

(iv) The value of $2 \tan 45^\circ - 2 \sin 30^\circ$ is _____.

(A) 2

(B) 1

(C) $\frac{1}{2}$

(D) $\frac{3}{4}$

Sol: We know that $\tan 45^\circ = 1$ and $\sin 30^\circ = \frac{1}{2}$

$$\begin{aligned}\text{Thus, we get } 2 \tan 45^\circ - 2 \sin 30^\circ &= 2 \times 1 - 2 \times \frac{1}{2} \\ &= 2 - 1 = 1\end{aligned}$$

1. (B)

(i) In $\triangle ABC$, $\angle ABC = 90^\circ$, $\angle BAC = \angle BCA = 45^\circ$. If $AC = 9\sqrt{2}$, then find the value of AB.

Sol: Given, in $\triangle ABC$

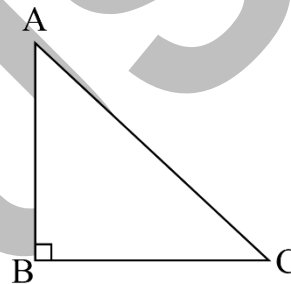
$$\angle ABC = 90^\circ, \angle BAC = \angle BCA = 45^\circ, AC = 9\sqrt{2}$$

$$\text{Now, } AB = \frac{1}{\sqrt{2}} \times AC$$

[Property of $45^\circ - 45^\circ - 90^\circ$ triangle]

$$\therefore AB = \frac{1}{\sqrt{2}} \times 9\sqrt{2}$$

$$\therefore AB = 9$$



(ii) Chord AB and chord CD of a circle with centre O are congruent. If $m(\text{arc AB}) = 120^\circ$, then find the $m(\text{arc CD})$.

Sol: Given, chord AB = Chord CD

$$m(\text{arc AB}) = 120^\circ$$

We know that,

$$\text{Arc AB} \cong \text{arc CD}$$

[Corresponding arcs of congruent chord of a circle are congruent]

$$\Rightarrow m(\text{arc AB}) = m(\text{arc CD})$$

$$\Rightarrow 120^\circ = m(\text{arc CD})$$

$$\therefore m(\text{arc CD}) = 120^\circ$$

(iii) Find the Y-coordinate of the centroid of a triangle whose vertices are (4, -3), (7, 5), and (-2, 1).

Sol: Vertices of the triangle,

(4, -3), (7, 5), and (-2, 1) [Given]

$$x_1 = 4, x_2 = 7, x_3 = -2$$

$$y_1 = -3, y_2 = 5, y_3 = 1$$

By using the centroid formula,

$$\text{Co-ordinate of centroid} = \left[\frac{x_1 + x_2 + x_3}{3}, \frac{y_1 + y_2 + y_3}{3} \right]$$

Now, Y – coordinate of centroid = $\frac{y_1 + y_2 + y_3}{3}$

$$= \frac{-3+5+1}{3} = \frac{3}{3} = 1$$

\therefore Y – coordinate of centroid = 1.

(iv) If $\sin \theta = \cos \theta$, then what will be the measure of angle θ ?

Sol: Given, $\sin \theta = \cos \theta$

We know that,

$$\sin \theta = \cos(90^\circ - \theta)$$

$$\therefore \cos \theta = \cos(90^\circ - \theta)$$

$$\Rightarrow \theta = 90^\circ - \theta$$

$$\Rightarrow \theta + \theta = 90^\circ$$

$$\Rightarrow 2\theta = 90^\circ$$

$$\Rightarrow \theta = \frac{90^\circ}{2}$$

$$\therefore \theta = 45^\circ$$

2. (A)

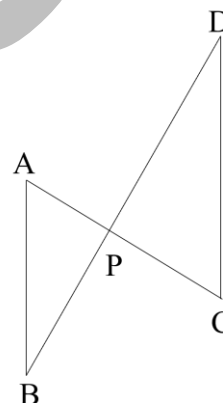
(i) In the given figure, seg AC and seg BD intersect each other in point P. If $\frac{AP}{CP} = \frac{BP}{DP}$, then complete the following activity to prove $\triangle ABP \sim \triangle CDP$.

