$\Rightarrow -1.56 - 2.260 < \mu_D < -1.56 + 2.260$

$\Rightarrow -3.820 < \mu_D < 0.70$

→ All the previous techniques are parametric (quantitative)

- Chi-Square (χ^2) test (Qualitative Techniques)

1) Test for goodness of fit : (if taken from population)
2) Test for independence (if data already exists)
3) Yate's correction χ^2 - test : (only for 2x2 contigency table) (i.e. not more than that)

→ Test for goodness of fit

observed values: O_i } $\forall i = 1, 2, \ldots, n$

expected values: E_i

H_0 : There is no association b/w observed & expected value i.e. $O_i = E_i$

H_A or H_1 : There is an association b/w observed & expected value i.e. $O_i \neq E_i$

LOS : $\alpha = 1\%, 5\%$ or 10%.

Test statistic :

Case 1: Let $x_1, x_2, \ldots x_n$ be a random sample drawn from any population then ~~χ^2~~ chi-square test ~~square~~ statistic is denoted by χ^2 & is defined as

$$\chi^2 = \sum_{i=1}^{n} \frac{(O_i - E_i)^2}{E_i} \sim \chi^2[(R-1)(C-1)]$$

d.f.

Case 2 : If the data 2x2 contigency table then

Total of row / column

$$\chi^2 = \frac{[|ad - bc|]^2 * n}{[(a+b)(c+d)(a+c)(b+d)]}$$

$\sim \chi^2(1)$ df: (here, d.f = 1)

since $R-1 = 2-1 = 1$ & $C-1 = 2-1 = 1$

Decision Rule :

If $\chi^2_{cal} < \chi^2_{tab}$ with $[(R-1)(C-1)]$ d.f. (#rows, #coln)

then accept H_0 otherwise reject H_0.

Conclusion : Summarise the result as per the decision rule.

Ques: A reader read that fireman related deaths for people 1 to 18 years distributed as follows.

74% works accidental

16% works home suicides

10% works other reason. In this district, there were 68 accidental, 27 home suicide, and 5 were other reason during the last year. At $\alpha = 10\%$ L.O.S. claim that %ages are equal.

Sol^n: $O_1 = 68$ $O_2 = 27$ & $O_3 = 5$

Expected values :

$E_1 = 74$ $E_2 = 16$ $E_3 = 10$

and df : $(R-1)(C-1) = (2-1)(3-1) = 2$

Scanned by CamScanner

$\therefore$ use $\chi^2 = \sum_{i=1}^{n} \frac{(O_i - E_i)^2}{E_i} \sim \chi^2(2)$ d.f.

O_i	E_i	$O_i - E_i$	$(O_i - E_i)^2$	$(O_i - E_i)^2/E_i$
68	74	−6	36	$36/74 = 0.486$
27	16	11	121	$121/16 = 7.562$
5	10	−5	25	$25/10 = 2.5$

$$\chi^2 = \sum_{i=1}^{3} \frac{(O_i - E_i)^2}{E_i} = \underline{0.486 + 7.562 + 2.5}$$

$$= 10.548$$

Since d.f. = 2 $\therefore$ $\chi^2(2)$ for 10% l.o.s. = 4.604 (using table)

$\because$ $\underset{(10.548)}{\chi^2_{cal}} > \underset{(4.604)}{\chi^2_{tab}}$ $\Rightarrow$ reject H_0.

Ques) A research to determine whether there is a relationship b/w gender of an individual and the amount of alcohol consumed. A sample of 68 people is selected & a following data is obtained. At $\alpha = 5\%$ l.o.s., can the researcher conclude that the alcohol consumption is related to gender.

Sol

Gender	Alcohol			Total
	low	moderate	high	
Male	10 O_{11}	9 O_{12}	8 O_{13}	27
Female	13 O_{21}	16 O_{22}	12 O_{23}	41
Total	23	24	20	68 = N

Scanned by CamScanner

$$d.f. = (R-1)(C-1) = (2-1)(3-1) = 2.$$

$$E_{11} = \frac{R_{11} C_{11}}{N} = \frac{27 * 23}{68} = 9.13$$

$$E_{12} = \frac{R_{11} C_{12}}{N} = \frac{27 * 25}{68} = 9.92$$

$\therefore O_{ij}$	E_{ij}	$(O_{ij} - E_{ij})$	$(O_{ij} - E_{ij})^2$	$\frac{(O_{ij} - E_{ij})^2}{E_{ij}}$
10	$\frac{27 \times 23}{68} = 9.13$	0.87	0.7569	0.082
9	$\frac{27 \times 25}{68} = 9.92$	−0.92	0.8464	0.085
8	$\frac{27 \times 20}{68} = 7.94$	0.06	0.0036	0.0004
13	$\frac{41 \times 23}{68} = 13.86$	0.86	0.7396	0.0533
16	$\frac{41 \times 25}{68} = 15.07$	0.93	0.8649	0.0573
12	$\frac{41 \times 20}{68} = 12.05$	−0.05	0.0025	0.0002

$$\therefore \chi^2 = \sum_{i=1}^{6} \left(\frac{(O_{ij} - E_{ij})^2}{E_{ij}} \right)$$

$$= 0.2782$$

Since $d.f. = 2$ and $\alpha = 5\% = 0.05$

$\therefore \chi^2_{tab} = 5.99$

$\therefore \underset{(0.27)}{\chi^2_{cal}} < 5.99$ i.e. $\chi^2_{cal} < \chi^2_{tab} \Rightarrow$ accept H_0.

Ques) Data on smoking habits and presence of lung cancer is given below.

Cancer	Smoking: Yes	Smoking: No	Total	
Yes	a 20	b 10	30	$R_{11} = a+b$
No	c 30	d 140	170	$R_{12} = c+d$
Total	C_{11} 50	C_{12} 150	200 = N (= $R_1 + R_2$ = $C_1 + C_2$)	

Since d.f. = 1, use formula $\chi^2 = \dfrac{[|ad-bc|]*N}{[(a+b)(c+d)(a+c)(b+d)]} \sim \chi^2(1)$ d.f.

$$\chi^2 = \frac{(|ad-bc|)*N}{R_1 * R_2 * C_1 * C_2}$$

$$= \frac{(20 \times 140 - 10 \times 30) \times 200}{38,250,000}$$

$$= \frac{(2800 - 300) \times \cancel{200}}{38250\cancel{000}}$$

$$= \frac{25\cancel{00} \times 2}{3825\cancel{00}} = \frac{50}{3825}$$

$$= 0.013$$

Since χ^2_{tab} at $\alpha = 5\%$ (1-0.5 default value) & d.f. = 1

$\chi^2_{tab} = 3.84$

Scanned by CamScanner

$\chi^2_{cal} < \chi^2_{tab}$ ⇒ accept H_0

pt) Chi square test for independence

ues) A sociologist wishes to see whether the no. of years of college a person has completed is related to his/her place of residence. A sample of 88 ppl is selected and classified as shown.

location \ college	No college	4-Year college	Advanced Degree
Urban	15	12	8
Suburban	8	15	9
Rural	6	8	7

At $\alpha = 5\%$ l.o.s, can the sociologist conclude that a person's location is dependent on the no. of years of college.

Soln H_0 : There is no association between no. of years of college and his/her residence.

H_a : There is an association b/w no. of years of college and his/her residence.

$\alpha = 5\%$

$$\chi^2 = \sum_{i=1}^{n} \frac{(O_i - E_i)^2}{E_i} \sim \chi^2[(R-1)(C-1)] \text{ d.f.}$$

Loc^n (Residence) / College	No college	4-Year clg	Ado. degree	Total
Urban	15	12	8	35
Suburban	8	15	9	32
Rural	6	8	7	21
Total	29	35	24	88 = N

Scanned by CamScanner

O_i	E_i	$O_i - E_i$	$(O_i - E_i)^2$	$(O_i - E_i)^2 / E_i$
15	$35 \times 29 / 88 = 11.53$	3.47	12.04	1.044
12	$35 \times 35 / 88 = 13.92$	−1.97	3.686	0.264
8	$35 \times 24 / 88 = 9.54$	−1.54	2.371	0.248
8	$32 \times 29 / 88 = 10.54$	−2.54	6.451	0.612
15	$32 \times 35 / 88 = 12.72$	2.28	5.198	0.408
9	$32 \times 24 / 88 = 8.72$	0.28	0.078	0.008
6	$21 \times 29 / 88 = 6.92$	−0.92	0.846	0.122
8	$21 \times 35 / 88 = 8.35$	−0.32	0.122	0.0146
7	$21 \times 24 / 88 = 5.72$	1.28	1.638	0.286
				3.0066

$$\chi^2 = \sum_{i=1}^{9} \frac{(O_i - E_i)^2}{E_i}$$

$$= 3.0066$$

$d.f. = (3-1)(3-1) = 4$ and $\alpha = 0.05$

$\therefore \chi^2_{tab} = 9.49$

$\chi^2_{cal} < \chi^2_{tab} \Rightarrow$ accept H_0.

★ Chi-Square (Yate's correction) Test for 2x2 contigency table. $(O_i \leq 5)$

a	b	R_1
c	d	R_2
C_1	C_2	2x2

$$\chi^2 = \frac{\left[\,|ad - bc| - N/2\,\right]^2 * N}{(a+b)(c+d)(a+c)(b+d)} \sim \chi^2(1) \text{ d.f.}$$

let $x_1, x_2, \ldots, x_n$ be a random sample drawn from any

population. If ~~assumed~~ observed frequency is less than or equal to 5 (≤ 5) then Yate's correction chi-square test statistic is denoted by χ^2_y and is defined as

$$\chi^2_y = \frac{\left[\,|ad - bc| - N/2\,\right]^2 * N}{\underbrace{(a+b)}_{R_1} * \underbrace{(c+d)}_{R_2} * \underbrace{(a+c)}_{C_1} * \underbrace{(b+d)}_{C_2}} \sim \chi^2(1) \text{ d.f.}$$

- Null hypothesis : $H_0 : O_i - E_i = 0 \Rightarrow O_i - E_i$ i.e. no association between observed and expected frequencies
- Alternate hypothesis : $H_a : O_i - E_i \neq 0$ i.e. there is an association b/w expected & observed frequences.

