**SSC** **MARCH 2020**

# MATHEMATICS
## GEOMETRY – PART II

**Time allowed: 2 hours** **Maximum marks: 40**

**General Instructions:**

(i) All questions are compulsory.
(ii) Use of calculator is not allowed.
(iii) The numbers to the right of the questions indicate full marks.
(iv) In case MCQ's Q. No. 1(A) only the first attempt will be evaluated and will be given credit.
(v) For every MCQ, the correct alternative (A), (B), (C) or (D) of answers with sub question number is:

**1. (A) Four alternative answer are given for every sub – question. Select the correct alternative and write the alphabet of that answer:** **[4]**

**(i) Out of the following which is the Pythagorean triple?**

**(A)** $(\mathbf{1,5,10})$ **(B)** $(\mathbf{3,4,5})$ **(C)** $(\mathbf{2,2,2})$ **(D)** $(\mathbf{5,5,2})$

**Answer:** (B) $(3,4,5)$

**(ii)Two circles of radii $\mathbf{5.5}$ cm and $\mathbf{3.3}$ cm respectively touch each other externally. What is the distance between their centres?**

**(A)** $\mathbf{4.4}$ **cm** **(B)** $\mathbf{2.2}$ **cm** **(C)** $\mathbf{8.8}$ **cm** **(D)**$\mathbf{8.9}$ **cm**

**Answer:** (C) 8.8 cm

**(iii) Distance of point (-3, 4) from the origin is ___________.**

**(A)** $\mathbf{7}$ **(B)** $\mathbf{1}$ **(C)** $\mathbf{-5}$ **(D)** $\mathbf{5}$

**Answer:** (D) 5

**(iv) Find the volume of a cube of side 3 cm: ***

**(A)** $\mathbf{27\ cm^3}$ **(B)** $\mathbf{9\ cm^3}$ **(C)** $\mathbf{81\ cm^3}$ **(D)** $\mathbf{3\ cm^3}$

**Answer:** Answer is not given due to the change in reduced syllabus.

**(B) Solve the following questions:** **[4]**

**(i) The ratio of corresponding sides of similar triangles is 3 : 5, then find the ratio of their areas.**

**Answer:** Ratio of areas of similar triangles =

$$(\text{Ratio of corresponding sides of similar triangles})^2$$

$$= \frac{3^2}{5}$$

Ratio of their areas $= \frac{9}{25}$

**(ii) Find the diagonal of a square whose side is $\mathbf{10}$ cm.**

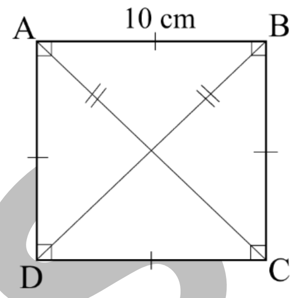
**Answer:** Let ☐ ABCD is a square

l(AB) = l(BC) = l(CD) = l(AD) = 10 cm

In $\Delta aBC$, (Given)

$AC^2 = AB^2 + BC^2$

(Pythagoras theorem)




$\therefore$ $AC^2 = AB^2 + AB^2$ $(\because AB = BC)$

$\therefore$ $AC^2 = 2AB^2$

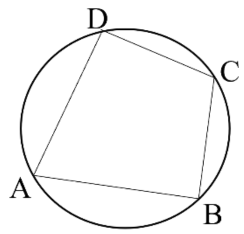
$\therefore$ $AC = \sqrt{2}\,AB$

$= \sqrt{2}\,(10)$ cm (AB = 10cm)

$\therefore$ $AC = 10 \times 1.414 = 14.14$ cm

$\therefore$ Diagonal of the square AC $= 14.14$ cm

**(iii) ☐ ABCD is cyclic. If $\angle B = \mathbf{110}$, then find measure of $\angle$D.**




**Answer:** ☐ ABCD is cyclic

$\therefore$ $m\angle B + m\angle D = 180^\circ$

$\therefore$ $110^\circ + m\angle D = 180^\circ$

(Given, $m\angle B = 110^\circ$)

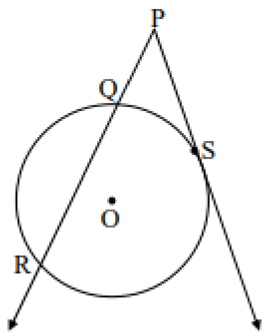
$\therefore$ $m\angle D = 180^\circ - 110^\circ$

