



# Artificial Intelligence

## Chapter 3: Solving Problems by Searching



# Overview

- Problem-Solving Agents
- Example Problems
- Searching for Solutions
- Uninformed Search Strategies
- Avoiding Repeated States



# Problem-Solving Agents

- Reach goals through sequences of actions
- Formulate the goal(s) and the problem
  - Abstraction: Should be easier than original problem
- Search for a sequence of actions to reach a goal state
  - A solution is a sequence of actions from initial state to a goal state
- Execute the sequence of actions

# Problem-Solving Agents

```
function SIMPLE-PROBLEM-SOLVING-AGENT(percept) returns an action
  static: seq, an action sequence, initially empty
           state, some description of the current world state
           goal, a goal, initially null
           problem, a problem formulation

  state ← UPDATE-STATE(state, percept)
  if seq is empty then
    goal ← FORMULATE-GOAL(state)
    problem ← FORMULATE-PROBLEM(state, goal)
    seq ← SEARCH(problem)
  action ← RECOMMENDATION(seq, state)
  seq ← REMAINDER(seq, state)
  return action
```

# Formulating the Problem

- Initial state = starting state for the agent
- Given a state and a set of actions, the successor function gives the possible next states
- Initial state + successor function yields the state space
  - State space = the set of all states reachable from the initial state

# Formulating the Problem, Continued

- The goal test determines if a state is a goal state
- A path is a particular sequence of states, connected by particular actions
- The path cost function assigns a cost to each path
  - Optimal solution: a solution with minimal path cost



# Example Problems

- “Toy” Problems: Simple, but useful for theory and analysis
  - Vacuum world, Sliding-block puzzles, N-Queens
- Real Problems: The solutions are more useful, but not as helpful to theory
  - Route-finding, touring, assembly sequencing, Internet searching