- Decision Rule (critical value) : $\alpha = 5\%$ L.O.S - If $\chi^2_{cal} < \chi^2_{tab}$ with '1' d.f. at 5% L.O.S, then accept H_0, otherwise reject H_0.
- Conclusion : Summarise the result based on the decision.

Ques) Data on smoking habit and presence of lung cancer in the study is given below. ($\chi^2_{tab} = 3.87$)

Cancer	Yes	No	Total
Yes	6	1	$7 = R_1$
No	4	19	$23 = R_2$
Total	$10 = C_1$	$20 = C_2$	$30 = N$

dimension: 2x2

$\chi^2_{tab} = 3.87$ (given)

Soln.

$$\chi^2_y = \frac{\left[(6\times19 - 1\times4) - 30/2\right]^2 \times 30}{7\times23\times10\times20}$$

$$= \frac{\left[(114-4) - 15\right]^2 \times 3\cancel{0}}{3220\cancel{0}}$$

$$= \frac{(110-15)^2 \times 3}{3220}$$

$$= \frac{\cancel{105}\ 95\times95\times3}{3220}$$

$$= \frac{27075}{3220} = 8.408$$

since $\chi^2_{cal} < \chi^2_{tab}$ ⇒ accept H_0.

19 Sept). Questions.

1. Scholar wishes to test the claim that the avg. cost of tuition fee at four year public clg. is greater than 5700. student select a random sample of 36 four year public college and find the mean to be 5950. The popn s.d. is 659. Is there evidence to support the claim at $\alpha = 5\%$.

Ans $n = 36$, $\bar{x} = 5950$, $\mu = 5700$, $\sigma = 659$ at $\alpha = 5\%$.

$H_0 : \mu = \mu_0$

$H_1 : \mu > \mu_0$

$$z = \frac{\bar{x} - \mu}{\sigma/\sqrt{n}} = \frac{5950 - 5700}{659/\sqrt{36}} = \frac{250\times6}{659}$$

Scanned by CamScanner

$$= \frac{1500}{659} = 2.27$$

at 5% L.O.S., $Z_{tab} = 1.96$

since $Z_{cal} > Z_{tab}$ ⇒ reject H_0,

hence, $\mu > \mu_0$ is true.

2. A telephone company representative estimate that 40% of its customer have a call waiting. Selected a sample of 100 customers and found that 37% had call waiting. At $\alpha = 1\%$ L.O.S. is there enough evidence to reject the claim,

Ans. $\hat{p} = 40\% = \frac{40}{100} = 0.4$

and $p = 37\% = 0.37$ ⇒ $q = 1 - p = 1 - 0.37 = 0.63$

$H_0 : \hat{p} = p$

$H_1 : \hat{p} \neq p$

$\hat{p} > p$

$\hat{p} < p$

population proportion

sample proportion

$$\therefore Z = \frac{\hat{p} - p}{\sqrt{pq/n}} = \frac{0.40 - 0.37}{\sqrt{\frac{0.37 \times 0.63}{100}}} = \frac{0.03}{\sqrt{\frac{0.2331}{100}}}$$

$$= \frac{0.3}{0.482} = 0.622$$

at 1% L.O.S, $Z_{tab} = 2.56$

$\because Z_{cal} < Z_{tab} \Rightarrow$ accept H_0, (i.e. not enough evidence)

A survey found that the average hotel room rate for New ~~York~~ Delhi is ₹882 and the average room rate in Bengaluru is ₹806. Assume that the data ~~used~~ were obtained from 2 samples of 50 hotels each and the std. deviation of population are 56.2 and 48.3 respectively. At $\alpha = 5\%$, L.O.S., can it be concluded that there is significance diff. b/w rates?

Since $n = 50 > 30$ hence we calculate 'z' value.

Given: $\bar{x}_1 = 882$, $\bar{x}_2 = 806$

$n = n_1 = n_2 = 50$

H_0: there is no significant diff. b/w the rates'

H_1: there is a significant diff. b/w the rates'

$$Z_{cal} = \frac{\bar{x}_1 - \bar{x}_2}{\sqrt{\frac{\sigma_1^2}{n_1} + \frac{\sigma_2^2}{n_2}}} = \frac{882 - 806}{\sqrt{\frac{(56.2)^2}{50} + \frac{(48.3)^2}{50}}}$$

$$= \frac{76}{\sqrt{\frac{3158.44 + 2332.89}{50}}} = \frac{76}{\sqrt{\frac{5491.33}{50}}}$$

$$= 76 / \sqrt{109.82} = \frac{76}{10.47} = 7.25$$

$\because Z_{tab}$ at $\alpha = 5\% = 1.96$

$Z_{cal} > Z_{tab} \Rightarrow$ reject H_0. i.e. there is no significant difference b/w rates'

Scanned by CamScanner

4.

Subject	1	2	3	4	5	6
Before	210	235	208	190	172	244
After	190	170	210	188	173	228

Test at 10% L.O.S and tabvalue 2.015,

Ans

D_i :	20	65	-2	2	-1	16	$\Sigma D_i = 100$
D_i^2 :	400	4225	4	4	1	256	$\Sigma D_i^2 = 10890$

$$\bar{D} = \frac{\Sigma D_i}{n} = \frac{20+65-2+2-1+16}{6} = \frac{100}{6}$$

$$= 16.66$$

Now, H_0 : ~~no. diff~~ $\mu_D = 0$

H_1 : $\mu_D \neq 0$, $\mu_D > 0$, $\mu_D < 0$

$$t_D = \frac{\bar{D} - \mu_D}{S_D/\sqrt{n}}$$

$$t_D = \frac{\bar{D} - 0}{S_D/\sqrt{n}} \quad (\text{since } \mu_D = 0)$$

since $$S_D = \sqrt{\frac{n(\Sigma D_i^2) - (\Sigma D_i)^2}{n(n-1)}}$$

$$= \sqrt{\frac{(\Sigma D_i^2) - \frac{(\Sigma D_i)^2}{n}}{n-1}}$$

$$\therefore \frac{S_D}{\sqrt{n}} = \sqrt{\frac{(\Sigma D_i^2) - (\Sigma D_i)^2/n}{n(n-1)}}$$

Scanned by CamScanner

$$\therefore \frac{S_D}{\sqrt{n}} = \sqrt{\frac{10890 - (100 \times 100)/6}{6 \times 5}} = \sqrt{\frac{9223.34}{30}}$$

$$= \sqrt{307.44} = 17.53$$

$$\therefore t_D = \frac{16.66}{17.53} = 0.95$$

and t_{tab} at d.f. $= n-1 = 6-1 = 5$ at $\alpha = 10\%$.

$t_{tab} = 2.015$

$\therefore$ $t_{Cal} < t_{D\,tab}$ $\Rightarrow$ accept H_0.

(5) Indian Airports: 36.8, 73.5, 72.5, 61.2, 60.5, 40.1

Other Airports: 60.7, 51.2, 42.7, 38.6

Test at $\alpha = 10\%$ LOS, $t_{tab} = 2.57$.

Ans $\bar{X} = \frac{\Sigma X_i}{n_1} = \frac{36.8 + 73.5 + 72.5 + 61.2 + 60.5 + 40.1}{6}$

$$= \frac{344.5}{6} = 57.41$$

$$\bar{Y} = \frac{\Sigma Y_i}{n_2} = \frac{60.7 + 51.2 + 42.7 + 38.6}{4} = \frac{193.2}{4} = 48.3$$

x_i	$x_i - \bar{x}$	$(x_i - \bar{x})^2$
36.8	-20.61	424.77
73.5	16.09	258.88
72.4	14.99	224.70
61.2	3.79	14.36
60.5	3.09	9.54
40.1	-17.31	299.63
		$S = 1231.88$

Scanned by CamScanner

$$\therefore S_x^2 = \frac{\Sigma (x_i - x)^2}{n_1 - 1}$$

$$S_x = \sqrt{\frac{\Sigma (x_i - x)^2}{n_1 - 1}} = \sqrt{\frac{1231.88}{5}} = \sqrt{246.37}$$

$$= 15.69$$

and

y_i	$y_i - \bar{y}$	$(y_i - \bar{y})^2$
60.7	12.4	153.76
51.2	2.9	8.41
42.7	−5.6	31.36
38.6	−9.7	94.09
		$\Sigma = 287.62$

$$\therefore S_y = \sqrt{\frac{\Sigma (y_i - \bar{y})^2}{n_2 - 1}} = \sqrt{\frac{287.62}{3}}$$

$$= \sqrt{95.87} = 9.79$$

$$\therefore t = \frac{\bar{x} - \bar{y}}{\sqrt{\frac{S_x^2}{n_1} + \frac{S_y^2}{n_2}}} = \frac{57.41 - 48.3}{\sqrt{\frac{246.37}{6} + \frac{95.87}{4}}}$$

$$= \frac{9.11}{\sqrt{41.06 + 23.96}} = \frac{9.11}{\sqrt{65.02}} = \frac{9.11}{8.06}$$

$$= 1.13$$

$$d.f. = n_1 + n_2 - 2 = 6 + 4 - 2 = 8$$

and $\alpha = 10\%$, $t_{tab} = 2.57$

Since $t_{cal} = 1.13 < t_{tab}$ hence accept H_0.

