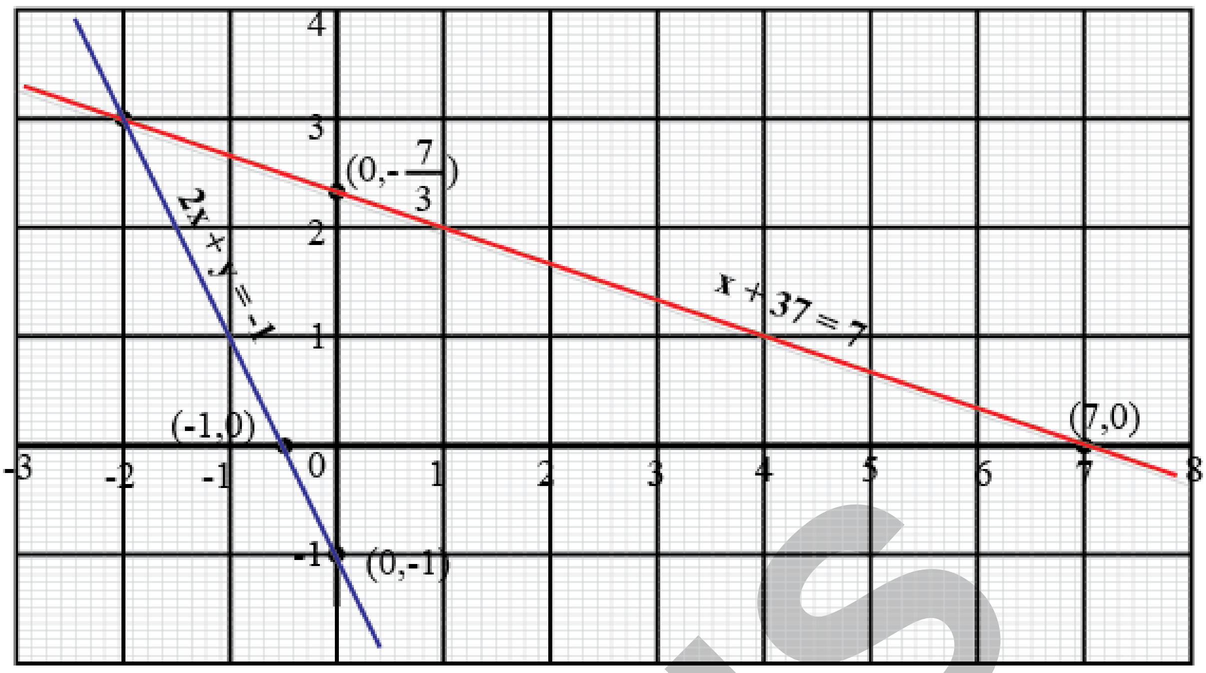
$\therefore P(A) = \frac{\underline{\hspace{1em}}}{52}$
$\therefore P(A) = \frac{\underline{\hspace{1em}}}{13}$
Event B: The card drawn is a spade.
$\therefore n\ (B) =$ ____
$$P(B) = \frac{n(B)}{n(S)} = \frac{13}{52}$$
$\therefore n\ (B) = \frac{\underline{\hspace{1em}}}{4}$

Solution:
Activity:
'S' is the sample space.
$\therefore (S) = 52$
Event A: The card drawn is an ace.
$\therefore$ (A) = 4
$\therefore P(A) = \frac{n(A)}{n(S)}$..........(formula)
$\therefore P(A) = \frac{4}{52}$
$\therefore P(A) = \frac{1}{13}$
Event B: The card drawn is a spade.
$\therefore n\ (B) = 13$
$$P(B) = \frac{n(B)}{n(S)} = \frac{13}{52}$$
$\therefore n\ (B) = \frac{1}{4}$

(B) Solve the following subquestions (any two): [6]

(i) Solve the simultaneous equations by using graphical method:
$\boldsymbol{x + 3y = 7,\ 2x + y = -1}$

Solution:
Plotting the points (7, 0), $(0, \frac{7}{3})$and $\left(\frac{-1}{2}, 0\right)$, $(0, -1)$, we get the graph. (next slide)
We observe that both the graphs are intersecting at −2, 3.
$\therefore x = -2$ and $y = 3$ is the solution.