$m\angle D = 70^\circ$

**(iv) Find the slope of the line passing through the points A $(\mathbf{2}, \mathbf{3})$ and B $(\mathbf{4}, \mathbf{7})$.***

**Answer:** Answer is not given due to the change is reduced syllabus.

**2. (A) Complete and write the following activities (Any two): [4]**



(i) **In the figure given, 'O' is the centre of the circle, seg PS is a tangent segment and S is the point of contact. Line PR is a secant. If $PQ = 3.6, QR = 6.4$, find PS.***

**Answer:** Answer is not given due to the change is reduced syllabus.

**(ii) If $\sec\theta = \frac{25}{7}$, find the value of $\tan\theta$.**

**Solution:** $1 + \tan^2\theta = \sec^2\theta$

$\therefore\ 1 + \tan^2\theta = \left(\frac{25}{7}\right)^{\square}$

$\therefore\ \tan^2\theta = \frac{625}{49} - \square$

$= \frac{625-49}{49}$

$= \frac{\square}{49}$

$\tan\theta = \frac{\square}{7}$ ...... (by taking square roots)

**Answer:** $1 + \tan^2\theta = \sec^2\theta$

$\therefore\ 1 + \tan^2\theta = \left(\frac{25}{7}\right)^{\boxed{2}}$

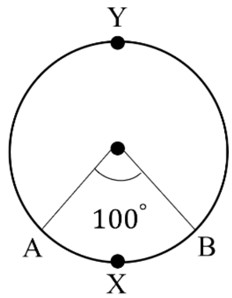
$\therefore\ \tan^2\theta = \frac{625}{49} - \boxed{1}$

$= \frac{625-49}{49} = \frac{\boxed{576}}{49}$

$\tan\theta = \frac{\boxed{24}}{7}$ .... by taking square roots

**(iii) In the figure given, O is the centre of the circle. Using given information complete the following table:***

| Type of arc | Name of the arc | Measure of the arc |
|---|---|---|
| **Minor arc** | $\square$ | $\square$ |
| **Major arc** | $\square$ | $\square$ |




**Answer:**

| Type of arc | Name of the arc | Measure of the arc |
|---|---|---|
| **Minor arc** | **Arc AXB** | **100°** |
| **Major arc** | **Arc AYB** | **260°** |

**(B) Solve the following sub – questions (Any four): [8]**

**(i) In $\Delta PQR, NM \parallel RQ$. If $PM = 15, MQ = 10, NR = 8$, then find PN.**

**Answer:** Given NM ∥ RQ

$\therefore \frac{PN}{NR} = \frac{PM}{MQ}$ (Basic proportionality theorem) …(i)

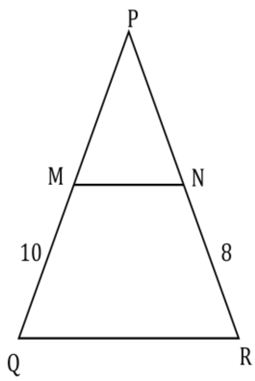
But PM = 15, MQ = 10, NR = 8 (Given)

$\therefore$ Equation (i) becomes,

$$\frac{PN}{8} = \frac{15}{10}$$

$\therefore PN = \frac{15\times 8}{10} = \frac{15\times 4}{5} = 3 \times 4$

$\therefore$ PN = 12 Unit




**(ii) In $\Delta MNP, \angle MNP = 90°$ seg NQ $\perp$ seg MP. If $MQ = 9, QP = 4$, then find NQ.**

**Answer:** In $\Delta MNP, \angle MNP = 90^\circ$, seg NQ $\perp$ seg MP

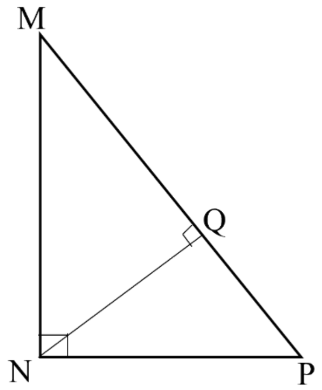
$\therefore$ According to right angled triangle geometric mean sub theorem

$NQ^2 = MQ \times QP$

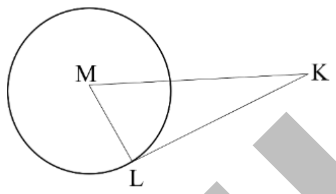
$= 9 \times 4 = 36$

$\therefore NQ = \sqrt{36}$

$= 6$ unit




**(iii) In the figure given above, M is the centre of the circle and seg KL is a tangent segment. L is a point of contact. If $\mathbf{MK = 12, KL = 6\sqrt{3}}$, then find the radius of the circle.**