23 Sept) CRD ⇒ one way ANOVA (analysis of variance) : powerful statistical tool

→ Basic principles of experiment

- Replication
- Randomisation
- Local control → all are under same condition (i.e homogeneous)

→ layout of ANOVA

(1) B	(2) C	(3) D	(4) A	(5) E
(6) C	(7) C	(8) A	(9) B	(10) E
(11) D	(12) A	(13) B	(14) E	(15) C
(16) A	(17) B	(18) E	(19) D	(20) D

A, B, C, D, E

4 replications

A → 4, 8, 12, 16

B → 1, 9, 13, 17

C → 2, 6, 7, 15

D → 3, 11, 19, 20

E → 5, 10, 14, 18

- Statistical Analysis Replication

→ H_0 : no significant difference b/w these treatment.

Scanned by CamScanner

Treatment	1	2	3	...	j	...	r
t_1	y_{11}	y_{12}	y_{13}	...	y_{1j}	...	y_{1r}
t_2	y_{21}	y_{22}	y_{23}	...	y_{2j}	...	y_{2r}
t_3	y_{31}	y_{32}	y_{33}	...	y_{3j}	...	y_{3r}
⋮	⋮	⋮	⋮		⋮		⋮
t_i	y_{i1}	y_{i2}	y_{i3}	...	y_{ij}	...	y_{ir}
⋮	⋮	⋮	⋮		⋮		⋮
t_k	y_{k1}	y_{k2}	y_{k3}	...	y_{kj}	...	y_{kr}

$$y_{ij} = \mu + \alpha_i + \varepsilon_{ij}$$

where y_{ij} = j^{th} of M i^{th} treatment

α_i = treatment effect

μ = general mean

ε_{ij} = error effect.

- ANOVA Table

$$H_0 : \alpha_1 = \alpha_2 = \alpha_3 = \dots = \alpha_k = 0$$

i.e. no significant difference between treatment effect.

$$\mu_1 = \mu_2 = \dots = \mu_k = 0$$

Source of variation	d.f.	Sum of Square	mean sum of square	F_{cal}	F_{tab}
Treatment	K-1	Tr.S.S.	$Tr.M.S = \frac{Tr.S.S}{K-1}$	$F_{cal} = \frac{TrMS}{EMS}$	$F_{tab}[K-1, N-K]$
Error	N-K	E.S.S	$EMS = \frac{E.S.S}{N-K}$		
Total	N-1	T.S.S.			

F_{tab}, Less than, Success otherwise there will be error.

Scanned by CamScanner

1. Correction factor (C.F.) = $\frac{(G.T.)^2}{N}$ → Grand Total; N → Total no. of replications.

2. $Tr.S.S. = \left\{ \frac{T_1^2}{n_1} + \frac{T_2^2}{n_2} + \dots + \frac{T_K^2}{n_K} \right\} - CF$

$= \sum_{i=1}^{K} \frac{T_i^2}{n_i} - CF$

3. $T.S.S. = \sum_{i=1}^{K} \sum_{j=1}^{r} (y_{ij})^2 - C.F.$

4. $Er.S.S. = T.S.S. - Tr.S.S.$

10 Oct '19

• Applications of one way ANOVA

Q1) A completely randomised design was conducted with 3 treatment A, B and C where treatment A is ~~required~~ replicated 6 times and B is replicated 4 times, & also C is replicated 5 times. Analyse the data & state your conclusion.

		Total
A:	16.5, 17.0, 16.0, 12.0, 18.0, 14.0	93.5 (= T_A)
B:	15.0, 13.8, 14.0, 17.9	60.7 (= T_B)
C:	18.2, 24.3, 25.0, 18.9, 21.0	107.4 (= T_C)
		G.T. = 261.6

Soln. $H_0 : \mu_1 = \mu_2 = \mu_3$ or $\alpha_1 = \alpha_2 = \alpha_3$

i.e. no significant difference b/w treatment of the population.

$H_1 : \mu_i \neq \mu_j \quad (i \neq j = 1, 2, 3)$

i.e. there is a significant difference b/w treatments of

the population.

critical Region - $\alpha = 5\%$ L.O.S. → grand total

C.F. (correction factor) $= \frac{(G.T.)^2}{N} = \frac{(261.6)^2}{15}$

(no. of observations)

$= \frac{68434.56}{15} = 4562.304$

We know, T.S.S. = Tr.S.S + Er.S.S. —— (1)

Total sum of square, $T.S.S = \sum_{i=1}^{K}\sum_{j=1}^{r}(y_{ij})^2 - C.F.$ —— (2)

$\therefore \sum_{i=1}^{K}\sum_{j=1}^{r}(y_{ij})^2 = (16.5)^2 + (17.0)^2 + (16)^2 + (12)^2 + (18)^2 + (14)^2 + (15)^2 + (13.8)^2 + \dots + (21.0)^2$

$= 272.25 + 289 + 256 + 144 + 324 + 196 + 225 + 190.44 + 196 + 320.41 + 331.24 + 590.49 + 625 + 357.21 + 441$

$= 4758.04$

Now, $T.S.S = 4758.04 - 4562.304$ (from 2)

$= 195.736$

and $Tr.S.S = \sum_{i=1}^{K}\frac{T_i^2}{n_i} - C.F.$

$= \left[\frac{T_A^2}{n_1} + \frac{T_B^2}{n_2} + \frac{T_C^2}{n_3}\right] - C.F.$

no. of replication
A have.

Scanned by CamScanner

$$= \left(\frac{(93.5)^2}{6} + \frac{(60.7)^2}{4} + \frac{(107.4)^2}{5} \right) - 4562.304$$

$$= \left(\frac{8742.25}{6} + \frac{3684.49}{4} + \frac{11534.76}{5} \right) - 4562.304$$

$$= (1457.04 + 921.12 + 2306.95) - 4562.304$$

$$= 4685.11 - 4562.304$$

$$= 122.806$$

From 1) $\quad Er.S.S = T.S.S - Tr.S.S$

$$= 195.736 - 122.806$$

$$= 72.93$$

$$Tr.M.S. = \frac{Tr.S.S}{K-1} = \frac{122.806}{3-1} = \frac{122.806}{2} = 61.403$$

# of treatment ← K

$$Er.M.S. = \frac{Er.S.S}{N-K} = \frac{72.93}{15-3} = \frac{72.93}{12} = 6.0775$$

Source of variation	d.f.	S.S.	M.S.S.	F ratio.	F_{tab}.
Treatment	$K-1=2$	Tr.S.S. 122.806	Tr.M.S. 61.403	$F_{cal} = \frac{Tr.M.S}{Er.M.S}$	$F_{tab}(K-1, N-K)$
Error	$N-K=15-3=12$	Er.S.S 72.93	Er.M.S. 6.0775	$=10.104$	$F_{tab}(2,12)$ at 5% lev
Total	$N-1=14$	195.736	67.4805		

$$F_{cal} = \frac{Tr.MS}{Er.M.S.} = 10.104 \left(= \frac{61.403}{6.077} \right)$$

$F_{tab}(2, 12)$ at 5% $= 3.8853$

row column

Scanned by CamScanner

$\therefore F_{cal} > F_{tab}$ hence, reject H_0.

i.e. there is a significant diff. b/w treatments of the population.

11 Oct

Q2) A research organisation tested microwave oven at $\alpha = 5\%$ L.O.S. If there is any significant different in the average prices of these 3 types of oven.

Soln.

$H_0 : \mu_1 = \mu_2 = \mu_3$ i.e. there is no significant difference b/w average prices of given 3 ovens.

$H_1 : \mu_i \neq \mu_j \neq \mu_3 \ \forall \ i = j = 1, 2, 3.$

given data: watts

1000	900	800
270, 230, 245	240, 135, 160,	180, 180, 155, 140,
190, 215, 250	230, 250, 200, 200	200, 130, 120, 140
(A)	210 (B)	(C)

$N = 8 + 8 + 6 = 22$

$T_A = 270 + 230 + \ldots + 250 = 1400$

$T_B = 240 + 135 + \ldots + 210 = 1625$

$T_C = 180 + 180 + \ldots + 140 = 1245.$

$G.T. = 4270$

$\therefore C.F. = \dfrac{(G.T.)^2}{N} = \dfrac{18232900}{22}$

$= 828768.18$

Scanned by CamScanner

and $Tr.S.S. = \left(\frac{(1400)^2}{6} + \frac{(1625)^2}{8} + \frac{(1245)^2}{8}\right) - C.F.$

$= (326666.66 + 330078.12 + 193753.12) - C.F.$

$= 850497.9 - 828768.18$

$= 21729.72$

$T.S.S. = (270)^2 + \ldots + (130)^2 - C.F$

$= (72900 + 60025 + 36100 + 46225 + 62500 + \cancel{62500}\ 52900$

$+ 57600 + 18225 + 25600 + 52900 + 62500 + 40000 +$

$40000 + 44100 + 32400 + 24025 + 40000 + 14400 + 19600 +$

$32400 + 19600 + 16900) - C.F.$

$= 870900 - 828768.18$

$= 42131.82$

$Er.S.S = T.S.S. - Tr.S.S.$

$= 42131.82 - 21729.72$

$= 20402.1$

$Tr.M.S. = \frac{21729.72}{3-1} = \frac{21729.72}{2} = 10864.86$

$Er.M.S. = \frac{20402.1}{22-3} = 1073.79$

$\therefore F_{cal} = \frac{Tr.M.S.}{Er.M.S.} = \frac{10864.86}{1073.79} = 10.11$

$F_{tab}[2, 19]$ at 5% l.o.s. $= 3.5219$

(column row in F-table)

Scanned by CamScanner

$\therefore F_{cal} > F_{tab}$ ⇒ reject H_0.

⇒ there is a significant difference b/w avg. prices of ovens.