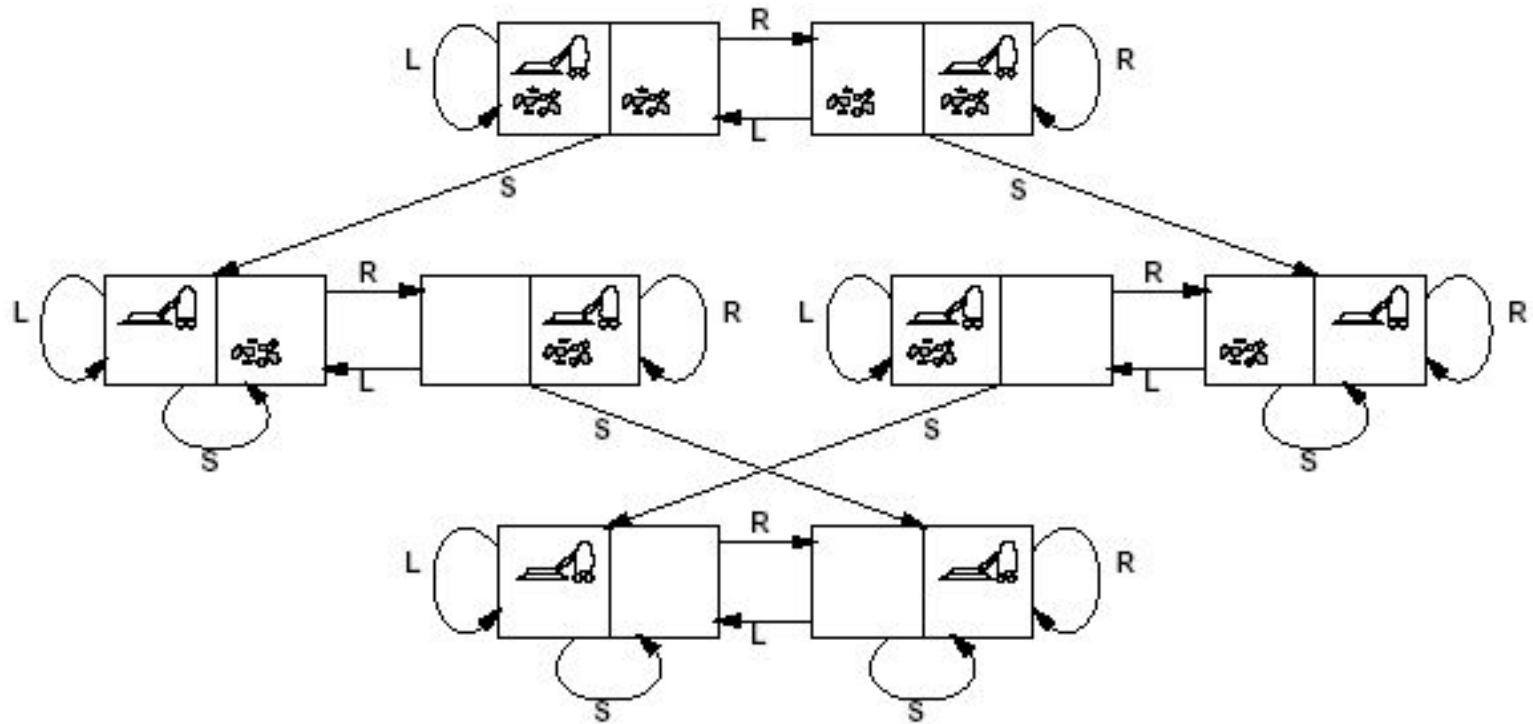


# Example: The Vacuum World

- States: Location, dirt or not
- Initial State: Given by problem
- Goal: No dirt in any location
- Actions: Left, Right, Vacuum, NoOp
- Path Cost: 1 per action (0 for NoOp)