**Answer:** In given figure,

radius ML $\perp$ tangent Segment KL … (Tangent theorem)

$\therefore m\angle MLK = 90^\circ$

In right – angled $\Delta$ MLK

$MK^2 = ML^2 + LK^2$ (According to Pythagoras theorem)

$\therefore (12)^2 = ML^2 + (6\sqrt{3})^2$

$\therefore 144 = ML^2 + 108$

$\therefore ML^2 = 144 - 108 = 36$

$\therefore ML = 6$

$\therefore$ Radius ML = 6 unit

**(iv) Find the co-ordinate of midpoint of the segment joining the points (22, 20) and (0, 16).**

**Answer:** Given points are (22, 20) and (0, 16)

Let, $x_1 = 22, x_2 = 0, y_1 = 20, y_2 = 16$

We know, Midpoint $= \left(\frac{x_1 + x_2}{2}, \frac{y_1 + y_2}{2}\right)$

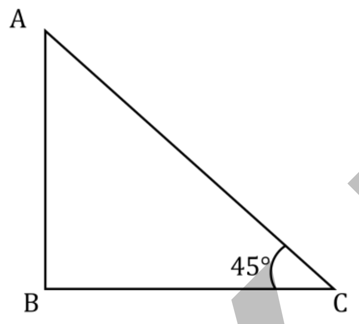
$= \left(\frac{22+0}{2}, \frac{20+16}{2}\right)$

$= \left(\frac{22}{2}, \frac{36}{2}\right)$

$= (11, 18)$

**(v) A person is standing at a distance of 80 metres from a Church and looking at its top. The angle of elevation is of 45. Find the height of the Church. ***

**Answer:**




Let, AB be the height of the church.

$\angle ACB = 45°, BC = 80$ m

In right angled $\triangle$ ABC, we have,

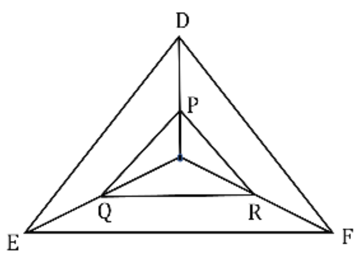
$\tan 45° = \frac{AB}{BC}$

$\Rightarrow 1 = \frac{AB}{80}$

$\Rightarrow AB = 80$ m

**3. (A) Complete and write the following activities (Any one): [3]**

**(i) In the given figure, X is any point in the interior of the triangle. Point X is joined to the vertices of triangle. seg PQ $\parallel$ seg DE, seg QR $\parallel$ seg EF. Complete the activity and prove that seg PR $\parallel$ seg DF.**



**Proof:** In $\Delta XDE$

$PQ \parallel DE$ ….. (Given)

$\therefore \frac{XP}{PD} = \frac{\square}{QE}$ …... (Basic propatianality theorem) …. (i)

$\therefore \frac{XP}{PD} = \frac{\square}{\square}$ …. [From (i) and (ii)]

$\therefore$ Seg PR ∥ seg DF … (By converse of basic proportionality throrem)

**Answer:** In $\Delta XEF$

$QR \parallel EF$ …. (Given)

$\therefore \frac{XQ}{\boxed{QE}} = \frac{XR}{\boxed{RF}}$ …. ($\boxed{\text{Basic proportionality theorem}}$) …. (ii)

$\therefore \frac{XP}{PD} = \frac{\boxed{XR}}{\boxed{RF}}$ …. [From (i) and (ii)]

$\therefore$ Seg PR ∥ seg DF … (By converse of basic proportionality throrem)

**(ii) If A $(6, 1)$, B $(8, 2)$, C $(9, 4)$ and D $(7, 3)$ are the vertices of □ ABCD, show that □ ABCD is a parallelogram. ***

**Answer:** Given A $(6, 1)$, B $(8, 2)$, C $(9, 4)$ and D $(7, 3)$

$AB = \sqrt{(8-6)^2 + (2-1)^2}$ [$\because$ Distance $= \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$]