Q3) In order to find out yielding abilities of 5 varieties of seeds and experiment was conducted in the green house using completely randomised design with 4 pots per variety. Analyse the data and state your conclusion

Varieties of seed yield (g/pot)

	A	B	C	D	E
	8	10	18	12	8
	8	12	17	10	11
	6	13	13	15	9
	10	9	16	11	8
Total:	32	44	64	48	36

G.T. = 224

$$\therefore Tr.S.S. = \left(\frac{T_A^2}{n_1} + \frac{T_B^2}{n_2} + \frac{T_C^2}{n_3} + \frac{T_D^2}{n_4} + \frac{T_E^2}{n_5}\right) - C.F$$

$$= \left(\frac{(32)^2 + (44)^2 + (64)^2 + (48)^2 + (36)^2}{4}\right) - C.F.$$

$$= \left(\frac{1024 + 1936 + 4096 + 2304 + 1296}{4}\right) - C.F.$$

$$= \left(\frac{10656}{4}\right) - C.F.$$

$$= 2664 - 2508.8 = 155.2$$

Scanned by CamScanner

Since $C.F. = \frac{(G.T.)^2}{N} = \frac{(224)^2}{20} = \frac{50176}{20} = 2508.8$

and $T.S.S. = \sum_{i=1}^{k} \sum_{j=1}^{r} (y_{ij})^2 - CF$

$\sum_{i=1}^{k} \sum_{j=1}^{r} y_{ij} = 64 + 64 + 36 + 100 + 100 + 144 + 169 + 81 + 324 + 289 + 169 + 256 + 144 + 100 + 225 + 121 + 64 + 121 + 64 + 81$

$= \underline{2716}$

$\therefore T.S.S = 2716 - 2508.8 = 207.2$

$ESS = T.S.S - Tr.S.S = 207.2 - 155.2 = 52$

$\therefore Tr.M.S = \frac{155.2}{5-1} = \frac{155.2}{4} = 38.8$

and $\underline{Er.M.S.} = \frac{52}{20-5} = \frac{52}{15} = 3.46$

$\therefore F_{cal} = \frac{Tr.M.S}{Er.M.S.} = \frac{38.8}{3.46} = 11.2138$

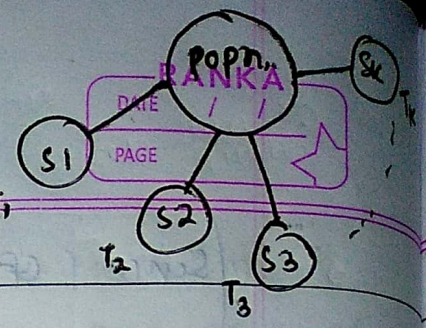
$F_{tab}[5-1, 20-5] = F_{tab}[4, 15]$ at 5% l.o.s.

(by default it not mentioned in question)

$= 3.0556$

$\therefore F_{cal} > F_{tab} \Rightarrow$ reject H_0.

- Two-way Analysis of Variance (or ANOVA) or Randomised



Block Design.

Treatment

Blocks					
B_1	A	B	C	D	E
B_2	B	C	D	E	A
B_3	C	D	E	A	B
B_4	A	E	D	B	C

? (Drawback of one-way ANOVA :- If any equality b/w any 2 treatment does not satisfy then all other treatment get rejected. i.e. If any one has significant difference with other → all are significantly different i.e. $1000 \neq 900 \neq 800$

⇒ reject H_0)

14 Oct) Two-way ANOVA or RBD (Repeated Measure Analysis)

→ Statistical Analysis

Let $t_1, t_2, t_3, \ldots, t_k$ are 'k' no. of treatments replicated in corresponding blocks $B_1, B_2, \ldots, B_r$ respectively.

Treatments	b_1	b_2	$\ldots$	b_j	$\ldots$	b_r	Total	Mean
t_1	y_{11}	y_{12}	$\ldots$	b_{1j}	$\ldots$	b_{1r}	T_1	$\bar{T}_1$
t_2	y_{21}	y_{22}	$\ldots$	b_{2j}	$\ldots$	b_{2r}	T_2	$\bar{T}_2$
$\vdots$	$\vdots$	$\vdots$		$\vdots$		$\vdots$	$\vdots$	$\vdots$
t_i	y_{i1}	y_{i2}	$\ldots$	b_{ij}	$\ldots$	b_{ir}	T_i	$\bar{T}_i$
$\vdots$	$\vdots$	$\vdots$		$\vdots$		$\vdots$	$\vdots$	$\vdots$
t_k	y_{k1}	y_{k2}	$\ldots$	b_{kj}	$\ldots$	b_{kr}	T_k	$\bar{T}_k$
Total	B_1	B_2	$\ldots$	B_j	$\ldots$	B_r	G.T.	
Mean	$\bar{B}_1$	$\bar{B}_2$	$\ldots$	$\bar{B}_j$	$\ldots$	$\bar{B}_r$		

Scanned by CamScanner

H_{0T_r} : $\mu_1 = \mu_2 = \mu_3 = \dots = \mu_n$ or $\alpha_1 = \alpha_2 = \dots = \alpha_k$

i.e. there is no significant difference b/w treatment means of a population.

H_{1T_r} : $\mu_i \neq \mu_j \quad \forall \; i = 1, 2, 3, \dots, k$

$j = 1, 2, \dots, r.$

i.e. there is a significance difference b/w treatment means of population.

H_{0B} : $V_1 = V_2 = \dots = V_r$ or $\beta_1 = \beta_2 = \dots = \beta_r$ i.e. there is no significant diff. b/w block means of the population.

H_{1B} : $V_i \neq V_j \quad \forall \; i = 1, 2, 3, \dots k$

$j = 1, 2, \dots, k$

- Statistical model :

$$y_{ij} = \mu + \alpha_i + \beta_j + \epsilon_{ij} \; ; \; \epsilon_{ij} \sim N(0, \sigma^2)$$

where μ : population mean

α_i : i^{th} treatment effect

β_j : j^{th} variety / block effect

ϵ_{ij} : error effect

y_{ij} : i^{th} treatment replicated in j^{th} block.

- TWO WAY ANOVA TABLE

Scanned by CamScanner

degree of freedom sum of square mean sum of square

Source of variation	d.f.	S.S	M.S.S	F ratio	F_{tab}
Treatment	$(K-1)$	Tr.S.S.	$Tr.M.S. = \frac{Tr.S.S}{K-1}$	$F_{cal\,Tr} = \frac{Tr.M.S}{Er.M.S}$	
Block	$(r-1)$	B.S.S.	$B.M.S = \frac{B.S.S}{r-1}$	$F_{cal\,B} = \frac{B.M.S}{Er.M.S}$	
Error	$(K-1)(r-1)$ ~~(N-ab)~~	Er.S.S.	$Er.M.S. = \frac{Er.S.S}{(K-1)(r-1)}$		
Total ~~Total~~	$rK-1$	T.S.S.			

$F_{Ttab} = F[(K-1),(K-1)(r-1)]$

$F_{Btab} = F[(r-1),(K-1)(r-1)]$

→ If F_{cal} value < F_{tab} value then accept H_0, otherwise reject H_0.

- Correction factor, $C.F. = \frac{(G.T.)^2}{rK}$
- $Tr.S.S. = \sum_{i=1}^{K} \frac{T_i^2}{r} - C.F.$
- $B.S.S. = \sum_{j=1}^{r} \frac{B_j^2}{K} - C.F.$
- $T.S.S. = \sum_{i=1}^{K} \sum_{j=1}^{r} (y_{ij})^2 - C.F.$
- $E.S.S. = T.S.S. - [Tr.S.S + B.S.S]$

★ Applications:

Ques 1) The yield of 6 varieties of a crop, alongwith the plan of the experiment are given below. The # of blocks is 5, plot of size 1/20 acres & the varieties have been represented

by A, B, C, D & E. Analyse the data & state your conclusion.

B-I	B 12	E 26	D 10	C 15	A 26	F 62
B-II	E 23	C 16	F 56	A 30	D 20	B 10
B-III	A 28	B 09	E 35	F 64	D 23	C 14
B-IV	F 75	D 20	E 30	C 14	B 07	A 23
B-V	D 17	F 70	A 20	C 12	B 09	E 28

Soln.

* First do the setting

	A	B	C	D	E	F	T_i	$\bar{T}_i = T_i/K$
B-I	26	12	15	10	26	62	151	25.16
B-II	30	10	16	20	23	56	155	25.83
B-III	28	09	14	23	35	64	173	28.83
B-IV	23	07	14	20	30	75	169	28.16
B-V	20	09	12	17	28	70	156	26
B_i	127	47	71	90	142	327	804	
$\bar{B}_i = B_i/r$	25.4	9.4	14.2	18	28.4	65.4	G.T.	