(ii) There is an auditorium with 27 rows of seats. There are 20 seats in the first row, 22 seats in second row, 4 seats in the third row and so on. Find how many total numbers of seats in the auditorium?

Solution:

The number of seats arranged row-wise is as follows:

20, 22, 24, ….

This sequence is an A. P. with $a = 20$, $d = 22 - 20 = 2$, and $n = 27$

We know, $S_n = \frac{n}{2}(2a + d \times (n - 1))$

$\Rightarrow S_{27} = \frac{27}{2}(2 \times 20 + 2 \times (27 - 1))$

$\Rightarrow S_{27} = \frac{27}{2}(40 + 52)$

$\Rightarrow S_{27} = \frac{27}{2}(92) = 1242$

Total seats in the auditorium are 1242.

(iii) Sum of the present ages of Manish and Savitha is 31 years. Manish's age 3 years ago was 4 times the age of Savitha at that time. Find their present ages.

Solution:

Suppose the present age of Manish is x years and Savitha be y years.

According to the first condition, the sum of their present ages is 31.

So, $x + y = 31$… (i)

Three years ago;

Age of Manish = $x - 3$ years

Age of Savitha = $y - 3$ years

∴ According to the second condition, 3 years ago Manish's age was 4 times the age of Savitha's. So,

$x - 3 = 4\ y - 3$.

$x - 3 = 4y - 12$

$\therefore x - 4y = -9$

$x - 4y = -9$ … … (ii)
Subtracting equation (ii) from (i), We get
$5y = 40$
$\Rightarrow y = 8$
Substituting $y = 8$ equation (i), We get
$x + y = 31$
$x + 8 = 31$
$\Rightarrow x = 23$
Therefore, present age of Manish is 23 years and Savitha is 8 years.

(iv) Solve the following quadratic equation using formula:
$\boldsymbol{x^2 + 10x + 2 = 0}$

Solution:
Comparing the given equation $x^2 + 10x + 2 = 0$ with $ax^2 + bx + c = 0$
$\therefore$ We get, $a = 1$, $b = 10$, $c = 2$
We know the quadratic formula: $x = \frac{-b\pm\sqrt{b^2-4ac}}{2}$
$\Rightarrow x = \frac{-10\pm\sqrt{100-4\times 2}}{2}$
$\Rightarrow x = \frac{-10\pm\sqrt{92}}{2}$
$\Rightarrow x = \frac{-10\pm\sqrt{23\times 4}}{2}$
$\Rightarrow x = \frac{-5\times 2\pm 2\sqrt{23}}{2}$
$\Rightarrow x = -5 \pm \sqrt{23}$
$\therefore$ Roots of the quadratic equation are $-5 + \sqrt{23}\ and - 5 - \sqrt{23}$.

4. Solve the following subquestions (any two): [8]

(i) If 460 is a natural number, then quotient is 2 more than nines times the divisor and remainder is 5. Find the quotient and divisor

Solution:
Let the divisor be x.
Them, according to question quotient = $9x + 2$
We know, Dividend = Divisor × Quotient + Remainder
$\Rightarrow 460 = x \times (9x + 2) + 5$
$\Rightarrow 460 = 9x^2 + 2x + 5$
$\Rightarrow 455 = 9x^2 + 2x$
$\Rightarrow 9x^2 + 2x - 4500 = 0$
$\Rightarrow 9x^2 + 65x - 63x - 455 = 0$
$\Rightarrow 9(x - 7) + 65\ (x - 7) = 0$
$\Rightarrow (9x + 65)\ (x - 7) = 0$
$\Rightarrow (9x + 65) = 0$ or $(x - 7) = 0$
$\Rightarrow x = \frac{-65}{9}\ or\ x = 7$
However, 460 is divided by a natural number so $x = 7$.