$= \sqrt{2^2 + 1^2} = \sqrt{5}$

$BC = \sqrt{(9-8)^2 + (4-2)^2}$

$= \sqrt{1^2 + 2^2} = \sqrt{5}$

$CD = \sqrt{(7-9)^2 + (3-4)^2}$

$= \sqrt{2^2 + (-1)^2} = \sqrt{5}$

$DA = \sqrt{(7-6)^2 + (3-1)^2}$

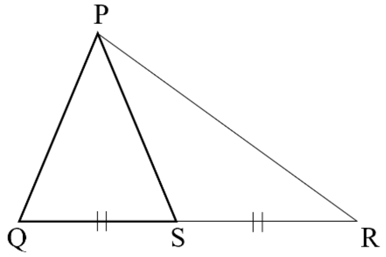
$= \sqrt{1^2 + (2)^2} = \sqrt{5}$

$\therefore AB = BC = CA = DA$

Hence, ABCD is a parallelogram.

**(B) Solve the following sub – questions (Any two): [8]**

**(i) If $\Delta$PQR, point S is the mid – point of side QR. If $PQ = 11, PR = 17, PS = 13$, find QR.**



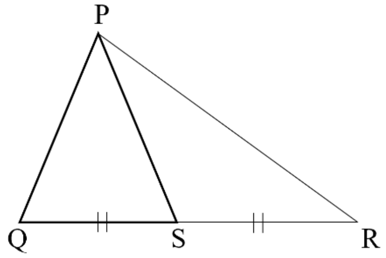

**Answer:** In $\Delta$PQR, point S is the mid – point of side QR.

$\therefore$ Segment PS is median of $\Delta$PQR

According to Apollonius's theorem

$$PQ^2 + PR^2 = 2PS^2 + 2QS^2$$

As per given values,




$\therefore (11)^2 + (17)^2 = 2(13)^2 + 2QS^2$

$\therefore$ 121 + 289 = 2(169) + $2QS^2$

$\therefore$ 410 = 338 + $2QS^2$

$\therefore 2QS^2 = 410 - 338 = 72$

$\therefore QS^2 = \frac{72}{2} = 36$

$\therefore$ QS = 6 unit …. (i)

We know, point S is the mid – point of side QR

$\therefore$ 2QS = QR ($\because$ QS = SR)

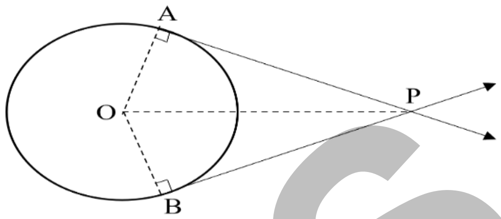
$\therefore QR = 2 \times (6)$ [From equation (i)]

$\therefore$ QR = 12 unit

∴ Length of side QR is 12 unit.

**(ii) Prove that, tangent segments drawn from an external point to the circle are congruent.**

**Answer:** Point O is the centre of the circle and point P is external to the circle. Segment PA and segment PB are tangent segments to the circle. Point A and point B are touch points of the tangent segments.




Prove: PA ≅ PB

Construction: Draw OA, OB and OP.

Proof: ∵ Each tangent of a circle is perpendicular to the radius drawn through the

point of contact .... (Theorem)

∴ Radius OA ⊥ AP and, Radius OB ⊥ BP .....(i)

∴ $m\angle PAO = 90^{\circ}$ and $m\angle PBO = 90^{\circ}$

∴ ΔPAO and ΔPBO are right – angled triangles.

Now in ΔPAO and ΔPBO,

$OA = OB$ (∵ Radius of same circle)

∠PAO = ∠PBO [Using (i)]

Hypotenuse OP = Hypotenuse OP (∵ common side)

∴ $\Delta PAO \cong \Delta PBO$ (RHS conguruency criterion)

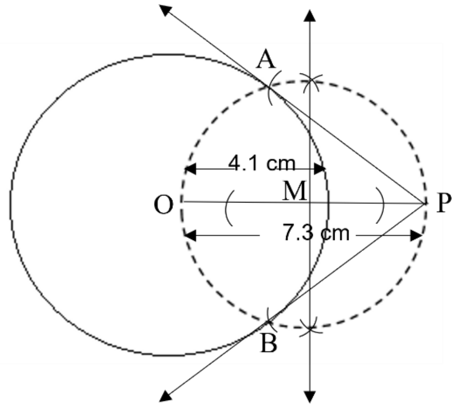
∴ line PA ≅ line PB (∵ corresponding sides of Congruent triangles)

Line PA and line PB are tangent. **Hence proved.**

**(iii) Draw a circle with radius $4.1$ cm. Construct tangents to the circle from a point at a distance $7.3$ cm from the centre.**

**Answer:**

**Steps of construction:**




Step 1: Draw a circle of radius 4.1 cm with centre O.