B_i = # of blocks

K = # of treatments

$$\therefore C.P = \frac{(G.T.)^2}{rK} = \frac{(804)^2}{5 \times 6} = \frac{646416}{30}$$

$$= 21547.2$$

$r = 5$ (# of blocks)

$K = 6$ (# of treatments)

DATE / /

PAGE

$$T.S.S. = \sum_{i=1}^{k} \frac{T_i^2}{r} - C.F.$$

$$= \left\{ \frac{(151)^2}{5} + \frac{(155)^2}{5} + \frac{(173)^2}{5} + \frac{(169)^2}{5} + \frac{(156)^2}{5} \right\} - C.F$$

$$= \frac{22801 + 24025 + 29929 + 28561 + 24336}{5} - C.F.$$

$$= \frac{129652}{5} - C.F. = 25930.4 - 21547.2$$

$$= 4383.2$$

$$B.S.S. = \sum_{j=1}^{r} \frac{B_j^2}{k} - C.F.$$

$$= \frac{(127)^2 + (47)^2 + (71)^2 + (90)^2 + (142)^2 + (322)^2}{6} - C.F$$

$$= \frac{16129 + 2209 + 5041 + 8100 + 20164 + 106829}{6} - C.F$$

$$= \frac{158572}{6} - C.F = 26428.66 - 21547.2$$

$$= 4881.46$$

$$T.S.S. = \sum_{i=1}^{k} \sum_{j=1}^{r} (y_{ij})^2 - C.F.$$

$$= \left((26)^2 + (12)^2 + \dots + (70)^2 \right) - C.F.$$

$$= (676 + 144 + 225 + 100 + 676 + 3844 + 900 + 100 +$$

Scanned by CamScanner

$256 + 400 + 529 + 3136 + 784 + 81 + 196 + 529 + 4096 + 529 + 49 + 196 + 400 + 900 + 5625 + 400 + 81 + 144 + 289 + 784 + 4900 + 1225) - C.F$

$= 32194 - 21547.2 = 10646.8$

$E.S.S. = T.S.S. - (Tr.S.S. + B.S.S)$

$= 10646.8 - (4383.2 + 4881.46)$

$= 10646.8 - 9264.66$

$= 1382.14$

$Tr.M.S. = \frac{Tr.S.S}{K-1} = \frac{4383.2}{(6-1)} = 876.64$

$B.M.S. = \frac{4881.46}{(5-1)} = 1220.365$

$E.M.S. = \frac{Er.S.S}{(K-1)(r-1)} = \frac{1382.14}{(5-1)(6-1)} = 69.107$

$\therefore F_{cal\,Tr} = \frac{876.64}{69.107} = 12.685 \quad \left(= \frac{Tr.M.S}{Er.M.S}\right)$

$F_{B\,cal} = \frac{B.M.S}{Er.M.S.} = \frac{1220.365}{69.107} = 17.659$

and $F_{tab\,Tr}[(K-1), (K-1)(r-1)] = F_{Tr\,tab}[5, 20]$

$= 2.7109$

$F_{tab\,B} = [(r-1), (K-1)(r-1)] = F_{B\,tab}[4, 20]$

$= 2.8661$

Scanned by CamScanner

$\therefore F_{cal} > F_{tab}$ in both case

hence ~~accept~~ reject H_0.

17/10/19 Ques 2) As a new type of environmental, natural air is being developed. It is tested that whether there is any significant difference b/w these factors.

treatment	Block I	Block II (Home type)	Block III	
A	25, 28, 26, 30, 31	30, 32, 35, 29, 31	43, 40, 42, 49, 48	T_A
B	15, 18, 22, 21, 17	21, 27, 18, 15, 19	23, 25, 24, 17, 13	T_B
	B_I	B_{II}	B_{III}	

Soln

$T_A = 140 + 157 + 222 = 519$

$T_B = 93 + 100 + 102 = 295$

$\Big\} = 814 = G.T.$

$B_I = 140 + 93 = 233$

$B_{II} = 157 + 100 = 257$

$B_{III} = 222 + 102 = 324$

$\Big\} = 814 = G.T.$

$$\therefore C.F. = \frac{(G.T.)^2}{r \cdot k} = \frac{(814)^2}{3 \cdot 2} = \frac{662596}{6}$$

# of block

treatment

$$= 110432.66$$

$$T.S.S. = \sum_{i=1}^{k} \frac{T_i^2}{r} - C.F$$

$$= \frac{(519)^2}{3} + \frac{(295)^2}{3} - 110432.66$$

$$= \left(\frac{269361 + 87025}{3}\right) - C.F.$$

Scanned by CamScanner

$$= \frac{356386}{3} - 110432.66 = 118795.33 - 110432.66$$

$$= 8362.67$$

$$B.S.S. = \sum_{j=1}^{r} \frac{B_j^2}{k} - C.F.$$

$$= \frac{(233)^2}{2} + \frac{(257)^2}{2} + \frac{(324)^2}{2} - C.F.$$

$$= \frac{54289 + 66049 + 104976}{2} - C.F.$$

$$= \frac{225314}{2} - 110432.66 = 112657 - 110432.66$$

$$= 2224.34$$

$$T.S.S = \sum_{i=1}^{k} \sum_{j=1}^{r} (y_{ij})^2 - C.F.$$

$$= \{(140)^2 + (157)^2 + (222)^2 + (93)^2 + (100)^2 + (102)^2\} - C.F.$$

$$= \{19600 + 24649 + 49284 + 8649 + 10000 + 10404\} - C.F.$$

$$= 122586 - 110432.66$$

$$= 12153.34$$

$$E.S.S = T.S.S. - (Tr.S.S. + B.S.S.)$$

$$= 12153.34 - (2224.34 + 8362.67)$$

$$= 1566.33$$

Scanned by CamScanner

$$Tr.M.S. = \frac{Tr.S.S.}{K-1} = \frac{8362.67}{2-1} = 8362.67$$

$$B.M.S. = \frac{B.S.S}{r-1} = \frac{2224.34}{3-1} = \frac{2224.34}{2}$$

$$= 1112.17$$

$$E.M.S. = \frac{1566.33}{(2-1)(3-1)} = \frac{1566.33}{2} = 783.165$$

$$F_{cal\,Tr.} = \frac{Tr.M.S}{E.M.S} = \frac{8362.67}{783.165} = 10.678$$

$$F_{Bcal} = \frac{B.M.S}{E.M.S} = \frac{1112.17}{783.165} = 1.4200$$

$$F_{tab\,Tr}([(K-1),(K-1)(r-1)]) = F_{tab\,Tr}(1,2) = 18.51$$

$$F_{tab\,B}([(r-1),(K-1)(r-1)]) = F_{tab}(2,2) = 19.00$$

at $\alpha = 5\%$ i.e. 0.05

$\therefore F_{cal} < F_{tab}$ in both the cases.

$\Rightarrow$ accept H_0.

Ques 3) A scholar wishes to see whether the type of gasoline used and the type of automobile driven have any effect on gasoline consumption. Two types of gasoline - regular & high octane will be used, and 2 types of automobile 2-wheeler & 4-wheeler drive will be used in each group. There will be 2 automobiles in

Scanned by CamScanner

each group for a total '8' automobiles used. Using 2 way 'anova', the scholar will perform following steps.

Treatment	Types of automobile → Block: two-wheeler	4-wheeler	
Regular	26.7, 25.2	28.6, 29.3	T_A
High octane	32.3, 32.8	26.1, 24.2	T_B
	B_I	B_{II}	

Soln. $K = 2$ and $r = 2$. $\therefore rK = 2 \cdot 2 = 4$

and $T_A = 51.9 + 57.9 = 109.8$

$T_B = 65.1 + 50.3 = 115.4$

$B_I = 51.9 + 65.1 = 117$

$B_{II} = 57.9 + 50.3 = 108.2$

$\therefore G.T. = T_A + T_B = B_I + B_{II} = 225.2$

$$\therefore C.F. = \frac{(G.T.)^2}{r \cdot K} = \frac{(225.2)^2}{4} = \frac{50715.04}{4}$$

$$= 12678.76$$

$$\therefore Tr.S.S = \frac{T_A^2}{r} + \frac{T_B^2}{r} - C.F.$$

$$= \frac{(109.8)^2}{2} + \frac{(115.4)^2}{2} - C.F.$$

$$= \frac{12056.04 + 13317.16}{2} - C.F.$$

Scanned by CamScanner

$$= \frac{25373.2}{2} - 12678.76$$

$$= 12686.6 - 12678.76$$

$$= 7.84$$

$$B.S.S = \frac{B_I^2}{K} + \frac{B_{II}^2}{K} - CF$$

$$= \frac{13689}{2} + \frac{11707.24}{2} - CF$$

$$= \frac{25396.24}{2} - CF = 12698.12 - 12678.72$$

$$= 19.4$$

$$\& \ T.S.S = \{(51.4)^2 + (57.9)^2 + (65.1)^2 + (50.3)^2\} - CF$$

$$= \{2693.61 + 3352.41 + 4238.01 + 2530.09\} - CF$$

$$= 12814.12 - 12678.72$$

$$= 135.4$$

$$Er.S.S. = T.S.S - (Tr.S.S. + B.S.S)$$

$$= 135.4 - (19.4 + 7.84)$$

$$= 108.16$$

$$Tr.M.S. = \frac{Tr.S.S}{K-1} = \frac{7.84}{2-1} = 7.84$$

Scanned by CamScanner

Note: $F_{table}[D_1, D_2]$: $D_2 \rightarrow$, D_2

$$B.M.S = \frac{B.S.S}{r-1} = \frac{19.4}{1} = 19.4$$

$$\& \; E.M.S = \frac{E.S.S}{(r-1)(K-1)} = \frac{108.16}{1 \cdot 1} = 108.16$$

$$\therefore F_{Tr\,cal} = \frac{Tr.M.S}{E.M.S} = \frac{7.84}{108.16} = 0.0724$$

$$F_{B\,cal} = \frac{B.M.S}{E.M.S} = \frac{19.4}{108.16} = 0.1793$$

$$F_{tab\,Tr}[(K-1),(K-1)(r-1)] = F_{Tr\,tab}[1,1] = 161.4$$

$$F_{B\,Tab}[(r-1),(K-1)(r-1)] = F_{B\,tab}[1,1] = 161.4$$

since $F_{cal} < F_{tab}$ in the both cases.

hence, accept H_0

18 Oct) CRD, RBD and LSD

CRD has no local control

RBD has local control (i.e. block)

- LSD (Latin Square Design) or Three-way ANOVA involves treatment, row and column.

layout:

A	B	C	D
B	C	D	A
C	D	A	B
D	A	B	C

Scanned by CamScanner

$i = j = k = 1, 2, \ldots, m$

here, $m = 4$

	C_1	C_2	$\cdots$	C_j	$\cdots$	C_m
R_1	y_{11}	y_{12}	$\cdots$	y_{1j}	$\cdots$	y_{1m}
R_2	y_{21}	y_{22}	$\cdots$	y_{2j}	$\cdots$	y_{2m}
$\vdots$	$\vdots$	$\vdots$		$\vdots$		$\vdots$
R_i	y_{i1}	y_{i2}	$\cdots$	y_{ij}		y_{im}
$\vdots$	$\vdots$	$\vdots$		$\vdots$		$\vdots$
R_m	y_{m1}	y_{m2}	$\cdots$	y_{mj}	$\cdots$	y_{mm}

$$\epsilon_{ijk} \sim N(0, \sigma^2)$$

where y_{ij} = i^{th} treatment j^{th} row k^{th} column observed value

μ - general / population mean

α_i - i^{th} treatment effect

β_j - j^{th} row effect

γ_k - k^{th} column effect

epsilon — ϵ_{ijk} - error effect

- ANOVA TABLE

Scanned by CamScanner

Source of variation	d.f.	S.S	M.S.S	F-ratio	F_{tab}
Treatment	$m-1$	Tr.S.S.	$Tr.M.S. = \frac{Tr.S.S}{m-1}$	$F_{Trcal} = \frac{Tr.M.S}{Er.M.S}$	$F_{tab}[(m-1)$
~~Column~~ Row	$m-1$	R.S.S.	$R.M.S = \frac{R.S.S}{m-1}$	$F_{Rcal} = \frac{R.M.S}{Er.M.S.}$	$,(m-1)(m-2)]$
Column	$m-1$	C.S.S	$C.M.S. = \frac{C.S.S}{m-1}$	$F_{Ccal} = \frac{C.M.S}{Er.M.S}$	"
Error	$(m-1)(m-2)$	Er.S.S	$Er.M.S. = \frac{Er.S.S}{(m-1)(m-2)}$		"
Total	m^2-1	T.S.S			

d.f. (T ~~S.S~~ otal) { d.f (Treatment) + df (Row) + df (column) }

d.f. of Error $= (m^2-1) - (m-1+m-1+m-1)$

$= m^2-1-3m+3$

$= m^2-3m+2 = (m-1)(m-2)$

Scanned by CamScanner

Source of variation	d.f.	S.S	M.S.S	F-ratio.	F_{tab}
Treatment	$m-1$	Tr.S.S.	$Tr.M.S. = \frac{Tr.S.S}{m-1}$	$F_{Trcal} = \frac{Tr.M.S}{Er.M.S}$	$F_{tab}[(m-$
~~column~~ ROW	$m-1$	R.S.S.	$R.M.S = \frac{R.S.S}{m-1}$	$F_{Rcal} = \frac{R.M.S}{Er.M.S.}$	$,(m-1)(m-$
Column	$m-1$	C.S.S	$C.M.S. = \frac{C.S.S}{m-1}$	$F_{Ccal} = \frac{C.M.S}{Er.M.S}$	"
Error	$(m-1)(m-2)$	Er.S.S	$Er.M.S. = \frac{Er.S.S}{(m-1)(m-2)}$		"
Total	m^2-1	T.S.S			

d.f.(T~~otal~~) { d.f (Treatment) + d.f (Row) + d.f(column)

d.f. of Error $= (m^2-1) - (m-1+m-1+m-1)$

$= m^2-1-3m+3$

$= m^2-3m+2 = (m-1)(m-2)$

- $T.S.S = Tr.S.S + R.S.S + C.S.S + Er.S.S$

$\Rightarrow$ $Er.S.S. = T.S.S - (Tr.S.S + R.S.S + C.S.S)$

Conclusion: If $F_{cal} < F_{tab}$ then accept H_0, otherwise reject H_0.

$H_{0T}: \alpha_1 = \alpha_2 = \dots = \alpha_k \quad (\forall\, k = 1, 2, \dots, m)$

i.e. there is no significant difference b/w treatment of the population.

Similarly, $H_{0R}: \beta_1 = \beta_2 = \dots = \beta_i \quad (\forall\, i = 1, 2, \dots, m)$

$H_{0C}: \gamma_1 = \gamma_2 = \dots = \gamma_j \quad (\forall\, j = 1, 2, \dots, m)$

$H_{1T}: \alpha_i \neq \alpha_j \quad (\forall\, i = j = 1, 2, \dots, m)$

Scanned by CamScanner

i.e. there is significant difference b/w treatments of the populations.

H_{IR} : $\beta_i \neq \beta_j$ $(\forall\, i=j=1,2,\ldots m)$

H_{IC} : $\gamma_i \neq \gamma_j$ $(\forall\, i=j=1,2,\ldots m)$

* $C.F. = \dfrac{(G.T.)^2}{m^2}$

* $T.S.S = \sum_{i=1}^{m} \dfrac{T_i^2}{m} - C.F.$

* $R.S.S. = \sum_{j=1}^{m} \dfrac{R_j^2}{m} - C.F.$

* $C.S.S. = \sum_{k=1}^{m} \dfrac{C_k^2}{m} - C.F.$

* $T.S.S. = \sum_{i=1}^{m} \sum_{j=1}^{m} \sum_{k=1}^{m} (y_{ijk})^2 - C.F.$

<u>Applications</u>

<u>Ques)</u> An experiment on cotton was conducted to study the effect of foliar application of urea in combination with sprays in the cotton yield. Five treatments were tried to 6/6 lattice square design. The layout plan & yield is given below. Analyse the data & state your conclusion.

4.9 T_2	6.4 T_4	3.3 T_5	9.5 T_1	11.8 T_3	
9.3 T_3	4.0 T_1	6.2 T_2	5.1 T_5	5.4 T_4	$R_1 = 35.9$
					$R_2 = 30$

Scanned by CamScanner

7.0 T_4	15.4 T_3	6.5 T_1	6.0 T_2	4.6 T_5
5.3 T_5	7.6 T_2	13.2 T_3	8.6 T_4	4.9 T_1
9.3 T_1	6.3 T_5	11.8 T_4	15.9 T_3	7.6 T_2
$C_1 = 35.8$	$C_2 = 39.7$	$C_3 = 41$	$C_4 = 45.1$	$C_5 = 34.3$

$R_3 = 39.5$

$R_4 = 39.6$

$R_5 = 50.9$

$195.9 = G.T.$

$$T_1 = 9.5 + 4.0 + 6.5 + 4.9 + 9.3 = 34.2$$
$$T_2 = 4.9 + 6.2 + 6 + 7.6 + 7.6 = 32.3$$
$$T_3 = 11.8 + 9.3 + 15.4 + 13.2 + 15.9 = 65.6$$
$$T_4 = 6.4 + 5.4 + 7.0 + 8.6 + 11.8 = 39.2$$
$$T_5 = 3.3 + 5.1 + 4.6 + 5.3 + 6.3 = 24.6$$
$$= (195.9)$$

$$\therefore C.F. = \frac{(G.T.)^2}{m^2} = \frac{(195.9)^2}{(5)^2} = \frac{38376.81}{25} = 1535.07$$

$$Tr.S.S. = \frac{(34.2)^2 + (32.3)^2 + (65.6)^2 + (39.2)^2 + (24.6)^2}{5} - C.F.$$

$$= \frac{8658.09}{5} - 1535.07$$

$$= 1731.61 - 1535.07 = 196.548$$

$$R.S.S = \frac{(35.9)^2 + (30)^2 + (39.5)^2 + (39.6)^2 + (50.9)^2}{5} - C.F.$$

$$= \frac{7908.03}{5} - CF = 1581.606 - 1535.07$$

$$= 46.536$$

Scanned by CamScanner

$$C.S.S = \frac{(35.8)^2 + (39.7)^2 + (41)^2 + (45.1)^2 + (34.3)^2}{5} - CF$$

$$= \frac{7749.23}{5} - 1595.07 = 1549.846 - 1535.07$$

$$= 14.77$$

$$T.S.S. = \{(4.9)^2 + (6.4)^2 + \dots + (7.6)^2\} - CF.$$

$$= 1821.07 - 1535.07 = 286$$

$$\therefore \text{Tr.M.S} = \frac{196.548}{5-1} = \frac{196.548}{4} = 49.137$$

$$\text{R.M.S.} = \frac{46.536}{4} = 11.634$$

$$\text{C.M.S} = \frac{14.77}{4} = 3.69$$

$$\text{E.S.S.} = 286 - (196.548 + 46.536 + 14.77)$$

$$= 28.146$$

$$\therefore \text{EMS} = \frac{28.146}{(4)(3)} = 2.3455$$

$$\therefore F_{Tr\,cal} = \frac{49.137}{2.34} = 20.99$$

$$F_{R\,cal} = \frac{11.634}{2.34} = 4.97$$

$$F_{C\,cal} = \frac{3.69}{2.34} = 1.567$$

Scanned by CamScanner

and $F_{tab}[(5-1), (5-1)(5-2)] = F_{tab}[4, 12] = 3.2592$ at 5% to.s.

$\therefore$ for column, H_0 is accepted & for rest two, H_0 is rejected.

24 Oct 19) LSD application..

The layout plan and yield is given below, Analyse the data, interpret the result.