# State Space for the Vacuum World

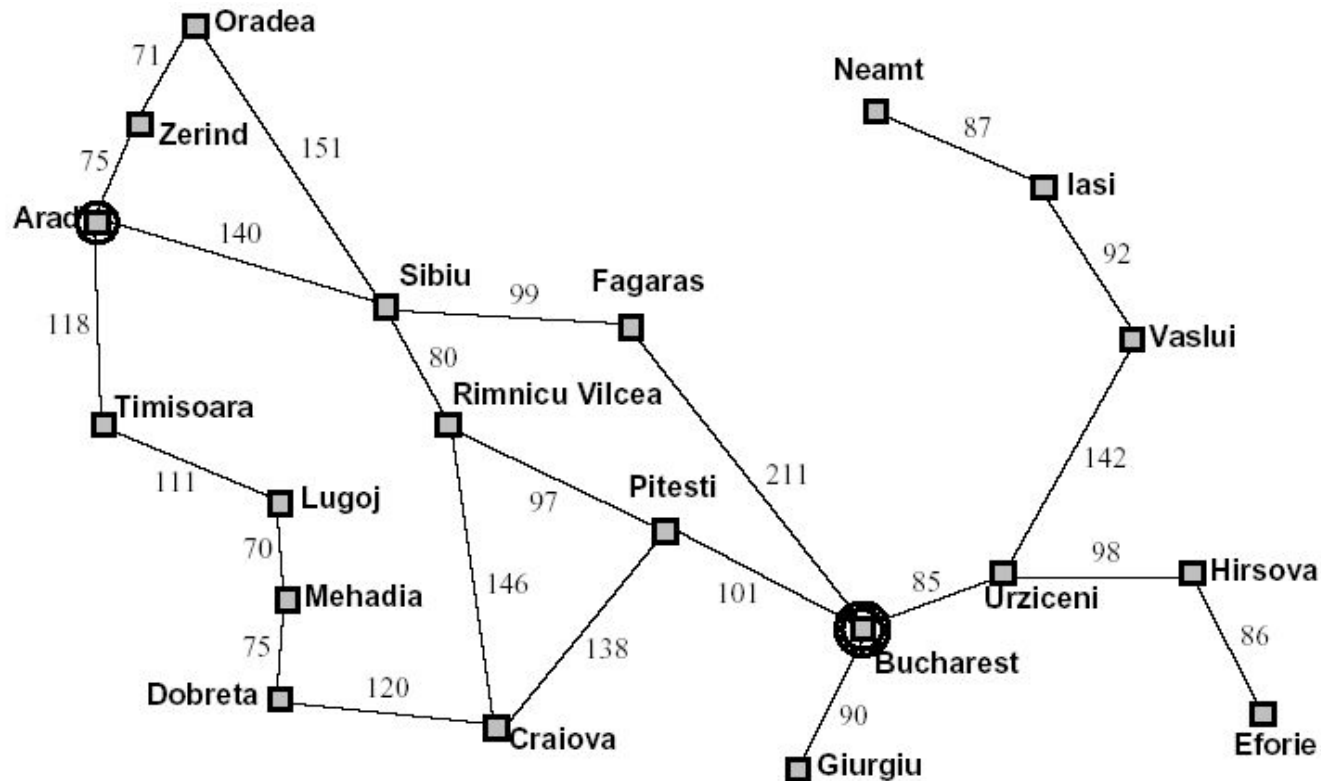




# Example: Driving in Romania

- States: In a city
- Goal: Arrive in Bucharest
- Initial State: in Arad
- Actions: Drive from one city to another

# Example: Driving in Romania





# Searching for Solutions

- State space can be represented as a search tree (or graph) of search nodes
- Each node represents a state
  - Many nodes can represent the same state
- Each arc represents a valid action from one state to another
- A solution is a path from initial to goal nodes

# Search Nodes

- A search node is a data structure that contains
  - The real-world state represented by the node
  - The parent node that generated it
  - The action that lead to it
  - The path cost to get to it (from the initial state)
  - Its depth (the number of steps to reach it from the initial state)

# Searching for Solutions

- The search tree starts with just the root node, which represents the initial state
- First, use the goal test to see if the root is a goal
- If not, expand this node, which generates new nodes
  - Use successor function to apply valid actions. The results of these actions lead to new states, represented by new nodes
- Nodes that are generated, but not yet expanded are in the fringe, (physically a queue data structure)



# Search Strategies

- Given several states, which to consider (i.e., goal test, expand, etc.) first?
  - This is determined by the particular search strategy being used in the algorithm

# General Idea

```
function TREE-SEARCH(problem, fringe) returns a solution, or failure  
  fringe  $\leftarrow$  INSERT(MAKE-NODE(INITIAL-STATE[problem]), fringe)  
  loop do  
    if fringe is empty then return failure  
    node  $\leftarrow$  REMOVE-FRONT(fringe)  
    if GOAL-TEST[problem] applied to STATE(node) succeeds return node  
    fringe  $\leftarrow$  INSERTALL(EXPAND(node, problem), fringe)
```