Sol: Activity: In $\triangle APB$ and $\triangle CDP$

$$\frac{AP}{CP} = \frac{BP}{DP} \dots\dots\dots \boxed{\text{Given}}$$

$$\therefore \angle APB \cong \angle DPC \dots\dots\dots \text{Vertically opposite angles}$$

$$\therefore \boxed{\angle ABP} \sim \triangle CDP \dots\dots\dots \text{test of similarity}$$



(ii) In the given figure, $\square ABCD$ is a rectangle. If $AB = 5$, $AC = 13$, then complete the following activity to find BC.

Sol: Activity:

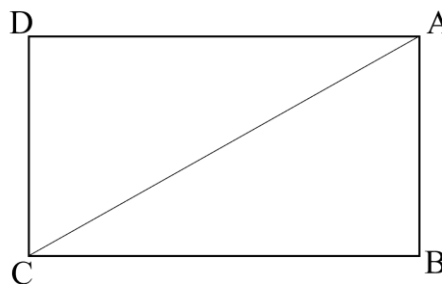
$\triangle APB$ is $\boxed{\text{right-angled}}$ triangle.

\therefore By Pythagoras theorem

$$AB^2 + BC^2 = AC^2$$

$$\therefore 25 + BC^2 = \boxed{169}$$

$$\therefore BC^2 = \boxed{144}$$



$$\therefore BC = \boxed{12}$$

(iii) Complete the following activity to prove: $\cot \theta + \tan \theta = \operatorname{cosec} \theta \times \sec \theta$

Sol: Activity:

$$\begin{aligned} \text{L.H.S} &= \cot \theta + \tan \theta \\ &= \frac{\cos \theta}{\sin \theta} + \frac{\sin \theta}{\cos \theta} = \frac{\boxed{\cos^2 \theta} + \sin^2 \theta}{\sin \theta \times \cos \theta} \\ &= \frac{1}{\sin \theta \times \cos \theta} \dots \dots \dots \boxed{\because \cos^2 \theta + \sin^2 \theta = 1} \\ &= \frac{1}{\sin \theta} \times \frac{1}{\cos \theta} \\ &= \boxed{\operatorname{cosec} \theta} \times \sec \theta \\ \therefore \text{L. H. S} &= \text{R. H. S} \end{aligned}$$

2. (B)

(i) If $\triangle ABC \sim \triangle PQR$, AB: PQ = 4 : 5 and A ($\triangle PQR$) = 125 cm^2 , then find A ($\triangle ABC$).

Sol: Given: $\triangle ABC \sim \triangle PQR$

We know that,

$$\frac{A(\triangle ABC)}{A(\triangle PQR)} = \frac{(AB)^2}{(PQ)^2} \dots \dots \dots [\text{Theorem of area of similar triangles}]$$

$$\Rightarrow \frac{A(\triangle ABC)}{125} = \frac{(4)^2}{(5)^2}$$

$$\Rightarrow \frac{A(\triangle ABC)}{125} = \frac{16}{25}$$

$$\Rightarrow A(\triangle ABC) = \frac{16}{25} \times 125$$

$$\therefore A(\triangle ABC) = 80 \text{ cm}^2$$

(ii) In the given figure, $m(\text{arc } DXE) = 105^\circ$, $m(\text{arc } AYC) = 47^\circ$ then find the measure of $\angle DBE$.

Sol: From Figure.

Chord AD and CE intersect externally at point B.