T_2 (5.4)	T_4 (6.3)	T_5 (3.6)	T_1 (5.9)	T_3 (8.11)	$R_1 = 29.31$
T_3 (9.2)	T_1 (6.2)	T_2 (6.3)	T_5 (5.4)	T_4 (5.8)	$R_2 = 32.9$
T_1 (7.9)	T_5 (7.2)	T_4 (6.3)	T_3 (9.0)	T_2 (6.4)	$R_3 = 36.8$
T_5 (5.9)	T_2 (7.11)	T_3 (11.3)	T_4 (3.9)	T_1 (8.2)	$R_4 = 41.41$
T_4 (3.6)	T_3 (3.9)	T_1 (8.3)	T_2 (5.9)	T_5 (9.0)	$R_5 = 30.7$

$C_1 = 32$ $C_2 = 30.71$ $C_3 = 35.8$ $C_4 = 35.1$ $C_5 = 37.57$ $G.T. = 171.12$

$T_1 = 5.9 + 6.2 + 7.9 + 8.2 + 8.3 = 36.5$

$T_2 = 5.4 + 6.3 + 6.4 + 7.11 + 5.9 = 31.11$

$T_3 = 8.11 + 9.2 + 9.0 + 11.3 + 3.9 = 41.51$

$T_4 = 6.3 + 5.8 + 6.3 + 8.9 + 3.6 = 30.9$

$T_5 = 3.6 + 5.4 + 7.2 + 5.9 + 9.0 = 31.1$

$G.T. = 171.12$

$m = 5$

$\therefore C.F. = \frac{(G.T.)^2}{m^2} = \frac{(171.12)^2}{25} = 1171.282$

$$Tr.S.S. = \frac{(36.5)^2 + (31.11)^2 + (41.51)^2 + (30.9)^2 + (31.1)^2}{5} - CF$$

$$= \frac{5945.18}{5} - CF = 1189.036 - 1171.282$$

$$= 17.75$$

$$R.S.S. = \frac{(29.31)^2 + (32.9)^2 + (36.8)^2 + (41.41)^2 + (30.7)^2}{5} - CF$$

$$= \frac{5953.0042}{5} - CF = 1190.600 - 1171.282$$

$$= 19.318$$

$$C.S.S. = \frac{(32)^2 + (30.71)^2 + (35.8)^2 + (35.1)^2 + (37.57)^2}{5} - CF$$

$$= \frac{5892.259}{5} - CF = 1178.451 - 1171.282$$

$$= 7.169$$

$$T.S.S. = \{(5.4)^2 + (6.3)^2 + \dots + (9.0)^2\} - CF.$$

$$= 1256.2342 - 1171.282$$

$$= 84.9522$$

$$Er.S.S. = T.S.S. - (Tr.S.S. + R.S.S. + C.S.S)$$

$$= 84.9522 - (17.75 + 19.318 + 7.169)$$

$$= 40.7152$$

$$Tr.M.S. = \frac{Tr.S.S}{m-1} = \frac{17.75}{4} = 4.4375$$

Scanned by CamScanner

$$R.M.S. = \frac{R.S.S}{m-1} = \frac{19.318}{4} = 4.8295$$

$$C.M.S. = \frac{7.169}{4} = 1.79225$$

and $$EMS = \frac{E.S.S}{(m-1)(m-2)} = \frac{40.7152}{4 \cdot 3} = 3.392$$

Now, $$F_{Tr\,cal} = \frac{Tr.MS}{EMS} = \frac{4.437}{3.392} = 1.308$$

$$F_{R\,cal} = \frac{4.829}{3.392} = 1.4236$$

$$F_{C\,cal} = \frac{1.792}{3.392} = 0.528$$

at 5% L.O.S, $F_{Tab}[m-1, (m-1)(m-2)] = F_{Tab}[4, 12]$
$= 3.259$

$\therefore$ $F_{cal} < F_{tab}$ for all 3 hence accept H_0

Q4 Now) Application of LSD

Ques) Four treatments were tried in a 4x4 layout latin square design. The layout plan & design is given below. Analyse the data & state your conclusions.

(8.1) T_2	(9.0) T_1	(7.2) T_3	(3.6) T_4
(8.1) T_3	(8.9) T_2	(4.5) T_4	(9.2) T_1
(6.3) T_4	(5.4) T_3	(9.9) T_1	(8.2) T_2
(9.5) T_1	(7.2) T_4	(8.3) T_2	(6.3) T_3

Scanned by CamScanner

Soln.

$T_1 = 9 + 9.2 + 9.9 + 9.5 = 37.6$

$T_2 = 8.1 + 8.9 + 8.2 + 8.3 = 33.5$

$T_3 = 7.2 + 8.1 + 5.4 + 6.3 = 27$

$T_4 = 3.6 + 4.5 + 6.3 + 7.2 = 21.6$

$G.T. = 119.7$

$R_1 = 27.9$, $R_2 = 30.7$, $R_3 = 29.8$, $R_4 = 31.3$

$C_1 = 32$, $C_2 = 30.5$, $C_3 = 29.9$, $C_4 = 27.3$

$\because m = 4 \quad \therefore CF = \frac{(G.T.)^2}{16} = 895.505$

$$Tr.S.S. = \frac{(37.6)^2 + (33.5)^2 + (27)^2 + (21.6)^2}{4} - CF$$

$$= \frac{3731.57}{4} - CF = 932.89 - CF$$

$$= 37.385$$

$R.S.S. = 897.15 - CF = 1.645$

$$\& \; C.S.S. = \frac{3593.55}{4} - CF = 898.387 - CF$$

$$= 2.882$$

$$T.S.S. = \{(8.1)^2 + (9)^2 + \dots + (6.3)^2\} - CF$$

$$= 945.89 - CF = 50.385$$

$$Er.S.S = 50.385 - (37.385 + 2.882 + 1.645)$$

$$= 8.4726$$

Scanned by CamScanner

$Tr.M.S. = (37.385)/3 = 12.461$

$C.M.S. = 2.882/3 = 0.960$

$R.M.S. = (1.645)/3 = 0.548$

$E.M.S. = 8.4726/(3,2) = 1.4121$

$F_{Tr} = 8.82$, $F_C = 0.679$, $F_R = 0.3881$

$F_{tab}[3, 6] = 4.757$ at $\alpha = 5\%$.

Since F_R & F_C < F_{tab} then accept H_{0R} & H_{0C}

but F_{Tr} > F_{tab} hence reject H_{0T}.

14 Nov) Non - parametric tests.

- **SIGN-TEST** : $S_{cal} < S_{tab}$ ⇒ reject H_0

  otherwise accept H_0.

  ~~cal~~ get S_{tab} value from sign-test table.

  H_0 : sample is randomly selected or not.

- To calculate S_{cal} = minimum of +ve sign or -ve sign

  i.e. S_{cal} = min (no. of +ve sign , no. of -ve sign)

For eg. 2 3 7 8 9

if median is 5

∴ − − + + +

no. of +ve sign = 3 and # of -ve sign = 2

∴ $S_{cal} = \min\{3, 2\} = 2$

Scanned by CamScanner

If median = 7

∴ − − 0 + +

~~hence~~ thus, $S_{cal} = \min(2, 2) = 2$

- Sign test — [one sample / two sample]

Let $x_1, x_2, x_3, \ldots x_n$ be a sample drawn from an arbitrary population respectively.

Step 1: Define Null and alternative hypothesis for the claim.

Step 2: Find out the critical value (tabulated value) at α % of L.O.S.

Step 3: Find out the median for a given data or choose arbitrary median for the given data.

→ If the observed value > median then make it as positive sign.

→ If the observed value is equal to the median value make it as zero, otherwise negative sign.

Step 4: If data is paired sample, calculate the deviation from before and after observed value.

Step 5: Calculate the minimum (or small) no. of +ve & −ve signs.

→ If the calculated value (i.e. S_{cal}) <= tabulated value (i.e. S_{tab}) then reject H_0 otherwise accept H_0

Step 6: If the sample size >= 26 (i.e. $n >= 26$) then

*

Scanned by CamScanner

the z-statistic is denoted by z_{cal} & is defined as

$$z_{cal} = \frac{(x+0.5) - (n/2)}{\sqrt{(n/2)}}$$

where x = smaller no. of +ve and -ve signs

n = sample size

Ques) A store owner hypothesis the median no. of snow ~~covers~~ cones sells per day is 40. A random sample of 20 days yields the following data for the no. of snow ~~cones~~ sold each day.

18, 43, 40, 16, 22, 30, 29, 32, 37, 36, 39, 34, 39, 45, 28, 36, 40, 34, 39, 52.

At $\alpha = 5\%$ L.O.S., test the owner's hypothesis.

Soln. Given that H_0 : median = 40

H_1 : median $\neq$ 40

no. of (+) = 3, # of (0) = 2, # of (−) = 15

$n = 20$ ∴ sample size = n − (# of zero(s))

= 20 − 2 = 18

∴ $S_{cal} = 3 \{= \min(no.(+), no.(-))\}$

S_{tab} for $n = 18$ and at 5% L.O.S. = 4

Since $S_{cal} < S_{tab}$ ⇒ reject H_0.

Ques) Based on info. from census data, the median age

of residence is 36.4 years. A scholar selects a sample of 50 residence in his area & finds that 21 are older than 36.4 years. At $\alpha = 5\%$ l.o.s. test the claim that the median of the residence is at least 36.4 years.

Soln

H_0 : median = 36.4 years

H_1 : median < 36.4 years

here, $n = 50$ and $x = 21$ (since $n \geq 26$ hence use Z-cal)

$$\therefore Z_{cal} = \frac{(x + 0.5) - (n/2)}{\sqrt{n/2}}$$

$$= \frac{(21 + 0.5) - 25}{\sqrt{25}} = \frac{-3.5}{5}$$

$$= -0.7$$

Now, $Z_{tab} = -1.645$ at $\alpha = 5\%$ for one-tail test.

Now, $Z_{cal} > Z_{tab} \Rightarrow$ accept H_0.

Ques A scholar believed that no. of ear infection in swimmer can be reduced if the swimmer use ear plugs. A sample of 10 pp is selected & no. of infections recorded for a 4-month period was ~~not used earplugs~~. During the 1st 2 months, the swimmers did not use ear plugs. During the 2nd two months they did. The data are shown below at $\alpha = 5\%$ l.o.s. can the scholar conclude

Scanned by CamScanner

that using ear plugs reduced the # of infections?