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```
function EXPAND(node, problem) returns a set of nodes  
  successors  $\leftarrow$  the empty set  
  for each action, result in SUCCESSOR-FN[problem](STATE[node]) do  
    s  $\leftarrow$  a new NODE  
    PARENT-NODE[s]  $\leftarrow$  node; ACTION[s]  $\leftarrow$  action; STATE[s]  $\leftarrow$  result  
    PATH-COST[s]  $\leftarrow$  PATH-COST[node] + STEP-COST(node, action, s)  
    DEPTH[s]  $\leftarrow$  DEPTH[node] + 1  
    add s to successors  
  return successors
```



# Search Algorithm Performance

- Performance measured along four dimensions:
  - **Completeness:** Will the algorithm always return a solution, if one exists?
  - **Optimality:** Will it always return the optimal solution, if one exists?
  - **Time complexity:** How long does it take?
  - **Space complexity:** How much memory does it take?
- Branching factor ( $b$ ), depth of shallowest goal node ( $d$ ), maximum length of any path in the state space ( $m$ )
- Search cost vs. total cost
  - Tradeoffs



# Uninformed Search

- Uninformed (or “blind”) search algorithms can only generate successor nodes and do the goal test
  - No other “problem insight”, common sense, etc. is used in finding a solution
- Search strategies differ in how they choose which node to check/expand next



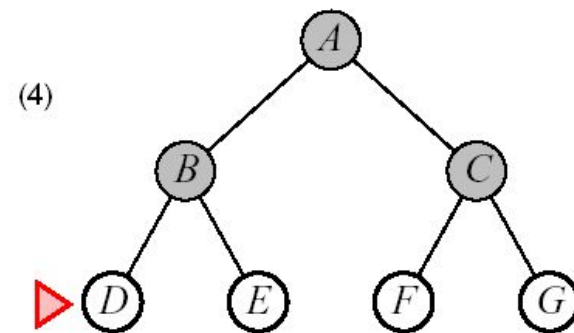
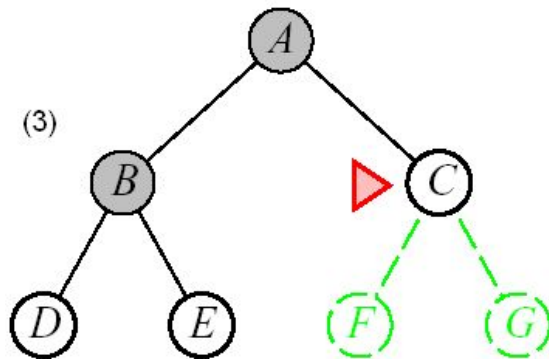
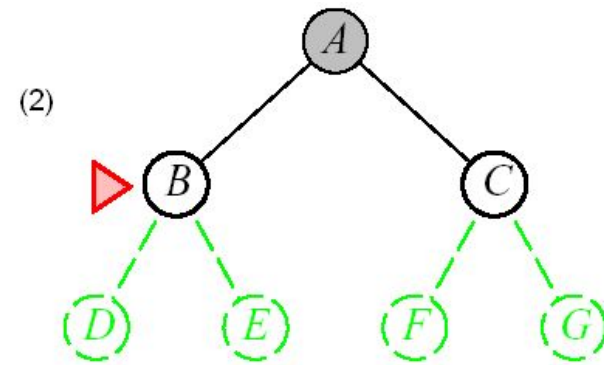
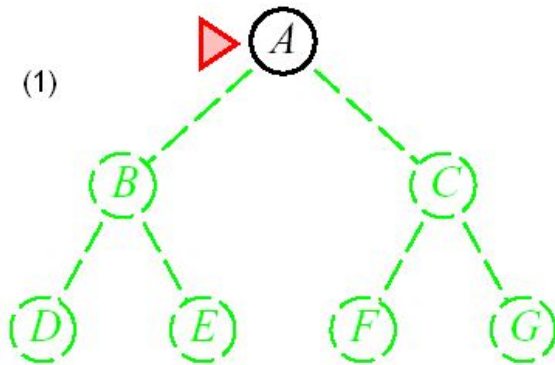
# Some Uninformed Search Algorithms

- Breadth-first search
- Uniform-cost search
- Depth-first search
- Depth-limited search