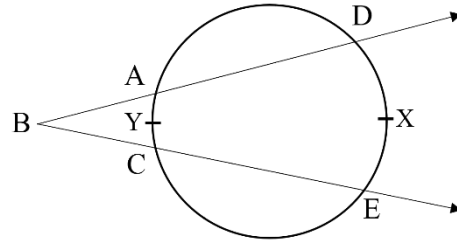
$$\therefore m(\text{arc } DEX) = \frac{1}{2} [m(\text{arc } DXE) - m(\text{arc } AYC)]$$

..... [By inscribed angle theorem]

$$\therefore m(\text{arc } DEX) = \frac{1}{2} [105^\circ - 47^\circ]$$

$$\therefore m(\text{arc } DEX) = \frac{1}{2} [58^\circ]$$

$$\therefore m(\text{arc } DEX) = 29^\circ$$



(iii) Draw a circle of radius 3.2 cm and centre O. Take any point P on it. Draw tangent to the circle through Point P using the centre of the circle.

Sol: **Given:** Radius of the circle = 3.2 cm

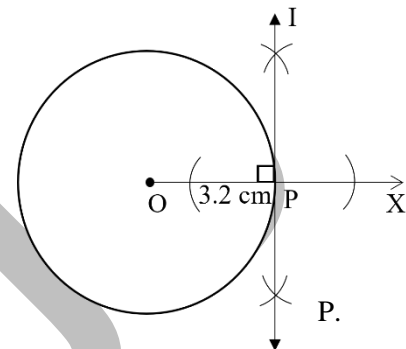
Construction:

(i) With O as the centre draw a circle of radius 3.2 cm.

(ii) Take a point P on the circle and draw ray OP.

(iii) Draw line l Perpendicular to ray OX through point

(iv) Line l is the required tangent to the circle at point P.



(iv) If $\sin \theta = \frac{11}{61}$, then find the value of $\cos \theta$ using trigonometric identity.

Sol: We know $\sin^2 \theta + \cos^2 \theta = 1$

$$\Rightarrow \cos^2 \theta = 1 - \sin^2 \theta$$

$$\Rightarrow \cos^2 \theta = 1 - \left(\frac{11}{61}\right)^2$$

$$\Rightarrow \cos^2 \theta = 1 - \left(\frac{121}{3721}\right) = \frac{3721-121}{3721} = \frac{3600}{3721}$$

$$\Rightarrow \cos^2 \theta = \sqrt{\left(\frac{60}{61}\right)^2} = \frac{60}{61}$$

Thus the value of $\cos \theta$ is $\frac{60}{61}$.

(v) In $\triangle ABC$, $AB = 9$ cm, $BC = 40$ cm, and $AC = 41$ cm. State whether $\triangle ABC$ is a right-angled triangle or not? Write reason.

Sol: Side of $\triangle ABC$ are $AB = 9$ cm, $BC = 40$ cm, $AC = 41$ cm

The triangle's longest side measures 41 cm.

$$\therefore 41^2 = 1681 \quad \dots\dots\dots (i)$$

Now, the sum of the square of the remaining sides is

$$9^2 + 40^2 = 81 + 1600$$

$$= 1681 \quad \dots\dots\dots (ii)$$

From equations (i) and (ii), as the square of the longest side equals the sum of the squares of the remaining two sides, by using converse of Pythagoras theorem the given sides form a right – angle triangle.

3. (A)

(i) In the given figure, chord PQ and chord RS intersect each other at point T. If $\angle STQ = 58^\circ$ and $\angle PSR = 24^\circ$, then complete the following activity to verify:

$$\angle STQ = \frac{1}{2} [m(\text{arc PR}) + m(\text{arc SQ})]$$

Sol: Activity:

In $\triangle PTS$,

$$\angle SPQ = \angle STQ - \boxed{\angle PSR} \quad \because \text{Exterior angle theorem}$$

$$\angle SPQ = 34^\circ$$

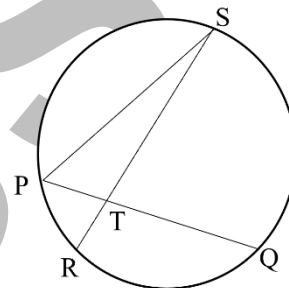
$$\therefore m(\text{arc QS}) = 2 \times \boxed{34^\circ} = 68^\circ \quad \dots\dots \because \text{Inscribed angle theorem}$$

$$\text{Similarly } m(\text{arc PR}) = 2 \angle PSR = \boxed{48^\circ}$$

$$\therefore \frac{1}{2} [m(\text{arc QS}) + m(\text{arc PR})] = \frac{1}{2} \times \boxed{116^\circ} = 58^\circ \quad \dots\dots\dots (I)$$