# of persons	1	2	3	4	5	6	7	8	9	10
before	3	0	5	4	2	4	3	5	2	1
after	2	1	4	0	1	3	1	3	2	3
Sign	+	−	+	+	+	+	+	+	0	−

Soln H_0: there is no significant diff. b/w before & after use of ear plugs.

H_1: there is a significant difference b/w before & after use of ear plugs.

no. of (+) = 7

no. of (−) = 2 ∴ sample size = n − # of zeros

no. of (0) = 1 = 10 − 1 = 9

and S_{cal} = min (+, −) = min {7, 2} = 2.

From sign table, $S_{tab} = 1$ at $\alpha = 5\%$ & $n = 9$

since $S_{cal} > S_{tab}$ ⇒ accept H_0

Ques) A faculty wants to investigate the relationship b/w a student mid term score & end term score. 8 students were selected and their score on 2 examinations are noted below. At 10% l.o.s. is there any significant evidence to conclude that there is a difference in score?

Soln. H_0: there is no difference in score

H_1: there is a significant difference in score.

mid term	75	92	68	85	65	80	75	60
end term	82	90	79	95	70	83	72	79
Sign	−	+	−	−	−	−	+	−

Scanned by CamScanner

of (+) = 2 & no. of (−) = 6

$\therefore$ Sample size $= n - 0 = 8 - 0 = 8$

$\therefore S_{cal} = 2 \quad (\min(6, 2))$

and $S_{tab} = 1$ $\quad \because S_{cal} > S_{tab} \Rightarrow$ accept H_0.

(*) Wilcoxon's rank sum test or mann whitney 'U' test

Let any 2 random samples drawn from an arbitrary population respectively.

Step 1. Define the H_0 and H_A and test the claim.

Step 2. Find out the critical value from a given table

Step 3. The z-statistic define as

$$\boxed{z_w = \frac{R - \mu_R}{\sigma_R}}$$

where R = sum of the ranks of smaller size sample (i.e. n_1)

* $\mu_R = \dfrac{n_1(n_1 + n_2 + 1)}{2}$

* $\sigma_R = \sqrt{\dfrac{n_1 n_2 (n_1 + n_2 + 1)}{12}}$

where $n_1 \geq 10$ and $n_2 \geq 10$

Note: If $n_1 = n_2$ then choose any one of sample size.

Step 4: If z-calculated value is less than z_{tab} value

then accept H_0. otherwise reject H_0 at α% l.o.s.

- Applications.

Ques) A random sample of men & women in prison was asked to give the length of sentence each reach for a certain type of crime. At $\alpha = 5\%$ l.o.s., test the claim that there is no difference in the sentence received by each gender.

Males: 8 12 6 14 22 27 32 24 26 19 15 13

Females: 7 5 2 3 21 26 30 9 4 17 23 12 11 16

Soln. $n_1 = 12$ $n_2 = 14$

Given that H_0: there is no significant difference b/w length of the sentence by each gender.

H_1: there is a significant difference b/w length of sentence b/w each gender.

- arrange the data in ascending order

$\frac{(10+11)}{2} = 10.5$

$= 5 + 7 + 10.5 + 12 + 13 + 14 + 17 + 19 + 21 + 22.5 + 24 + 26 = \underline{\underline{191}}$

$$Z_w = \frac{R - \mu_R}{\sigma_R}$$

$$\mu_R = \frac{n_1(n_1 + n_2 + 1)}{2} = \frac{\cancel{12}^{6}(12 + 14 + 1)}{\cancel{2}} = 6 \times 27$$

$$= 162$$

$$\sigma_R = \sqrt{\frac{n_1 n_2 (n_1 + n_2 + 1)}{12}} = \sqrt{\frac{\cancel{12} \times 14(12 + 14 + 1)}{\cancel{12}}}$$

$$= \sqrt{14 \times 27} = \sqrt{378} = 19.44$$

$$\therefore Z_w = \frac{191 - 162}{19.44} = \frac{29}{19.44} = 1.49$$

since at $\alpha = 5\%$, $Z_{tab} = 1.96$

$\therefore Z_{cal} < Z_{tab} \Rightarrow$ accept H_0.

Ques) A sample of "count" from the 2 region is selected, and the no. of hunting accidents are shown. At $\alpha = 5\%$ level of significance, is there difference b/w in the no. of accident in 2 areas?

North: 10, 21, 11, 11, 9, 17, 13, 8, 15, 17

South: 14, 3, 7, 13, 11, 2, 8, 5, 5, 6

Soln $n_1 = n_2 = 10$

arrange in ascending order

S S S S S S S N N N N S N

observations : 2, 3, 5, 5, 6, 7, 8, 8, 9, 10, 11, 11, 11, 13,

S S N N N N

13, 14, 15, 17, 17, 21

rank : 1, 2, 3.5, 3.5, 5, 6, 7.5, 7.5, 9, 10, 12, 12,

12, 14.5, 14.5, 16, 17, 18.5, 18.5, 20

'Take S'

R = small sum of rank = 1 + 2 + 3.5 + 3.5 + 5 + 6 + 7.5 + 12 + 14.5 + 16 = 71

$$\mu_R = \frac{n_1(n_1+n_2+1)}{2} = \frac{\cancel{10}^{5} \cdot 21}{\cancel{2}} = 105$$

$$\sigma_R = \sqrt{\frac{n_1 n_2 (n_1+n_2+1)}{12}} = \sqrt{\frac{\cancel{10}^{5} \cdot \cancel{10}^{5} \cdot \cancel{21}^{7}}{\cancel{12}_{3}}} = 5\sqrt{7}$$

$$= 13.22$$

$$\therefore |Z_\omega| = \left|\frac{R-\mu_R}{\sigma_R}\right| = \left|\frac{64-105}{13.22}\right| = \left|\frac{-41}{13.22}\right|$$

$$= 3.101$$

and $Z_{tab} = 1.96$ at $\alpha = 5\%, 1\%, 0.1$

$\therefore Z_{cal} > Z_{tab} \Rightarrow$ reject H_0.

★ Wilcoxon Signed Rank Test

Let a sample is drawn from any arbitrary populati-on respectively.

Step 1.) Define H_0 & H_A and test the claim.

Step 2.) Find the critical value for the given test.

Step 3.) Define $D_i = R_i - R_j \quad \forall\, i = j = 1, 2, \ldots n$

$|D| \longrightarrow$ assign the rank.

Scanned by CamScanner

W_{cal} = smaller no. of +ve or -ve signed ranks.

→ $W_{cal} < W_{tab}$ ⇒ accept H_0 otherwise reject H_0.

Step 4) If $n \geq 30$ then z-statistic defined as

$$\boxed{Z_{WS} = \frac{R - \mu_{WS}}{\sigma_{WS}}}$$

where $\mu_{WS} = \frac{n(n+1)}{4}$

R = smaller no. of +ve or -ve signed ranks.

$\sigma_{WS} = \sqrt{\frac{n(n+1)(2n+1)}{24}}$

n = sample size

Step 5) If $Z_{WS\,cal} < Z_{WS\,tab}$ ⇒ accept H_0, otherwise reject H_0 at α% L.O.S.

Ques) In a corporation, female & male workers were matched a/c to years of experience working for a company. ~~This~~ Their salary were compared. At α = 10% L.O.S, is there a difference ~~bt~~ in salaries of males & females.

Males : 18 43 32 27 15 45 21 22

Female : 16 38 35 29 15 46 25 28

Soln. H_0 : there is no significant b/w ~~both~~ salaries. ~~integer~~

Males	Female	$D = R_i - R_j$	$\lvert D \rvert$	Rank.
18	16	2	2	2.5
43	38	+5	5	~~n = 5 / 6~~
32	35	−3	3.	~~n = 8~~ 4
27	29	−2	2	~~n = 8~~ 2.5
15	15	0	0	X
41	46	−1	1	1
22	25	−4	4	5
22	28	−6	6	7

~~$W_{scal} = 2.5 + 6 = 8.5$~~

~~$(W_{cal} M_a = 2.5 + 68.5)$~~

~~$\therefore W_{cal} > W_{stab} \Rightarrow$ reject H_0 & at $n=7$ & $\alpha = 10\%$ L.O.S, $W_{stab} = 5$~~

~~$\therefore W_{scal} > W_{stab} \Rightarrow$ reject H_0.~~

Since # of (+ve) signed < # of (-ve) signed

$\therefore W_{scal}$ = sum of ranks of the signed

$= 2.5 + 6$

$= 8.5$

$\therefore W_{cal} > W_{stab} \Rightarrow$ reject H_0

Ques) In a large department store, the owner wishes to see whether ~~also~~ the no. of incident per day will change. If the no. of uniform society is increased and 7 days after the increase is shown in below table,

Day 1:	M	T	W	Th	Fri	Sat	Sun
Before:	7	2	3	6	5	8	12
After:	5	3	4	3	1	6	4

at $\alpha = 5\%$ L.O.S. test the claim.

Sol:

Before	After	D	$\lvert D_i \rvert$	Rank	
7	5	2	2	3.5	
2	3	−1	1	1.5	
3	4	−1	1	1.5	
6	3	3	3	5	# of -ve sign is less.

Scanned by CamScanner

Before	After	|D|	Rank
5	1	4	6
8	6	2	3.5
12	4	8	7

~~W_{scal} = 1.5 + 1.5 tables~~

$W_{scal} = 1.5 + 1.5 = 3$ (add the ranks of -ve signed numbers)

$W_{tab} = 2$ at $\alpha = 5\%$ & 0.5 & $n = 7$

~~W_{scal} (Z_R) W_{stan}~~

$\therefore W_{scal} > W_{stab}$ ⇒ reject H_0

Scanned by CamScanner