- Iterative deepening depth-first search
- Bidirectional search

# Breadth-First Search (BFS)

- BFS considers all nodes at a given depth before moving on to the next depth
  - Always expands the shallowest node first
- Implemented in TREE-SEARCH using a first-in first-out (FIFO) queue
  - State to consider is pulled from front of queue
  - Generated states are added at the back of the queue
  - Check all successors of the root node. Then check all of *their* successors, etc.

# Breadth-First Search (BFS)





# Performance of Breadth-First Search

## ■ Good news

- It is complete
- It is optimal, as long as the path cost function is non-decreasing (e.g., if all actions have the same cost)

## ■ Bad news

- Time and (especially) space complexity can be prohibitively high
  - Has to keep all nodes in memory: queue gets large
  - If  $d$  is large, takes a long time to reach a goal

# Uniform-Cost Search (UCS)

- UCS expands the closest (in terms of path cost) node first
  - If all step costs are equal, this is equivalent to BFS
  - “Closest” means total path cost so far, which is not necessarily the same as the number of steps
- Implemented in TREE-SEARCH using a first-in first-out (FIFO) queue
  - State to consider is pulled from front of queue
  - Generated states are added to the queue, then contents of queue are ordered according to path cost



# Performance of Uniform-Cost Search

- Good news

- It is complete and optimal, as long as step costs are positive

- Bad news

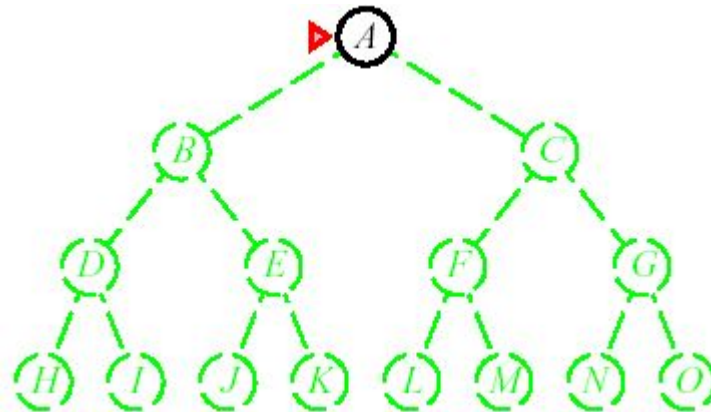
- Time and (especially) space complexity can be prohibitively high, just as with BFS