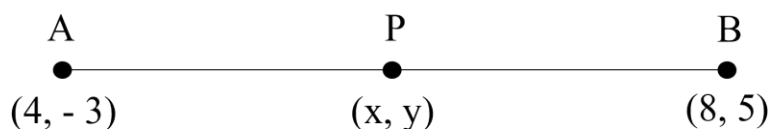
But $\angle STQ = 58^\circ \dots\dots\dots (II)$ given

$$\therefore \frac{1}{2} [m(\text{arc PR}) + m(\text{arc QS})] = \boxed{\angle STQ} \dots\dots\dots [\text{From (I) and (II)}]$$



(ii) Complete the following activity to find the co – ordinates of point P which divides seg AB in the ratio 3 : 1 where A (4, -3) and B (8, 5).

Sol. Activity:



\therefore By section formula,

$$x = \frac{mx_2 + nx_1}{m+n}, y = \frac{my_2 + ny_1}{m+n}$$

$$\therefore x = \frac{3 \times 8 + 1 \times 4}{3+1}, y = \frac{3 \times 5 + 1 \times (-3)}{3+1}$$

$$\therefore x = \frac{24+4}{4}, y = \frac{15-3}{4},$$

$$\therefore x = 7 \therefore y = 3$$

3. (B)

(i) In $\triangle ABC$, seg $XY \parallel$ side AC . If $2AX = 3BX$ and $XY = 9$, then find the value of AC .

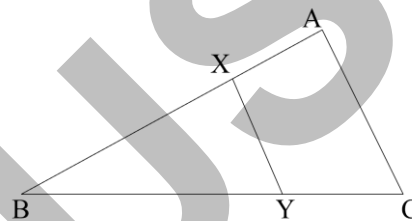
Sol: Given seg $XY \parallel$ seg AC , $2AX = 3BX$ and $XY = 9$

Consider, $2AX = 3BX$

$$\therefore \frac{AX}{BX} = \frac{3}{2}$$

$$\Rightarrow \frac{AX+BX}{BX} = \frac{3+2}{2} \quad \dots\dots \text{(By Componendo)}$$

$$\Rightarrow \frac{AB}{BX} = \frac{5}{2} \quad \dots\dots\dots \text{(I)}$$



$\triangle ABC \sim \triangle BXY \dots\dots\dots$ SAS test of similarity

$$\therefore \frac{BA}{BX} = \frac{AC}{XY} \dots\dots\dots \text{corresponding sides of similar triangles}$$

$$\therefore \frac{5}{2} = \frac{AC}{9} \dots\dots\dots \text{from (I)}$$

$$\therefore AC = 22.5$$

(ii) Prove that, “Opposite angles of cyclic quadrilateral are supplementary”.

Sol: Let O be the centre of the circle. Join O to B and D .

Let the angle subtended by the minor arc and the major arc at

the centre be x and y respectively.

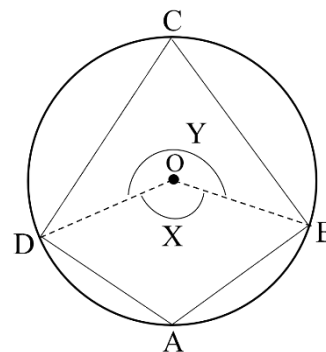
$$\text{Proof: } x = 2\angle C \quad [\text{Angle at centre theorem}] \dots\dots\dots \text{(i)}$$