Step 2: Take a point P in the exterior of the circle such that $OP = 7.3$ cm

Step 3: Draw segment OP, draw perpendicular bisector of segment OP to get its midpoint M.

Step 4: Draw a circle with radius OM and centre M.

Step 5: Name the point of intersection of the two circles as A and B.

Step 6: Join PA and PB.

Thus, PA and PB are required tangents.

**(iv) A metal cuboid of measures $16$cm $11$cm $10$cm was melted to make coins. How many coins were made, if the thickness and diameter of each coin was 2 mm and 2 cm respectively? $(\pi = 3.14)$ ***

**Answer:** Radius of each coin, r $= \frac{2}{2} = 1$ cm

Thickness of each coin, $h = 2$ mm $= \frac{2}{10} = 0.2$ cm (1 cm = 10 mm)

Let the number of coins made be n.

It is given that a metal parallelopiped is melted to make the coins.

$\therefore\ n \times \text{Volume of metal in each coin} = \text{Volume of the metal cuboid}$

$\Rightarrow n = \frac{\text{Volume of the metal cuboid}}{\text{Volume of metal in each coin}}$

$\Rightarrow n = \frac{16 \times 11 \times 10}{\pi r^2 h}$

$\Rightarrow n = \frac{16 \times 11 \times 10}{\frac{22}{7} \times 1 \times 1 \times 0.2} = 2800$

Thus, the number of coins made are 2800.

**4. Solve the following sub – questions (Any two): [8]**

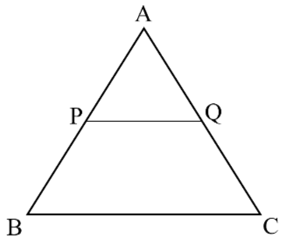
**(i) In $\Delta ABC$, PQ is a line segment intersecting AB at P and AC at Q such that seg PQ ∥ seg BC. If PQ divides Δ ABC into two equal parts having equal areas, find $\frac{BP}{AB}$.**

**Answer:** In above figure $\Delta ABC$, PQ ∥ BC

$A - P - B$ and $A - Q - C$

and ar $(\Delta APQ) = $ ar $(\square\ PBCQ)$ ...

In $\Delta APQ$ and $\Delta ABC$




$\angle A = \angle A$ ….. (common angle)

$\angle APQ = \angle ABC$ …. (corresponding angle)

$\therefore$ $\Delta APQ \sim \Delta ABC$ ….. (A – A similarity test)

$\therefore \frac{\text{ar}\,(\Delta\,ABC)}{\text{ar}\,(\Delta APQ)} = \frac{AB^2}{AP^2}$ (Theorem of areas of similar triangles) ….. (i)

Now,

$\text{ar}\,(\Delta APQ) = ar\,(\square\ PBCQ)$ (Given)

$\therefore$ $\frac{\text{ar}\,[\square PBCQ]}{\text{ar}\,[\Delta APQ]} = \frac{1}{1}$

Adding 1 on both sides,

$\therefore$ $\frac{\text{ar}\,[\square PBCQ] + \text{ar}\,(\Delta APQ)}{\text{ar}\,(\Delta APQ)} = \frac{1+1}{1} = \frac{2}{1}$

$\therefore$ $\frac{\text{ar}(\Delta ABC)}{\text{ar}\,(\Delta APQ)} = \frac{2}{1}$ …. (ii) [$\because$ ar $(\Delta APQ)$ + ar $(\square\ PBCQ)$ = ar $(\Delta ABC)$]

$\therefore$ From (i) and (ii)

$\therefore$ $\frac{\text{ar}\,(\Delta ABC)}{\text{ar}\,(\Delta APQ)} = \frac{2}{1} = \frac{AB^2}{AP^2}$

$\frac{AB}{AP} = \frac{\sqrt{2}}{1}$ (by taking square roots on both sides)

Let $AB = \sqrt{2}x$ …. (iii)

and $AP = 1\,x$

Now, $BP = AB - AP$

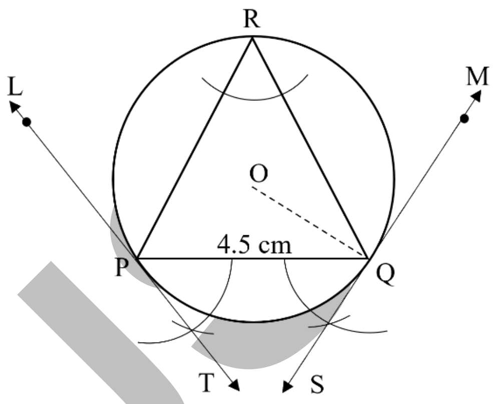
$\therefore$ $BP = \sqrt{2}\,x - 1x = (\sqrt{2} - 1)x$ …. (iv)