- Can get stuck in an infinite loop: zero-cost steps, repeated states



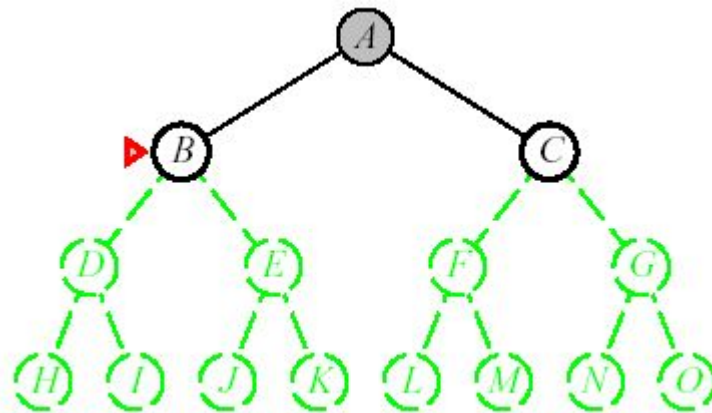
# Depth-First Search (DFS)

- DFS expands the deepest node first
- Implemented in TREE-SEARCH using a last-in first-out (LIFO) queue
  - State to consider is pulled from front of queue
  - Generated states are added at the front of the queue
  - Check first successor of the root node. Then check *its* successor, etc.
  - If a non-goal node is reached with no successors, back up just until an unexpanded node is reached, then continue.

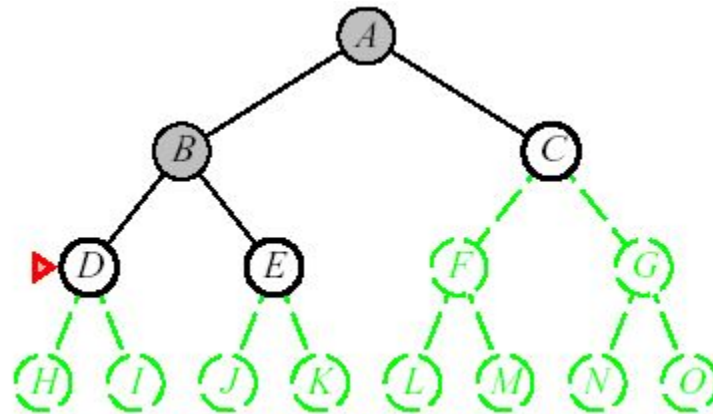
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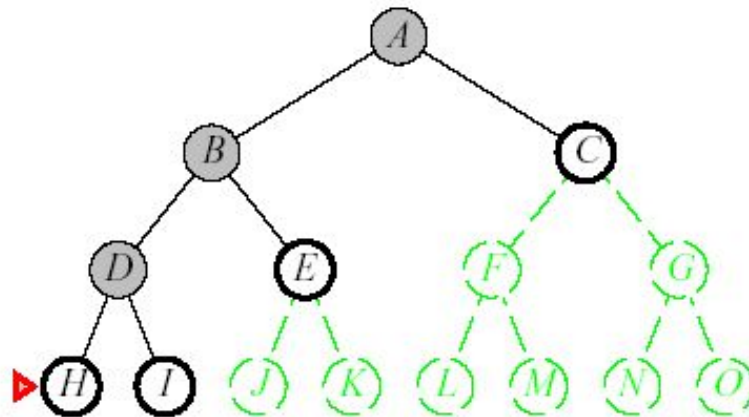
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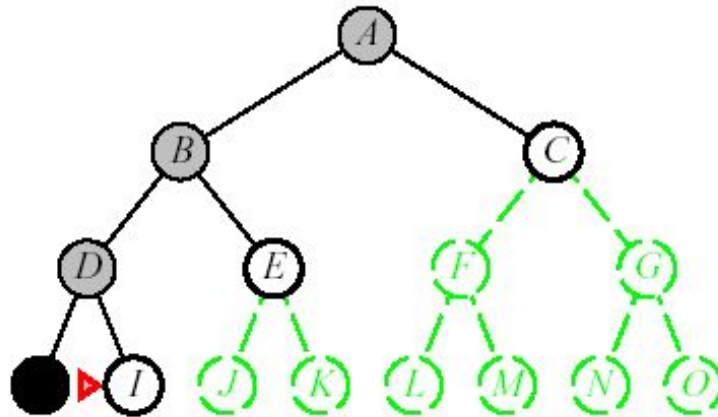
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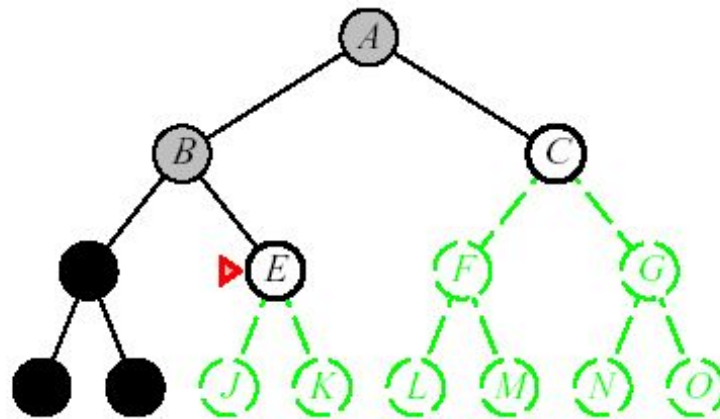
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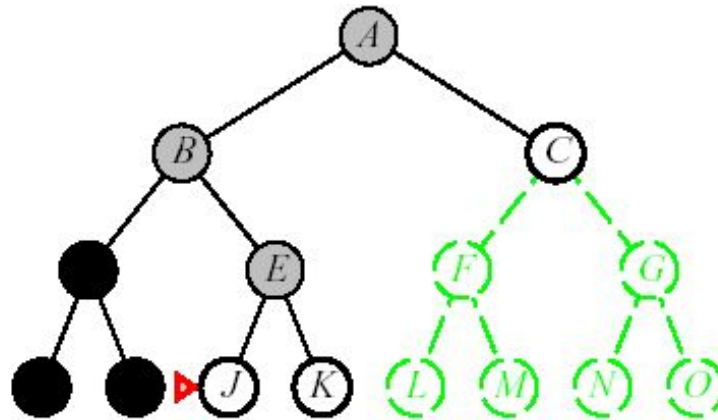
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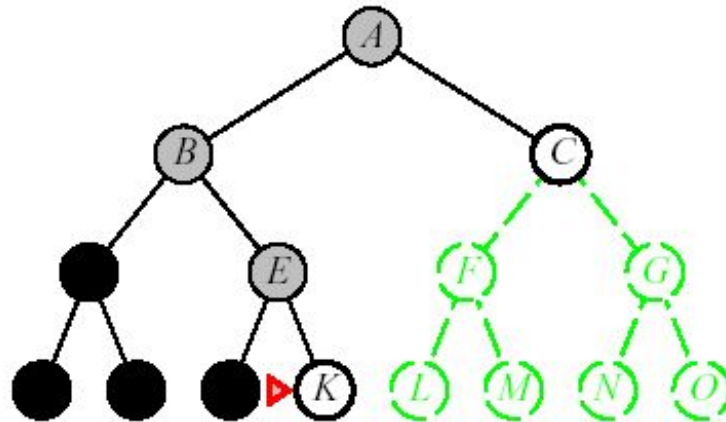


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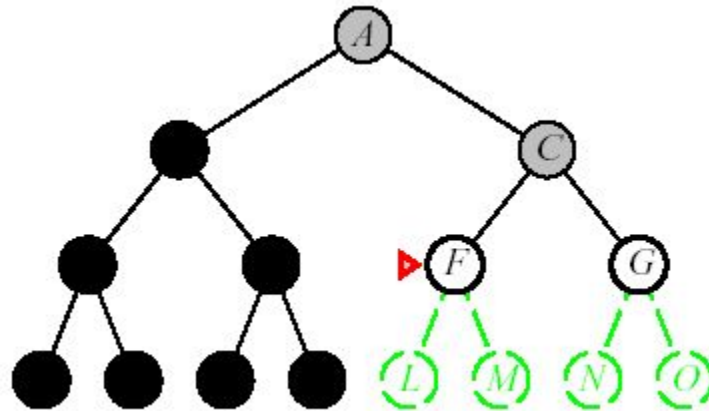




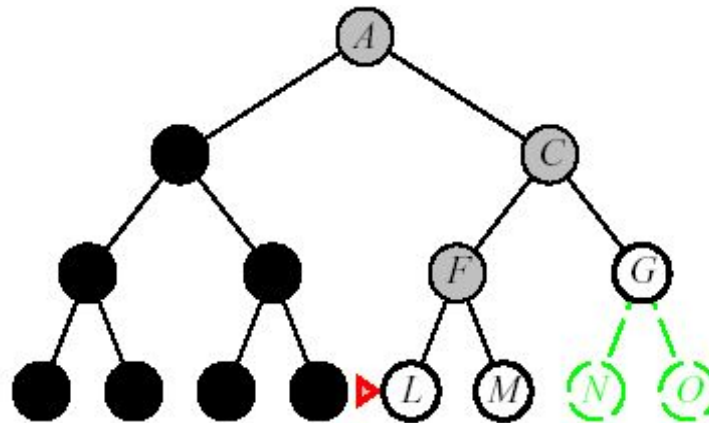
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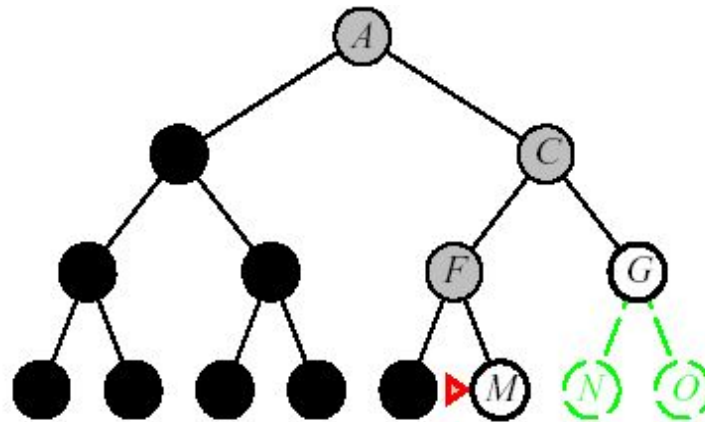
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# Depth-First Search (DFS)





# Performance of Depth-First Search

- Good news
  - Space complexity is small
- Bad news
  - Neither complete nor optimal
  - Time complexity can be large
  - Can get stuck in an infinite loop: repeated states