$$\text{and } y = 2\angle A \dots\dots\dots \text{(ii)}$$

Adding (i) and (ii), we get

$$x + y = 2\angle C + 2\angle A \dots\dots\dots \text{(iii)}$$

$$\text{But, } x + y = 360^\circ \dots\dots\dots \text{(iv)}$$



From (iii) and (iv), we get

$$2\angle C + 2\angle A = 360^\circ$$

$$\Rightarrow \angle C + \angle A = 180^\circ$$

But we know that angle sum property of quadrilateral

$$\angle A + \angle B + \angle C + \angle D = 360^\circ$$

$$\Rightarrow \angle B + \angle D + 180^\circ = 360^\circ$$

$$\Rightarrow \angle B + \angle D = 180^\circ$$

Hence proved that opposite angles of cyclic quadrilateral are supplementary.

(iii) $\triangle ABC \sim \triangle PQR$. In $\triangle ABC$, $AB = 5.4$ cm, $BC = 4.2$ cm, $AC = 6.0$ cm, and $AB : PQ = 3 : 2$, then construct $\triangle ABC$ and $\triangle PQR$.

Sol: $\triangle ABC \sim \triangle PQR$[Given]

We know that corresponding sides of triangle which are similar are in proportion.

$$\therefore \frac{AB}{PQ} = \frac{BC}{QR} = \frac{AC}{PR} = \frac{3}{2}$$

$$\Rightarrow \frac{AB}{PQ} = \frac{3}{2}$$

$$\text{Also, } \frac{BC}{QR} = \frac{3}{2}$$

$$\text{Also, } \frac{AC}{PR} = \frac{3}{2}$$

$$\Rightarrow \frac{5.4}{PQ} = \frac{3}{2}$$

$$\Rightarrow \frac{4.2}{QR} = \frac{3}{2}$$

$$\Rightarrow \frac{6}{PR} = \frac{3}{2}$$

$$\Rightarrow PQ = \frac{5.4 \times 2}{3} = 3.6 \text{ cm}$$

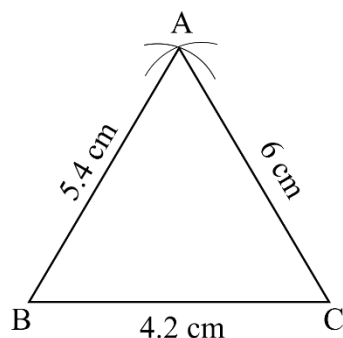
$$\Rightarrow QR = \frac{4.2 \times 2}{3}$$

$$\Rightarrow PR = \frac{6 \times 2}{3} = 4 \text{ cm}$$

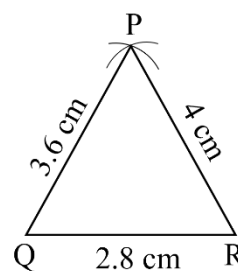
$$\Rightarrow QR = 2.8 \text{ cm}$$

Now, draw angle $\triangle ABC$ with sides $AB = 5.4$ cm, $BC = 4.2$ cm and $AC = 6$ cm.

Also draw triangle $\triangle PQR$ with sides $PQ = 3.6$ cm, $QR = 2.8$ cm and $PR = 4$ cm.



[1]



[1]

(iv) Show that: $\frac{\tan A}{(1 + \tan^2 A)^2} + \frac{\cot A}{(1 + \cot^2 A)^2} = \sin A \times \cos A$

Sol: $\frac{\tan A}{(1 + \tan^2 A)^2} + \frac{\cot A}{(1 + \cot^2 A)^2}$

$$= \frac{\tan A}{(\sec^2 A)^2} + \frac{\cot A}{(\operatorname{cosec}^2 A)^2} \quad [\because 1 + \tan^2 A = \sec^2 A \text{ and } 1 + \cot^2 A = \operatorname{cosec}^2 A]$$

$$= \frac{\sin A}{\cos A} \times \cos^4 A + \frac{\cos A}{\sin A} \times \sin^4 A$$

$$= \sin A \cos^3 A + \cos A \sin^3 A$$

$$= \sin A \cos A$$

Hence proved.