$\therefore$ Divisor = 7
And quotient = 9(7) + 2 = 65.

(ii) If the $9th$ term of an A. P. is zero, then prove that the $29th$ term is double the $19th$ term.

Solution:
We know, n
th term of a sequence is $tn = a + (n - 1)$d
$\therefore t_9$ = ninth term = $a + 9 - 1\ d = a + 8d = 0$ (Given)
And t_{29} = 29th term = $a + 29 - 1\ d = a$ + 28d
= $a + 8d$ + 20d = 0 + $20d$ = 20d ($\because a + 8d = 0$)
$\Rightarrow t_{29} = 20d$

And t_{19} = 19th term = $a + 19 - 1\ d = a$ + 18d
= $a + 8d$ + 10d = 0 + $10d$ = 10d ($a + 8d = 0$)
$\Rightarrow t_{19} = 10d$

So, we have, $t_{29} = 20d\ and\ t_{19} = 10d$
Observe that

$$t_{29} = 2 \times t_{19} as\ 20d = 2 \times 10d$$

Hence proved that if the $9th$ term of an $A.P.$ is zero, then prove that the $29th$ term is double the $19th$ term.

(iii) The perimeter of an isosceles triangle is $24\ cm$. The length of its congruent sides is $13\ cm$ less than twice the length of its base. Find the lengths of all sides of the triangle.

Solution:
Let the length of the base of isosceles triangle = $x\ cm$
Length of congruent sides = $2x - 13\ cm$ (Given)
Perimeter of isosceles triangle = 24 cm (Given)

Perimeter = Length of base + length of congruent sides
$\Rightarrow 24 = x + 2x - 13 + 2x - 13$
$\Rightarrow 24 = 5x - 26$
$\Rightarrow 50 = 5x$

$\Rightarrow x = 10$
So, the length of Base = 10 cm
Congruent side = $2x - 13 = 20 - 13 = 7$

$\therefore$ The length of base is 10 cm and the length of congruent side are 7 cm and 7 cm.

5. Solve the following subquestions (any one): [3]

(i) A bag contains 8 red and some blue balls. One ball is drawn at random from the bag. If ratio of probability of getting red ball and blue ball is 2:5, then find the

number of blue balls.

Solution:
Suppose the number of blue balls = x
⇒ (Blue ball) = x
Number of red balls = 8
⇒ (Red ball) = 8
Total number of balls = 8 + x
⇒ (Total) = 8 + x

∴ P(Blue ball drawn) $= \frac{n(Blue\ ball)}{n\ (Total)} = \frac{x}{8+x}$
According to the given condition,
$$\frac{P(Blue\ ball\ drawn)}{P(Red\ ball\ drawn)} = \frac{5}{2}$$
$$\frac{P(Blue\ ball\ drawn)}{P(Red\ ball\ drawn)} = \frac{\left(\frac{x}{8+x}\right)}{\left(\frac{8}{8+x}\right)} = \frac{x}{8}$$
$\Rightarrow \frac{x}{8} = \frac{5}{2}$
⇒ x = 20
Hence, the number of blue balls is 20.

(ii) Measures of angles of a triangle are in $A.P$. The measure of smallest angle is five times of common difference. Find the measures of all angles of a triangle. (Assume the measures of angles as a, $a + d$, $a + 2d$.)

Solution:
Let the angles of triangles be a, $a + d$, $a + 2d$.
We know that, sum of angles of a triangle = 180°
⇒ $a + a + d + a + 2d$ = 180°
⇒ $3a + 3d$ = 180°
⇒ $a + d$ = 60°
According to the given conditions,
Smallest angle, $a = 5d$
Putting $a = 5d$ in $a + d$ = 60°
⇒ $6d$ = 60°
⇒ d = 10°

Putting d = 10° in $a + d$ = 60°
⇒ a + 10 = 60°
⇒ a = 50°

As, angles of triangles are a, $a + d$, $a + 2d$,
Hence, a = 50°
And $a + d$ = 60°
And $a + 2d$ = 70°.
∴ Angles of the given triangle are 50°, 60°, and 70°.