From (iii) and (iv)

$\therefore \quad \frac{BP}{AB} = \frac{(\sqrt{2}-1)}{\sqrt{2}}$

**(ii) Draw a circle of radius $2.7$cm and draw a chord PQ of length $4.5$ cm. Draw tangents at point P and Q without using centre.**

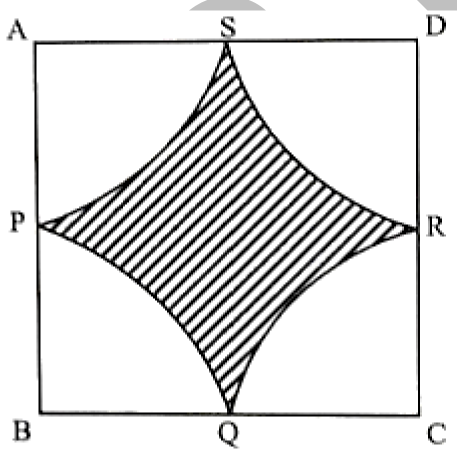
**Answer:**
**Step of construction:**




Step 1 : Draw a circle of with centre O and radius 2.7 cm
Step 2 : Draw a chord PQ of length 4.5 cm
Step 3 : Taking a point R on the major arc QP, join PR and QR.
Step 4 : Make ∠ QPT = ∠ PRQ and ∠ PQS = ∠ PRQ.
Step 5 : Produce TP to L and SQ to M.

Hence, TPL and SQM are the required tangents.

**(iii) In the figure given □ ABCD is a square of side $50$ m. Points P, Q, R, $S$ are midpoints of side AB, side BC, side CD, side CD, side AD respectively. Find area of shaded region.***



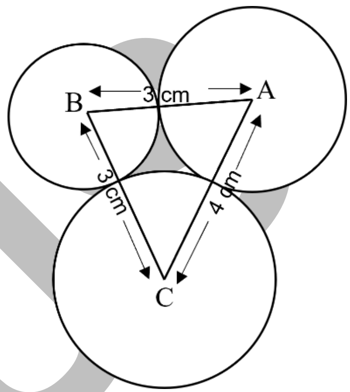

**Answer:** Answer is not given due to the change in reduced syllabus.

**5. Solve the following sub – questions (Any one):** **[3]**
**(i) Circles with centres A, B and C touch each other externally. If $AB = 3$ cm, $BC = 3$ cm, $CA = 4$ cm, then find the radii of each circle.**

**Answer:** Suppose radius of circle with centre A is x cm

∴ Radius of circle with centre B $= (3 - x)$ cm (∵ AB = 3cm)
and radius of circle with centre C $= (4 - x)$ cm (∵ CA = 4cm)

∴ $(3 - x) + (4 - x) = BC = 3$
∴ $3 - x + 4 - x = 3$
∴ $7 - 2x = 3$
∴ $2x = 7 - 3$
∴ $2x = 4$
∴ $x = 2$
∴ Radius of circle with centre A = 2cm
∴ Radius of circle with centre B $= (3 - x)$
$= (3 - 2)$
$= 1$ cm
∴ Radius of circle with centre C $= (4 - x)$
$= (4 - 2)$
$= 2$ cm




**(ii) If $\sin\theta + \sin^2\theta = 1$**
**Show that: $\cos^2\theta + \cos^4\theta = 1$**

**Answer:** $\sin\theta + \sin^2\theta = 1$ …. (Given)

But $\sin^2\theta + \cos^2\theta = 1$ …. (Standard result)

∴ Putting the value 1 in given relation we get.

$$\sin\theta + \sin^2\theta = \sin^2\theta + \cos^2\theta$$

∴ $\sin\theta = \cos^2\theta$ …. (i)

Now as per given relation

$$\sin\theta + \sin^2\theta = 1$$

∴ $\cos^2\theta + (\cos^2\theta)^2 = 1$ […. From equation (i)]

∴ $\cos^2\theta + \cos^4\theta = 1$ Hence proved.