4.

(i) $\square ABCD$ is parallelogram. Point P is the midpoint of side CD. Seg BP intersects diagonal AC at point X, then prove that: $3AX = 2AC$

Sol: From the figure, in $\triangle ABX$ and $\triangle CPX$

As, $AB \parallel CD$

$\angle BAX = \angle PCX$ [Alternate angle]

$\angle BXA = \angle PXC$ [Vertically opposite angles]

$\therefore \triangle ABX \sim \triangle CPX$ [By AA similarity theorem]

We know that,

Similar triangles have comparable side ratios that are similar to or equal.

$$\therefore \frac{AX}{CX} = \frac{AB}{CP}$$

But $CD = AB$ and P is mid – point of CD.

$$\therefore AB = 2CP$$

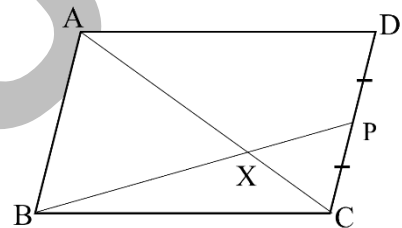
$$\Rightarrow \frac{AX}{AC - AX} = \frac{2CP}{CP} = 2$$

$$\Rightarrow AX = 2(AC - AX)$$

$$\Rightarrow AX = 2AC - 2AX$$

$$\Rightarrow AX + 2AX = 2AC$$

$$\Rightarrow 3AX = 2AC$$



Hence proved.

(ii) In the given figure, seg AB and seg AD are tangent segments drawn to a circle with centre C from exterior point A, then prove that:

$$\angle A = \frac{1}{2} [m(\text{arc BYD}) - m(\text{arc BXD})]$$

Sol: Proof: From figure

Seg AB \perp seg BC and seg AD \perp seg CD

[By tangent theorem]

$$\therefore \angle ABC = \angle ADC = 90^\circ$$

In \square ABCD,

$$\angle A + \angle B + \angle C + \angle D = 360^\circ \dots\dots [\text{Angle of the square}]$$

$$\therefore \angle A + 90^\circ + \angle C + 90^\circ = 360^\circ$$

$$\therefore \angle A + \angle C = 360^\circ - 180^\circ$$

$$\therefore \angle A + \angle C = 180^\circ$$

$$\therefore \angle A + m(\text{arc BXD}) = 180^\circ \text{ [Central angle]} \dots\dots (i)$$

$$\text{Now, } m(\text{arc BXD}) + m(\text{arc BYD}) = 360^\circ$$

$$\dots\dots [\text{Two arcs contribute a complete circle}] \dots\dots (ii)$$

Now, multiply equation (i) by 2 on both sides

$$2[\angle A + m(\text{arc BXD})] = 2 \times 180^\circ$$

$$\Rightarrow 2\angle A + 2 \times m(\text{arc BXD}) = 360^\circ$$

$$\Rightarrow 2\angle A = 360^\circ - 2 \times m(\text{arc BXD})$$

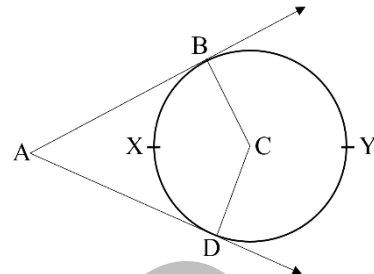
$$\Rightarrow 2\angle A = m(\text{arc BXD}) + m(\text{arc BYD}) - 2m(\text{arc BXD})$$

$$\Rightarrow 2\angle A = m(\text{arc BYD}) - m(\text{arc BXD})$$

..... [From (ii)]

$$\Rightarrow \angle A = \frac{1}{2} [m(\text{arc BYD}) - m(\text{arc BXD})]$$

Hence proved.