# Depth-Limited Search (DLS)

- DLS is DFS with a depth limit,  $L$ 
  - Nodes at depth limit are treated as leaves
  - Depth limits can come from domain knowledge
    - E.g.,  $L = \text{diameter}$  of state space
- Implemented same as DFS, but with depth limit
- Performance
  - Incomplete if  $L < d$
  - Nonoptimal if  $L > d$
  - Time and space complexity  $\leq$  DFS

# Iterative Deepening Depth-First Search (IDDFS)

- IDDFS performs DLS one or more times, each time with a larger depth limit,  $l$ 
  - Start with  $l = 0$ , perform DLS
  - If goal not found, repeat DLS with  $l = 1$ , and so on
- Performance
  - Combines benefits of BFS (completeness, optimality) with benefits of DFS (relatively low space complexity)
- Not as inefficient as it looks
  - Repeating low-depth levels: these levels do not have many nodes

# Bidirectional Search

- Perform two searches simultaneously
  - One search from the initial state to a goal state
  - Another search from a goal state to the initial state
  - Stop searching when either search reaches a state that is in the fringe of the other state (i.e., when they “meet in the middle”)
- Performance
  - Complete and optimal if both searches are BFS
  - Not easy to generate predecessors for some goals (e.g., checkmate)



# Avoiding Repeated States

- Reaching a node with a repeated state means that there are at least two paths to the same state
  - Can lead to infinite loops
- Keep a list of all nodes expanded so far (called the closed list; the fringe is the open list)
  - If a generated node's state matches a state in the closed list, discard it rather than adding it to the fringe