(iii) Find the co-ordinates of centroid of a triangle if points D (-7, 6), E (8, 5), and F(2, -2) are the mid - points of the sides of the that triangle.

Sol: Suppose A (x_1, y_1), B (x_2, y_2) and C(x_3, y_3) are the vertices of triangle. D (-7, 6), E (8, 5) and F (2, -2) are the midpoints of sides BC, AC and AB respectively. Let G be the centroid

of $\triangle ABC$. D is the midpoint of seg BC.

By midpoint formula,

$$\text{Co-ordinates of D} = \left(\frac{x_2 + x_3}{2}, \frac{y_2 + y_3}{2} \right)$$

$$\Rightarrow (-7, 6) = \left(\frac{x_2 + x_3}{2}, \frac{y_2 + y_3}{2} \right)$$

$$\Rightarrow \frac{x_2 + x_3}{2} = -7, \frac{y_2 + y_3}{2} = 6$$

$$\Rightarrow x_2 + x_3 = -14 \dots (i), y_2 + y_3 = 12 \dots (ii),$$

E is the midpoint of seg AC;

$$\text{Co-ordinates of E} = \left(\frac{x_1 + x_3}{2}, \frac{y_1 + y_3}{2} \right)$$

$$\Rightarrow (8, 5) = \left(\frac{x_1 + x_3}{2}, \frac{y_1 + y_3}{2} \right)$$

$$\Rightarrow \frac{x_1 + x_3}{2} = 16, \frac{y_1 + y_3}{2} = 10$$

$$\Rightarrow x_1 + x_3 = 16 \dots (iii), y_1 + y_3 = 10 \dots (iv)$$

Similarly, as F is the midpoint of seg AB;

$$\text{Co-ordinates of F} = \left(\frac{x_1 + x_2}{2}, \frac{y_1 + y_2}{2} \right)$$

$$\Rightarrow x_1 + x_2 = 4 \dots (v), y_1 + y_2 = -4 \dots (vi)$$

Adding (i), (iii) and (v),

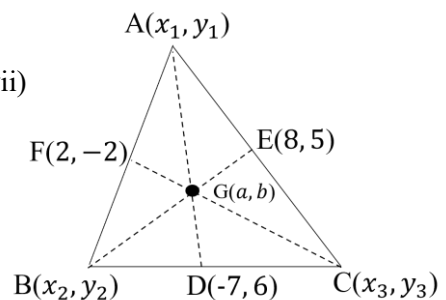
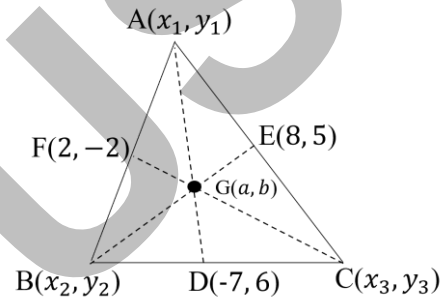
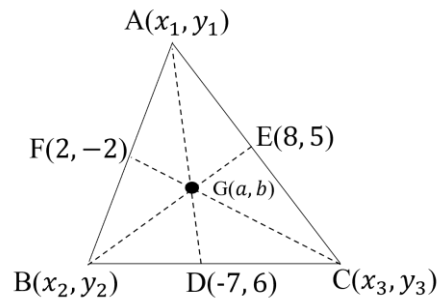
$$x_2 + x_3 + x_1 + x_3 + x_1 + x_2 = -14 + 16 + 4$$

$$\Rightarrow 2x_1 + 2x_2 + 2x_3 = 6 \Rightarrow x_1 + x_2 + x_3 = 3 \dots (vii)$$

Adding (ii), (iv) and (vi),

$$y_2 + y_3 + y_1 + y_3 + y_1 + y_2 = 12 + 10 - 4$$

$$\Rightarrow 2y_1 + 2y_2 + 2y_3 = 18 \Rightarrow y_1 + y_2 + y_3 = 9 \dots (viii)$$



G is the centroid of ΔABC . By centroid formula,

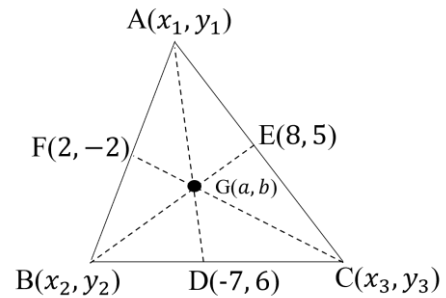
$$\text{Co-ordinates of } G = \left(\frac{x_1 + x_2 + x_3}{3}, \frac{y_1 + y_2 + y_3}{3} \right)$$

$$= \left(\frac{3}{3}, \frac{9}{3} \right) \dots \dots \text{From (vii) and (viii)}$$

$$= (1, 3)$$

\therefore The Co-ordinates of the centroid of the triangle are

(1, 3)



5.

(i) If a and b are natural numbers and $a > b$ If $(a^2 + b^2)$, $(a^2 - b^2)$ and $2ab$ are the sides of the triangle, then prove that the triangle is right angled. Find out two pythagorean triplets by taking suitable values of a and b .

Sol: $a^2 + b^2$, $a^2 - b^2$, $2ab$ are sides of triangle.

By Pythagoras' theorem,

$$(a^2 + b^2)^2 = (a^2 - b^2)^2 + (2ab)^2$$

$$a^4 + b^4 + 2a^2b^2 = a^4 + b^4 - 2a^2b^2 + 4a^2b^2$$

$$a^4 + b^4 + 2a^2b^2 = a^4 + b^4 + 2a^2b^2$$

AS L.H.S. = R.H.S.

\therefore Triangle is a right – angle triangle as it follows Pythagorean triplets As $a > b$
..... [Given]

Let $a = 4$, $b = 3$

$$a^2 + b^2 = 4^2 + 3^2 = 16 + 9 = 25$$

$$a^2 - b^2 = 16 - 9 = 7$$

$$2ab = 2 \times 4 \times 3 = 24$$

$\therefore (25, 7, 24)$ is a Pythagorean triplet.

Let $a = 2$, $b = 1$

$$a^2 + b^2 = 2^2 + 1^2 = 4 + 1 = 5$$

$\therefore (5, 3, 4)$ is another Pythagorean triplet.

(ii). Construct two concentric circles with centre O with radii 3 cm and 5 cm. construct tangent to a smaller circle from any point A on the larger circle. Measure and write the length of tangent segment. Calculate the length of tangent segment using Pythagoras theorem.

Sol: Following are the steps to draw tangents on the given circle:

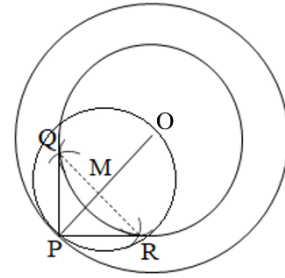
Step 1: Draw a circle of 3 cm radius with centre O on the given plane.

Step 2: Draw a circle of 5 cm radius, taking O as its centre.
Locate a point P on this circle and join OP.

Step 3: Bisect OP. Let M be the midpoint of PO.

Step 4: Taking M as its centre and MO as its radius, draw a circle. Let it intersect the given circle at points Q and R.

Step 5: Join PQ and PR. PQ and PR are the required tangents.



It can be observed that PQ and PR are of length 4 cm each.

Since PQ is a tangent,

$$\therefore \angle PQO = 90^\circ \text{ and } PO = 5 \text{ cm and } QO = 3 \text{ cm}$$

Applying Pythagoras theorem in ΔPQO , we obtain

$$PQ^2 + QO^2 = PO^2$$

$$\Rightarrow PQ^2 + (3)^2 = (5)^2$$

$$\Rightarrow PQ^2 + 9 = 25$$

$$\Rightarrow PQ^2 = 25 - 9 = 16$$

$$\Rightarrow PQ = 4 \text{ cm}$$